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WELFARE-ENHANCING DISTRIBUTIONAL EFFECTS OF CENTRAL BANK ASSET PURCHASES

ANDREAS SCHABERT

Department of Economics
University of Cologne
Albertus-Magnus-Platz
D-50923 Köln
Germany

http://www.wiso.uni-koeln.de

Welfare-Enhancing Distributional Effects of Central Bank Asset Purchases[1]

Andreas Schabert[2]
University of Cologne

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Abstract

This paper shows that central bank interventions in secondary markets for private debt can enhance social welfare. We apply a model with idiosyncratic risk and limited contract enforcement, while abstracting from unusually large disruptions in financial market. By purchasing debt at above-market prices the central bank induces an increase in credit supply, by which rather borrowers than debt holders gain. We show that asset purchases can not only replicate a tax/subsidy that addresses pecuniary externalities induced by a collateral constraint, but can even improve upon the constrained efficient allocation. We further demonstrate that countercyclical asset purchases are desirable under aggregate risk, which reduce the build-up of debt in favorable times.

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[2] University of Cologne, Center for Macroeconomic Research, Albertus-Magnus-Platz, 50923 Cologne, Germany, Phone: +49 221 470 2483, Email: schabert@wiso.uni-koeln.de.

1 Introduction

Should central banks intervene in secondary markets for private debt securities? While central banks have traditionally traded money in exchange for (short-term) treasuries, the US Federal Reserve (Fed) and the European Central Bank (ECB) have recently included large-scale purchases of private debt securities in secondary markets into their set of policy instruments.[3] Theoretical studies on unconventional monetary policy have focussed on their effects under stressed financial markets, showing that other types of interventions, like direct central bank lending and purchases of equity or treasuries, can be beneficial when agents face extraordinary high costs of financial intermediation or of asset liquidation.[4] Yet, there is neither well-established empirical evidence nor a common theoretical view on the transmission mechanism of central bank interventions in secondary markets for private debt securities, leaving open the question if this type of instrument should also be applied when there are no unusually large disruptions in financial markets. Moreover, distributional consequences of private debt purchases, in particular, the effects on holders and issuers of debt, which have so far not been addressed in the literature,[5] are entirely unclear.

If, for example, a central bank offers a favorable price for specific assets in secondary markets, one might suppose that primarily agents who hold these assets (i.e. savers or lenders) gain from this intervention. Given that these agents tend to be relatively wealthy and to be characterized by a relatively low marginal valuation of funds, a redistributive policy that is particularly beneficial for them is unlikely to enhance social welfare. This argument, however, neglects that these agents, who receive liquid funds (central bank money) in exchange for less liquid assets, might further use/invest the proceeds, such that prices and other participants in financial markets are also affected. Thus, in tranquil times, when asset liquidation or liquidity hoarding is not urgent, the pass-through of price effects from central bank asset purchases becomes relevant and their distributional consequences are per se not unambiguous. Specifically, when borrowers, who are likely to be constrained and tend to have a relatively high marginal valuation of funds, sufficiently gain from price effects induced by central bank interventions, social welfare can in principle increase.

This paper demonstrates that central bank interventions in secondary markets can enhance social welfare, while abstracting from particular gains under stressed financial markets. Our analysis is nevertheless based on the view that financial markets are characterized by imperfections, such that policy interventions that alter prices and quantities in financial markets can in principle affect the allocation in a favorable way. The main novel contribution is that the pass-through of price

[3]For example, the Fed purchased large positions of MBS in 2009 and 2012, while the ECB is currently still purchasing private debt securities. For example, purchases of asset-backed securities by the ECB were introduced in 2014 and expected to "facilitate credit conditions" (see ECB press release of 2nd Oct, 2014).

[4]See, e.g. Curdia and Woodford (2011), Gertler and Karadi (2011), Del Negro et al., (2016), or Woodford (2016).

[5]Studies on distributional effects of monetary policy have until now focussed on conventional policies (see, e.g., Berentsen et. al, 2005, Algan and Ragot, 2010, Lippi et al., 2015, Auclert, 2016, or Garriga et al., 2016), which will not be examined in this paper.

effects triggered by secondary market interventions can particularly benefit constrained borrowers. Specifically, we show that lenders (i.e. the holders of eligible assets) can be incentivized to increase their supply of funds by the central bank offering an above-market price for existing debt securities. This however benefits borrowers and their consumption, as the interest rate on debt falls (in accordance with empirical evidence on recent asset purchases programmes).[6] Simultaneously, the lenders' effective real return on debt holdings increases with central bank interventions, implying that they save more and that their consumption declines. Thus, private debt purchases in secondary market can induce a redistribution of funds from lenders to borrowers. In an environment where agents are constrained and tend to borrow less than under an efficient allocation, the central bank can thereby enhance social welfare.

We apply a simple incomplete market model where private agents face idiosyncratic preference shocks and borrow/lend among each other in terms of secured debt. To isolate the effects of financial market interventions and to present the main novel results in a transparent way, we abstract from financial intermediation, endogenous production, and aggregate risk (for the baseline scenario), implying that conventional monetary policy actions do not affect the equilibrium allocation. Agents differ with regard to their valuation of non-durable consumption goods, for which money serves as a mean of payment (as in Lucas and Stokey, 1987), giving rise to borrowing/lending in terms of money. As the main friction, we consider that contract enforcement is limited, such that lending relies on the borrower's ability to pledge collateral (as in Kiyotaki and Moore, 1997). Likewise, the central bank supplies money only against eligible assets, which would solely consist of treasury securities under a conventional monetary policy regime. Here, we further account for the possibility of central bank purchases of collateralized loans in secondary markets, which are non-neutral due to asymmetric effects on lenders and borrowers. Individually rational lenders participate in asset purchases programs when the central bank offers an above-market price, while they do not internalize that this leads to financial market conditions that are ultimately more favorable for borrowers.

To facilitate aggregation and to enable the derivation of analytical results, we apply linear-quadratic preferences and define a competitive equilibrium with a representative lender and a representative borrower. We first look at a financial market intervention that can be examined in a more straightforward way than an asset purchase regime, where the central bank has several instruments at its disposal. Specifically, we show that a borrowing subsidy, which is financed by a lump-sum tax on borrowers and, therefore, only affects marginal costs of borrowing, induces a welfare-enhancing increase in borrowing. The reason for the subsidy to be welfare-enhancing is a pecuniary externality induced by the collateral constraint, i.e. agents do not internalize how their

[6] Specifically, the loan rate falls by a reduction in the liquidity premium, which accords to empirical evidence on price effects of US Federal Reserve asset purchases (see Gagnon et al., 2011). The behavior of the liquidity premium is further consistent with Krishnamurthy and Vissing-Jorgensen's (2012) findings.

behavior affects access to external funds via the relative price of collateral.[7] We then establish that a central bank can replicate the optimal borrowing subsidy by purchasing secured loans at an above-market price. Furthermore, we show that – compared to the constrained efficient allocation under the borrowing subsidy – central bank asset purchases can implement welfare-dominating allocations. The reason is that the central bank does not only alter relative prices (like the borrowing subsidy), but can further raise borrowers' consumption by increasing the amount of funds available for lending. Notably, these beneficial effects for borrowing agents arise even though the central bank does – as in reality – not directly trade with ultimate borrowers.

To provide numerical examples for welfare-enhancing policies, we apply a CRRA utility function. While the latter facilitates the calibration of the model, we rely on pooled end-of-period funds within households (as in Lucas and Stokey, 1987, or Shi, 1997) when defining a competitive equilibrium with representative borrowers and lenders. For this version, we confirm the main results derived for the previous version with linear-quadratic preferences. We further introduce aggregate risk in form of a stochastic aggregate endowment and examine welfare effects of state-contingent purchases of secured loans. Specifically, we consider monetary policy to depend on the aggregate state of the economy and show that – conditional on aggregate endowment shocks – a countercyclical share of secured loans that are purchased by the central bank can enhance social welfare. The reason is that under adverse shocks borrowers suffer not only from a reduction in endowment, but also from a decline in the price of collateral (i.e. the price of housing), which reduces their borrowing capacity. Countercyclical asset purchases can therefore stimulate (dampen) borrowing and thus borrowers' consumption in situations where their marginal utility of consumption is particularly high (low). Thus, the central bank can in this way support prudential policies that aim at reducing the economy's vulnerability in crisis times by reducing debt ex-ante (see, for example, Stein, 2012); an analysis of this interaction being an issue for future research.

Our analysis of central bank purchases of private debt securities in secondary markets relates to studies on other types of unconventional monetary policies by Curdia and Woodford (2011) and Gertler and Karadi (2011), who show that direct central bank lending to ultimate borrowers can be effective if financial market frictions are sufficiently severe. Using an estimated preferred habitat model, Chen et al. (2012) find that changing the composition of treasury debt as under US Federal Reserve large scale asset purchase programs during the financial crisis had moderate GDP growth and inflation effects. Del Negro et al. (2016) examine government purchases of equity in response to an adverse shock to resaleability and show that the introduction of this type of policy after 2008 have prevented the US economy from a repeat of the Great Depression. Woodford (2016) applies a model with fire sale externalities and a positive probability of crisis states to assess the impact of central bank purchases of long-term treasuries on financial market stability. In contrast

[7] Davila and Korinek (2017) provide a comprehensive analysis of this type of externality and describe cases where either overborrowing or underborrowing (as in our framework) arises.

to our paper, these studies do not examine private debt purchases in secondary markets, focus on the case of stressed financial markets (like in the recent financial crisis), and they do not derive distributional consequences of central bank interventions. Our paper further relates to Araújo et al. (2013), who show that asset purchases can exert ambiguous welfare effects under endogenous collateral constraints. In contrast to our paper, where asset purchases affect prices via a liquidity premium stemming from the role of money as a means of payment, there is no special role of currency in their model. The liquidity premium effects, by which central bank debt purchases alters asset prices in our model, are similar to the effects of central bank treasury purchases on the term premium in Williamson (2016). The specification of central bank operations in our paper closely relates to Schabert (2015), who examines welfare gains from money rationing in a New Keynesian model without idiosyncratic shocks and with frictionless financial markets. Our analysis of borrowing subsidies relates to Correia et al. (2015), who apply a representative agent model with frictional intermediation due to costly enforcement and show that credit subsidies are desirable and they are – in contrast to our analysis – superior to monetary policy measures.

In Section 2 we present the model. In Section 3, we provide analytical results on welfare-enhancing financial market interventions. In Section 4, we present some numerical examples and analyze state-contingent asset purchases under aggregate risk. Section 5 concludes.

2 The model

In this Section, we develop a simple incomplete markets model with idiosyncratic preference shocks and limited contract enforcement. Debt contracts are only available in nominal terms and money is assumed to serve as a means of payment for non-durable goods. To focus on the effects of asset purchases, we disregard endogenous production, price rigidities, and aggregate risk (in this Section), such that conventional monetary policy actions are neutral. We further abstract from financial intermediation, for convenience, while we model monetary policy implementation in a more detailed way (as in Schabert, 2015), allowing for a realistic specification of central bank interventions. In particular, we specify unconventional monetary policy by purchases of private debt in secondary markets, which is not equivalent to direct central bank lending to ultimate borrowers in our model.[8]

2.1 Overview

The economy consists of households, a central bank, and a government. Households enter a period with money holdings and government bonds, and dispose of an exogenously given endowment of a non-durable good. They can further hold a durable good, which is supplied at a fixed amount. At the beginning of each period, open market operations are conducted, where the central bank sells or

[8] Analyses of direct central bank lending can be found in Curdia and Woodford (2011) and Gertler and Karadi (2011).

purchases assets outright or supplies money under repurchase agreements (repos) against treasury securities at the policy rate. Then, idiosyncratic preference shocks are realized and, subsequently, housing is traded. Households with a high realization of the preference shock tend to consume more than households with a low realization of the preference shock. Given that money serves as a means of payment for cash goods (non-durable goods), the former tend to borrow money from the latter. We assume that loan contracts cannot be enforced, such that only collateralized loans are feasible. As the primary object of our analysis, we consider that these collateralized loans might be purchased by the central bank from lenders, such that the proceeds are available to extend loan supply. After cash goods are traded, repos are settled and subsequently the asset market opens. In the asset market, borrowing agents repay secured loans, the government issues new bonds, and the central bank reinvests earnings from maturing bonds.

The central bank sets the price of money (i.e. the policy rate), decides on the amount of money that is supplied against treasuries in open market operations and via purchases of loans, and it transfers interest earnings to the government. The government issues one period bonds, has access to lump-sum transfers, and might introduce a borrowing tax/subsidy as a means of financial market intervention, which will be examined in Section 3.2. The effects of asset purchases will rely on rationed money supply, i.e. on money being supplied by the central bank only against eligible assets that are not unboundedly available.[9] By setting the price of money below agents' marginal valuation of money, the central bank can induce a scarcity of money as well as of eligible assets (i.e. treasuries and collateralized loans), and can influence asset prices via changes in liquidity premia.

2.2 Details

Private sector There are infinitely many and infinitely lived households i of measure one, which are characterized by identical initial stocks of wealth. Their utility increases with consumption $c_{i,t}$ of a non-durable good and holdings of a durable good, i.e. housing $h_{i,t}$; the supply of the latter being normalized to one. Each household is endowed with y_i, where $y_{i,t} = y_t$ and y_t denotes aggregate endowment that is exogenously determined with mean one. Households can differ with regard to their marginal valuation of consumption of the non-durable good due to preference shocks $\epsilon_i > 0$, which are i.i.d. across households and time. The instantaneous utility function $u_{i,t}$ of a household i is given by

$$u_{i,t} = u(\epsilon_i, c_{i,t}, h_{i,t}), \tag{1}$$

where $h_{i,t}$ denotes the end-of-period stock of housing. We assume that $u_{i,t}$ is strictly increasing, concave, and separable in consumption and housing. The idiosyncratic shock ϵ_i exhibits two possible realizations, $\epsilon_i \in \{\epsilon_l, \epsilon_b\}$, with mean one, equal probabilities $\pi_\epsilon = 0.5$, and $\epsilon_l < \epsilon_b$.

[9] Under money rationing, the central bank can simultaneously control the price and the amount of money, and can thereby implement welfare dominating allocations compared to policy regimes that satiate money demand (see Schabert, 2015).

Households rely on money for purchases of non-durable goods, whereas we treat housing as a "credit good" (see Lucas and Stokey, 1987). They hold money $M_{i,t-1}^H$ at the beginning of each period and they can acquire additional money $I_{i,t}$ from the central bank, for which they hold eligible assets. Specifically, households can get money $I_{i,t}$ from the central bank in open market operations, where money is supplied against treasury securities $B_{i,t-1}$ discounted with the policy rate R_t^m:

$$0 \leq I_{i,t} \leq \kappa_t^B B_{i,t-1}/R_t^m. \tag{2}$$

The central bank supplies money against a fraction $\kappa_t^B \geq 0$ of randomly selected bonds under repurchase agreements and outright. Notably, purchases of public debt can affect the allocation only via an increase in the supply of money, while associated effects on the interest rate on treasuries will be irrelevant for the equilibrium allocation. When household i draws the realization ϵ_b (ϵ_l), which materializes after treasury open market transactions are conducted,[10] it is willing to consume more (less) than households who draw ϵ_l (ϵ_b). Hence, ϵ_b-type households tend to borrow an additional amount of money from ϵ_l-type households. We assume that borrowing and lending among households only takes place in form of short-term nominal debt at the price $1/R_t^L$. Following Jermann and Quadrini (2012), we assume – for convenience – that loan contracts are signed at the beginning of the period and repaid at the end of each period. We account for the fact that debt repayment cannot be guaranteed and that enforcement of debt contracts is limited. Thus, agents have to pledge collateral to be able to borrow, i.e. loans are collateralized by the stock of borrowers' housing. Specifically, households can borrow the amount $-L_{i,t} > 0$ up to the liquidation value of their housing stock at the end of the period $h_{i,t}$ (when loans mature),

$$-L_{i,t} \leq z P_t q_t h_{i,t}, \tag{3}$$

where P_t denotes the aggregate price level, q_t the real housing price, and $z \in (0,1)$ the liquidation share of collateral. As the main object of our analysis, we consider that the possibility that the central bank purchases secured loans in addition to treasuries. In particular, after the preference shocks are realized and loan contracts are signed, the central bank offers money in exchange for a randomly selected fraction $\kappa_t \in [0,1]$ of secured loans at the price $1/R_t^m$:

$$0 \leq I_{i,t}^L \leq \kappa_t L_{i,t}/R_t^m. \tag{4}$$

By purchasing loans, the central bank can thus influence lenders' valuation of secured loans and can induce an increase in the amount of money that is available for loan supply. For this, the price $1/R_t^m$ that the central bank pays and its relation to the market price $1/R_t^L$ are obviously decisive. We assume that loan purchases are conducted in form of repos, where loans are repurchased by

[10] The assumption that preference shocks are realized after money is supplied in open market operations against treasuries is only relevant for the case where the money supply constraint (2) is not binding.

lenders before they mature (such that lenders earn the interest on loans). After loans are issued and asset purchases are conducted, the market for non-durables opens. Money is assumed to serve as the means of payment for non-durable goods, for which household i can use money holdings $M^H_{i,t-1}$ as well as new injections $I_{i,t}$ and $I^L_{i,t}$ plus/minus loans, such that the cash-in-advance constraint for household i is

$$P_t c_{i,t} \leq I_{i,t} + I^L_{i,t} + M^H_{i,t-1} - L_{i,t}/R^L_t. \tag{5}$$

Before the asset market opens, repurchase agreements are settled, i.e. agents buy back loans and treasuries under repos from the central bank, and transfers are paid. In the asset market, households repay intraperiod loans, invest in treasuries, and might trade assets among each other. Thus, the budget constraint of household i is

$$\begin{aligned} &M^H_{i,t-1} + B_{i,t-1} + L_{i,t}\left(1 - 1/R^L_t\right) + P_t y_{i,t} + P_t \tau_{i,t} \\ &\geq M^H_{i,t} + (B_{i,t}/R_t) + \left(I_{i,t} + I^L_{i,t}\right)(R^m_t - 1) + P_t c_{i,t} + P_t q_t \left(h_{i,t} - h_{i,t-1}\right), \end{aligned} \tag{6}$$

where $1/R_t$ denotes the price of treasuries and $\tau_{i,t}$ the lump-sum government transfer. Maximizing $E \sum_{t=0}^{\infty} \beta^t u_{i,t}$, where the discount factor satisfies $\beta \in (0,1)$, subject to (1)-(6) taking prices as given, leads to the following first order conditions for consumption, holdings of treasuries and money, and additional money from treasury open market operations $\forall i \in \{b, l\}$:

$$u'(\epsilon_i, c_{i,t}) = \lambda_{i,t} + \psi_{i,t}, \tag{7}$$

$$\lambda_{i,t} = \beta R_t E_t \left[\left(\lambda_{i,t+1} + \kappa^B_{t+1}\eta_{i,t+1}\right)/\pi_{t+1}\right], \tag{8}$$

$$\lambda_{i,t} = \beta E_t \left[\left(\lambda_{i,t+1} + \psi_{i,t+1}\right)/\pi_{t+1}\right], \tag{9}$$

$$\overline{E}_t \psi_{i,t} = (R^m_t - 1)\,\overline{E}_t \lambda_{i,t} + \overline{E}_t R^m_t \eta_{i,t}, \tag{10}$$

where π_t denotes the inflation rate and $\overline{E}_t$ the expectations at the beginning of period t before individual shocks are drawn. Further, $\lambda_{i,t} \geq 0$ is the multiplier on the asset market constraint (6), $\eta_{i,t} \geq 0$ the multiplier on the money supply constraint (2), and $\psi_{i,t} \geq 0$ the multiplier on the cash-in-advance constraint (5), where all constraints are expressed in real terms. Condition (8) indicates that the interest rate on government bonds is affected by a liquidity premium, stemming from the possibility to exchange a fraction κ^B_t of bonds in open market operations (see 2). Condition (10) for money supplied against treasuries reflects that idiosyncratic shocks are not revealed before treasury open market operations are initiated. Further, the following type-specific first order conditions for loans and housing have to be satisfied, for borrowers

$$\lambda_{i,t}\left(1 - 1/R^L_t\right) - (\psi_{i,t}/R^L_t) + \zeta_{i,t} = 0, \tag{11}$$

$$u'(h_{i,t}) + \zeta_{i,t} z q_t + \beta E_t q_{t+1}\lambda_{i,t+1} - q_t \lambda_{i,t} = 0, \tag{12}$$

and for lenders, where we additionally consider the first order condition for money acquired from loan purchases $I^L_{l,t}$,

$$\lambda_{i,t}\left(1-1/R^L_t\right)-(\psi_{i,t}/R^L_t)+\mu_{i,t}\kappa_t=0, \tag{13}$$

$$u'(h_{i,t})+\beta E_t q_{t+1}\lambda_{i,t+1}-q_t\lambda_{i,t}=0, \tag{14}$$

$$-\lambda_{i,t}\left(1-1/R^m_t\right)+(\psi_{i,t}/R^m_t)-\mu_{i,t}=0, \tag{15}$$

Note that differences between the first order conditions for borrowers and lenders are due to the multiplier $\zeta_{i,t}\geq 0$ on the collateral constraint (3), which is only relevant for borrowers, and the multiplier $\mu_{i,t}\geq 0$ on the money supply constraint (4), which restricts loan purchases and is therefore only relevant for lenders. Condition (15), which describes the agents' willingness to sell loans to the central bank, is evidently also exclusively relevant for lenders. The conditions (11) and (13) further show that the multiplier on the cash-in-advance constraint (5) is positive if the loan rate R^L_t exceeds one, as the latter measures the relative price of cash goods. Further, the associated complementary slackness conditions,[11] as well as (2)-(5), (6) as an equality, and the associated transversality conditions hold.

Combining (7) and (9) to $\frac{\psi_{i,t}}{u'(\epsilon_i,c_{i,t})}=1-\beta\frac{E_t[u'(\epsilon_i,c_{i,t+1})/\pi_{t+1}]}{u'(\epsilon_i,c_{i,t})}$ shows that the liquidity constraint (5) is binding when the nominal marginal rate of intertemporal substitution $\frac{u'(\epsilon_i,c_{i,t})}{\beta E_t(u'(\epsilon_i,c_{i,t+1})/\pi_{t+1})}$ exceeds one. Further, the money supply constraint (4) is binding, $\mu_{i,t}>0$, implying that lenders are willing to refinance loans at the central bank to the maximum amount, when this allows to extract rents. This is the case when the policy rate R^m_t is lower than the loan rate R^L_t, which can be seen from combining (15) with (7), (9), and (18) to

$$\frac{\mu_{i,t}}{u'(\epsilon_i,c_{i,t})}=\frac{1}{1-\kappa}\left(\frac{1}{R^m_t}-\frac{1}{R^L_t}\right). \tag{16}$$

If, however, the policy rate equals the loan rate, $R^m_t=R^L_t$, lenders have no incentive to refinance loans at the central bank and (4) becomes slack (see 16). Thus, only if the central bank offers a price for loans $1/R^m_t$ that exceeds the market price $1/R^L_t$, lenders are willing to sell secured loans until the money supply constraint (4) is binding ($\mu_{i,t}>0$). Notably, lenders can thereby be incentivized to raise their supply of loans, while they do not take into account the impact of asset purchases on market prices.

The conditions for loan demand (11) and loan supply (15) reveal that the credit market allocation can be affected by the borrowing constraint (for $\zeta_{i,t}>0$) as well as by central bank loan purchases (for $\mu_{i,t}>0$). The borrowers' demand condition for loans (11) can – by using (7), (9),

[11] Specifically, complementary slackness conditions are given by $\eta_{i,t}[\kappa^B_t b_{i,t-1}(\pi_t R^m_t)^{-1}-i_{i,t}]=0$, $\zeta_{i,t}\left[zq_t h_{i,t}+l_{i,t}\right]=0$, $\mu_{i,t}\left[\kappa_t l_{i,t}/R^m_t-i^L_{i,t}\right]=0$, and $\psi_{i,t}\left[i_{i,t}+i^L_{i,t}+m^H_{i,t-1}-(l_{i,t}/R^L_t)-c_{i,t}\right]=0$, where the real variables are given by $b_{i,t}=B_{i,t}/P_t$, $l_{i,t}=L_{i,t}/P_t$, $m^H_{i,t}=M^H_{i,t}/P_t$, $i_{i,t}=I_{i,t}/P_t$, $i^L_{i,t}=I^L_{i,t}/P_t$.

and (15) – be rewritten as

$$\frac{1}{R_t^L} = \beta \frac{E_t \left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1} \right]}{u'(\epsilon_i, c_{i,t})} + \frac{\zeta_{i,t}}{u'(\epsilon_i, c_{i,t})}. \tag{17}$$

Hence, a positive multiplier $\zeta_{i,t}$ tends to raise the RHS of (17), implying a relative increase in current marginal utility of consumption, which can be mitigated by a lower loan rate. Put differently, under a binding borrowing constraint (3) the borrowers' nominal marginal rate of intertemporal substitution exceeds the loan rate. Further, the lenders' loan supply condition (13) can – by using (7) and (9) – be written as $\frac{1}{R_t^L} \cdot \frac{1-\kappa_t R_t^L/R_t^m}{1-\kappa_t} = \beta \frac{E_t[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}]}{u'(\epsilon_i, c_{i,t})}$ or

$$\frac{1}{R_t^L} = \kappa_t \cdot \frac{1}{R_t^m} + (1-\kappa_t) \cdot \beta \frac{E_t \left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1} \right]}{u'(\epsilon_i, c_{i,t})}. \tag{18}$$

Condition (18) implies that the loan rate depends on the lender's nominal marginal rate of intertemporal substitution as well as on the policy rate R_t^m, if the central bank purchases loans, $\kappa_t > 0$. According to (18), a higher share of purchased loans κ_t for a given policy rate $R_t^m < R_t^L$, or a lower policy rate R_t^m for a given share of purchased loans, $\kappa_t > 0$, tend to reduce the loan rate, while the loan rate approaches the policy rate, $R_t^L \to R_t^m$, for $\kappa_t \to 1$. Further note that (7), (9), and (10) imply $\frac{\overline{E}_t \eta_{i,t}}{\overline{E}_t u'(\epsilon_i, c_{i,t})} = \frac{1}{R_t^m} - \beta \frac{\overline{E}_t[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}]}{\overline{E}_t u'(\epsilon_i, c_{i,t})}$, where the term $\frac{\beta \overline{E}_t[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}]}{\overline{E}_t u'(\epsilon_i, c_{i,t})}$ cannot be larger than the inverse of the loan rate $1/R_t^L$ (see 17 and 18). Thus, a policy rate satisfying $1 \leq R_t^m < R_t^L$ ensures that money is scarce, such that the liquidity constraint (5) is binding, and that agents liquidate all available bonds, such that the money supply constraint (2) is binding as well as (4). Given that money supply is then effectively constrained by the available amount of eligible assets, i.e. bonds and secured loans, this type of monetary policy implies <u>money rationing</u>. The central bank can then control the price of money (by setting R_t^m) as well as the amount of money by setting κ_t and κ_t^B. If however the central bank supplies money in an unrestricted way at the policy rate R_t^m, the nominal marginal rate of intertemporal substitution will be equal to the latter and asset purchases are irrelevant.

Public sector The government issues nominal bonds at the price $1/R_t$ and pays lump-sum transfers τ_t, while we abstract from government spending and issuance of long-term debt. In Section 3.2, we further introduce a borrowing tax/subsidy as a means of financial market intervention, which is not specified here, for convenience. For simplicity, we assume that the supply of government bonds, which are held by households and the central bank, is assumed to be exogenous to the state of the economy. Specifically, the total amount of short-term government bonds B_t^T grows at a rate $\Gamma > 0$,

$$B_t^T = \Gamma B_{t-1}^T, \tag{19}$$

given $B_{-1}^T > 0$. The government further receives seigniorage revenues τ_t^m from the central bank, such that its budget constraint reads $(B_t^T/R_t) + P_t \tau_t^m = B_{t-1}^T + P_t \tau_t$. Due to the existence of lump-sum transfers/taxes, which balance the budget, fiscal policy will be irrelevant for the equilibrium allocation, except for the supply of treasuries (19).[12]

The central bank supplies money in open market operations either outright or temporarily via repos against treasuries, M_t^H and M_t^R. It can further increase the supply of money by purchasing secured loans from lenders, I_t^L, i.e. it conducts repos where secured loans serve as collateral. At the beginning of each period, its holdings of treasuries and the stock of outstanding money are given by B_{t-1}^c and M_{t-1}^H. It then receives treasuries and loans in exchange for money. Before the asset market opens, where the central bank rolls over maturing assets, repos in terms of treasuries and secured loans are settled. Hence, its budget constraint reads $(B_t^c/R_t) - B_{t-1}^c + P_t\tau_t^m = R_t^m\left(M_t^H - M_{t-1}^H\right) + (R_t^m - 1)\left(I_t^L + M_t^R\right)$, showing that the central bank earns interest from bonds purchased outright and by supplying money in open market operations. The central bank transfers its interest earnings from asset holdings and from open market operations to the government, $P_t\tau_t^m = (1 - 1/R_t)\, B_t^c + R_t^m\left(M_t^H - M_{t-1}^H\right) + (R_t^m - 1)\left(I_t^L + M_t^R\right)$. Thus, the budget constraint implies that central bank asset holdings evolve according to $B_t^c - B_{t-1}^c = M_t^H - M_{t-1}^H$. Further assuming that initial values for its assets and liabilities satisfy $B_{-1}^c = M_{-1}^H$, gives the central bank balance sheet

$$B_t^c = M_t^H. \tag{20}$$

The central bank has four instruments at its disposal. It sets the policy rate $R_t^m \geq 1$ and can decide how much money to supply against a randomly selected fraction of treasuries, for which it can adjust $\kappa_t^B \in (0, 1]$. The central bank can further decide whether it supplies money in exchange for treasuries either outright or temporarily via repos. Specifically, it controls the ratio of treasury repos to outright purchases $\Omega_t > 0 : M_t^R = \Omega_t M_t^H$, where a sufficiently large value for Ω_t ensures that injections are always positive, $I_{i,t} > 0$. Finally, the central bank can decide to purchase loans, i.e. to supply money temporarily against secured loans under repos. In each period, it therefor decides on a randomly selected share of secured loans $\kappa_t \in [0, 1]$ that it is willing to exchange for money under repos.

2.3 Competitive equilibrium and first best

In equilibrium, agents' optimal plans are satisfied and prices adjust such that all markets clear: $0 = \sum_i l_{i,t}$, $h = \sum_i h_{i,t}$, $y = \sum_i c_{i,t}$, $m_t^H = \sum_i m_{i,t}^H$, $m_t^R = \sum_i m_{i,t}^R$, $b_t = \sum_i b_{i,t}$, and $b_t^T = b_t^c + b_t$, where $l_{i,t} = L_{i,t}/P_t$, $m_{i,t}^H = M_{i,t}^H/P_t$, $m_t^R = M_t^R/P_t$, $b_{i,t} = B_{i,t}/P_t$, $b_t = B_t/P_t$, $b_t^c = B_t^c/P_t$, and $b_t^T = B_t^T/P_t$. A definition of a competitive equilibrium is given in Appendix A. Before we examine policy effects on the equilibrium allocation, we describe the first best allocation, which maximizes

[12] Note that the growth rate Γ might affect the long-run inflation rate if the money supply constraint (2) is binding. As shown in Appendix C, the central bank can nonetheless implement a desired inflation target by suited long-run adjustments of its money supply instruments.

ex-ante social welfare

$$E\sum_{t=0}^{\infty}\beta^t\sum_i u_{i,t}, \tag{21}$$

s.t. $h=\sum_i h_{i,t}$, and $y=\sum_i c_{i,t}$ and serves as a reference case for the subsequent analysis. For the following proposition we apply the law of large numbers and we index all agents drawing ϵ_l (ϵ_b) in period t with l (b).

Proposition 1 *The first best allocation* $\{c^*_{b,t}, c^*_{l,t}, h^*_{b,t}, h^*_{l,t}\}_{t=0}^{\infty}$ *satisfies* $u_c(\epsilon_b, c^*_{b,t}) = u_c(\epsilon_l, c^*_{l,t})$, $h^*_{b,t} = h^*_{l,t}$, $h^*_{b,t} + h^*_{l,t} = h$, *and* $c^*_{l,t} + c^*_{b,t} = y$.

Under the first best allocation, the marginal utilities of consumption and of the end-of-period stock of housing are identical for borrowers and lenders. This will typically not be the case in a competitive equilibrium where the borrowing constraint (3) is binding. In the subsequent sections, we examine how policy interventions can influence the private credit market. For asset purchases to be relevant, money has to be supplied at a favorable price (see 16), which implies that access to money is effectively rationed by the available amount of assets eligible for central bank operations. Specifically, the central bank has to set the policy rate below the lender's marginal rate of intertemporal substitution, implying $R^m_t < R^L_t$ (see 18), to ration money supply. Under a non-rationed money supply, which is equivalent to the case where the central bank supplies money in a lump-sum way (as typically assumed in the literature), the money supply constraints (2) and (4) are slack and the loan rate is identical to the policy rate $R^L_t = R^m_t$. In this case, asset purchases are irrelevant (see 16). For the subsequent analysis, we will therefore separately examine the two cases where money supply is rationed and where money supply is not rationed; the latter being the case under a conventional monetary policy regime.

3 Welfare enhancing financial market interventions

In this Section, we analyze policy interventions that influence the credit market, where agents borrow less than under first and second best. In the first part of this Section, we impose some further assumptions, which facilitate aggregation and the derivation of analytical results, and we define a competitive equilibrium in terms of a representative borrower and a representative lender. In the second part of this Section, we analyze the constrained efficient allocation that a social planer can implement by a borrowing tax/subsidy, which can be examined in a more straightforward way than an asset purchase regime, where the central bank has more instruments at its disposal. We show that a borrowing subsidy can correct for inefficiencies induced by a pecuniary externality stemming from the collateral constraint. In the last part, we introduce asset purchases and show that the central bank can replicate the constrained efficient allocation. We further show that the set of feasible allocations is larger under asset purchases than under the borrowing tax/subsidy, including welfare-dominating allocations.

3.1 Aggregation under a conventional monetary policy regime

Here, we examine the benchmark case of a conventional monetary policy, where the central bank sets the policy rate equal to the loan rate, $R_t^m = R_t^L$, such that both money supply constraints (2) and (4) are not binding ($\eta_{i,t} = \mu_{i,t} = 0$), implying that asset purchases are irrelevant (see 16). Even when the central bank were willing to buy loans, lenders would then not gain from selling loans and prices would not be affected. Given that we aim at disclosing the distributional and welfare effects of financial market interventions, we apply three assumptions that allow to derive the main results in an analytical way. We assume that preferences are given by a linear-quadratic form, which enables aggregation over individual choices. Once the competitive equilibrium is defined in terms of aggregate variables, we analytically derive the main results on policy interventions, which will be shown also to hold for alternative preferences (see Section 4).[13]

Assumption 1 *Instantaneous utility of households satisfies*

$$u(\epsilon_i, c_{i,t}, h_{i,t}) = \epsilon_i(\delta c_{i,t} - (1/2)c_{i,t}^2) + (\gamma h_{i,t} - (1/2)h_{i,t}^2), \tag{22}$$

where $\partial u/\partial c_{i,t} = u'(\epsilon_i, c_{i,t}) > 0$ *and* $\partial u/\partial h_{i,t} = u'(h_{i,t}) > 0$*.*

According to Assumption 1 the marginal utilities of consumption and housing are linear, $u_c(\epsilon_i, c_{i,t}) = \epsilon_i(\delta - c_{i,t})$ and $u_h(\epsilon_i, c_{i,t}, h_{i,t}) = \gamma - h_{i,t}$, where the parameters $\delta > 0$ and $\gamma > 0$ guarantee that the marginal utilities of consumption and housing are strictly positive in equilibrium. Under Assumption 1, the set of conditions that describe the behavior of agents' who draw ϵ_l in period t – indexed with (l, i, t) – is given by (6) holding as an equality, (7), (9), (14), $\epsilon_l(\delta - c_{l,i,t})/R_t^L = \beta E_t[0.5(\epsilon_l(\delta - c_{l,i,t+1}) + \epsilon_b(\delta - c_{b,i,t+1}))]/\pi_{t+1}$, and $c_{l,i,t} \leq 0.5\,(i_{l,i,t} + i_{b,i,t}) + m_{l,i,t-1}^H \pi_t^{-1} - l_{l,i,t}/R_t^L$, where the last condition, i.e. the cash-in-advance constraint, accounts for treasury open market operations being conducted before idiosyncratic shocks are drawn. Due to the linear-quadratic utility function, all conditions are linear in the agents' choice variables. The cash-in-advance constraint might, however, not be binding, which would be the case when the nominal interest rate equals one, $R_t^m = R_t^L = 1$ (see 7 for $\mu_{l,i,t} = 0$). Nevertheless, the latter policy is not sufficient to implement the first best allocation, which would require agents to hoard money to a sufficiently large amount to ensure the borrowing constraint never to be binding (see Section 2.2). To avoid indeterminacies due to a slack cash-in-advance constraint, we assume that the latter is just binding even when the nominal interest rate equals one and the associated multiplier equals zero, $\psi_{l,i,t} = 0$. Alternatively, one can assume that the policy rate remains infinitesimally away from one.

Assumption 2 *Agents will hold money equal to the amount of planned nominal consumption expenditures even when the multiplier on the cash-in-advance constraint equals zero.*

[13] For the case of CRRA preferences, which are introduced in Section 4, aggregation will be enabled by pooling funds within households at the end of each period.

It should be noted that Assumption 2 is made for convenience only and does not affect the main conclusions: If cash-in-advance constraints were not binding, monetary policy would apparently be irrelevant. As will be shown below, a conventional monetary policy will in fact also be irrelevant if the cash-in-advance constraint is binding (see Corollary 2). Assumption 2 will therefore not be decisive for the assessment of monetary policy. Under both Assumptions 1 and 2, we can easily aggregate by summing over all agents who draw ϵ_l in period t. Let $c_{l,t} = 2\sum_{l,i} c_{l,i,t}$, $h_{l,t} = 2\sum_{l,i} h_{l,i,t}$, $l_{l,t} = 2\sum_{l,i} l_{l,i,t}$, $\lambda_{l,t} = 2\sum_{l,i} \lambda_{l,i,t}$, and $i_{l,t} = 2\sum_{l,i} i_{l,i,t}$. Then, we get the following set of conditions for a representative lender: $0.5m^H_{t-1}\pi_t^{-1} + 0.5b_{t-1}\pi_t^{-1} + l_{l,t}\left(1 - 1/R^L_t\right) + 0.5y_t + 0.5\tau_t = m^H_{l,t} + (b_{l,t}/R_t) + 0.5\left(i_{l,t} + i_{b,t}\right)\left(R^m_t - 1\right) + c_{l,t} + q_t\left(h_{l,t} - 0.5h\right)$,

$$\lambda_{l,t} = \epsilon_l(\delta - c_{l,t})/R^L_t, \tag{23}$$

$$q_t\lambda_{l,t} = \gamma - h_{l,t} + \beta E_t q_{t+1}\lambda_{l,t+1}, \tag{24}$$

$$\frac{\epsilon_l(\delta - c_{l,t})}{R^L_t} = \beta E_t\left[\frac{0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))}{\pi_{t+1}}\right], \tag{25}$$

$$c_{l,t} = 0.5\left(i_{l,t} + i_{b,t}\right) + 0.5m^H_{t-1}\pi_t^{-1} - l_{l,t}/R^L_t, \tag{26}$$

where we used that all agents face the same probability (0.5) of drawing ϵ_l in period t, such that average money holdings of these agents at the beginning of each period must satisfy $m^H_{l,t-1} = \Sigma_{l,i}m^H_{l,i,t-1} = 0.5m^H_{t-1}$. The same argument, which is based on the law of large numbers, has been used for bond holdings and housing, $b_{l,t-1} = \sum_{l,i} b_{l,i,t-1} = 0.5b_{t-1}$ and $h_{l,t-1} = \sum_{l,i} h_{l,i,t-1} = 0.5h$.

Given that we are interested in analyzing policy interventions we restrict our attention to cases where the equilibrium allocation is inefficient due to a relevant distortion, which is here given by the collateral requirement originating from limited contract enforcement. For the equilibrium allocation to be inefficient, the borrowing constraint (3) therefore has to be binding, which is apparently more likely for a larger difference in the agents' valuation of consumption and for a lower liquidation value of collateral. To further facilitate aggregation, we restrict our attention to the case where the associated multiplier is strictly positive for all agents drawing ϵ_b, $\zeta_{b,i,t} > 0$, which can be guaranteed by a sufficiently large difference in agents' valuation of consumption relative to the liquidation value of collateral, $(\epsilon_b - \epsilon_l)/z$.

Assumption 3 *The ratio $(\epsilon_b - \epsilon_l)/z$ is sufficiently large such that the borrowing constraint (3) is binding for all agents drawing ϵ_b.*

Applying the Assumptions 1 and 3, we can easily derive the corresponding set of conditions describing the behavior of a representative borrower. The following conditions describe the behavior of a representative agent drawing ϵ_b in period t : $0.5m^H_{t-1}\pi_t^{-1} + 0.5b_{t-1}\pi_t^{-1} + l_{b,t}\left(1 - 1/R^L_t\right) +$

$$0.5y_t + 0.5\tau_t = m^H_{b,t} + (b_{b,t}/R_t) + 0.5\left(i_{l,t} + i_{b,t}\right)\left(R^m_t - 1\right) + c_{b,t} + q_t\left(h_{b,t} - 0.5h\right),$$

$$\lambda_{b,t} = \beta E_t\left[0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))/\pi_{t+1}\right], \tag{27}$$

$$q_t\lambda_{b,t} = \gamma - h_{b,t} + \zeta_{b,t} z q_t + \beta E_t q_{t+1}\lambda_{b,t+1}, \tag{28}$$

$$\frac{\epsilon_b(\delta - c_{b,t})}{R^L_t} = \beta E_t\left[\frac{0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))}{\pi_{t+1}}\right] + \zeta_{b,t}, \tag{29}$$

$$c_{b,t} = 0.5\left(i_{l,t} + i_{b,t}\right) + 0.5m^H_{t-1}\pi_t^{-1} - l_{b,t}/R^L_t, \tag{30}$$

$$-l_{b,t} = zq_t h_{b,t}, \tag{31}$$

where aggregate variables are defined as before and the last condition follows from (3) and $\zeta_{b,i,t} > 0$. Using that $h = h_{l,t} + h_{b,t}$, $l_t = l_{l,t} = -l_{b,t}$, and that (23), (25), and (27) imply $\lambda_t = \lambda_{b,t} = \lambda_{l,t}$, and substituting out $\zeta_{b,t}$, λ_t, and l_t, we can define a competitive equilibrium in terms of a representative borrower and a representative lender as follows.

Definition 1 *A competitive equilibrium of the economy with a representative borrower and a representative lender under a conventional monetary policy regime is a set of sequences* $\{c_{b,t},\ c_{l,t},\ h_{b,t},\ q_t,\ \pi_t\}_{t=0}^{\infty}$ *satisfying*

$$\epsilon_l(\delta - c_{l,t}) = \beta 0.5E_t\left[(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))\left\{R^L_t/\pi_{t+1}\right\}\right], \tag{32}$$

$$\left\{R^L_t/q_t\right\}(2h_{b,t} - h)/z = \epsilon_b(\delta - c_{b,t}) - \beta 0.5E_t[(\epsilon_b(\delta - c_{b,t+1}) + \epsilon_l(\delta - c_{l,t+1}))\left\{R^L_t/\pi_{t+1}\right\}], \tag{33}$$

$$\epsilon_l(\delta - c_{l,t})\left\{q_t/R^L_t\right\} = \gamma - (h - h_{b,t}) + \beta E_t[\epsilon_l(\delta - c_{l,t+1})\left\{q_{t+1}/R^L_{t+1}\right\}], \tag{34}$$

$$c_{b,t} - c_{l,t} = 2zh_{b,t}\left\{q_t/R^L_t\right\}, \tag{35}$$

$$y_t = c_{b,t} + c_{l,t}, \tag{36}$$

and $R^L_t = R^m_t$, *for* $\{y_t\}_{t=0}^{\infty}$ *and a sequence* $\{R^m_t \geq 1\}_{t=0}^{\infty}$ *set by the central bank.*

An agent who draws a preference shock ϵ_b in period t, borrows money from other agents to increase its consumption possibilities. Given that the loan has to be repaid at the end of the period, it has less funds available at the beginning of period $t+1$. While idiosyncratic histories of shock realizations matter for individual net wealth positions, they do not matter for the aggregate behavior of borrowers/lenders, given that all agents – regardless of their net wealth position – face the same probability of drawing ϵ_b (ϵ_l) and their behavioral relations are linear. Further note that the multiplier on the borrowing constraint satisfies

$$\zeta_{b,t} = [\epsilon_b(\delta - c_{b,t}) - \epsilon_l(\delta - c_{l,t})]/R^L_t = \left(2h_{b,t} - h\right)/(zq_t) \geq 0, \tag{37}$$

indicating that both, the housing and the consumption choice (that would ideally satisfy $h_b = h_l$ and $\epsilon_l(\delta - c_{b,t}) = \epsilon_l(\delta - c_{l,t})$, see Proposition 1), are distorted by a binding borrowing constraint ($\zeta_{b,t} > 0$). On the one hand, the marginal utility of consumption is then larger for borrowers than for lenders, $\epsilon_b(\delta - c_{b,t}) > \epsilon_l(\delta - c_{l,t})$. On the other hand, borrowers' housing exceeds lenders' housing, $h_{b,t} > h/2$, as the former is characterized by a relatively higher valuation of housing due to its ability to serve as collateral. Given that the supply of non-durables and durables is exogenous, such that $h = h^*_{b,t} + h^*_{l,t}$, and $y = c_{l,t} + c_{b,t}$, (37) implies that the equilibrium allocation equals the first best allocation (see Proposition 1) when the borrowing constraint gets irrelevant, $\zeta_{b,t} \to 0$.

Corollary 1 *For the limiting case where the multiplier on the borrowing constraint approaches zero, the equilibrium allocation is identical with the first best allocation.*

Notably, Corollary 1 implies that the distortion due to the liquidity constraint (5) alone does not lead to an allocative inefficiency. Accordingly, Definition 1 reveals that the nominal interest rate and thus the policy rate only matters jointly with either the housing price or the inflation rate. Precisely, the conditions (32)-(36) impose restrictions on the allocation, $c_{b,t}, c_{l,t}$, and $h_{b,t}$, the ratio R^L_t/q_t, and the real interest rate R^L_t/π_{t+1} (see curly brackets in 32-35), but not on q_t, π_t, and R^L_t. Thus, monetary policy measures, i.e. changes in the policy rate $R^m_t = R^L_t$, leave the allocation unaffected, while they affect the inflation rate and the relative price of housing. The latter effect is due to the liquidity constraint and the well-known inflation tax on cash goods (here, non-durables), which implies that higher interest rates reduce the demand for consumption and raise the demand for housing.

Corollary 2 *Under a conventional monetary policy regime, changes in the monetary policy rate do not affect the equilibrium allocation, while the housing price and the inflation rate increase with the nominal interest rate.*

The reason for the neutrality summarized in Corollary 2 is that conventional monetary policies can only affect equilibrium prices that are equally relevant for both agents, while the aggregate endowment with durable and non-durable goods is exogenously determined. Notably, asset purchases will instead drive a wedge between prices that are either relevant for borrowers or for lenders. Given that changes in the monetary policy instrument R^m_t under a conventional monetary policy regime do not affect the equilibrium allocation, the latter is time-invariant if there is no aggregate risk. To facilitate comparisons between the different policy experiments, we restrict our attention to the case of time-invariant policies in the subsequent analysis. In Section 4.3, where we introduce aggregate risk, we extend the analysis by considering state-contingent and thus time-varying policies.

3.2 Constrained efficiency under a borrowing subsidy

For the remainder of this section, we abstract from aggregate risk, $y_t = y$, and focus on time-invariant financial market interventions, such that neither the allocation nor prices are time-varying. While we are primarily interested in the effects of asset purchases, where the central bank has several instruments at its disposal, we first examine a financial market intervention with one instrument, which can be analyzed in a more straightforward way and which facilitates the analysis of efficiency gains. We consider a simple policy intervention that alters agents' incentives and thereby prices in financial markets, while abstracting from purely redistributive policies (which typically fall into the domain of fiscal policies). Specifically, we suppose that a planer can influence private borrowing by a tax/subsidy on debt τ^L and transfers the funds in cash to the taxed agents in a lump-sum way, such that the borrower's loan price net of taxes is $(1-\tau^L)/R_t^L$ and the lump-sum transfer/tax equals $\tau_t^R = \tau^L l_t / R_t^L$.[14] Thus, this intervention affects the marginal costs of borrowing and can thereby correct for inefficiencies induced by externalities associated with financial market frictions, implementing a constrained efficient allocation.

Consider the competitive equilibrium as given in Definition 1 under time-invariant endogenous variables and with the lump-sum financed tax/subsidy. The borrowers' consumption Euler equation (33) then changes to $(1-\tau^L)\epsilon_b(\delta - c_b) = \beta 0.5[(\epsilon_b(\delta-c_b)+\epsilon_l(\delta-c_l))\left\{R^L/\pi\right\} + \left\{R^L/q\right\}(2h_b - h)/z$, and condition (34) implies the price of housing relative to consumption q/R_L to be positively related to borrowers' housing and to lenders' consumption,

$$\frac{q}{R^L} = \frac{\gamma - h + h_b}{(1-\beta)\epsilon_l(\delta - c_l)}, \tag{38}$$

which is not internalized by private agents. Notably, an increase in the ratio q/R^L tends to raise the difference between consumption of borrowers and lenders, as it relaxes the impact of the collateral constraint (3) on borrowers' consumption. Using (38) to substitute out q/R^L in the borrowers' consumption Euler equation and in (35), and (32) to substitute out the real interest rate, yields

$$(1-\tau^L)\epsilon_b(\delta - c_b) - \epsilon_l(\delta - c_l) = (2h_b - h)\,\frac{(1-\beta)\epsilon_l(\delta - c_l)}{z(\gamma - h + h_b)}, \tag{39}$$

Given that there is no time variation, the problem of a social planer, who maximizes social welfare (21) by controlling the tax/subsidy rate τ^L and respecting the borrowing constraint, can then be summarized as

$$\max_{c_l, c_b, h_b} \{u(\epsilon_b, c_b, h_b) + u(\epsilon_l, c_l, h - h_b)\}/(1-\beta), \tag{40}$$
$$\text{s.t.} \quad y = c_l + c_b,\ c_b - c_l \leq 2zh_b \cdot \left(q/R^L\right), \text{ and } (38).$$

In contrast to private agents, the social planer takes into account that changes in the allocation alter the relative price q/R^L (see 38) and might correct for pecuniary externalities using the tax/subsidy. Thereby, the solution to this problem, where the social planer decides on private borrowing subject to the competitive equilibrium conditions, leads to a constrained efficient allocation. Given this constrained efficient allocation, (39) determines the associated tax/subsidy rate τ^L. The following proposition summarizes how the social planer can implement the constraint efficient allocation.

Proposition 2 *The implementation of a constrained efficient allocation of the representative*

[14] Thus, the tax/subsidy and the lump-sum transfers/taxes enter the budget constraint (6) and the goods market constraint (5) of borrowers.

agents economy without aggregate risk requires a subsidy on borrowing, $\tau^L < 0$, if but not only if $z \geq 1 - \beta$. Compared to the laissez-faire case ($\tau^L = 0$), the borrowing subsidy raises borrowers' consumption and housing as well as the real interest rate, which is associated with a decline in lenders' consumption and housing.

Proof. See Appendix B. ■

Proposition 2 implies that a financial market intervention that stimulates borrowing can enhance social welfare if the liquidation value of collateral is sufficiently large, $z \geq 1 - \beta$. In this case, which is likely to be satisfied by reasonable values for the parameters β and z, the positive impact of borrowers' housing dominates the associated negative (positive) impact of borrowers' (lenders') consumption on the terms of borrowing via the relative price q/R^L (see 38). Put differently, borrowing is then inefficiently low in a competitive equilibrium, given that the private agents do not internalize the favorable effects of increased housing demand on the relative price q/R^L. The social planer can then correct for this pecuniary externality by a borrowing subsidy $\tau^L < 0$ (financed by a lump-sum tax on borrowers, $\tau_t^R = -\tau^L l_t/R_t^L > 0$), which induces agents to internalize changes in the relative price q/R^L. As summarized in Proposition 2, the subsidy causes agents to borrow more, leading to an increase in borrowers' consumption and housing. Notably, the subsidy reduces the costs of borrowing, while it simultaneously raises the real interest rate R^L/π. This induces lenders to increase their supply of funds, such that their consumption and housing decreases. In total, the borrower's gains outweigh the lender's losses, implying that social welfare is enhanced by distributional effects of the policy intervention.

Given that $z \geq 1 - \beta$ is just a sufficient condition, a violation does not necessarily imply the opposite result. Nevertheless, at very low values for z the positive impact of an increase in borrowers' consumption and housing on the terms of borrowing (q/R^L) can be reversed, which would require rather a tax on debt than a subsidy. Hence, the competitive equilibrium is characterized by "under-borrowing" compared to a constrained efficient allocation if $z \geq 1 - \beta$, while "over-borrowing" might prevail, when the latter constraint is violated.[15]

3.3 Money rationing and asset purchases

We now turn to the effects of central bank purchases of secured loans, $\kappa_t > 0$ (see 4). Firstly, we will show that the central bank can implement the constrained efficient allocation, as derived in Proposition 2. Put differently, it will be shown that a monetary policy intervention can be equivalent to a lump-sum financed borrowing subsidy that supports the constrained efficient allocation. Secondly, we will show that compared to the case of a lump-sum financed borrowing tax/subsidy (as described in Section 3.2) the central bank can enlarge the set of feasible equilibria and can even implement allocations that welfare-dominate the constrained efficient allocation.

[15] See also Davila and Korinek (2017) for a discussion of conditions under which pecuniary externalities implied by borrowing constraints imply either "under-borrowing" or "over-borrowing" in a competitive equilibrium.

Given that a conventional monetary policy does not affect the allocation (see Corollary 2), the central bank thereby relies on asset purchases under money rationing. We therefore assume that the central bank ensures that the policy rate satisfies $1 \leq R_t^m < R_t^L$. Then, the central bank offers an above-market price for loans and the money supply constraints (2) and (4) are binding (see 16)

$$i_{b,t} = \kappa_t^B 0.5 b_{t-1}/(\pi_t R_t^m), \;\; i_{l,t} = \kappa_t^B 0.5 b_{t-1}/(\pi_t R_t^m), \; i_{l,t}^L = \kappa_t l_t / R_t^m, \tag{41}$$

such that money supply is effectively rationed by holdings of eligible collateral (i.e. treasuries and secured loans). Notably, the liquidity constraint (5) is binding as well, since $1 \leq R_t^m < R_t^L$ implies the nominal marginal rate of intertemporal substitution to exceed one. Under money rationing, purchases of loans $\kappa_t > 0$ increase the supply of money against eligible assets and can affect the allocation, as they drive a wedge between the borrowers' and the lenders' effective real loan rate. Specifically, the effective return for a lender is then distorted by the term $\frac{1-\kappa}{1-\kappa R^L/R^m}$ (see 18). Further, using (41) to rewrite the binding liquidity constraints (5) and taking differences yields

$$c_{b,t} - c_{l,t} = z q_t h_{b,t} \frac{2 - \kappa_t R_t^L / R_t^m}{R_t^L}, \tag{42}$$

where we substituted out loans with the binding borrowing constraint (31). Comparing (35) with (42), reveals that asset purchases can further affect the amount of loans and thereby the equilibrium allocation. Applying the same aggregation procedure as in Section 3.1, using (19), (20), and $B_t^T = B_t^c + B_t$, and eliminating the multiplier $\lambda_{l,t}$ with $\lambda_{l,t} = \beta E_t[0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))/\pi_{t+1}]$ in (24), we can summarize a competitive equilibrium under Assumptions 1-3 and money rationing ($R_t^m \in [1, R_t^L)$) as follows.

Definition 2 *A competitive equilibrium of the representative agents economy under money rationing is a set of sequences* $\{c_{b,t}, c_{l,t}, h_{b,t}, m_t^H, b_t, q_t, R_t^L, \pi_t\}_{t=0}^{\infty}$ *satisfying (33), (36), (42),*

$$\epsilon_l(\delta - c_{l,t}) = \beta 0.5 E_t[(\epsilon_b(\delta - c_{b,t+1}) + \epsilon_l(\delta - c_{l,t+1}))(R_t^L/\pi_{t+1})] \frac{1-\kappa}{1-\kappa R_t^L/R_t^m}, \tag{43}$$

$$\begin{aligned} & q_t \beta E_t \left[0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))/\pi_{t+1}\right] && (44) \\ & = \gamma - (h - h_{b,t}) + \beta^2 E_t q_{t+1} \left[0.5(\epsilon_l(\delta - c_{l,t+2}) + \epsilon_b(\delta - c_{b,t+2}))/\pi_{t+2}\right], \\ c_{b,t} &= 0.5(1+\Omega_t) m_t^H + z q_t h_{b,t} / R_t^L && (45) \\ (1+\Omega_t) m_t^H &= \kappa_t^B b_{t-1} \pi_t^{-1} / R_t^m + m_{t-1}^H \pi_t^{-1}, && (46) \\ b_t + m_t^H &= \Gamma \left(b_{t-1} + m_{t-1}^H\right)/\pi_t, && (47) \end{aligned}$$

and the transversality conditions, for $\{y_t\}_{t=0}^{\infty}$ *and sequences* $\{\kappa_t \geq 0, \; \kappa_t^B > 0, \; \Omega_t > 0, \; R_t^m \in [1, R_t^L)\}_{t=0}^{\infty}$ *set by the central bank, given* $m_{-1}^H > 0$*, and* $b_{-1} > 0$.

Evidently, there are more instruments available for the central bank when it supplies money in a rationed way. In fact, the fraction of bonds eligible for open market operations κ_t^B and the repo share Ω_t can be adjusted by the central bank to support a particular equilibrium allocation and associated prices, such that (45) is not a relevant binding constraint for the policy maker. It should further be noted that the long-run inflation rate π can in principle depend on the growth rate of treasuries Γ (see 47). Yet, the central bank can implement a desired inflation rate by suited adjustments of its instruments κ_t^B and Ω_t as shown in Appendix C. Thus, the inflation rate can actually be treated as a choice variable of the central bank, which we also demonstrate for numerical examples in Section 4.3. Under money rationing, the central bank can therefore manipulate the loan rate and the allocation not only via the inflation rate but also by setting the policy rate R_t^m and the share of purchased loans κ_t. It can then easily be shown that the constrained efficient allocation can be implemented by a time-invariant asset purchase regime.

Suppose that there is no aggregate risk and that monetary policy is time-invariant, $\pi_t = \pi \geq 0$, $R_t^m = R^m \in [1, R^L)$ and $\kappa_t \geq 0$. A competitive equilibrium under money rationing consists of a set $\{c_{b,t}, c_{l,t}, h_{b,t}, q_t, R_t^L\}$ satisfying (33), (36), (42), (43), and (44) for a monetary policy setting $\{\kappa,\, R^m,\, \pi\}$. Using that all variables are time-invariant, substituting out q with (44) in (33) and (42), and applying (43), the competitive equilibrium under money rationing can be reduced to a set $\{c_b, c_l, h_b, R^L\}$ satisfying $y = c_l + c_b$,

$$\left[\frac{\pi}{R^L}\right] = \beta\frac{Eu_c}{u_{c,b}} + \frac{1-\beta}{z}\frac{(u_{h,l}-u_{h,b})u_{c,l}}{u_{h,l}u_{c,b}} \cdot \left[\frac{\pi}{R^L}\frac{1-\kappa R^L/R^m}{1-\kappa}\right], \tag{48}$$

$$c_b - c_l = h_b\frac{z}{1-\beta}\frac{u_{h,l}}{u_{c,l}} \cdot \left[\frac{(1-\kappa)\left(2-\kappa R^L/R^m\right)}{(1-\kappa R^L/R^m)}\right], \tag{49}$$

$$\left[\frac{\pi}{R^L}\frac{1-\kappa R^L/R^m}{1-\kappa}\right] = \beta\frac{Eu_c}{u_{c,l}}, \tag{50}$$

(where $Eu_c = 0.5(\epsilon_b(\delta - c_b) + \epsilon_l(\delta - c_l))$, $u_{c,b} = \epsilon_b(\delta - c_b)$, $u_{c,l} = \epsilon_l(\delta - c_l)$, $u_{h,l} = \gamma - (h - h_b)$, and $u_{h,b} = \gamma - h_b$) given R^m, κ, and π. Now, we compare the latter with the corresponding conditions under the subsidy $\widetilde{\tau}^L < 0$, which implements the constrained efficient allocation, given by $y = c_l + c_b$,

$$\left[(1-\widetilde{\tau}^L)\frac{1}{\widetilde{r}}\right] = \beta\frac{Eu_c}{u_{c,b}} + \frac{1-\beta}{z}\frac{(u_{h,l}-u_{h,b})u_{c,l}}{u_{h,l}u_{c,b}} \cdot \left[\frac{1}{\widetilde{r}}\right], \tag{51}$$

$$c_b - c_l = h_b\frac{z}{1-\beta}\frac{u_{h,l}}{u_{c,l}} \cdot [2], \tag{52}$$

$$\left[\frac{1}{\widetilde{r}}\right] = \beta\frac{Eu_c}{u_{c,l}}, \tag{53}$$

where (51) stems from combining (32) with (39), and (52) from (35) and (38), while $\widetilde{r}$ denotes the real interest rate R^L/π under the constrained efficient allocation implemented with the optimal subsidy $\widetilde{\tau}^L$. The comparison of the terms in square brackets in (48)-(50) with the corresponding terms in (51)-(53) reveals that the implementation of the constrained efficient allocation requires the monetary policy instruments, R^m, κ, and π, to satisfy $\pi/R^L = (1-\widetilde{\tau}^L)/\widetilde{r}$, $(\pi/R^L)\left(1-\kappa R^L/R^m\right)/(1-\kappa) = 1/\widetilde{r}$, and $(1-\kappa)\left(2-\kappa R^L/R^m\right)/\left(1-\kappa R^L/R^m\right) = 2$, and therefore

$$\kappa = -\widetilde{\tau}^L > 0,\ \pi = (1 - \widetilde{\tau}^L)\left(R^L/\widetilde{r}\right) \geq 0,\ R^m = (1+\kappa)\, R^L/2, \tag{54}$$

where R^L is endogenously determined in a competitive equilibrium. Thus, the central bank can implement the constrained efficient allocation by setting the instruments according to (54), which is feasible for $\pi \geq 2/\widetilde{r}$, which ensures $R^m = (1+\kappa)\, R^L/2 \geq 1$, and for $\kappa < 1$, which ensures $R^m = (1+\kappa)\, R^L/2 < R^L$. This result is summarized in the following proposition.

Proposition 3 *Suppose that money supply is rationed and $z \geq 1-\beta$. Then, the constrained efficient allocation without aggregate risk can be implemented by the central bank via asset purchases.*

As shown above, the central bank can replicate a subsidy on borrowing by purchasing a positive share of loans κ at a price R^m that is lower than the loan rate R^L and by setting a suited inflation rate π, such that its instruments satisfy (54). It thereby effectively reduces the equilibrium loan rate compared to the case without loan purchases, which tends to stimulate borrowing. Asset purchases then reduce the borrower's real rate, R^L/π, relative to the lender's effective real rate, $\frac{1-\kappa}{1-\kappa R^L/R^m}(R^L/\pi)$, such that the representative borrower (lender) consumes more (less) than without asset purchases (see Proposition 2 and Section 4.2 for numerical examples).

The central bank can, however, implement a larger set of allocations than the borrowing subsidy. As shown above, asset purchases do not only influence the price of loans, which is relevant for (48) and (50), but can also affect the tightness of the borrowing constraint (see 49). Given that the central bank has three instruments at its disposal, i.e. κ, R^m, and π, it can, on the one hand, replicate the relative price effects of a borrowing subsidy and, on the other hand, relax the constraint imposed on borrowers' consumption by the collateral requirement. Precisely, replicating the price effects requires setting κ and π to satisfy the first two conditions in (54), while the policy rate R^m can be set to increase the amount of funds available for lending, such that the RHS of (49) exceeds the RHS of (52). It can thereby implement allocations that welfare-dominate the constrained efficient allocation under a borrowing subsidy described in Proposition 2. This result is summarized in the following proposition.

Proposition 4 *Suppose that money supply is rationed and there is no aggregate risk. Then, the central bank can implement allocations via asset purchases that welfare-dominate allocations that are implementable under a lump-sum financed tax/borrowing subsidy.*

Proof. See Appendix D. ■

The borrowing subsidy alters the effective borrowers' real interest rate, $(1-\tau^L)(R^L/\pi)$, whereas asset purchases change the effective lenders' real interest rate, $\frac{1-\kappa}{1-\kappa R^L/R^m}(R^L/\pi)$. As shown in Proposition 3, asset purchases can thereby mimic a borrowing subsidy. However, the central bank can by purchasing loans further increase the amount of money that are available for loans by choosing a policy rate that is sufficiently low compared to the loan rate. Given that an increase in loans can raise the consumption differential $c_b - c_l$, which is under a binding borrowing constraint inefficiently small, asset purchases can even implement allocations that welfare-dominate allocations under a lump-sum financed borrowing subsidy. Yet, the increase in loans has to be accompanied by an increase in collateral, which further distorts the housing allocation. As a consequence, first best can in general not be implemented by asset purchases. In the subsequent analysis, we will provide numerical examples for different regimes, which confirm the results presented in the Propositions 2-4.

4 Numerical examples

In this section, we provide numerical examples illustrating the theoretical results derived in the previous section. To facilitate the parametrization of the model, we introduce a more standard (CRRA) utility function. Applying such a utility function, however, implies that we cannot easily aggregate over individual households as in Section 3. To focus on the effects of central bank asset purchases, we simplify the analysis and abstract from implications of an endogenous distribution of agents' net wealth. We therefor introduce the assumption of pooling funds within households at the end of each period, such that household members are ex-ante identical before they split up into borrowers and lenders. An equilibrium in terms of a representative borrower and a representative lender then only differs from the previous version by non-linear – instead of linear – marginal utilities. In the last part of this section, we further introduce aggregate risk via a random aggregate endowment and demonstrate that countercyclicality of asset purchases can reduce short-run welfare losses.

4.1 A version with CRRA preferences

We consider infinitely many households of measure one, which consist of infinitely many members i. In each period, ex-ante identical household members draw the idiosyncratic preference shock, which induces some members to borrow and others to lend. Like Lucas and Stokey (1987) or Shi (1997), we assume that at the end of each period (after loans are repaid) household members obtain equal shares of total household wealth, such that they are again equally endowed before new preference shocks are drawn in the next period. Thus, the representativeness of agents is induced by a redistribution of wealth within each household (rather than resulting from the linearity of agents' behavioral relations). We assume that period utility of household member i is given by a separable CRRA utility function

$$u^{CRRA}(\epsilon_i, c_{i,t}, h_{i,t}) = \epsilon_i \frac{c_{i,t}^{1-\sigma} - 1}{1-\sigma} + \gamma \frac{h_{i,t}^{1-\sigma} - 1}{1-\sigma}, \text{ where } \gamma > 0 \text{ and } \sigma > 0, \tag{55}$$

such that Assumption 1 (and thus 22) does not apply. We further allow for aggregate risk in form of a random process for aggregate endowment, which will be examined in Section 4.3. Specifically, we assume that the log of aggregate endowment follows an AR1 process

$$\log y_t = \rho \log y_{t-1} + \varepsilon_t, \tag{56}$$

where the $\varepsilon_t' s$ are i.i.d. with mean zero and $\rho \in [0, 1)$. Otherwise, the model presented in Section 2 is unchanged, such that the competitive equilibrium in terms of a representative borrower and a representative lender is identical to those given in Definitions 1 and 2, except for marginal utilities being non-linear in this version (see Definition 4 in Appendix E). As in the case of linear-quadratic preferences, it can be shown that a constrained efficient allocation without aggregate risk can be implemented by a subsidy on borrowing, $\tau^L < 0$ (see Appendix F), while the constrained efficient allocation can again be implemented via asset purchases, given that Proposition 3 apparently holds for both types of preferences (see 48-53).

To solve the model numerically, we have to assign values for the elasticity of intertemporal substitution σ, the discount factor β, the utility weight for housing γ, the liquidation value of collateral z, the degree of household heterogeneity $\Delta\epsilon = \epsilon_b - \epsilon_l$, the autocorrelation coefficient ρ of the endowment process (56), and the standard deviation of the innovations σ_ε.[16] We interpret a model period as one year and calibrate the model consistent with postwar US data. We estimate the process (56) using (linearly detrended) annual US data for real gdp per capita (for 1947-2008), leading to $\rho = 0.752$ and $\sigma_\varepsilon = 0.0216$. The value for the elasticity of intertemporal substitution σ set equal to 2, which is a typical value applied in business cycle studies. The liquidation value of collateral z is set equal to 0.6, which is similar to values applied in related studies (see Iacoviello, 2005, or Garriga et al., 2015). For the remaining three parameters, β, γ, and $\Delta\epsilon$, we apply values that allow to match three targets for the reference case without financial market interventions. These targets are the mean share of installment loans to income of 21% (for 1998-2004, see Survey of Consumer Finances), the mean yield on MBS of 6.6% for pre-2009 US data taken from Hancock and Passmore (2011), which corresponds to the rate on secured loans $R^L - 1$, and the cross sectional standard deviation of real log consumption of 0.64 (see De Giorgi and Gambetti, 2012).[17] While it is not possible to exactly match all three targets, our choice $\beta = 0.8$, $\gamma = 0.002$, and $\Delta\epsilon = 0.76$ yields to a reasonable match given by $R^L = 1.06$, $(l/R^L)/y = 0.2$, and a standard deviation of real log consumption of 0.6.

4.2 Welfare gains of asset purchases without aggregate risk

We first consider the case without aggregate risk ($\sigma_\varepsilon = 0$) and compute the equilibrium allocation and associated interest rates for different types of policy regimes, specifically, a regime with the lump-sum financed borrowing subsidy and regimes with asset purchases. As references cases, we

[16] We further have to assign values to the growth rate of treasuries Γ and for the repo share Ω. Given that both are not relevant for the equilibrium allocation under the current set of central bank instruments, we apply the values $\Gamma = \pi$ and $\Omega = 1$, for convenience.

[17] Notably, the data samples are not alligned due to limited data availability.

further present a laissez-faire case, i.e. where monetary policy is conducted in a conventional way and does therefore not affect the equilibrium allocation (see Corollary 2), as well as the first best allocation. The first column of Table 1 lists the policy instruments and endogenous variables, where we also consider the effective real rates of the representative borrower r_b and of the representative lender r_l,

$$r_b = (1-\tau)R^L/\pi \text{ and } r_l = (R^L/\pi)\frac{1-\kappa}{1-\kappa R^L/R^m}, \tag{57}$$

as well as welfare of the representative borrower and lender, $v_b = u^{CRRA}(\epsilon_b, c_b, h_b)/(1-\beta)$ and $v_l = u^{CRRA}(\epsilon_l, c_l, h_l)/(1-\beta)$, and social welfare, $v_b + v_l$.[18] The second column shows the results for the laissez-faire case, where neither a subsidy nor asset purchases are considered and prices are constant. Under this regime, consumption of the representative borrower equals 0.699, while the effective real rates are identical and equal to 1.06. Individual welfare indicates that the representative lender is better off than under first best, given that the representative borrower consumes less due to the borrowing constraint.

The third column of Table 1 presents results for the case where a lump-sum financed subsidy implements the constrained efficient allocation (CEA), as shown in Appendix F. The optimal borrowing subsidy equals $\tau^L = -0.306$ and stimulates borrowing by reducing the effective real rate of borrowers r_b to 0.937. The increase in borrowing, which is accompanied by a larger housing share of borrowers and an increased housing price, induces consumption of the representative borrower to increase by roughly 4% to 0.726. The corresponding decline in lender's consumption is associated by a substantial increase in the effective real rate of the representative lender ($r_l = 1.224$). The individual welfare measures indicate that the representative borrower (lender) gains (loses) under the constrained efficient allocation. Social welfare increases from -6.670 under laissez-faire to -6.612 under the constrained efficient allocation, which reduces the total welfare loss compared to first best by 39%. While the consumption allocation under first best can almost be replicated by the subsidy, the allocation of housing, which is distorted by its role as collateral for borrowers, substantially differs between both regimes.

As shown in Proposition 3, the central bank can in principle implement the constrained efficient allocation by purchasing assets, $\kappa > 0$, under money rationing, $R^m < R^L$, i.e. by setting its instruments according to (54). This however requires that the zero lower bound is not binding, specifically, that $(1+\kappa)\,R^L/2 \geq 1$. Under the parameter values applied in this section, the central bank cannot exactly replicate the optimal subsidy without violating the zero lower bound.[19] We therefore examine four different regimes with asset purchases under money rationing, where we set the policy rate equal to its lowest possible value ($R^m = 1$), and show that they nevertheless

[18] Social welfare in terms of consumption equivalents is not reported, for convenience, since agents differ with regard to their consumption valuation.

[19] Precisely, implementing the constrained efficient allocation would require setting the instruments according to $\kappa = 0.306$, $\pi = 1.13$, and $R^m = 0.62$; the latter not being feasible with the zero lower bound.

Table 1: Distributional and wefare effects under different regimes without aggregate risk

		Laissez-faire	CEA	AP I	AP II	AP III	AP IV	First best
Instruments	τ^L	–	-0.306	–	–	–	–	–
	κ	–	–	0.306	0.306	0.900	0.250	–
	$\pi - 1$	0.0%	0.0%	0.0%	2.0%	5.0%	100%	–
	R^m	–	–	1.000	1.000	1.000	1.000	–
Variables	c_b	0.699	0.726	0.701	0.702	0.719	0.732	0.730
	c_l	0.301	0.274	0.299	0.298	0.281	0.268	0.270
	h_b	0.903	0.917	0.911	0.911	0.929	0.917	0.500
	R^L	1.060	1.224	1.050	1.067	1.019	1.825	–
	q	0.425	0.550	0.502	0.514	0.807	1.086	–
	r_b	1.060	0.937	1.050	1.046	0.971	0.916	–
	r_l	1.060	1.224	1.074	1.078	1.178	1.259	–
Welfare	v_b	-3.790	-3.318	-3.749	-3.735	-3.443	-3.229	-3.260
	v_l	-2.880	-3.294	-2.919	-2.931	-3.198	-3.382	-3.260
	$v_b + v_l$	-6.670	-6.612	-6.668	-6.666	-6.641	-6.611	-6.520

enhance welfare compared to laissez-faire.

The forth column of Table 1 shows the results for an asset purchase regime with a fraction of purchased assets κ equal to minus the optimal subsidy rate, as implied by (54), and with constant prices (AP I). Compared to the laissez-faire case, consumption of the representative borrower is stimulated, which is induced by a reduction in the effective borrower's real rate and an increase in the effective lender's real rate. This tendency is even more pronounced when the central bank raises the inflation rate to a moderate level ($\pi = 1.02$), as under the second asset purchase regime (AP II), which is presented in the fifth column.[20] Consequently, welfare of the representative borrower (lender) increases (decreases) from laissez-faire over AP I to AP II, where the borrower's gains dominate the lender's losses, such that social welfare increases. Under the third asset purchase regime (AP III, see sixth column), which exhibits a larger fraction of purchased loans ($\kappa = 0.9$) as well as a higher inflation rate ($\pi = 1.05$), the previous effects are more pronounced and the social welfare gain compared to the laissez-faire case is half as large as under the constrained efficient allocation.[21] As shown in Proposition 4, asset purchases even enable the implementation of allocations that welfare-dominate the constrained efficient allocation under lump-sum financed subsidies, if the difference between the loan rate and the policy rate is sufficiently large. This is demonstrated by the fourth asset purchase regime (AP IV, see seventh column), which is char-

[20]Further, the borrowers' housing share and the housing price are increased by asset purchases; the latter effect being more pronounced under higher inflation (see also Corollary 1).

[21]This regime's mean values for the policy instruments are also applied in the subsequent analysis under aggregate risk.

acterized by a smaller fraction of purchased loans ($\kappa = 0.25$) and a hyperinflation ($\pi = 2$). This policy induces an even more pronounced gap between the borrower's and the lender's effective real interest rates than under the borrowing subsidy and (slightly) welfare-dominates the latter regime.

4.3 Aggregate risk and state contingent asset purchases

In the final part of the analysis, we introduce aggregate risk, by considering a positive standard deviation σ_ε for the aggregate endowment process (56), and examine state contingent asset purchases. Due to aggregate endowment shocks, welfare losses stemming from credit market imperfections can be amplified in short-run, i.e. when the economy deviates from a stationary equilibrium due to $\varepsilon_t \neq 0$. For example, constrained borrowers might suffer more in downturns, where house prices slump and the borrowing constraint becomes tighter, than lenders. Thus, there might be a beneficial role for policy interventions that allow to enhance consumption smoothing, in particular, of constrained borrowers.

As discussed above, the central bank disposes of several instruments under an asset purchase regime, namely, the policy rate R_t^m, the share of purchased loans κ_t, and the inflation rate π_t (or, alternatively, κ_t^B and Ω_t). Thus, a welfare-enhancing asset purchase policy under aggregate risk will, in general, be characterized by state-contingency of all instruments. To keep the analysis transparent and to allow for a straightforward interpretation of the welfare effects stemming from state-contingent policy interventions, we fix the policy rate and the inflation rate, while we consider the share κ_t to be a function of the variable state of the economy, i.e. the aggregate endowment y_t. In particular, we apply a policy regime with money rationing, a positive mean value for the share of purchased assets $\kappa > 0$, and a log-linear feedback function for the share of purchased loans

$$\log\left(\kappa_t/\kappa\right) = \kappa_y \cdot \log\left(y_t/y\right). \tag{58}$$

Suppose first that the share of purchased assets is held constant, $\kappa_y = 0$. When the economy is hit by an adverse endowment shock, $\varepsilon_t < 0$, both types of agents (borrowers and lenders) have less non-durable goods available for consumption.[22] As the lenders' demand for housing shifts downward (see 38), the housing price and thereby the value of collateral fall, which tends to tighten the borrowing capacity of agents. Thus, borrowers particularly suffer from the adverse shock, implying that their marginal utility of consumption increases relatively more than the lenders' marginal utility of consumption. In such a situation, a policy of stimulating borrowing can be welfare enhancing, when it mitigates the decline in borrowers' consumption. This can in principle be achieved by an asset purchase regime with a state-contingent fraction of purchased loans, as induced by a non-zero elasticity of asset purchases with regard to aggregate endowment $\kappa_y \neq 0$ (see 58).

[22] This scenario is also displayed by the solid lines in Figure 1.

Table 2: Welfare gains of countercyclical asset purchases for AP III

	$\kappa_y = +4$	$\kappa_y = +2$	$\kappa_y = +1$	$\kappa_y = 0$	$\kappa_y = -1$	$\kappa_y = -2$	$\kappa_y = -4$
st.dev.(c_b)	0.0229	0.0224	0.0222	0.0220	0.0217	0.0215	0.0210
st.dev.(c_l)	0.0099	0.0103	0.0106	0.0108	0.0111	0.0113	0.0118
c_b^p	0.7182	0.7181	0.7181	0.7183	0.7185	0.7188	0.7197
c_l^p	0.2728	0.2730	0.2729	0.2728	0.2726	0.2723	0.2714
$\widetilde{v}_b + \widetilde{v}_l$	-6.6514	-6.6512	-6.6510	-6.6507	-6.6504	-6.6500	-6.6490

Table 2 presents results for different values for the elasticity κ_y for the asset purchase regime AP III with a mean share of purchased assets equal to $\kappa = 0.9$ and constant values for inflation and the policy rate, $\pi = 1.05$ and $R^m = 1$.[23] The columns show standard deviations of consumption and welfare measures for κ_y-values ranging between +4 and -4. For positive values for κ_y, implying procyclical asset purchases, the central bank stimulates borrowing in high income state more than in low income state, which tends to raise the volatility of borrowers' consumption compared to the case of a constant share, $\kappa_y = 0$. Simultaneously, the standard deviation of lenders' consumption is reduced by procyclical asset purchases. The overall impact of both effects on social welfare $\widetilde{v}_b + \widetilde{v}_l$ is negative compared to a constant share κ, as indicated by social welfare decreasing from $\kappa_y = 0$ to $\kappa_y = 4$ (see last row).[24] The central bank can revert these effects by conducting asset purchases with negative values for the elasticity κ_y. Specifically, for $\kappa_y = -4$ the standard deviation of borrowers' consumption is reduced by 4.5% compared to the case of a constant share ($\kappa_y = 0$) and borrowers' welfare in terms of consumption equivalents c_b^p increases by 0.2%. Though, welfare of the representative lender decreases, the total impact of state-contingent asset purchases on social welfare is positive when they are conducted in a countercyclical way, $\kappa_y < 0$.

To illustrate these effects, Figure 1 presents impulse responses to a negative endowment shock by one standard deviation under asset purchase regime AP III with the elasticities $\kappa_y = 0$ (black solid line), $\kappa_y = 4$ (red dotted crossed line), and $\kappa_y = -4$ (blue circled line). In all cases, the adverse endowment shock reduces consumption of borrowers and lenders. Under a constant or a procyclical share of asset purchases, the loan rate increases and the amount of loans falls. In case of countercyclical asset purchases, the central bank increases the share of purchased loans to $\kappa = 0.98$ on impact in response to the adverse endowment shock. This policy intervention tends to stimulate debt issuance via a reduction in the real loan rate of borrowers r_b, while the lenders' effective loan rate r_l increases by more than under constant (or procyclical) asset purchases. Given

[23] To implement a constant inflation rate, the central bank adjusts the share of eligible treasuries κ_t^B in a state-contingent way, which alters agents acces to central bank (see 46).

[24] Welfare under aggregate risk $\widetilde{v}_i$ is computed as $\widetilde{v}_i = E_{-1}\widetilde{v}_{i,t}$, where $\widetilde{v}_{i,t} = u^{CRRA}(\epsilon_i, c_{i,t}, h_{i,t}) + \beta E_t \widetilde{v}_{i,t+1}$ for $i \in \{b, l\}$, using a second order perturbation method, and the permanent consumption equivalents are computed as $c_i^p = ((1-\beta)(1-\sigma)(1/\epsilon_i)\widetilde{v}_i + 1)^{1/(1-\sigma)}$.

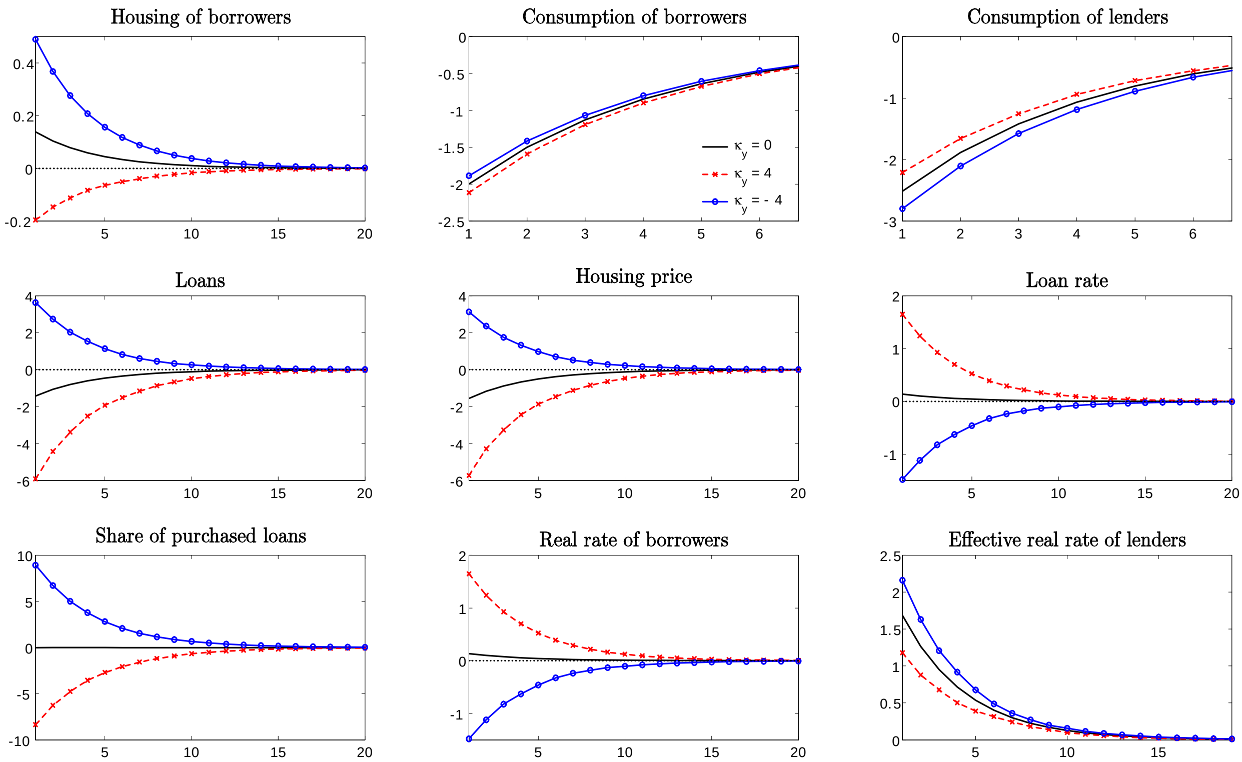

Figure 1: Responses to a minus one st.dev. aggregate endowment shock (in % deviations from a non-stochastic mean) for $\kappa = 0.9$ and $\pi = 1.05$ (black solid line: $\kappa_y = 0$, red dashed crossed line: $\kappa_y = 4$, blue circled line: $\kappa_y = -4$)

that the effects of countercyclical asset purchases allow borrowers to issue more debt, this tends to increase housing demand, which induces a rise in the housing price. The latter effect is also opposed to the case with a constant or a procyclical share of purchased assets. By stimulating borrowing a countercyclical asset purchase regime can therefore mitigate the decline in borrowers' consumption, while implying a more pronounced decline in lenders' consumption. Thus, the volatility of borrowers' consumption is reduced and the volatility of lenders' consumption is raised under countercyclical asset purchases, which leads to an overall increase in social welfare (see Table 2), given that borrowers exhibit a higher marginal utility of consumption than lenders. Given that this policy mitigates the build-up of debt in favorable states of the economy, it can support prudential financial regulations that aim at reducing the vulnerability in crisis times by reducing debt ex-ante.

5 Conclusion

This paper examines distributional effects of unconventional monetary policy. It is shown that an exchange of private debt securities against central bank money can enhance social welfare by stimulating the private debt market, which is particularly beneficial for borrowers facing relevant borrowing limits. We show that the central bank can incentivize (individually rational) lenders to enhance the supply of funds by purchasing debt securities at an above-market price. This causes the borrower's real interest rate to fall relative to the effective real interest of lenders, such that borrowers consume more and lenders less. Asset purchases can thus enhance welfare by inducing a redistribution of funds from lenders to borrowers who are characterized by a higher marginal valuation of funds (i.e. a higher marginal utility of consumption). These results are derived without referring to stressed financial markets or to a crisis scenario, implying that purchases of private debt securities can be a useful monetary policy instrument even in non-crisis times. Specifically, our analysis suggests that asset purchases shall be conducted in a countercyclical way.

To present the results in a transparent way, we consider an endowment economy with limited contract enforcement and two states for idiosyncratic shocks, which facilitates the derivation of analytical results. Yet, the downside of these assumptions is that the overall welfare effects, in particular, of state-contingent asset purchases, are small compared to welfare gains based on other/more macroeconomic frictions. It would therefore be desirable to examine the effects of asset purchases in a model with endogenous production (e.g. productive labor), a more realistic heterogeneity of agents (e.g. variable population shares), and further distortions (e.g. nominal rigidities), which is however beyond the scope of this paper. The analysis might further be extended by considering times of financial market stress (e.g. a variable liquidation value), where the interaction between ex-post interventions and prudential policies can be addressed.

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Appendix

A Competitive equilibrium

Definition 3 *A competitive equilibrium is a set of sequences* $\{c_{i,t},\, l_{i,t},\, i_{i,t},\, i^L_{i,t},\, \zeta_{i,t},\, \lambda_{i,t},\, h_{i,t},\, m^H_{i,t},\, b_{i,t},\, m^H_t,\, b_t,\, b^T_t,\, \pi_t,\, R^L_t,\, q_t\}_{t=0}^{\infty}$ *satisfying for all* $i \in [0,1]$

$$\begin{aligned}
\lambda_{i,t} &= \beta E_t[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}],\\
\frac{1}{R^L_t} &= \beta \frac{E_t\left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right]}{u'(\epsilon_i, c_{i,t})} + \frac{\zeta_{i,t}}{u'(\epsilon_i, c_{i,t})} \;\; or \;\; \frac{1}{R^L_t} = \beta \frac{E_t\left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right]}{u'(\epsilon_i, c_{i,t})} \cdot \frac{1-\kappa_t}{1-\kappa_t R^L_t/R^m_t},\\
c_{i,t} &= i_{i,t} + i^L_{i,t} + m^H_{i,t-1}\pi_t^{-1} - l_{i,t}/R^L_t \;\; if \; \psi_{i,t} > 0,\\
&\quad or \; c_{i,t} \le i_{i,t} + i^L_{i,t} + m^H_{i,t-1}\pi_t^{-1} - l_{i,t}/R^L_t \;\; if \; \psi_{i,t} = 0,\\
R^m_t i_{i,t} &= \kappa^B_t b_{i,t-1}\pi_t^{-1} \;\; if \; \eta_{i,t} > 0, \;\; or \; R^m_t i_{i,t} < \kappa^B_t b_{i,t-1}\pi_t^{-1} \;\; if \; \eta_{i,t} = 0,\\
R^m_t i^L_{i,t} &= \kappa_t l_{i,t} \;\; if \; \mu_{i,t} > 0 \;\; or \; R^m_t i^L_{i,t} \le \kappa_t l_{i,t} \;\; if \; \mu_{i,t} = 0,\\
-l_{i,t} &= z q_t h_{i,t} \;\; if \; \zeta_{i,t} > 0, \;\; or \; -l_{i,t} \le z_t q_t h_{i,t} \;\; if \; \zeta_{i,t} = 0,\\
q_t \lambda_{i,t} &= u_{h,i,t} + \zeta_{i,t} z q_t + \beta E_t q_{t+1}\lambda_{i,t+1},\\
i_{i,t} &= (1+\Omega_t) m^H_{i,t} - m^H_{i,t-1}\pi_t^{-1},\\
b^T_t &= b_t + m^H_t,\\
b^T_t &= \Gamma b^T_{t-1}/\pi_t,
\end{aligned}$$

$0 = \sum_i l_{i,t}$, $h = \sum_i h_{i,t}$, $y = \sum_i c_{i,t}$, $b_t = \sum_i b_{i,t}$, *and* $m^H_t = \sum_i m^H_{i,t}$, *where the multipliers* $\psi_{i,t}$, $\mu_{i,t}$, *and* $\eta_{i,t}$ *satisfy* $\psi_{i,t} = u'(\epsilon_i, c_{i,t}) - \beta E_t\left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right] \ge 0$, $\mu_{i,t} = [(1/R^m_t) - (1/R^L_t)]u'(\epsilon_i, c_{i,t})/(1-\kappa) \ge 0$, *and* $\sum_i \eta_{i,t} = (\sum_i u'(\epsilon_i, c_{i,t})/R^m_t) - \beta E_t \sum_i \left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right] \ge 0$, *the transversality conditions, a monetary policy setting* $\{R^m_t \ge 1,\, \kappa^B_t > 0,\, \kappa_t \in [0,1],\, \Omega_t > 0\}_{t=0}^{\infty}$, *given* $\Gamma > 0$, $\{y_t\}_{t=0}^{\infty}$, *and initial values* $m^H_{i,-1} = m^H_{-1} > 0$, $b_{i,-1} = b_{-1} > 0$, $h_{i,-1} = h_{-1} = 1$ *and* $b^T_{-1} > 0$.

B Proof of Proposition 2

The problem of the social planer problem (40), which aims at maximizing ex-ante social welfare, can be written as a static

$$\begin{aligned}
&\max_{c_b, c_l, h_b} \min_{\chi_1, \chi_2} \{\epsilon_l(\delta c_l - (1/2)c_l^2) + \epsilon_b(\delta c_b - (1/2)c_b^2) + (\gamma h - (1/2)((h-h_b)^2 + h_b^2)\}/(1-\beta)\\
&\qquad + \chi_1\left[y - c_b - c_l\right] + \chi_2\left[2zh_b \cdot (q/R^L) - c_b + c_l\right],
\end{aligned}$$

where $(q/R^L) = \frac{1}{(1-\beta)}\frac{\gamma-(h-h_b)}{\epsilon_l(\delta-c_l)}$. The first order conditions are $\epsilon_b(\delta-c_b)/(1-\beta) = \chi_1 + \chi_2$,

$$\begin{aligned}
\epsilon_l(\delta - c_l)/(1-\beta) &= \chi_1 - \chi_2\left(1 + 2zh_b \cdot \left[\partial(q/R^L)/\partial c_l\right]\right),\\
(2h_b - h)/(1-\beta) &= \chi_2\left(2z \cdot \left(q/R^L\right) + 2zh_b \cdot \left[\partial(q/R^L)/\partial h_b\right]\right),
\end{aligned}$$

where $\partial(q/R^L)/\partial c_l = \frac{\gamma-h+h_b}{(1-\beta)\epsilon_l}\frac{1}{(\delta-c_l)^2} > 0$ and $\partial(q/R^L)/\partial h_b = \frac{1}{(1-\beta)\epsilon_l(\delta-c_l)} > 0$. Substituting out the multipliers χ_1 and χ_2 as well as (q/R^L), we get the following condition for the constrained efficient allocation

$$\frac{\epsilon_b(\delta - c_b) - \epsilon_l(\delta - c_l)}{(2h_b - h)\,(1-\beta)\epsilon_l(\delta - c_l)(1/z)} = \frac{1}{\gamma - (h - h_b)} \cdot \Delta, \tag{59}$$
$$\text{where } \Delta \equiv \frac{1 + zh_b \cdot \left[\partial(q/R^L)/\partial c_l\right]}{1 + [\partial(q/R^L)/\partial h_b] \cdot h_b/(q/R^L)}.$$

To disclose the implications for the tax/subsidy rate, which is associated with this policy, we compare (59) with the competitive equilibrium condition (39), which can be rewritten as

$$\frac{(1 - \boldsymbol{\tau}^L) \cdot \epsilon_b(\delta - c_b) - \epsilon_l(\delta - c_l)}{(2h_b - h)\,(1-\beta)\epsilon_l(\delta - c_l)(1/z)} = \frac{1}{\gamma - h + h_b}, \tag{60}$$

Apparently, the LHS of (60) differs from the LHS of (59) only by the tax rate τ^L, while the RHSs differ by the term Δ in (59), which depends on the derivatives of the relative price (q/R^L). Inserting the derivatives and using used the constraints $c_b - c_l = 2zh_b \frac{\gamma-(h-h_b)}{(1-\beta)\epsilon_l(\delta-c_l)}$ and $y = c_l + c_b$, Δ can be rewritten as

$$\Delta = \frac{(\gamma - h + h_b) + h_b \cdot \Xi}{(\gamma - h + h_b) + h_b}, \quad \text{where } \; \Xi \equiv \frac{(1-\beta)}{z}\epsilon_l \left(\frac{y - 2c_l}{2h_b}\right)^2$$

Hence, if the term Ξ is smaller than one, Δ is also smaller than one, implying $\tau^L < 0$. Using that (37) implies $h_b \geq 0.5h$, $\epsilon_l < 1$, and that $y = h = 1$, we get the following inequality

$$\Xi = \frac{1-\beta}{z}\epsilon_l \left(\frac{y - 2c_l}{2h_b}\right)^2 < \frac{1-\beta}{z}$$

such that $z \geq 1-\beta$ is a sufficient condition for $\Delta < 1$ and thus for borrowing subsidy to be required for the implementation of the constrained efficient allocation $\tau^L < 0$.

We further seek to identify the impact of a subsidy on consumption and housing of the representative borrower. For this, we apply the competitive equilibrium conditions (36), (60), and $c_b - c_l = 2zh_b\frac{\gamma-(h-h_b)}{(1-\beta)\epsilon_l(\delta-c_l)}$, and substitute out c_l with $c_l = y - c_b$ to get $F(\boldsymbol{\tau}^L, h_b, c_b) = 0$ and $G(h_b, c_b) = 0$, where

$$F(\boldsymbol{\tau}^L, h_b, c_b) = \frac{(1-\boldsymbol{\tau}^L) \cdot \epsilon_b(\delta - c_b) - \epsilon_l(\delta - y + c_b)}{(2h_b - h)\,(1-\beta)\epsilon_l(\delta - y + c_b)(1/z)} - \frac{1}{\gamma - h + h_b},$$
$$G(h_b, c_b) = 2zh_b \frac{\gamma - (h - h_b)}{(1-\beta)\epsilon_l(\delta - y + c_b)} - 2c_b + y.$$

The partial derivatives of $G(h_b, c_b)$, where G_x abbreviates $\partial G/\partial x$, are given by

$$G_{h_b} = 2z\frac{2h_b - h + \gamma}{\epsilon_l\,(\delta - y + c_b)\,(1-\beta)} > 0, \quad G_{c_b} = -2\left(\frac{z}{\epsilon_l}\frac{h_b\,(\gamma - h + h_b)}{(1-\beta)\,(c_b - y + \delta)^2} + 1\right) < 0,$$

implying $\partial h_b/\partial c_b = -G_{c_b}/G_{h_b} > 0$. The partial derivatives of $F(\boldsymbol{\tau}^L, h_b, c_b)$ are given by

$$F_{\boldsymbol{\tau}^L} = -\frac{\epsilon_b\left(\delta - c_b\right)}{\epsilon_l\left(1-\beta\right)\left(2h_b - h\right)\left(\delta - y + c_b\right)\left(1/z\right)} < 0,\ F_{h_b} = -\frac{2\gamma - h}{\left(2h_b - h\right)\left(\gamma - h + h_b\right)^2} < 0,$$
$$F_{c_b} = -\frac{2\epsilon_b\left(\delta - y/2\right)\left(1-\tau^L\right)}{\epsilon_l\left(\delta - y + c_b\right)^2\left(1-\beta\right)\left(2h_b - h\right)\left(1/z\right)} < 0.$$

Thus, consumption of the representative borrower decreases with the tax rate, since

$$\partial c_b/\partial \tau^L = -(G_{h_b}F_{\tau^L})/(F_{c_b}G_{h_b} - F_{h_b}G_{c_b}) < 0.$$

Hence, introducing a subsidy $\tau^L < 0$ increases consumption and housing of the representative borrower (by $\partial h_b/\partial c_b > 0$). Given that consumption (housing) of lenders decreases for a given endowment (stock of housing), the lenders' consumption Euler equation (32), which can be written as $1 = \beta 0.5[1 + \epsilon_b(\delta - c_b)/(\epsilon_l(\delta - c_l))]\left(R^L/\pi\right)$, further implies that the real interest rate increases with the subsidy. ■

C Monetary policy and inflation

Suppose that government bonds are supplied at a rate that is not identical to the inflation target, $\Gamma \neq \pi^*$. Then, the total stock of bonds $b_t^T = b_t + m_t^H$ might grow or shrink in a long-run equilibrium at a constant rate Γ/π (see 47). The money demand condition (45) then requires for constant steady state values c_b, R^L, h_b, q, and z, that the term $\widetilde{m}_t = (1+\Omega_t)m_t^H$ is also constant in the long-run. Combining (45), (46), and (47), leads to $\kappa_t^B b_t = R_t^m \pi_t[\widetilde{m}_t - \widetilde{m}_{t-1}(1+\Omega_{t-1})^{-1}\pi_t^{-1}]$ and $[b_t + \widetilde{m}_t/(1+\Omega_t)] = \Gamma\left[b_{t-1} + \widetilde{m}_{t-1}/(1+\Omega_{t-1})\right]/\pi_t$. Further, substituting out b_t, gives

$$\left[\frac{R_t^m \pi_t}{\kappa_t^B}\left(\widetilde{m}_t - \frac{\widetilde{m}_{t-1}\pi_t^{-1}}{1+\Omega_{t-1}}\right) + \frac{\widetilde{m}_t}{1+\Omega_t}\right] = \frac{\Gamma}{\pi_t}\left[\frac{R_{t-1}^m \pi_{t-1}}{\kappa_{t-1}^B}\left(\widetilde{m}_{t-1} - \frac{\widetilde{m}_{t-2}\pi_{t-1}^{-1}}{1+\Omega_{t-2}}\right) + \frac{\widetilde{m}_{t-1}}{1+\Omega_{t-1}}\right]. \quad (61)$$

Taking the limit $t \to \infty$ of both sides of (61), we can use that for a constant long-run inflation rate π and a constant policy rate R^m a steady state is characterized by a constant value for $\widetilde{m}_t$. The term in the square brackets in (61) grows/shrinks with the constant rate Γ/π. When the growth rate of bonds exceeds the inflation rate, $\Gamma > \pi$, this can be guaranteed by a permanently shrinking value for κ_t^B. Thus, the central bank can let κ_t^B grow at the rate π/Γ and can let the share of money supplied outright go to zero in the long-run, i.e. it can set κ_t^B and $1/\Omega_t$ according to $\lim_{t\to\infty}\kappa_t^B/\kappa_{t-1}^B = \pi/\Gamma < 1$ and $\lim_{t\to\infty} 1/\Omega_t = 0$ if $\Gamma > \pi$. For $\Gamma < \pi$, the term in the square bracket in (61) permanently shrinks, which can not be supported by a growing value κ_t^B without violating the restriction $\kappa_t^B \leq 1$. In this case, the central bank can let κ_t^B go to zero and can let the share $1/\Omega_t$ of money supplied outright grow in a long-run equilibrium. For $\pi = 1$ and $\Gamma < 1$, it can thus set κ_t^B and $1+1/\Omega_t$ in a steady state according to $\lim_{t\to\infty}\left(1+1/\Omega_t\right)/\left(1+1/\Omega_{t-1}\right) = 1/\Gamma > 1$ and $\lim_{t\to\infty}\kappa_t^B = 0$.

D Proof of Proposition 4

Consider a competitive equilibrium under money rationing and without aggregate risk, which consists of a set $\{c_b, c_l, h_b, R^L\}$ satisfying $y = c_l + c_b$, (48), and (49), (50), where R^m, κ, and π are set by the central bank. As revealed by a comparison of (48)-(50) with (51)-(53), the central bank can implement identical effective real rates for borrowers and lenders as the borrowing subsidy by setting its instruments κ and π according to the conditions $\frac{\pi}{R^L}\frac{1-\kappa R^L/R^m}{1-\kappa} = \frac{1}{\widetilde{r}}$ and $\frac{\pi}{R^L} = (1-\widetilde{\tau}^L)\frac{1}{\widetilde{r}}$, which can be rewritten as $1 - \kappa\frac{\widetilde{r}}{1-\widetilde{\tau}^L}\pi/R^m = (1-\kappa)/(1-\widetilde{\tau}^L)$ and $\pi = R^L(1-\widetilde{\tau}^L)/\widetilde{r}$, for an endogenously determined loan rate R^L and a given R^m, which is also set by the central bank. If the latter is set according to $R^m < (1+\kappa)R^L/2 = \frac{1+\kappa}{2}\frac{\widetilde{r}}{1-\widetilde{\tau}^L}\pi$, the following inequality holds:

$$\frac{(1-\kappa)\left(2-\kappa R^L/R^m\right)}{(1-\kappa R^L/R^m)} > 2,$$

such that the RHS of (49) is strictly larger than the RHS of (52), which implies that the difference $c_b - c_l$ is also larger under asset purchases than under a borrowing subsidy. Given that this difference is under a binding borrowing constraint smaller than under first best, asset purchases can enlarge the set of feasible allocations in a way that allocations can be implemented that welfare-dominate allocations under a borrowing subsidy. ■

E A CRRA version with representative agents

Definition 4 *A competitive equilibrium of the economy with preferences satisfying (55) and wealth redistribution within households consists of a set of sequences* $\{c_{b,t},\ c_{l,t},\ \pi_t,\ R_t^L,\ h_{b,t},\ q_t,\ b_t,\ b_t^T,\ m_t^H\}_{t=0}^{\infty}$ *satisfying*

$$\begin{aligned}
(1-\tau^L)\epsilon_b c_{b,t}^{-\sigma}/R_t^L &= \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}] + \gamma((h-h_{b,t})^{-\sigma} - h_{b,t}^{-\sigma})/[q_t z],\\
\epsilon_l c_{l,t}^{-\sigma}/R_t^L &= \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]\frac{1-\kappa_t}{1-\kappa_t R_t^L/R_t^m},\\
q_t \epsilon_l c_{l,t}^{-\sigma}\frac{1/R_t^L - \kappa_t/R_t^m}{1-\kappa_t} &= \gamma\,(h-h_{b,t})^{-\sigma} + \beta E_t\left[q_{t+1}\epsilon_l c_{l,t+1}^{-\sigma}\frac{1/R_{t+1}^L - \kappa_{t+1}/R_{t+1}^m}{1-\kappa_{t+1}}\right],\\
c_{b,t} - c_{l,t} &\leq z q_t h_{b,t}\left[(2/R_t^L) - (\kappa_t/R_t^m)\right],\\
0.5(1+\Omega_t)m_t^H &\geq c_{b,t} + z_t q_t h_{b,t}/R_t^L,\\
\kappa_t^B b_{t-1}\pi_t^{-1}/R_t^m &\geq (1+\Omega_t)m_t^H - m_{t-1}^H\pi_t^{-1},\\
c_{l,t} + c_{b,t} &= y_t,\\
b_t^T &= \Gamma b_{t-1}^T/\pi_t,\\
b_t^T &= b_t + m_t^H,
\end{aligned}$$

the transversality conditions, a monetary policy setting $\{R_t^m \geq 1,\ \kappa_t \in [0,1],\ \kappa_t^B > 0,\ \Omega_t > 0\}_{t=0}^{\infty}$, *a tax/subsidy* τ^L, *given* $\{y_t\}_{t=0}^{\infty}$, $\Gamma > 0$, $b_{-1}^T > 0$, $b_{-1} > 0$, *and* $m_{-1}^H > 0$.

The first best allocation apparently satisfies $\epsilon_b c_{b,t}^{-\sigma} = \epsilon_l c_{l,t}^{-\sigma}$ and $h_{b,t} = h_{l,t} = 2h$. Under binding borrowing, liquidity, and money supply constraints, a competitive equilibrium without aggregate risk consists of a set $\{c_l,\, c_b,\, R^L,\, h_b,\, q\}$ satisfying

$$1/R^L = \beta\,(c_l^{\sigma}/\epsilon_l)\,0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})\pi^{-1}\frac{1-\kappa}{1-\kappa R^L/R^m}, \tag{62}$$

$$(1-\tau^L)\epsilon_b c_b^{-\sigma} = R^L\beta 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})\pi^{-1} + R^L\,(\gamma/qz)\,((h-h_b)^{-\sigma} - h_b^{-\sigma}), \tag{63}$$

$$\gamma(h-h_b)^{-\sigma} = q\,(1-\beta)\,\epsilon_l c_l^{-\sigma}\frac{1/R^L - \kappa/R^m}{1-\kappa}, \tag{64}$$

$$c_b - c_l = zqh_b[(2/R^L) - (\kappa/R^m)], \tag{65}$$

$$y = c_l + c_b, \tag{66}$$

for a monetary policy setting $\{1 \leq R^m < R^L,\, \kappa \in [0,1),\, \pi > \beta\}$, and a tax/subsidy τ^L. Once the set $\{c_l,\, c_b,\, R^L,\, h_b,\, q\}$ is determined, the values m^H and b are given by $m^H = \left(c_b - zqh_b/R^L\right)\frac{1}{0.5(1+\Omega)}$ and $b = \frac{R^m\pi}{\kappa^B}\left(1+\Omega-\pi^{-1}\right)m^H$ given κ^B and Ω.

F Constrained efficiency under CRRA preferences

In this Appendix, we consider an economy under CRRA preferences and pooling of wealth within households as summarized in Definition 4. We will show that a constrained efficient allocation is again associated with a lump-sum financed borrowing subsidy, as already shown for the case of linear-quadratic preferences (see Proposition 2).

Proposition 5 *Consider an economy without aggregate risk, with preferences satisfying (55), and wealth redistribution within households. The constrained efficient allocation can be implemented by a subsidy on borrowing, if but not only if $\epsilon_b/\epsilon_l < 3^{\sigma}$.*

Proof. Consider the economy as given in Definition 4 for $y_t = y$, $R^m = R^L$, and $\pi_t = \pi$. Given that conventional monetary policy measures do not affect the allocation and we restrict the tax/subsidy rate also to be constant, the equilibrium allocation and prices are time-invariant. Hence, the set $\{c_l,\, c_b,\, R^L,\, h_b,\, q\}$ has to satisfy (63), (66)

$$\epsilon_l c_l^{-\sigma}/R^L = \beta 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})/\pi, \tag{67}$$

$$c_b - c_l \leq zqh_b 2/R^L, \tag{68}$$

$$\gamma(h-h_b)^{-\sigma} = q\beta(1-\beta)0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})/\pi, \tag{69}$$

given $\{\tau^L, \pi\}$. Substituting out the housing price q with (69) in (68), leads to

$$0 \leq zh_b 2\frac{\gamma(h-h_{b,t})^{-\sigma_h}}{(1-\beta)\epsilon_l c_l^{-\sigma}} - c_b + c_l, \tag{70}$$

where we further used (67) to substitute out the real rate R^L/π. Then, the problem of a social planer, who aims at maximizing social welfare (21) by setting the tax/subsidy rate τ^L, can be summarized by

$$\max_{\{c_{b,t},c_{l,t},h_{b,t}\}_{t=0}^{\infty}} [(1-\beta)(1-\sigma)]^{-1}\cdot\left[\epsilon_b(c_{b,t}^{1-\sigma}-1)+\epsilon_l(c_{l,t}^{1-\sigma}-1)+\gamma(h_{b,t}^{1-\sigma}-1)+\gamma((h-h_{b,t})^{1-\sigma}-1)\right],$$
$$\text{s.t. (66) and (70).}$$

The first order conditions for the policy problem are given by

$$0 = 0.5\gamma\left(h_b^{-\sigma}-((h-h_b)^{-\sigma}\right)+\chi_2\,(c_b-c_l)\,h_b^{-1}\left(1-\sigma(1-(h/h_b))^{-1}\right), \tag{71}$$
$$\chi_1 = 0.5\epsilon_l c_l^{-\sigma}+\chi_2\sigma c_l^{-1}\,(c_b-c_l)+\chi_2, \tag{72}$$
$$\chi_1 = 0.5\epsilon_b c_b^{-\sigma}-\chi_2, \tag{73}$$

where χ_1 and $\chi_2 \geq 0$ are the multipliers on (66) and (70). Substituting out χ_1 with (73) in (72), gives $0.5\epsilon_b c_b^{-\sigma}-0.5\epsilon_l c_l^{-\sigma}=\chi_2\left(\sigma c_l^{-1}(c_b-c_l)+2\right)$, and substituting out χ_2 with (71),

$$\frac{\epsilon_b c_b^{-\sigma}-\epsilon_l c_l^{-\sigma}}{\gamma\left((h-h_b)^{-\sigma}-h_b^{-\sigma}\right)}=\frac{\sigma c_l^{-1}(c_b-c_l)+2}{(c_b-c_l)\,h_b^{-1}\,(1-\sigma(1-(h/h_b))^{-1})}. \tag{74}$$

Now combine (63) and (67) to $(1-\tau^L)\epsilon_b c_b^{-\sigma}/R^L-\epsilon_l c_l^{-\sigma}/R^L=\gamma((h-h_b)^{-\sigma}-h_b^{-\sigma})/(qz)$, and substitute out the housing price with $q=\frac{\gamma(h-h_{b,t})^{-\sigma}}{\epsilon_l c_l^{-\sigma}/R^L}$ (see 67 and 69), yielding

$$\frac{(1-\tau^L)\epsilon_b c_b^{-\sigma}-\epsilon_l c_l^{-\sigma}}{\gamma((h-h_b)^{-\sigma}-h_b^{-\sigma})}=\frac{2h_b}{c_b-c_l}, \tag{75}$$

where we further used that $\frac{z\gamma(h-h_{b,t})^{-\sigma}h}{(1-\beta)\epsilon_l c_l^{-\sigma}}=\frac{c_b-c_l}{h_b 2}$ holds in a competitive equilibrium under a binding borrowing constraint (see 67-69). A comparison between (74) and (75) immediately shows that the LHSs only differ by the tax/subsidy rate τ^L. Let Δ be the difference between the RHSs of (74) and (75), which satisfies

$$\begin{aligned}
\Delta &= \frac{\sigma c_l^{-1}(c_b-c_l)+2}{(c_b-c_l)\,h_b^{-1}\,(1-\sigma(1-(h/h_b))^{-1})}-\frac{2h_b}{c_b-c_l}\\
&= \frac{h_b}{c_b-c_l}\frac{\sigma}{1-\sigma(1-(h/h_b))^{-1}}\left((c_b c_l^{-1}-1)-\frac{2}{(h/h_b)-1}\right)\\
&\leq \frac{h_b}{c_b-c_l}\frac{\sigma}{1-\sigma(1-(h/h_b))^{-1}}\left((c_b c_l^{-1}-1)-2\right)\\
&\leq \frac{h_b}{c_b-c_l}\frac{\sigma}{1-\sigma(1-(h/h_b))^{-1}}\left((\epsilon_b/\epsilon_l)^{1/\sigma}-3\right),
\end{aligned}$$

where we used $h/2\leq h_b \Leftrightarrow (h/h_b)-1\leq 1$ for the first inequality and that the ratio c_b/c_l is smaller under a binding borrowing constraint than under first best, $(c_b^*/c_l^*)=(\epsilon_b/\epsilon_l)^{1/\sigma}$, for the second inequality. Given that $1-\sigma(1-(h/h_b))^{-1}>0$ (see 74), the difference Δ is strictly negative, $\Delta<0$, such that the constrained efficient allocation can be implemented by a subsidy, $\tau^L<0$, if but not only if the preference shock satisfies $\epsilon_b/\epsilon_l<3^\sigma$. ■