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# Do Price-Matching Guarantees with Markups Facilitate Tacit Collusion? Theory and Experiment 

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#### Abstract

This paper studies how competitive prices are affected by price-matching guarantees allowing for markups on the lowest competing price. This new type of low-price guarantee was recently introduced in the German retail gasoline market. Using a sequential Hotelling model, we show that such guarantees, similar to perfect pricematching guarantees, can induce collusive prices. In particular, this occurs if the first mover provides a price guarantee with a markup which is below a threshold value. In these cases, prices are on average set at the monopoly level. A laboratory experiment supports the theoretical predictions.


Keywords: price-matching guarantee, tacit collusion, Hotelling, spatial competition, sequential pricing, laboratory experiment
JEL Codes: C92, D21, D22, D43, L11, L13, L41

[^0]
## 1 Introduction

Since summer 2015, Shell promotes a new kind of low-price guarantee for standard gasoline: a price-matching guarantee with a markup on the lowest competing price within the regional market. In order to benefit from this guarantee, Shell's customers have to register once, which is free of charge. Hereafter, Shell automatically checks for any purchase whether the posted gasoline price exceeds the lowest competing price by more than 2 Cents per liter, and, if this is the case, reduces its selling price to the lowest price plus the markup of 2 Cents. ${ }^{1}$

The introduction of the guarantee followed a change in the design of the gasoline retail market, implemented by the German antitrust authority in 2013. More precisely, the Bundeskartellamt established a real-time database for standard gasoline and diesel, called the Markttransparenzstelle für Kraftstoffe or market transparency unit, and forced almost all gasoline retailers to keep their prices in the database up to date. ${ }^{2}$ The market transparency unit is accessible for anyone free of charge via various websites or smart-phone apps. The purpose of its introduction was to increase competition in the German gasoline retail market, as this market was found to be prone to (tacit) price coordination. ${ }^{3}$ However, it also enabled Shell to introduce this kind of guarantee, by providing the data for its automatic price comparisons. This made the guarantee especially attractive to customers, because they do not incur any costs of invoking the guarantee.

The question arising from this motivating example is whether this new kind of low-price guarantee might have an anti-competitive effect. Previous theoretical, empirical and experimental literature suggests that perfect price-matching guarantees are anti-competitive if the costs of invoking the guarantee are low. In contrary, other forms of low-price guarantees, especially price-beating guarantees, can even be procompetitive. In a nutshell, the anti-competitive effect of the perfect price-matching guarantees results from making it virtually impossible to effectively undercut a rival's price. However, this argument does not apply if the guarantee comes with a markup, since effective undercutting within the markup is possible. To the authors best knowledge, no previous theoretical or experimental paper studied the effect of a price guarantee with a maximal markup on competing prices, except for a recent empirical study by Dewenter and Schwalbe (2015), who find evidence for an anti-competitive effect of Shell's guarantee.

[^1]This paper intends to close this gap. First, it analyzes the effects of pricematching guarantees with non-negative markups on competition in a theoretical framework inspired by the motivating example. Second, the obtained theoretical predictions are tested in a laboratory experiment. Both, theoretical and experimental results show that the guarantee with a non-negative markup can indeed induce price coordination and leads to (on average) monopoly prices in these cases.

The remainder of the paper is structured as follows. The second section provides a brief overview of previous theoretical, experimental and empirical literature on low-price guarantees. The third section theoretically analyzes the price guarantee with a markup in a sequential Hotelling framework with two symmetric firms competing in prices and producing homogeneous goods. The fourth section presents an experimental design which is used to test the main theoretical predictions. Finally, the last section summarizes and discusses the results.

## 2 Previous literature

The effects of low-price guarantees have been discussed extensively in the economics and law literature since the early 1980s. ${ }^{4}$

Salop (1986) was the first to intuitively point out that perfect price-matching guarantees potentially lead to inefficient and anti-competitive market outcomes. The basic idea is that, when a firm faces a competitor with a perfect price-matching guarantee, its incentive to undercut the competitor's price is dampened since his rebate mechanism effectively creates a penalty (Salop, 1986, p.16), as individual price cuts become mutual. Accordingly, whenever all firms offer price-matching in markets with simultaneous price competition, new equilibria arise with prices above the competitive level. This was later formalized by Doyle (1988). A further study by Logan and Lutter (1989) shows that under certain conditions, it is sufficient for a collusive market outcome if at least one firm offers a perfect price-matching guarantee. The authors endogenize the adoption of perfect price-matching guarantees in a model with asymmetric costs, differentiated goods and simultaneous price competition. They find that only the high-cost firm offering a guarantee can induce an anti-competitive market outcome. In particular, if cost asymmetries are small, it adopts the guarantee and hereby creates incentives for supra-competitive pricing, whereas under large asymmetries it does not offer price-matching.

Additional literature focuses on further potentially negative effects of price matching guarantees. Edlin and Emch (1999) study the role of market entry and find

[^2]that in markets with perfect price-matching, new entrants are attracted by collusive profits and also adopt the given pricing strategy. Hence, these entries only create inefficiencies, due to their entry and fixed costs, without making prices more competitive. Furthermore, Corts (1996) and Chen et al. (2001) suggest that low-price guarantees can be a tool to facilitate price discrimination between informed and uninformed customers, as only the former can invoke the guarantee. Consequently, in most cases uninformed customers loose whereas informed customers gain. ${ }^{5}$

In line with the previous argumentations Hay (1982), Sargent (1993) and Edlin (1997) advocate in favor of legislative prohibition of low-price guarantees and advise anti-trust authorities to at least carefully monitor markets in which they are used.

Further theoretical literature points out restrictions of the previous arguments against low-price guarantees. Hviid and Schaffer (1999) introduce the term hassle costs, which subsumes all non-pecuniary costs of invoking the guarantee. They show that whenever hassle costs exist, a perfect price-matching guarantee does not prevent a competitor from undercutting within the hassle costs, since customers would not enforce the guarantee in these cases. This reasoning implies that in the presence of hassle costs, price-matching guarantees do not give rise to collusive equilibria in symmetric markets, while in asymmetric markets the potential for collusive outcomes is limited. ${ }^{6}$ Moorthy and Winter (2006) show that in highly asymmetric markets with costly information, low-costs firms adopt price guarantees not to foster collusion, but rather as a signaling device. In these cases, under certain conditions low-price guarantees can increase welfare.

A different strand of literature studies price-beating guarantees, i.e. promises to strictly underbid the lowest competing price to a certain percentage or amount. Hviid and Schaffer (1994) as well as Corts (1995) find that these guarantees do not lead to collusive market outcomes and in turn can be used to offset perfect price-matching guarantees. The reason is that price-beating guarantees reestablish the firms' ability to unilaterally undercut prices, even if the competitors offer pricematching or beating. Intuitively, by posting a higher price, a firm offering a pricebeating guarantee forces itself to effectively undercut the competitors' prices, while at the same time the guarantees of the competitors are not activated. Kaplan (2000) criticizes these findings by pointing out that these results are restricted to price guarantees which pertain to posted prices, although admitting that these form of guarantees are empirically more relevant.

[^3]Empirical studies qualitatively confirm most of the theoretical results. For example, Hess and Gerstner (1991) study the price development of five supermarket chains in North Carolina in the mid 1980s. They find that after the first chain adopted a perfect price-matching guarantee for specific goods, the others followed suit by adopting similar guarantees. Consequently, prices of the goods included in the guarantees rose significantly in comparison to those excluded, while the differences in the former prices almost vanished completely. Arbatskaya et al. (2004) study over 500 price guarantees by using data from newspaper advertisements. They find that 56 percent of the perfect price-matching guarantees and only about 10 percent of the price-beating guarantees led to pricing above the competitive level. In addition, they find that most of the latter referred to posted instead of effective prices. A further study by Arbatskaya et al. (2006) comes to a similar conclusion when reviewing low-price guarantees in the retail tire market. Moorthy and Winter (2006) as well as Moorthy and Zhang (2006) find support for the usage of price-matching guarantees as signaling device by low-cost firms. A recent paper by Dewenter and Schwalbe (2015) studies the effect of low-price guarantees in the German gasoline market. With a difference-in-difference panel regression, controlling for exogenous effects and using data from the market transparency unit, they examine changes in pricing of two chains which recently started offering low-price guarantees. For the chain HEM, which are offering a non-automatic prefect price-matching guarantee, they do not find any significant price effect. The authors speculate, that this is a result of the relative high hassle costs customer faces for invoking the guarantee. For Shell's hassle cost free price-matching guarantee with a markup, the authors find a significant price increase by Shell of 2.4-2.8 Cent per liter standard gasoline after the introduction of the guarantee.

Furthermore, experimental literature also supports most of the theoretical implications. Dugar (2007) and Mago and Pate (2009) consider perfect price-matching guarantees and focus on the resulting equilibrium selection in symmetric and asymmetric markets with homogeneous goods and simultaneous pricing. They find evidence for the selection of the most collusive equilibrium, as long as the asymmetries in costs are sufficiently small. In addition, Fatas and Manez (2007) and Fatas et al. (2013) results support the prediction that perfect price-matching guarantees lead to a collusive outcome when symmetric firms compete simultaneously in a market with differentiated goods. Finally, Fatas et al. (2005) find no evidence that price-beating guarantees cause an anti-competitive market outcome.

## 3 Theory

### 3.1 Framework

The model is based on the Hotelling duopoly framework with linear transportation costs. There are two firms producing a homogeneous good, which are located at the opposite ends of a road and compete in prices (Hotelling, 1929).

Customers are uniformly distributed along the road, normalized to a mass of 1 , and have a valuation of $v>0$ for a unit of the good. They behave as price takers, since they are infinitely many. Customers have full transparency about prices and incur linear transportation costs of $t>0$ times the distance to their dealer. The transportation costs are assumed to be moderate, i.e. $t<\frac{v}{3}$, which keeps the analysis simple and assures that the firms serve the entire road in equilibrium. All customers behave rationally and have a single unit demand. Thus, they buy at the best deal they can get whenever their net benefit is positive, otherwise they do not buy at all.

Firms do not face capacity constraints and incur neither fixed nor variable costs. They are allowed to set any non-negative price, i.e. dumping is prohibited. Firm A is located at the left end of the road, at position $x_{A}=0$. The main new element of the model is that it provides a price-matching guarantee with an (exogenous) markup $m \geq 0 .{ }^{7}$ That is, it guarantees to customers that it never exceeds the competitor's price by more than $m .^{8}$ Firm B is located at the right end of the road, i.e. at $x_{B}=1$, and offers no price guarantee. Restricting to only Firm A offering a price guarantee is sufficient to show the collusive effect of the price guarantee. In Appendix B we prove that any version of the model where Firm B additionally has an arbitrary price guarantee with a non-negative markup leads to identical prices in equilibrium, compared to the game with only Firm A offering such a guarantee.

The timing of the game is as follows. In the first stage, Firm A chooses its posted price $p_{A}^{p}$ (i.e. its initially announced price). After observing $p_{A}^{p}$, Firm B chooses its price $p_{B}$ in the second stage. Based on these posted prices the effective price of Firm A, denoted as $p_{A}$, results by applying the guarantee, i.e.

$$
\begin{equation*}
p_{A}:=\min \left\{p_{A}^{p}, p_{B}+m\right\} \quad \text { with } \quad m \in[0, t[. \tag{1}
\end{equation*}
$$

[^4]The effective price of Firm B always equals its posted price. Once the effective prices are determined, customers make their purchasing decisions, and the game ends.

The game is solved by backward induction, i.e. the solution concept is a sub-game perfect Nash equilibrium.

### 3.2 Market demand in equilibrium

Now, we derive the market demand function of Firm $i$. The location of the customer who is indifferent between purchasing at Firm A or Firm B is denoted by $\tilde{x}^{A B}$ (i.e. $\tilde{x}^{A B} \in[0,1]$ is the share of customers located to the left of this customer on the road). If this customer has a non-negative net benefit from consumption, his position determines market shares, since all customers to the left of him will buy at Firm A whereas all customers to the right of him will find it more profitable to buy at Firm B.

In general, a customer located at position $x$ gets a net benefit of $u_{x}^{A}$ from buying at Firm A, which equals his valuation minus the price and the incurred transportation costs:

$$
\begin{equation*}
u_{x}^{A}=v-p_{A}-x \cdot t . \tag{2}
\end{equation*}
$$

The same customer receives a net benefit of $u_{x}^{B}$ if he instead buys at Firm B:

$$
\begin{equation*}
u_{x}^{B}=v-p_{B}-(1-x) \cdot t . \tag{3}
\end{equation*}
$$

Consequently, the location of the customer who is indifferent between Firm A and Firm B is

$$
\tilde{x}^{A B}=\frac{1}{2}+\frac{p_{B}-p_{A}}{2 t} .
$$

This position is interior (i.e., between 0 and 1 ) if and only if

$$
\begin{equation*}
p_{A}-t<p_{B}<p_{A}+t . \tag{4}
\end{equation*}
$$

Naturally, being indifferent between buying at Firm A and Firm B does not necessarily assure that the customer is willing to buy at all. This is only the case if his net benefit of purchasing is non-negative, i.e. $u_{\tilde{x}^{A B}}^{A}=u_{\tilde{x}^{A B}}^{B} \geq 0$. This is equivalent to:

$$
\begin{equation*}
p_{B} \leq 2 v-t-p_{A} . \tag{5}
\end{equation*}
$$

Next, we consider four possible cases depending on the location and preferences of the indifferent customer.

Case 1: Condition (5) is not satisfied, while the indifferent customer does not exist along the road. The non-existence of the indifferent customer implies that condition (4) does not hold, i.e. the price difference between the firms exceeds the highest possible transportation cost. Then, all customers on the road prefer the firm with the lower price over the other firm (whose demand is then 0 anyway). The former firm hence faces a monopolistic demand function:

$$
D_{M}\left(p_{i}\right)= \begin{cases}1 & \text { if } \quad p_{i} \leq v-t  \tag{6}\\ \frac{v-p_{i}}{t} & \text { if } \quad v-t<p_{i}<v \\ 0 & \text { else }\end{cases}
$$

Case 2: Condition (5) is not satisfied, while the indifferent customer exists along the road. In this case, there exists a range of customers along the road who do not buy from any of the firms. Then, firms do not effectively compete with each other, since the price of one firm does not affect the demand of the other, and hence again face a monopolistic demand function given by (6), except that the first segment with $D_{M}\left(p_{i}\right)=1$ does not exist in this case.

Case 3: Condition (5) is satisfied, while the indifferent customer does not exist along the road. Then, as in Case 1, the firm with a higher price has a demand of zero, while the firm with a lower price faces monopolistic demand. However, one can show that under considered conditions it always holds for the latter firm that $p_{i} \leq v-t$, which by (6) implies that it demand is 1 .

Case 4: Condition (5) is satisfied, while the indifferent customer exists along the road. In this case, the indifferent customer prefers to buy the good over not buying. Hence, Firm A (B) faces competitive demand given by the fraction of customers positioned to the left (right) from the indifferent customer. That is, the market demand for Firm $i \in\{A, B\}$, denoted as $D_{i}$, is a function of the effective prices $p_{i}$ and $p_{-i}$ :

$$
\begin{equation*}
D_{i}\left(p_{i}, p_{-i}\right)=\frac{1}{2}+\frac{p_{-i}-p_{i}}{2 t} . \tag{7}
\end{equation*}
$$

Finally, note that a firm gets a demand of 1 if and only if the following condition is satisfied:

Lemma 1. Firm $i$ receives the whole demand if and only if $p_{i} \leq v-t$ and $p_{i}<p_{-i}-t$.
Proof. A given firm receives the whole demand if and only if the following two incentive constraints for the customers are satisfied: 1) all customers prefer buying from this firm over not buying; 2) all customers prefer buying from this firm over buying from the other firm. Given (2) and (3), these conditions are equivalent to the conditions stated in the lemma.

Thus, summing up all four cases and taking Lemma 1 into account, the market demand for Firm $i \in\{A, B\}$ is:

$$
D_{i}\left(p_{i}, p_{-i}\right)= \begin{cases}1 & \text { if } p_{i} \leq v-t \wedge p_{i}<p_{-i}-t  \tag{8}\\ \frac{1}{2}+\frac{p_{-i}-p_{i}}{2 t} & \text { if } p_{i} \leq 2 v-t-p_{-i} \wedge p_{i} \in\left[p_{-i}-t, p_{-i}+t\right] \\ \frac{v-p_{i}}{t} & \text { if } \left.p_{i}>2 v-t-p_{-i} \wedge p_{i} \in\right] v-t, v[ \\ 0 & \text { else. }\end{cases}
$$

### 3.3 Equilibrium pricing without guarantees The competitive benchmark case

To begin, we relax the assumption that Firm A offers a price guarantee and look what happens in the competitive benchmark case, i.e. $p_{A}^{p}=p_{A} .{ }^{9}$ In the next section, we will then consider the model with Firm A having a price-matching guarantee with a non-negative markup, as described above.

In stage 2, Firm B knows $p_{A}^{p}$ and maximizes $\pi_{B}$ by choosing the optimal $p_{B}$. Since Firm A's posted price is also its effective price and given (8), we get the following piecewise defined profit function:

$$
\pi_{B}^{N o P G}\left(p_{B}, p_{A}^{p}\right)= \begin{cases}p_{B} & \text { if } p_{B} \leq v-t \wedge p_{B}<p_{A}^{p}-t,  \tag{9}\\ p_{B} \cdot\left[\frac{1}{2}+\frac{p_{A}^{p}-p_{B}}{2 t}\right] & \text { if } p_{B}<2 v-t-p_{A}^{p} \wedge p_{B} \in\left[p_{A}^{p}-t, p_{A}^{p}+t\right], \\ p_{B} \cdot\left[\frac{v-p_{B}}{t}\right] & \text { if } \left.p_{B} \geq 2 v-t-p_{A}^{p} \wedge p_{B} \in\right] v-t, v[, \\ 0 & \text { else. }\end{cases}
$$

[^5]This implies the following result:

Proposition 1. Firm B's reaction function, when Firm A does not provide a price guarantee, is given by:

$$
R_{B}^{N o P G}\left(p_{A}^{p}\right)= \begin{cases}v-t & \text { if } p_{A}^{p}>v \\ p_{A}^{p}-t & \text { if } \quad 3 t \leq p_{A}^{p} \leq v \\ \frac{p_{A}^{p}+t}{2} & \text { if } p_{A}^{p}<3 t\end{cases}
$$

Proof. See Appendix B.

Thus, Firm B's best response depends on $p_{A}^{p}$ being in one of three different cases. Now, we discuss the intuition for B's best response in each of these cases.

Case 1 - Firm A posts a prohibitively high price, i.e. $p_{A}^{p}>v$. In this case, Firm B is de facto a monopolist. Since the transportation costs are moderate, a monopolist wants to serve the entire road and sets a price of $v-t$.

Case 2 - Firm A posts a price between $3 t$ and $v$. In this interval, $p_{A}^{p}$ is not prohibitive, but high enough to make it profitable for Firm B to serve the full market on its own. Thus, Firm B undercuts Firm A's price just to the extent of the transportation costs.

Case 3 - Firm A posts a price between 0 and $3 t$. For this interval of $p_{A}^{p}$, it is not optimal, even in some cases not possible, for Firm B to serve the entire road. Hence, Firm B shares the market with Firm A. The price $p_{B}=\frac{1}{2}\left(p_{A}^{p}+t\right)$ solves Firm B's trade-off between gaining a higher market share and charging a higher price.

In the first stage, Firm A anticipates Firm B's reaction function given by Proposition 1 and hence faces the following maximization problem:

$$
\underset{p_{A}^{p}}{\operatorname{argmax}} \pi_{A}^{N o P G}\left(p_{A}^{p}\right) \left\lvert\, R_{B}^{N o P G}=\left\{\begin{array}{ll}
0 & \text { if }
\end{array} p_{A}^{p} \geq 3 t, ~\left\{\begin{array}{ll}
0 &  \tag{10}\\
p_{A}^{p} \cdot\left[\frac{1}{2}+\frac{t-p_{A}^{p}}{4 t}\right] & \text { if }
\end{array} p_{A}^{p}<3 t, ~ \$\right.\right.\right.
$$

If $p_{A}^{p}$ is at least $3 t$, Firm B, according to Proposition 1, undercuts Firm A's price at least by $t$. In these cases, the demand of Firm A will be zero, as even the closest customer at $x=0$ would prefer to buy from Firm B.

If $p_{A}^{p}$ is smaller than $3 t$, Firm B undercuts, if at all, to a lesser extent than $t$ by setting $p_{B}=\frac{p_{A}^{p}+t}{2}$. In these cases, all conditions of the second case of the demand function in (8) are fulfilled: First,

$$
\begin{aligned}
p_{A}^{p}-p_{B} & =\frac{p_{A}^{p}-t}{2}<t, \\
p_{B}-p_{A}^{p} & =\frac{t-p_{A}^{p}}{2}>-t,
\end{aligned}
$$

where the inequalities follow from $p_{A}^{p}<3 t$. Second,

$$
\left.\begin{array}{rl}
p_{B}=\frac{p_{A}^{p}+t}{2} & \leq 2 v-t-p_{A}^{p} \\
\Leftrightarrow & p_{A}^{p}
\end{array}\right) \frac{4}{3} v-t,
$$

which holds for $p_{A}^{p}<3 t$ because $t \leq \frac{v}{3}$. Thus, by (8), whenever $p_{A}^{p}$ is smaller than $3 t$ the demand for Firm A is

$$
\frac{1}{2}+\frac{p_{B}-p_{A}^{p}}{2 t}=\frac{1}{2}+\frac{\frac{1}{2} p_{A}^{p}+\frac{1}{2} t-p_{A}^{p}}{2 t}=\frac{1}{2}+\frac{t-p_{A}^{p}}{4 t}
$$

which implies the above profit maximization problem of Firm A. That is, if Firm A posts a price higher than $3 t$, Firm B will serve the market on its own and consequently A's profits are zero, whereas for lower prices Firm A has to share the market with Firm B and the profits are equal to its market share multiplied by its charged price.

Solving the maximization problem of Firm A gives the optimal price of $p_{A}^{p}=\frac{3}{2} t$. Using Firm B's reaction function and the demand function in (8), we obtain the equilibrium characterization of the competitive benchmark case in which Firm A does not provide a price guarantee:

$$
p_{A}^{p}=\frac{3}{2} t, \quad p_{B}=\frac{5}{4} t, \quad D_{A}=\frac{3}{8}, \quad D_{B}=\frac{5}{8}, \quad \pi_{A}=\frac{18}{32} t, \quad \pi_{B}=\frac{25}{32} t .
$$

Note that Firm B is better off than Firm A in equilibrium, which results from the sequential structure of the game. Since prices are strategic complements, Firm B has a second mover advantage. It can profitably undercut Firm A's price and hereby gain a higher market share as well as higher profits in equilibrium.

### 3.4 Equilibrium pricing with guarantees

In this subsection we assume that Firm A provides a guarantee with a non-negative markup on the competitor's price. ${ }^{10}$ This includes, if $m$ is zero, also a perfect pricematching guarantee.

In the second stage Firm B maximizes its profit function

$$
\pi_{B}^{P G}\left(p_{B}, p_{A}\left(p_{A}^{p}, p_{B}\right)\right)= \begin{cases}\pi_{B}^{N o{ }^{P G}}\left(p_{B}, p_{A}^{p}\right) & \text { if } \quad p_{B} \geq p_{A}^{p}-m \\ \pi_{B}^{P G-A c t i v e}\left(p_{B}, p_{B}+m\right) & \text { else }\end{cases}
$$

That is, only if it undercuts the price of Firm A by no more than m, Firm A's guarantee will not be activated and profits are defined by $\pi_{B}^{N o}{ }^{P G}$. For any lower $p_{B}$, the guarantee will be activated and the profits of Firm B are defined by (given the demand function (8)):

$$
\pi_{B}^{P G-\text { Active }\left(p_{B}, p_{B}+m\right)=\left\{\begin{array}{ll}
p_{B} \cdot\left[\frac{1}{2}+\frac{m}{2 t}\right] & \text { if } \quad p_{B} \leq v-\frac{t+m}{2}  \tag{11}\\
p_{B} \cdot\left[\frac{v-p_{B}}{t}\right] & \text { if } \quad v-\frac{t+m}{2}<p_{B}<v \\
0 & \text { else. }
\end{array} \text { l } l\right.}
$$

Whenever Firm A's price guarantee is active, the effective price difference between $p_{A}$ and $p_{B}$ equals the markup, independently of $p_{B}$. Consequently, the position of the customer being indifferent between buying at Firm A or B exists along the road, as the price difference $m$ is by assumption smaller than $t$ (see condition 4). Hence, by the demand function in (8), whenever this customer finds it profitable to purchase a good, i.e. if condition (5) holds (which is then equivalent to $p_{B} \leq$ $v-\frac{t+m}{2}$ ), the market demands are fixed to $D_{A}=\frac{1}{2}-\frac{m}{2 t}$ and $D_{B}=\frac{1}{2}+\frac{m}{2 t}$. This is plausible, as customers balance the trade-off between better (effective) prices and higher transportation costs. However, if $p_{B}$ exceeds $v-\frac{t+m}{2}$ (i.e. the indifferent customer prefers not to buy), the demand is calculated with the demand function of a monopolist, stated in (6). Thus, for $p_{B} \leq v-\frac{t+m}{2}$ profits are linearly increasing in $p_{B}$, whereas for higher prices profits are decreasing, so that the profits are maximized at $p_{B}=v-\frac{t+m}{2}$.

[^6]The following proposition derives the reaction function of Firm B maximizing $\pi_{B}^{P G}$ :
Proposition 2. Firm B's reaction function, when Firm A provides a price guarantee with a markup $m$ on the competitor's price, is given by:

$$
R_{B}^{P G}\left(p_{A}^{p}\right)= \begin{cases}v-\frac{t+m}{2} & \text { if } \quad p_{A}^{p}>v-\frac{t-m}{2},  \tag{12}\\ p_{A}^{p}-m & \text { if } \quad t+2 m<p_{A}^{p} \leq v-\frac{t-m}{2}, \\ \frac{p_{A}^{p}+t}{2} & \text { if } \quad p_{A}^{p} \leq t+2 m .\end{cases}
$$

Proof. See Appendix B.

Thus, Firm B's reaction is dependent on $p_{A}^{p}$ being in a specific interval. In the following paragraphs the intuition for the optimal choice of $p_{B}$ is briefly discussed with the help of a graphical illustration for each of the three intervals.


Figure 1: $\pi_{B}^{P G}$ if $p_{A}^{p}>v-\frac{t-m}{2}$
Figure 1 depicts the profit function of Firm B for $p_{A}^{p}>v-\frac{t-m}{2}$. Whenever $p_{B}$ is below $v-\frac{t+m}{2}$ Firm A's price guarantee is active while the market is fully covered. Hence, by (8), Firm B's market demand is $D_{B}=\frac{1}{2}+\frac{m}{2 t}$, and thus constant. Therefore, profits are linearly increasing in this interval. For any higher $p_{B}$, condition (5) is violated, and thus, independently of whether $p_{B}$ might activate Firm A's guarantee or not, Firm B faces monopolistic demand. Since the monopolist prefers to serve the entire road, Firm B's profits are monotonically decreasing in this segment. Consequently, it is optimal to set $p_{B}=v-\frac{t+m}{2}$ for any $p_{A}^{p}>v-\frac{t-m}{2}$, since any higher $p_{B}$ would lead to an unprofitable loss in market share and any lower price would trigger a harmful automatic reduction of Firm A's price. The latter precludes Firm B from gaining any higher market demand.

Figure 2 illustrates Firm B's profits for posted prices in the interval $[t+2 m$, $\left.v-\frac{t-m}{2}\right]$. Analogously to the reasoning above, due to the price guarantee, it can


Figure 2: $\pi_{B}^{P G}$ if $t+2 m<p_{A}^{p} \leq v-\frac{t-m}{2}$
not be optimal to set $p_{B}<p_{A}^{p}-m$, since this would result in decreased profits for both firms while leaving market shares unaffected. For any higher $p_{B}$, Firm B's trade-off between charging at a higher price and gaining a higher demand is in favor of the demand, independently of whether $p_{B}$ would violate condition (5) or not. As a result $\pi_{B}^{P G}$ is decreasing in this segment. In summary, $p_{A}^{p}$ is still sufficiently high so that Firm B has an incentive to undercut Firm A's price just to the extent of the markup.

Figure 3 finally refers to the states where Firm A posted a price lower or equal $2 m+t$. Here, the posted price of Firm A is so low, that Firm B would not want to undercut it by more than $m$, even if it could effectively do so. Thus, the optimal reaction for these posted prices is, similar to the competitive benchmark case, to set $p_{B}=\frac{p_{A}^{p}+t}{2}$.


Figure 3: $\pi_{B}^{P G}$ if $p_{A}^{p} \leq t+2 m$

In summary, Firm B ensures in all cases that the market is fully covered and shared with Firm A, with a maximal market share of Firm B of $\frac{1}{2}+\frac{m}{2 t}$.

Given Firm B's reaction, given by Proposition 2, and the demand function in (8), Firm A faces the following maximization problem:

$$
\underset{p_{A}^{p}}{\operatorname{argmax}} \pi_{A}\left(p_{A}^{p}\right) \left\lvert\, R_{B}^{P G}= \begin{cases}{\left[v-\frac{t-m}{2}\right] \cdot\left[\frac{1}{2}-\frac{m}{2 t}\right]} & \text { if } \quad p_{A}^{p}>v-\frac{t-m}{2},  \tag{13}\\ p_{A}^{p} \cdot\left[\frac{1}{2}-\frac{m}{2 t}\right] & \text { if } \quad t+2 m<p_{A}^{p} \leq v-\frac{t-m}{2}, \\ p_{A}^{p} \cdot\left[\frac{1}{2}+\frac{t-p_{A}^{p}}{4 t}\right] & \text { if } \quad p_{A}^{p} \leq t+2 m\end{cases}\right.
$$

That is, for any posted price higher than $t+2 m$, Firm A will serve a market demand of $\frac{1}{2}-\frac{m}{2 t}$, since Firm B will either undercut just to the extent of the markup or activate Firm A's price guarantee (see Proposition 2). Moreover, its profits strictly increase in the interval $\left[t+2 m, v-\frac{t-m}{2}\right]$ because its effective price will be the posted price as $p_{B}$ will be set to just $p_{A}^{p}-m$. All posted prices higher than $v-\frac{t-m}{2}$ will result in an effective price of Firm A of $v-\frac{t-m}{2}$, due to the activation of the price guarantee. Only for $p_{A}^{p} \leq t+2 m$ the maximization problem is identical to the problem of the competitive benchmark case.

The following proposition shows the optimal price of Firm A:
Proposition 3. In equilibrium, Firm $A$ will set

$$
p_{A}^{p}= \begin{cases}p_{A}^{p} \geq v-\frac{t-m}{2} & \text { if } m<\phi  \tag{14}\\ \frac{3}{2} t & \text { if } m \geq \phi\end{cases}
$$

with $\phi=\frac{1}{2} \sqrt{4 v^{2}-9 t^{2}}-v+t$.
Proof. See Appendix B.

According to Proposition 3 Firm A's optimal price depends on the markup being above or below the critical threshold value $\phi$. Whenever $m<\phi$, Firm A can maximize its profits by setting any collusive arbitrary high price which is at least $v-\frac{t-m}{2}$, since Firm B will then take care, that the market is jointly covered with the highest possible prices. For $m \geq \phi$ Firm A's optimal price coincides with the equilibrium price in the competitive benchmark case.

Figure 4 shows an example of Firm A's profit function if $m$ is below the critical threshold value. Here, the local profit maximum in the competitive section, reached with the competitive price $p_{A}^{p}=\frac{3}{2} t$ in the example, is clearly below the maximum which can be achieved by setting a collusive price. This argument also holds if the


Figure 4: $\pi_{A}^{P G}$ if $m<\phi$
local maximum of the parabola is to the right of the competitive section, i.e. if $\frac{3}{2} t<2 m+t$.

Figure 5 portrays Firm A's profit function when $m$ exceeds the critical threshold. Here, the profits from collusion are lower than the profits which can be achieved in the competitive segment. This results from the fact that the market division with collusion is increasingly disadvantageous for Firm A when $m$ gets larger.


Figure 5: $\pi_{A}^{P G}$ if $m \geq \phi$

Finally, Proposition 2 and Proposition 3 together imply the following result, fully characterizing the effective prices in equilibrium.

Proposition 4. If Firm A offers a price guarantee with a non-negative markup of
(a) $m<\phi$, the effective price of Firm $A$ is $v-\frac{t-m}{2}$ and the effective price of Firm $B$ is $v-\frac{t+m}{2}$, and hence prices equal on average the monopoly price of $v-\frac{t}{2}$.
(b) $m \geq \phi$, the effective price of Firm $A$ is $\frac{3}{2} t$, and the effective price of Firm $B$ is $\frac{5}{4}$ t, and hence prices are the same as in the competitive benchmark case where no firm provides a guarantee.

Proof. See Appendix B.

The intuition for Proposition 4 is simply that Firm A's market share is decreasing in the size of the markup. Hence, if $m$ exceeds $\phi$, Firm A's profit from setting a collusive price would be too small, so that it prefers to post the competitive price. In contrast, if $m$ is below $\phi$, the guarantee induces a collusive outcome.

### 3.5 Summary of results

Table 1 summarizes the results and compares equilibrium market outcomes across different kinds of competition.

It shows that whenever Firm A offers perfect price-matching, the market outcome is identical to a case where both firms are owned by a monopolist. This type of price guarantee is most attractive for Firm A, because it neutralizes the second mover advantage of Firm B, i.e. both firms share the monopoly profit of $v-\frac{t}{2}$ equally.

Table 1: Equilibrium Prices and Consumer Rent

|  | $p_{A}$ | $p_{B}$ | Consumer rent |
| :--- | :---: | :---: | :---: |
| Monopoly, <br> Perfect price-matching $(m=0)$ | $v-\frac{t}{2}$ | $v-\frac{t}{2}$ | $\frac{1}{4} t$ |
| Price-matching with $m \in(0, \phi]$ | $v-\frac{t-m}{2}$ | $v-\frac{t+m}{2}$ | $\frac{1}{4} t+\frac{m^{2}}{4 t}$ |
| Price-matching with $m \geq \phi$, | $\frac{3}{2} t$ | $\frac{5}{4} t$ | $v-\frac{81}{64} t$ |
| No price-matching |  |  |  |

According to Proposition 4, whenever Firm A is offering a price guarantee with a positive small markup, the average price level of both firms is still the monopoly price. However, due to the unequal market division, the profit of Firm A decreases when the markup gets bigger, whereas the profits of Firm B increase. The consumer rent in this case is slightly higher in comparison with perfect price-matching, as customers close to Firm B benefit from its lower prices and their gains overcompensate the losses of customers close to Firm A.

If Firm A offers a price guarantee with a high markup, the guarantee will be virtually ignored. Both firms set prices as in the competitive benchmark case, and accordingly rents and profits are unaffected by the guarantee.

## 4 Experiment

We conducted a laboratory experiment using the model-framework discussed in the previous section. We aim to investigate the collusive effects of guarantees with different maximum markups on the competitor's price.

Background information. All treatments were programmed with the software z-Tree (Fischbacher, 2007) and all sessions were conducted in the Cologne Laboratory for Economic Research at the University of Cologne in August 2016. Participants were randomly recruited from a sample of 1,500 students, enrolled in business administration or economics, via email with the Online Recruitment System ORSEE (Greiner, 2015). We conducted in total six sessions with 30 participants each. Each subject was only allowed to participate in one session. The share of males and females, $53.3 \%$ and $46.7 \%$ respectively, was almost equal. The average age was 24.7 years. Payments to subjects consisted of a 4 Euro lump-sum payment for showing up, another 4 Euro for completing a short questionnaire and additional money which could be earned in every period, based on achieved profits. The currency used was Experimental Currency Units (ECU), which was converted to Euro at the end of the experiment at an exchange rate of 1 EUR per 14,000 ECU. Average individual payments including the lump-sum payments were 13.56 Euro. Each session took about one hour.

### 4.1 Design and hypotheses

The lab experiment was designed to test the extend of tacit collusion in the presence of guarantees. Three treatments were conducted: A baseline treatment without a price guarantee, a treatment with a markup below and a treatment with a markup above the threshold value $\phi$, which determines whether a guarantee is expected to lead to collusive prices or not. In each treatment subjects were in role of either Firm A or Firm B and faced a computerized equilibrium demand function.

Besides the markup, all parameters were kept constant across treatments. The valuation of customers for a good was set to $v=200$ and the transportation costs were set to $t=35$. Given these parameters, the threshold value $\phi$ predicts that a price guarantee with a markup below 27.99 results in collusive prices, whereas guarantees with higher markups are expected to result in competitive prices. Additionally, potential customers along the road were set to a mass of 100 instead of 1 in the previous section. This does not qualitatively change theoretical predictions, but scales up demand and profits and thus makes the experiment less artificial and easier
to explain in the instructions. In order to gain sufficient statistical power for the analysis, all treatments consisted of two sessions with 30 participants each. Since we used a matching group size of six, this resulted in 10 independent observations for each role in every treatment.

Table 2 summarizes the treatment design and states theoretical point predictions for posted as well as effective prices, and the corresponding equilibrium profits.

Table 2: Treatment Design and Point Predictions

|  | Treatment 1 <br> (No guarantee) | Treatment 2 <br> (Small markup) | Treatment 3 <br> (High markup) |
| :--- | :---: | :---: | :---: |
| $m^{p}$ | - | 2 | 33 |
| $p_{A}$ | 52.50 | $\geq 183.50$ | 52.50 |
| $p_{A}$ | 52.50 | 183.50 | 52.50 |
| $p_{B}$ | 43.75 | 181.50 | 43.75 |
| $\pi_{A}$ | $1,968.75$ | $8,650.71$ | $1,968.75$ |
| $\pi_{B}$ | $2,734.38$ | $9,593.57$ | $2,734.38$ |

All values are stated in ECU.

In Treatment 1 (T1) Firm A has no price guarantee. This treatment serves as the competitive benchmark. In equilibrium Firm A sets a price of 52.50 . Firm B, due to its second mover advantage undercuts this by setting a price of 43.75 . Consequently, Firm A's price exceeds Firm B's price by $20 \%$, leading to a market coverage of $37.5 \%$ for Firm A compared to $62.5 \%$ for Firm B. Due to the higher market share, Firm B gets a profit of 2,734.38, which exceeds Firm A's profit of $1,968.25$.

In Treatment 2 (T2) Firm A has a price guarantee with a markup of $m=2$. Since this markup is below the threshold value $\phi$, it induces a collusive market outcome in theory. In any equilibrium of T2, Firm A posts the collusive price of 183.50 or higher. Firm B undercuts the posted price but only to the extent of $m$ plus the amount by which Firm A's price exceeds 183.50. Put differently, Firm B sets 181.50 in any equilibrium and thus assures that the effective price of Firm A is 183.50. Consequently, effective prices are close to another and equilibrium market shares and profits are only in slight favor of Firm B, which serves $52.86 \%$ of the market and earns 9,593.57 compared to $47.14 \%$ and $8,650.71$ for Firm A.

In Treatment 3 (T3) Firm A has a price guarantee with a markup of $m=33$. Since the markup is higher than $\phi$, theory predicts that the guarantee does not affect effective prices, market shares and profit levels compared to a setting where no guarantee is in place. Thus, chosen price levels in this treatment are expected to coincide with the price levels of T1.

Hypotheses. In summary, we get two hypotheses from the treatment comparisons. First, we expect the price guarantee with the small markup in T2 to lead to a collusive market outcome. That is, we expect Firm A and Firm B to set higher prices in T2 compared to T1. Second, we do not expect the price guarantee with the high markup to have any effect on competition. Consequently, the prices of Firm A and Firm B in T3 are expected not to differ compared to T1 but to be lower than in T2.

Procedures within the experiment. All treatments consisted of an individual trial stage, followed by an interaction stage consisting of 15 periods of the sequential pricing game.

Prior to the start of the experiment, subjects were randomly allotted to computer terminals. Then they received identical written instructions, explaining general lab rules, all treatment specific information, including the equilibrium demand function as well as the matching procedure in the interaction stage. ${ }^{11}$ Whenever subjects had questions, these were answered privately by referring to the relevant section in the instructions.

The trial stage, which lasted approximately five minutes, started roughly ten minutes after the instructions were distributed. This stage was not payoff relevant, did not involve any interaction between subjects and consisted of a simple scenario-calculator which used continuous posted prices as inputs and showed resulting effective prices, market shares and profit levels as outputs. This calculator was identical for all subjects within a treatment, independently of the role a subject was assigned to in the subsequent interaction stage, where it was also accessible. The purpose for providing the calculator was to allow subjects to deal with complex demand and profit calculations. By using a calculator with empty default values, it could be avoided to set anchoring points in contrast to providing payoff tables or examples, which inevitably put focus on certain price combinations. The scenario calculator could be used for any continuous price combination between 0 and 200. ${ }^{12}$

Finally, subjects proceeded to the interaction stage, consisting of 15 identical periods of the sequential pricing game. Each subject was assigned to a specific role, either Firm A or Firm B, and a matching group consisting of 6 subjects. These

[^7]assignments remained constant for the course of the experiment. In order to avoid reciprocal behavior, a stranger matching was used to determine pairs of Firm A and Firm B. That is, in every period each subject was randomly rematched within its matching group, while we ensured that the same pair was never matched in two consecutive periods. This matching procedure was clearly stated in the instructions. Only the size of the matching group was not mentioned. At the beginning of each period Firm A chooses its posted price. Thereafter Firm B, being aware of the posted price, chooses its effective price. Afterwards subjects received complete feedback on posted and effective prices, market shares and profit levels and proceeded to the next period.

Once the experiment was over, a short questionnaire appeared on the screen asking subjects for their age, field of study and gender. In addition to the collection of demographic data, the questionnaire justified the higher than usual total lump-sum payment of 8 Euro. The latter was needed for easing equilibrium payoff differences across treatments, as the exchange rate of ECU to EUR was identical across treatments.

### 4.2 Results

Table 3 summarizes the descriptive results of the experiment.

Table 3: Average Price and Profit Levels

|  | Treatment 1 <br> (no guarantee) |  | Treatment 2 <br> (small markup) |  | Treatment 3 <br> (high markup) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Prediction | Experiment |  |  |  |  | Prediction | Experiment |
| :---: | Prediction | Experiment |
| :---: |

All values are stated in ECU. Standard deviations are reported in parentheses and calculated at the matching group level.

In Treatment 1 the average price level for Firm A (Firm B) was 79.82 (68.99), which is higher than the predicted equilibrium price of 52.50 (43.75). In a sense, this could be interpreted as a form of collusion, although these price levels are far from perfect collusion. ${ }^{13}$ A closer look at the data reveals that Firm A sometimes attempted to establish (almost) perfect collusion, by posting prices higher than 180, in 76 out of 450 observations, often in early periods. However, in 39 of these cases the posted price was undercut such that Firm A was not able to sell any good. Only in 24 cases the collusive attempt was profitable in the sense that the resulting profit exceeded the equilibrium profit. As a consequence, only Firm B benefited from the higher than predicted price levels and received on average higher profits than in equilibrium.

In Treatment 2 Firm A posted on average a price of 180.52 , slightly below the equilibrium price of $p_{A}^{p} \geq 183.5$. More precisely, in 100 out of 450 observations an equilibrium strategy was played, either by posting a price of 183.5 ( 23 obs.) or higher ( 77 obs.), in 312 cases the posted prices were between 180 and 183.5 and in 38 cases a price lower than 180 was posted. Firm B set on average prices of 177.17. That is, in the 100 observations where an equilibrium price was posted, Firm B reacted on average by setting a price of 181.08 which is close to the predicted value of the reaction function of 181.5 for those cases. Firm B's average undercutting of 1.89 in the 371 observations where Firm A's posted price was between 70 and 183.5 is also close to the prediction of 2 for this interval of $p_{A}^{p}$. Hence, the descriptive data suggests that Firm A's price guarantee prevented Firm B from harsh undercutting. Consequently, the resulting effective prices of 178.96 (177.17) for Firm A (Firm B) and the hereby resulting profit levels were close to the point predictions of Table 2.

In Treatment 3 the posted price of Firm A was on average 73.86, which is, similar like to T 1 , a bit higher than the predicted 52.50 . An attempt for (almost) perfect collusion was observed in 61 of 450 cases, but only in 6 observations this attempt was profitable. The average undercutting of Firm B for posted prices in the interval [70,199] was 27.63 compared to the prediction of 33 for these cases ( 132 obs.). The average price of Firm B was 62.63 and, since the Firm A's price guarantee was almost never activated, the average effective price of Firm A rarely differed from the posted price. ${ }^{14}$ In terms of profit Firm B was better off than in equilibrium because it could benefit from Firm A's attempts for collusion whereas Firm A was worse off.

[^8]Test of hypotheses. In order to test the hypotheses, price levels across treatments are compared on the matching group level by using the non-parametric Mann-Whitney-Wilcoxon test (MWW test, all hereafter stated p-values refer to the twosided version of the test).

The first hypothesis states that the price guarantee with the small markup of 2 has a collusive effect. This hypothesis can be confirmed, as we find that the posted price of Firm A as well as the price of Firm B are significantly higher in T2 compared to T1 ( $p<0.001$ in both comparisons).

In addition, the experiment data is also consistent with the second hypothesis, which states that the price guarantee with a high markup of 33 does not lead to collusion. The comparison of T3 with T1 cannot detect any significant differences neither for the price of Firm A $(p=0.4057)$ nor the price of Firm B $(p=0.4983)$, whereas both prices are lower compared to T 2 ( $p<0.001$ for both comparisons).

Dynamic effects. For robustness, we control for dynamic effects. To this end, Table 4 provides the results of a random-effects GLS regression with the posted price of Firm A as the dependent variable and Treatment 2, where collusion is predicted, as baseline.

Table 4: Determinants of Firm A's posted price

|  | $(1)$ | $(2)$ |
| :--- | :---: | :---: |
| Constant | $180.5^{* * *}$ | $174.2^{* * *}$ |
|  | $(1.044)$ | $(3.600)$ |
| Treatment 1 | $-100.7^{* * *}$ | $-82.68^{* * *}$ |
|  | $(6.761)$ | $(8.780)$ |
| Treatment 3 | $-106.7^{* * *}$ | $-86.42^{* * *}$ |
|  | $(8.502)$ | $(9.160)$ |
| Period |  | $0.794^{*}$ |
|  |  | $(0.358)$ |
| Treatment $1 \times$ Period |  | $-2.251^{* *}$ |
|  |  | $(0.855)$ |
| Treatment $3 \times$ Period |  | $-2.530^{*}$ |
|  |  | $(0.995)$ |
| Observations | 1,350 | 1,350 |

The table shows the results of Random-Effects GLS model. Robust standard errors clustered on the level of experimental cohorts are listed in parentheses. ${ }^{* * *}$, ${ }^{* *}$ and ${ }^{*}$ indicate significance on the $1 \%, 1 \%$ and $5 \%$-level, respectively.


Figure 6: Price development across treatments

The regression results show that the coefficients for Treatment 1 and Treatment 3 are negative compared to Treatment 2, i.e. posted prices are lower in these treatments. The effects are highly statistically significant, independent of whether it is controlled for period effects or not, which reconfirms the results of the nonparametric analysis of a collusive price level in T2. Only the size of the (initial) treatment effect varies slightly, once period is taken into account. That is in T1 (T3) the size changes from $-100.7(-106.7)$ to $-82.68(-86.42)$. The coefficients on period and corresponding interaction terms show that subjects refrain over time from making collusive offers in T1 and T3 whereas the period effect in T2 goes in the opposite direction. More precisely posted prices decrease per period on average by 1.457 in T1 and 1.736 in T3 whereas posted prices increase slightly by 0.794 in $\mathrm{T} 2 .{ }^{15}$ This indicates that subjects behave in line with the theoretical predictions more frequently over time. Finally, Figure 6 illustrates the development for posted prices of Firm A and Firm B for T1 and T2.

[^9]
## 5 Conclusion

This paper studies whether a price-matching guarantee with a markup on the lowest competing price has the potential to induce tacit collusion. It shows both theoretically and experimentally that in a Hotelling duopoly framework with fixed locations and sequential pricing this can be indeed the case. Thus, these kind of price guarantees should be reviewed by antitrust authorities with the same skepticism as perfect price-matching guarantees.

In particular, whenever the guaranteed maximum markup on the lowest competing price does not exceed a threshold value, the guarantee leads to on average monopoly pricing. If instead the markup exceeds this threshold, the market outcome is unaffected. For the former results it suffices that the price leader, i.e. the first moving firm, offers a guarantee. However, if the level of the markup would be endogenously chosen by the first moving firm, it would offer the smallest possible markup, in other words, a perfect price-matching guarantee. The reason is that the latter neutralizes the second mover advantage of its competitor completely.

Apart from studying the role of the markup, the paper connects to the existing literature on perfect price-matching guarantees. It shows that even in a setting where competitors are symmetric with regard to their cost structure, a collusive market outcome can result. The reason is, that with sequential price competition the first mover has a disadvantageous position, similar to a firm which is disadvantaged by its cost structure in a simultaneous move game. Thus, it extends the findings of Logan and Lutter (1989).

With regard to the motivating example of Shell, which, to the best of the author's knowledge, is the first to use the considered type of guarantee, the paper does not claim to provide a final answer on whether their conduct intends to establish tacit collusion. One reason is that the model setup used does not cover all characteristics of the German gasoline retail market, e.g. it does not take repeated interaction or the heterogeneity of customers into account. Yet, it is an open question why Shell, which possesses price leadership according to Bundeskartellamt (2011, 2014), adopted a guarantee with a theoretically suboptimal markup of 2 Cents. A rationale for this could be to avoid suspicion about anti-competitive conduct, as perfect price-matching guarantees have been criticized extensively in the economics and law literature. At the same time, a recent empirical paper of Dewenter and Schwalbe (2015) found that Shell's prices increased after the introduction of its guarantee. This finding, in combination with the paper's result that such a guarantee in general has the potential to induce tacit collusion, suggest that Shell's conduct should be carefully monitored.

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## B Proofs

## Proof of Proposition 1

Let us consider all possible cases of $p_{A}$, and derive separately the best responses of Firm B. Since Firm B's profit function is piecewise defined, it is important to check which interval of $p_{B}$, for a given $p_{A}^{p}$, refers to which segment of the profit function $\pi_{B}^{N o P G}$. The four segments are therefore labeled with capital roman numbers, i.e.

$$
\pi_{B}^{N o P G}= \begin{cases}I: p_{B} & \text { if } p_{B} \leq v-t \wedge p_{B}<p_{A}^{p}-t \\ I I: p_{B} \cdot\left[\frac{1}{2}+\frac{p_{A}^{p}-p_{B}}{2 t}\right] & \text { if } p_{B}<2 v-t-p_{A}^{p} \wedge p_{B} \in\left[p_{A}^{p}-t, p_{A}^{p}+t\right], \\ I I I: p_{B} \cdot\left[\frac{v-p_{B}}{t}\right] & \text { if } \left.p_{B} \geq 2 v-t-p_{A}^{p} \wedge p_{B} \in\right] v-t, v[ \\ I V: 0 & \text { else. }\end{cases}
$$

In the following cases we never check for IV, since the zero profit is always strictly dominated by other segments (with at least one of these segments being nonempty for any $p_{A}^{p}$, as shown below).

Case 1 - Firm A sets $p_{A}^{p}>v$.
Segment $I$ is equivalent to $p_{B} \in[0, v-t]$. This follows from $p_{A}^{p}-t>v-t>0$, where the first inequality is by assumption on $p_{A}^{p}$, and the second follows from our parametric assumption $t<\frac{v}{3}$. Consequently, the highest reachable profit level in segment I is $v-t$, independently of $p_{A}^{p}$, which is reached by setting $p_{B}=v-t$.

Segment II is empty in the considered case. Assume by contradiction that it is non empty. Then there exists a $p_{B}$ such that

$$
\begin{gathered}
p_{B}<2 v-t-p_{A}^{p} \wedge p_{B} \geq p_{A}^{p}-t \\
\Rightarrow p_{A}^{p}-t<2 v-t-p_{A}^{p} \\
\Leftrightarrow p_{A}^{p}<v,
\end{gathered}
$$

which is a contradiction to the considered interval of $p_{A}^{p}$.
Segment III is equivalent to $\left.p_{B} \in\right] v-t, v[$. This follows from

$$
\left.p_{B} \geq 2 v-t-p_{A}^{p} \wedge p_{B} \in\right] v-t, v[
$$

$$
\left.\Leftrightarrow p_{B} \in\right] v-t, v[
$$

since $v-t>2 v-t-p_{A}^{p}$ by $p_{A}^{p}>v$. The highest reachable profit in this segment is in the limit $v-t$, because

$$
\frac{\partial\left(p_{B} \cdot \frac{v-p_{B}}{t}\right)}{\partial p_{B}}=\frac{v-2 p_{B}}{t}<0 \text { since } p_{B} \geq v-t \wedge t<\frac{v}{3}
$$

Best response - Case 1: Thus, the best response to $p_{A}^{p}>v$ is to set $p_{B}=v-t$.

Case $2-$ Firm A sets $p_{A}^{p}$ in the interval $\left.] v-t, v\right]$.
Segment $I$ is, given that $p_{A}^{p}<v-t$, defined in the interval equivalent to $p_{B}<$ $p_{A}^{p}-t$. Since $t<\frac{v}{3}$, the interval $\left[0, p_{A}^{p}-t\left[\right.\right.$ is non-empty for the given interval of $p_{A}^{p}$. Then, the highest reachable profit level in I is in the limit $p_{A}^{p}-t$.

Segment II is defined for

$$
\begin{gathered}
p_{B}<2 v-t-p_{A}^{p} \wedge p_{B} \in\left[p_{A}^{p}-t, p_{A}^{p}+t\right] \\
\Leftrightarrow p_{B} \in\left[p_{A}^{p}-t, 2 v-t-p_{A}^{p}[ \right.
\end{gathered}
$$

because $2 v-t-p_{A}^{p}<p_{A}^{p}+t$ due to assumed $p_{A}^{p}>v-t$. This interval of $p_{B}$ is non-empty, since $p_{A}^{p}-t<2 v-t-p_{A}^{p}$ is equivalent to $p_{A}^{p}<v$ which is satisfied in the considered interval of $p_{A}^{p}$.

The highest reachable profit level in II is dependent on $p_{A}^{p}$. For $p_{A}^{p}>3 t$ it is $p_{A}^{p}-t$, and is reached with $p_{B}=p_{A}^{p}-t$. For $p_{A}^{p} \leq 3 t$ it is $\frac{\left(p_{A}^{p}+t\right)^{2}}{8 t}$, and is reached with $p_{B}=\frac{p_{A}^{p}+t}{2}$. The reason is, that the profit function in segment II has a parabolic shape, but the maximum at $p_{B}=\frac{1}{2}\left(p_{A}^{p}+t\right)$ can be to the left of the interval of $p_{B}$ allowed by II, which is the case if:

$$
\begin{gathered}
\frac{1}{2}\left(p_{A}^{p}+t\right)<p_{A}^{p}-t \\
\Leftrightarrow p_{A}^{p}>3 t .
\end{gathered}
$$

At the same time, the right boundary of the interval of $p_{B}$ equal to $2 v-t-p_{A}^{p}$ is always above $\frac{p_{A}^{p}+t}{2}$ within the considered interval of $p_{A}^{p}$. Thus, when $p_{A}^{p}>3 t$, the lower bound of the interval of $p_{B}$, i.e. $p_{A}^{p}-t$, is the best response, whereas if $p_{A}^{p} \leq 3 t$ the best response is $\frac{p_{A}^{p}+t}{2}$.

Segment III is defined if the following conditions are met

$$
\left.p_{B} \geq 2 v-t-p_{A}^{p} \wedge p_{B} \in\right] v-t, v[
$$

$$
\Leftrightarrow v>p_{B} \geq 2 v-t-p_{A}^{p}
$$

because $2 v-t-p_{A}^{p} \geq v-t$ due to $p_{A}^{p} \leq v$. This interval of $p_{B}$ is non-empty, because $v>2 v-t-p_{A}^{p}$ is equivalent to $p_{A}^{p}>v-t$ which is satisfied in the considered interval of $p_{A}^{p}$.

The highest reachable profit level in III is $\frac{\left(p_{A}^{p}+t-v\right)\left(2 v-t-p_{A}^{p}\right)}{t}$ reached with the lowest $p_{B}$ within the interval, i.e. $2 v-t-p_{A}^{p}$. The reason is, that the profit function in III has a parabolic shape, but the price maximizing the parabola $p_{B}=\frac{v}{2}$ is always to the left of the interval $\left[2 v-t-p_{A}^{p}, v[\right.$ :

$$
\begin{aligned}
& \frac{v}{2}<2 v-t-p_{A}^{p} \\
& \Leftrightarrow p_{A}^{p}<\frac{3}{2} v-t
\end{aligned}
$$

which holds for the considered interval of $p_{A}^{p}$ since $t \leq \frac{v}{3}$.
Best response - Case 2: In summary, the best response to $\left.\left.p_{A}^{p} \in\right] v-t, v\right]$ is to set $p_{B}=p_{A}^{p}-t$ if $p_{A}^{p}>3 t$ and $p_{B}=\frac{p_{A}^{p}+t}{2}$ if $p_{A}^{p} \leq 3 t$.
The reason is, that the highest profit in II is always at least as high as the highest profit in I and III. The former comparison directly follows from the analysis of segments I and II above, given that the profit level $p_{A}^{p}-t$ is always achievable in II. The latter comparison results from the following:
(a) $p_{A}^{p}>3 t$ :

$$
\begin{gathered}
\frac{\left(p_{A}^{p}+t-v\right)\left(2 v-t-p_{A}^{p}\right)}{t} \leq p_{A}^{p}-t \\
\Leftrightarrow\left(p_{A}^{p}+t-v\right)\left(2 v-t-p_{A}^{p}\right)-t^{2}-p_{A}^{p} \cdot t \leq 0 .
\end{gathered}
$$

The left hand side is a parabola in $t$, with a global maximum. Thus, for the claim to be true the inequality needs to hold for the maximum at $t^{*}=\frac{3}{4}\left(p_{A}^{p}-v\right)$. Plugging $t^{*}$ into the inequality gives:

$$
\frac{1}{8}\left(\left(p_{A}^{p}\right)^{2}+6 p_{A}^{p} \cdot v-7 v^{2}\right) \leq 0
$$

which is fulfilled as long as $p_{A}^{p} \leq v$, which holds in the considered case.
(b) $p_{A}^{p} \leq 3 t$ :

$$
\begin{gathered}
\frac{\left(p_{A}^{p}+t-v\right)\left(2 v-t-p_{A}^{p}\right)}{t} \leq \frac{\left(p_{A}^{p}+t\right)^{2}}{8 t} \\
\Leftrightarrow \frac{\left(3 p_{A}^{p}+3 t-4 v\right)^{2}}{t} \geq 0
\end{gathered}
$$

which is fulfilled since $t>0$.

Case 3 - Firm A sets $p_{A}^{p}$ in the interval $\left.] t, v-t\right]$.
Segment $I$ is defined when the following conditions are met

$$
\begin{gathered}
p_{B} \leq v-t \wedge p_{B}<p_{A}^{p}-t \\
\Leftrightarrow p_{B}<p_{A}^{p}-t
\end{gathered}
$$

because $p_{A}^{p}$ is smaller than $v$ in the considered interval of $p_{A}^{p}$. This interval of $p_{B}$ is non-empty since $p_{A}^{p}>t$. The highest reachable profit level in I is in the limit $p_{A}^{p}-t$.

Segment $I I$ is defined under the conditions

$$
\begin{gathered}
p_{B}<2 v-t-p_{A}^{p} \wedge p_{B} \in\left[p_{A}^{p}-t, p_{A}^{p}+t\right] \\
\Leftrightarrow p_{B} \in\left[p_{A}^{p}-t, p_{A}^{p}+t\right]
\end{gathered}
$$

because $p_{A}^{p}+t \leq 2 v-t-p_{A}^{p}$, which equals $\Leftrightarrow p_{A}^{p} \leq v-t$ and is fulfilled for the given interval of $p_{A}^{p}$. The interval of $p_{B}$ stated above is non-empty because $p_{A}^{p}>t$ and $t \geq 0$. The highest reachable profit level in II is dependent on $p_{A}^{p}$. For $p_{A}^{p}>3 t$ it is $p_{A}^{p}-t$, and is reached with $p_{B}=p_{A}^{p}-t$. For $p_{A}^{p} \leq 3 t$ it is $\frac{\left(p_{A}^{p}+t\right)^{2}}{8 t}$, and is reached with $p_{B}=\frac{p_{A}^{p}+t}{2}$. The reasoning is identical to Case 2.

Segment III is not defined for the given interval of $p_{A}^{p}$. This is shown by contradiction. For III to be defined the following conditions have to be satisfied:

$$
\begin{gathered}
\left.p_{B}>2 v-t-p_{A}^{p} \wedge p_{B} \in\right] v-t, v[ \\
\Rightarrow v>2 v-t-p_{A}^{p} \\
\Leftrightarrow p_{A}^{p}>v-t,
\end{gathered}
$$

which is a contradiction.
Best response - Case 3: In summary, the best response to $\left.\left.p_{A}^{p} \in\right] t, v-t\right]$ is $p_{B}=p_{A}^{p}-t$ if $p_{A}^{p} \geq 3 t$ and $p_{B}=\frac{p_{A}^{p}+t}{2}$ if $p_{A}^{p}<3 t$. This is true because in the considered case the highest profit of II always exceeds the highest profit in I. Indeed, for $p_{A}^{p}>3 t$ the profit maximum of I is only in the limit as high as the profit maximum of II. For $p_{A}^{p} \leq 3 t$ the profit of II is at least as high as in I, because:

$$
\begin{gathered}
\frac{\left(p_{A}^{p}+t\right)^{2}}{8 t} \geq p_{A}^{p}-t \\
\Leftrightarrow\left(p_{A}^{p}\right)^{2}-6 p_{A}^{p} \cdot t+9 t^{2} \geq 0 .
\end{gathered}
$$

The left hand side is a parabola with a global minimum of 0 at $p_{A}^{p}=3 t$. Thus the condition is always met.

Case $4-$ Firm A sets $p_{A}^{p}$ in the interval $[0, t]$.
Segment $I$ does not exist. Assume by contradiction there exists $p_{B}<p_{A}^{p}-t$. This however cannot be true since $p_{A}^{p} \leq t$ is assumed in the considered case.

Segment II has the following conditions

$$
\begin{gathered}
p_{B}<2 v-t-p_{A}^{p} \wedge p_{B} \in\left[p_{A}^{p}-t, p_{A}^{p}+t\right] \\
\Leftrightarrow p_{B} \in\left[p_{A}^{p}-t, p_{A}^{p}+t\right],
\end{gathered}
$$

since $2 v-t-p_{A}^{p}>p_{A}^{p}+t$, which is in turn equivalent to $p_{A}^{p}<v-t$ and hence fulfilled in the given interval of $p_{A}^{p}$ due to the parametric assumption $t<\frac{v}{3}$. But since $p_{B}$ has to be non-negative by assumption, II is eventually equivalent to

$$
0 \leq p_{B} \leq p_{A}^{p}+t
$$

which is clearly a non-empty interval, because $p_{A}^{p} \geq 0$ and $t>0$.
The highest reachable profit level in II is $\frac{\left(p_{A}^{p}+t\right)^{2}}{8 t}$, which is achieved by setting $p_{B}=$ $\frac{p_{A}^{p}+t}{2}$, which in turn is always within the allowed boundaries of $p_{B}$, i.e. in $\left[0, p_{A}^{p}+t\right]$.

Segment III is not defined for the given interval of $p_{A}^{p}$. The argumentation is analogous to Case 3.

Best response - Case 4: In summary, the best response to $p_{A}^{p} \in[0, t]$ is $\frac{p_{A}^{p}+t}{2}$.

## Best response function - Summary of the four cases:

Let us now sum up the best responses for the considered intervals of $p_{A}^{p}$ together. This yields

$$
R_{B}^{N o P G}\left(p_{A}^{p}\right)= \begin{cases}v-t & \text { if } \quad p_{A}^{p}>v, \\ p_{A}^{p}-t & \text { if } \quad 3 t \leq p_{A}^{p} \leq v, \\ \frac{p_{A}^{p}+t}{2} & \text { if } \quad p_{A}^{p}<3 t .\end{cases}
$$

## Proof of Continuity of $\pi_{B}^{P G}$

Lemma 2. $\pi_{B}^{P G}$ is continuous in $p_{B}$.

Proof. The proof consists of three parts: First we prove that $\pi_{B}^{N o P G}$ is continuous, second we show that $\pi_{B}^{P G-A c t i v e}$ is continuous and finally the continuity of $\pi_{B}^{P G}$, which is a combination of $\pi_{B}^{N o{ }^{P G}}$ and $\pi_{B}^{P G-A c t i v e}$, is proven.

## 1.) Continuity of $\pi_{B}^{N o}{ }^{P G}$.

Here it is sufficient to show, that $D_{B}$ is continuous with respect to $p_{B}$ for any given $p_{A}=p_{A}^{p}$. The reason is that $\pi_{B}^{N o ~}{ }^{P G}=p_{B} \cdot D_{B}\left(p_{A}=p_{A}^{p}, p_{B}\right)$ and $p_{A}$ is not a function of $p_{B}$ since the price guarantee is inactive.
We first reformulate the demand function in the following way. From the argumentation of Cases 1 and 2 on page 8 it follows that whenever condition (5) is not satisfied, the firm with a lower price faces a monopolistic demand function given by (6). At the same time, since for the firm with the higher price it should hold that $p_{i} \geq v$ (which follows from $p_{B} \geq 2 v-t-p_{A}$ and $p_{i}>p_{-} i+t$ ), the monopolistic demand function is formally applicable to this firm as well in the considered case. Hence,

$$
\begin{equation*}
D_{B}\left(p_{A}, p_{B}\right)=\min \left\{\max \left\{\frac{v-p_{B}}{t}, 0\right\}, 1\right\} \quad \text { if } \quad p_{B}>2 v-t-p_{A} \tag{15}
\end{equation*}
$$

Next, note that $\frac{1}{2}+\frac{p_{A}-p_{B}}{2 t}>1$ iff $p_{A}>p_{B}+t$, and that $\frac{1}{2}+\frac{p_{A}-p_{B}}{2 t}<0$ iff $p_{A}<p_{B}-t$. This implies that whenever condition (5) is satisfied (given the argumentation in Case 3 and Case 4 on page 8), the demand function can be represented as

$$
\begin{equation*}
D_{B}\left(p_{A}, p_{B}\right)=\min \left\{\max \left\{\frac{1}{2}+\frac{p_{A}-p_{B}}{2 t}, 0\right\}, 1\right\} \quad \text { if } \quad p_{B} \leq 2 v-t-p_{A} \tag{16}
\end{equation*}
$$

Summing up (15) and (16) together, we obtain

$$
D_{B}\left(p_{A}, p_{B}\right)= \begin{cases}\min \left\{\max \left\{\frac{1}{2}+\frac{p_{A}-p_{B}}{2 t}, 0\right\}, 1\right\} \quad \text { if } \quad p_{B} \leq 2 v-t-p_{A}  \tag{17}\\ \min \left\{\max \left\{\frac{v-p_{B}}{t}, 0\right\}, 1\right\} \quad \text { else. }\end{cases}
$$

Because both sections of the demand function are continuous in $p_{B}$ for any $p_{B}>0$, it remains to show that at $p_{B}=2 v-t-p_{A}$ both sections of the demand function
yield the same level of demand. Indeed, at this value of $p_{B}$ :

$$
\frac{1}{2}+\frac{p_{A}-p_{B}}{2 t}=\frac{t+2 v-t-p_{B}-p_{B}}{2 t}=\frac{v-p_{B}}{t}
$$

## 2.) Continuity of $\pi_{B}^{P G-A c t i v e . ~}$

Since under an active price guarantee $p_{A}=p_{B}+m$, we have

$$
\pi_{B}^{P G-\text { Active }}=p_{B} \cdot D_{B}\left(p_{B}+m, p_{B}\right) .
$$

Given (17), the former demand function can be rewritten as

$$
D_{B}\left(p_{B}+m, p_{B}\right)=\left\{\begin{array}{l}
\min \left\{\max \left\{\frac{1}{2}+\frac{m}{2 t}, 0\right\}, 1\right\} \quad \text { if } \quad p_{B} \leq v-\frac{t+m}{2}  \tag{18}\\
\min \left\{\max \left\{\frac{v-p_{B}}{t}, 0\right\}, 1\right\} \quad \text { else. }
\end{array}\right.
$$

Because both sections of the demand function are continuous in $p_{B}$ for any $p_{B}>0$, it remains to show that at $p_{B}=v-\frac{t+m}{2}$ both sections yield the same demand. Indeed at this value of $p_{B}$

$$
\frac{v-p_{B}}{t}=\frac{v-v+\frac{t+m}{2}}{t}=\frac{1}{2}+\frac{m}{2 t} .
$$

## 3.) Continuity of $\pi_{B}^{P G}$.

The combined profit function $\pi_{B}^{P G}$ is defined as

$$
\pi_{B}^{P G}\left(p_{A}^{p}, p_{B}\right)= \begin{cases}\pi_{B}^{N o P G}\left(p_{A}^{p}, p_{B}\right) & \text { if } \quad p_{B} \geq p_{A}^{p}-m \\ \pi_{B}^{P G-\text { Active }}\left(p_{B}\right) & \text { else }\end{cases}
$$

Since $\pi_{B}^{P G-A c t i v e}$ and $\pi_{B}^{N o P G}$ are both continuous in $p_{B}$ for any $p_{B}>0$, as proven above, it only needs to be shown that at $p_{B}=p_{A}^{p}-m$ both functions give the same profit. Indeed, for this posted price

$$
\pi_{B}^{N o ~}{ }^{P G}=p_{B} \cdot D_{B}\left(p_{A}=p_{B}+m, p_{B}\right)
$$

and hence coincides with $\pi_{B}^{P G-A c t i v e}$. Consequently, $\pi_{B}^{P G}$ is continuous in $p_{B}$.

## Proof that $\pi_{B}^{P G}$ is hump-shaped

Lemma 3. There exists a $p_{B}^{*}$ such that $\pi_{B}^{P G}\left(p_{B}\right)$ is increasing for $p_{B}<p_{B}^{*}$ and decreasing for $p_{B} \geq p_{B}^{*}$.

Proof. The proof has three parts. First, we prove that $\pi_{B}^{P G-A c t i v e ~ i s ~ h u m p-s h a p e d . ~}$ Second, we show that $\pi_{B}^{N o P G}$ is hump-shaped. Finally, it is proven that the combined profit function $\pi_{B}^{P G}$ is hump-shaped.
1.) $\pi_{B}^{P G-A c t i v e}\left(p_{B}\right)$ is hump-shaped.

It is easy to see, that $\pi_{B}^{P G-A c t i v e}$ is linearly increasing in $p_{B}$ for values of $p_{B}$ smaller than $v-\frac{t+m}{2}$ (see (11)). For higher values, it has a parabolic shape up to $p_{B}=v$ and then stays flat at 0 . However, the maximum of the parabola is located at $p_{B}=\frac{1}{2} v$ which is lower than the left border of the parabolic interval $v-\frac{t+m}{2}$ since $t+m<v$, because $m<t \leq \frac{v}{3}$. Hence, $\pi_{B}^{P G-A c t i v e}\left(p_{B}\right)$ is decreasing in $p_{B}$ for $p_{B}>v-\frac{t+m}{2}$. Consequently, given that $\pi_{B}^{P G-A c t i v e}\left(p_{B}\right)$ is continuous (see proof of Lemma 2), it is hump-shaped with respect to $p_{B}$ with a global maximum at $p_{B}=v-\frac{t+m}{2}$.
2.) $\pi_{B}^{N o{ }^{P G}\left(p_{B}\right) \text { is hump-shaped. }}$

Since the composition of $\pi_{B}^{N o{ }^{P G}}\left(p_{B}\right)$ is dependent on $p_{A}^{p}$, the proof is given separately for each of the four cases of intervals of $p_{A}^{p}$ described in the proof of Proposition 1. In what follows, we refer to the results of the analysis in the proof of Proposition
 (see proof of Lemma 2).
Case 1 - Firm A sets $p_{A}^{p}>v$.
For $p_{B} \leq v-t$ the profit function is defined by Segment I of $\pi_{B}^{N o P G}$, and thus linearly increasing in $p_{B}$.

For $v-t<p_{B}<v$, the profit function is defined by Segment III of $\pi_{B}^{N o P G}$, which is a parabola with the maximum at $\frac{v}{2}$. Since this maximum is reached with a lower price than $v-t$, because $t<\frac{v}{3}$, the profit function is decreasing for $\left.p_{B} \in\right] v-t, v[$. For any higher $p_{B}$ the profit is zero. Hence, for $p_{A}^{p}>v$ the function $\pi_{B}^{N o ~}{ }^{P G}\left(p_{B}\right)$ is hump-shaped with respect to $p_{B}$ with a global maximum at $p_{B}=v-t$.
Case $2-\operatorname{Firm} \mathbf{A}$ sets $p_{A}^{p}$ in the interval $\left.] v-t, v\right]$.
For $p_{B}<p_{A}^{p}-t$ the profit function is defined by Segment I of $\pi_{B}^{N o ~}{ }^{P G}\left(p_{B}\right)$, and thus linearly increasing in $p_{B}$.

For $p_{A}^{p}-t \leq p_{B}<2 v-t-p_{A}^{p}$ the profit function is defined by Segment II of $\pi_{B}^{N o P G}$, which is a parabola with the maximum at $\frac{p_{A}^{p}+t}{2}$. If additionally $p_{A}^{p}>3 t$ this maximum is located at a price lower than the left interval border $p_{A}^{p}-t$ and the profit function is hence decreasing for $p_{B} \in\left[p_{A}^{p}-t, 2 v-t-p_{A}^{p}[\right.$. Otherwise, i.e. if $p_{A}^{p} \leq 3 t$, the profit function is increasing in $p_{B} \in\left[p_{A}^{p}-t, \frac{p_{A}^{p}+t}{2}[\right.$ and decreasing in $p_{B} \in\left[\frac{p_{A}^{p}+t}{2}, 2 v-t-p_{A}^{p}[\right.$.

For $2 v-t-p_{A}^{p} \leq p_{B}<v$ the profit function is defined by Segment III of $\pi_{B}^{N o ~ P G}$, which is a parabola with the maximum at $p_{B}=\frac{v}{2}$. Since this maximum is reached with a lower price than the left border of the interval $2 v-t-p_{A}^{p}$, because $t<\frac{v}{3}$ and $p_{A}^{p} \leq v$, the profit function is decreasing for $p_{B} \in\left[2 v-t-p_{A}^{p}, v[\right.$.

For any higher $p_{B}$ the profit is zero.
Hence, for $v-t<p_{A}^{p} \leq v$ the function $\pi_{B}^{N o ~}{ }^{P G}\left(p_{B}\right)$ is hump-shaped with a global maximum at $p_{B}=p_{A}^{p}-t$ if $p_{A}^{p}>3 t$ and at $p_{B}=\frac{p_{A}^{p}+t}{2}$ if $p_{A}^{p} \leq 3 t$.
Case $3-\operatorname{Firm} \mathbf{A}$ sets $p_{A}^{p}$ in the interval $\left.] t, v-t\right]$.
For $p_{B}<p_{A}^{p}-t$ the profit function is defined by Segment I of $\pi_{B}^{N o ~ P G}$, and thus linearly increasing in $p_{B}$.

For $p_{A}^{p}-t \leq p_{B} \leq p_{A}^{p}+t$ the profit function is defined by Segment II of $\pi_{B}^{N o ~}{ }^{P G}$, which is a parabola with the maximum at $\frac{p_{A}^{p}+t}{2}$. Whether this maximum is in the given interval for $p_{B}$ depends on $p_{A}^{p}$. If $p_{A}^{p}$ is larger than $3 t$ the maximum is located at $p_{B}$ lower than the left border of the interval $p_{A}^{p}-t$, and hence the profit function is decreasing for $p_{B} \in\left[p_{A}^{p}-t, p_{A}^{p}+t\right]$. If $p_{A}^{p} \leq 3 t$ the maximum is reachable, and the profit function is increasing for $p_{B} \in\left[p_{A}^{p}-t, \frac{p_{A}^{p}+t}{2}\right.$ [and decreasing for $\left.p_{B} \in \frac{p_{A}^{p}+t}{2}, p_{A}^{p}+t\right]$.

For any higher $p_{B}$ the profit is zero.
 imum at $p_{B}=\frac{p_{A}^{p}+t}{2}$ if $t<p_{A}^{p} \leq 3 t$ and at $p_{B}=p_{A}^{p}-t$ if $3 t<p_{A}^{p} \leq v-t$.
Case $4-$ Firm A sets $p_{A}^{p}$ in the interval $[0, t]$.
For $p_{B}<p_{A}^{p}+t$ the profit function is defined by Segment II of $\pi_{B}^{N o ~}{ }^{P G}$, which is a parabola with the maximum at $\frac{p_{A}^{p}+t}{2}$, which is within the given interval of $p_{B}$.

For any higher $p_{B}$ the profit is zero.
Hence, for $p_{A}^{p} \leq t$ the function $\pi_{B}^{N o ~}{ }^{P G}\left(p_{B}\right)$ is hump-shaped with a global maximum at $p_{B}=\frac{p_{A}^{p}+t}{2}$.

## Summing up all four cases.


3.) $\pi_{B}^{P G}\left(p_{B}\right)$ is hump-shaped.

The function $\pi_{B}^{P G}\left(p_{B}\right)$ consists of $\pi_{B}^{P G-A c t i v e}\left(p_{B}\right)$ for $p_{B}<p_{A}^{p}-m$ and of $\pi_{B}^{N o ~}{ }^{P G}\left(p_{B}\right)$ for any higher $p_{B}$. Since it is already proven that $\pi_{B}^{P G-A c t i v e}\left(p_{B}\right)$ and $\pi_{B}^{N o P G}\left(p_{B}\right)$ are both hump-shaped, while $\pi_{B}^{P G}\left(p_{B}\right)$ is continuous by Lemma 2 , it is sufficient to show that whenever $\pi_{B}^{P G-A c t i v e}\left(p_{B}\right)$ is hump-shaped on the interval $p_{B} \in\left[0, p_{A}^{p}-m[\right.$, the slope of $\pi_{B}^{N o{ }^{P G}}\left(p_{B}\right)$ is negative at $p_{A}^{p}-m$. Indeed, if $\pi_{B}^{P G-A c t i v e}\left(p_{B}\right)$ is humpshaped on the interval $p_{B} \in\left[0, p_{A}^{p}-m[\right.$, then, given that it has a global maximum
at $p_{B}=v-\frac{t+m}{2}$ as shown above, we must have

$$
\begin{equation*}
p_{A}^{p}-m>v-\frac{t+m}{2} . \tag{19}
\end{equation*}
$$

This yields $p_{A}^{p}>v-t$. Consequently, given the arguments in Case 1 and Case 2 in the proof above, $\pi_{B}^{N o{ }^{P G}\left(p_{B}\right) \text { is hump-shaped with a global maximum at } p_{B}=}$ $\min \left\{v-t, p_{A}^{p}-t\right\}$ if $p_{A}^{p}>3 t$ and at $p_{B}=\frac{p_{A}^{p}+t}{2}$ if $p_{A}^{p} \leq 3 t$. Let us show that both of these maximum points are below $p_{A}-m$ once (19) holds, in which case the slope of $\pi_{B}^{N o ~}{ }^{P G}\left(p_{B}\right)$ must be negative at $p_{B}=p_{A}^{p}-m$. For the first possible point, we have that $\min \left\{v-t, p_{A}^{p}-t\right\} \leq p_{A}^{p}-t$ is always below $p_{A}-m$ since $m<t$. Consider the second possible maximum point and assume by contradiction

$$
\frac{p_{A}^{p}+t}{2}>p_{A}^{p}-m \Leftrightarrow p_{A}^{p}<t+2 m .
$$

Given (19), we then have

$$
t+2 m>m+v-\frac{t+m}{2} \Leftrightarrow 3 t+3 m>2 v,
$$

which is a contradiction given that $m<t$ and $t<\frac{v}{3}$.
Thus, whenever $\pi_{B}^{P G-A c t i v e}\left(p_{B}\right)$ is hump-shaped on the interval $p_{B} \in\left[0, p_{A}^{p}-m[\right.$, the slope of $\pi_{B}^{N o{ }^{P G}}\left(p_{B}\right)$ is negative at $p_{B}=p_{A}^{p}-m$.
Consequently, $\pi_{B}^{P G}\left(p_{B}\right)$ is hump-shaped with respect to $p_{B}$ for any value of $p_{A}^{p}$.

## Proof of Proposition 2

Firm B's profit function $\pi_{B}^{P G}$ is given by $\pi_{B}^{P G-A c t i v e ~ f o r ~} p_{B}<p_{A}^{p}-m$, whereas for higher $p_{B}$ it is defined by $\pi_{B}^{N o P G}$. The proof is split in three different cases and uses the fact that $\pi_{B}^{P G}$ is continuous by Lemma 2, and is hump-shaped by Lemma 3. Hence, whenever either $\pi_{B}^{N o P G}$ or $\pi_{B}^{P G-A c t i v e ~ a r e ~ h u m p-s h a p e d ~ o n ~ t h e ~}$ corresponding interval, their peaks are the global maximum of $\pi_{B}^{P G}$. If none of them are hump-shaped on the corresponding interval, the maximum of $\pi_{B}^{P G}$ is located at the intersection of both functions, i.e. at $p_{B}=p_{A}^{p}-m$.
Case 1 - Firm A sets $p_{A}^{p} \geq v-\frac{t-m}{2}$.
Then, function $\pi_{B}^{P G-A c t i v e}\left(p_{B}\right)$ is hump-shaped on the corresponding interval, i.e., its peak, which is located at $p_{B}=v-\frac{t+m}{2}$ (see proof of Lemma 3) is reachable within $p_{B} \in\left[0, p_{A}^{p}-m\left[\right.\right.$. Indeed, $v-\frac{t+m}{2}<p_{A}^{p}-m$ is equivalent to $p_{A}^{p}>v-\frac{t-m}{2}$ which is fulfilled in the given interval of $p_{A}^{p}$.

Hence, the best response to $p_{A}^{p} \geq v-\frac{t-m}{2}$ is $p_{B}=v-\frac{t+m}{2}$.

Case $2-\operatorname{Firm} \mathbf{A}$ sets $p_{A}^{p}$ in the interval $\left.] t+2 m, v-\frac{t-m}{2}\right]$.
The maximum of $\pi_{B}^{P G-\operatorname{Active}}\left(p_{B}\right)$ is not reachable on the corresponding interval $p_{B} \in\left[0, p_{A}^{p}-m\left[\right.\right.$. Indeed, $v-\frac{t+m}{2} \geq p_{A}^{p}-m$ is equivalent to $p_{A}^{p} \leq v-\frac{t-m}{2}$ which is fulfilled in the given interval of $p_{A}^{p}$. Hence, $\pi_{B}^{P G-\operatorname{Active}}\left(p_{B}\right)$ monotonically increases in $p_{B}$ for $p_{B} \in\left[0, p_{A}^{p}-m\right]$ (given that it is hump-shaped over the whole interval of $p_{B}$, see proof of Lemma 3).

The maximum of $\pi_{B}^{N o P G}$ is also not reachable on the corresponding interval $p_{B} \in\left[p_{A}^{p}-m, \infty\left[\right.\right.$. Indeed, given that $\left.p_{A}^{p} \in\right] t, v[$ in the considered case, the maximum is located either at $p_{B}=\frac{p_{A}^{p}+t}{2}$ or $p_{B}=p_{A}^{p}-t$ (see Cases 2 and 3 in the proof of Lemma 3). Both values are smaller than $p_{A}^{p}-m$, since $\frac{p_{A}^{p}+t}{2}<p_{A}^{p}-m$ is equivalent to $p_{A}^{p}>t+2 m$ and $p_{A}^{p}-t<p_{A}^{p}-m$ is equivalent to $m<t$. The former is true for the given interval of $p_{A}^{p}$ and the latter is true due to the parametric assumption $m<t$. Since the maximum of $\pi_{B}^{N o P G}$ is to the left of $p_{A}^{p}-m$ and $\pi_{B}^{N o{ }^{P G}}$ is hump-shaped (see proof of Lemma 3), $\pi_{B}^{P G}$ has a decreasing slope to the right of $p_{A}^{p}-m$, where it is defined by $\pi_{B}^{N o P G}$. Since to the left of $p_{A}^{p}-m$ the function $\pi_{B}^{P G}$ is defined by $\pi_{B}^{P G-A c t i v e}$, which monotonically increases in $p_{B}$ for $p_{B} \in\left[0, p_{A}^{p}-m\right]$ in the considered case as shown above, the maximum of $\pi_{B}^{P G}$ is located at $p_{B}=p_{A}^{p}-m$.

Hence, the best response to $\left.\left.p_{A}^{p} \in\right] t+2 m, v-\frac{t-m}{2}\right]$ is $p_{B}=p_{A}^{p}-m$.
Case 3 - Firm A sets $p_{A}^{p}$ in the interval $[0, t+2 m]$.

Since $p_{A}^{p}<3 t$ (due to $m<t$ ), the maximum of $\pi_{B}^{N o P G}$ is reachable in the relevant interval of $p_{B}$ with $p_{B}=\frac{p_{A}^{p}+t}{2}$ (see Cases 2-4 in the proof of Lemma 3).

Hence, the best response to $p_{A}^{p} \in[0, t+2 m]$ is $p_{B}=\frac{p_{A}^{p}+t}{2}$.

## Best response function - Summary of the three cases.

In summary of all three cases the reaction function is given by

$$
R_{B}^{P G}\left(p_{A}^{p}\right)= \begin{cases}v-\frac{t+m}{2} & \text { if } \quad p_{A}^{p}>v-\frac{t-m}{2}, \\ p_{A}^{p}-m & \text { if } \quad t+2 m<p_{A}^{p} \leq v-\frac{t-m}{2}, \\ \frac{p_{A}^{p}+t}{2} & \text { if } \quad p_{A}^{p} \leq t+2 m .\end{cases}
$$

## Proof of Proposition 3

From (13) one can see that Firm A, in order to maximize its profits, chooses either $p_{A}^{p}$ from the "collusive" interval $\left.] v-\frac{t-m}{2}, \infty\right]$ or from the "competitive" interval $[0, t+2 m]$. The reason is, that the profit of Firm A, if $p_{A}^{p}$ is set in the interval $\left.] t+2 m, v-\frac{t-m}{2}\right]$, is strictly dominated by the constant profit level which results when $p_{A}^{p}$ is in the collusive interval.
In the competitive interval Firm A's profit function is defined by a parabola which reaches a maximum of $\frac{9}{16} t$ at $p_{A}^{p}=\frac{3}{2} t$. Now, we consider two cases depending on whether this maximum is reachable within the corresponding interval $p_{A}^{p} \in$ $[0, t+2 m]$.

Case 1: $\frac{3}{2} t \leq t+2 m$.
In this case, the maximum of the parabola is reached in the competitive interval. In this case, Firm A finds it only optimal to set a collusive $p_{A}^{p}$ if the constant collusive profit level is at least $\frac{9}{16} t$ :

$$
\begin{gathered}
\frac{9}{16} t \leq\left[v-\frac{t-m}{2}\right] \cdot\left[\frac{1}{2}-\frac{m}{2 t}\right] \\
\Leftrightarrow \frac{9 t^{2}}{16 t}-\left[\frac{8 m t-8 m v+8 t v-4 m^{2}-4 t^{2}}{16 t}\right] \leq 0 \\
\Leftrightarrow m^{2}-2 m(t-v)-2 t v+\frac{13}{4} t^{2} \leq 0 .
\end{gathered}
$$

The left-hand side is a parabola with respect to $m$. It has a global minimum, and only one positive root equal to $\frac{1}{2} \sqrt{4 v^{2}-9 t^{2}}-v+t$. Hence, the condition above is met whenever

$$
m \leq \frac{1}{2} \sqrt{4 v^{2}-9 t^{2}}-v+t
$$

In this case, Firm A's profit is maximized at any collusive price, i.e. at any $p_{A}^{p} \geq v-\frac{t-m}{2}$. Otherwise, the profit is maximized at the competitive price $p_{A}^{p}=\frac{3}{2} t$.

Case 2: $\frac{3}{2} t>t+2 m$.
Here, the maximum of Firm A's profit is to the right of the competitive interval. Consequently, the slope of the profit function is positive at $p_{A}^{p}=t+2 m$. Since the profit function is monotonically increasing for $p_{A}^{p}>t+2 m$, while being continuous over the whole domain (see (13)), a collusive $p_{A}^{p}$ (i.e., any $p_{A}^{p}$ above $v-\frac{t-m}{2}$ ) yields the highest profit.

## Summary of both cases.

Firm A's profit is maximized at any $p_{A}^{p} \geq v-\frac{t-m}{2}$ whenever

$$
m \leq \frac{1}{2} \sqrt{4 v^{2}-9 t^{2}}-v+t \vee\left(\frac{3}{2} t>t+2 m \Leftrightarrow m<\frac{t}{4}\right) .
$$

Since $\frac{t}{4}<\frac{1}{2} \sqrt{4 v^{2}-9 t^{2}}-v+t$ for $t \leq \frac{v}{3}$, the condition is equivalent to:

$$
m \leq \frac{1}{2} \sqrt{4 v^{2}-9 t^{2}}-v+t
$$

Otherwise, Firm A's profit is maximized under competitive price $p_{A}^{p}=\frac{3}{2} t$.

## Proof of Proposition 4

(a) In this case, by Propositions 2 and 3 it follows that $p_{A}^{p} \geq v-\frac{t-m}{2}$ while $p_{B}=$ $v-\frac{t+m}{2}$. Hence, $p_{A}^{p}-p_{B} \geq v-\frac{t-m}{2}-\left(v-\frac{t+m}{2}\right)=m$. Consequently, the guarantee is activated and the effective price of Firm A is $p_{B}+m=v-\frac{t-m}{2}$.
(b) From Proposition 3 it follows that $p_{A}^{p}=\frac{3}{2} t$ in the considered case. Consider the best response of Firm B described in Proposition 2. Let us show that the condition for a competitive reaction from Firm B, i.e. $p_{A}^{p} \leq t+2 m$, is fulfilled. Given that $m \geq \phi$, the sufficient condition for this is $p_{A}^{p} \leq t+2 \phi$, which is equivalent to $\phi \geq \frac{1}{4} t$. One can show that this always holds for $t \leq \frac{v}{3}$. Consequently, by Proposition 2, $p_{B}=\frac{p_{A}^{p}+t}{2}=\frac{5}{4} t$. Then we have, $p_{A}^{p}-p_{B}=\frac{3}{2} t-\frac{5}{4} t=\frac{1}{4} t \leq \phi \leq m$. Consequently, the effective price of Firm A is equal to its posted price: $p_{A}=p_{A}^{p}=\frac{3}{2} t$.

## Proof that Firm B's guarantee does not affect equilibrium prices

It is to show, that in comparison to a game where only Firm A offers a price guarantee with an arbitrary non-negative markup on the competitors' price (1PG-game), a game in which both firms offer such a price guarantee does not change equilibrium prices (2PG-game). In order to show this, it is important to recall that in any equilibrium of the $1 P G$-game Firm B undercuts Firm A.

In the $2 P G$-game Firm B is restricted in overbidding the price of Firm A by more than $m$. This restriction is binding for the best response function of Firm B in the $1 P G$-game only off the equilibrium path. Hence, it is sufficient to show
that Firm A's optimal strategy does not change between the 1PG-game and the 2PG-game.

Since Firm A's profit is weakly increasing in the price of Firm B, all posted prices of Firm A which lead to a less drastic overbidding of Firm B in the $2 P G$ game compared to the $1 P G$-game are becoming less attractive. However, all other profits, including the profit of the equilibrium action of the $1 P G$-game, are identical to the $2 P G$-game. Consequently, Firm A posts the same prices in equilibrium in both games, and Firm B reacts by undercutting to the same extent. ${ }^{16}$

[^10]
## C English Instructions (translated)

The following pages contain a translated version of the instructions. Curley brackets indicate the treatment variation of the instructions. Naturally subjects only saw their treatment variation.

## Instructions - Experiment Rules

## Welcome to the experiment!

In this experiment you can earn money. How much you will earn, depends on your decisions and on the decisions of other participants. Irrespectively of the decisions during the experiment, you will additionally receive an amount of 4.00 Euro for your appearance as well as another 4.00 Euro for the completion of a questionnaire at the end of the experiment.

During the experiment the currency "Experimental Currency Units" (ECU) is used. At the end of the experiment all ECU amounts, which you earned during the experiment, are converted into Euro and are paid to you in cash. The exchange rate for 14,000 ECU is 1 Euro.

All decisions during the experiment are anonymous. The payments at the end of the experiment are treated confidentially.

From now on, please do not communicate with other participants. If you have any questions, now or during the experiment, please raise your hand. We will come to you and answer your question. Moreover, during the experiment we ask you to switch off your mobile phone. Documents (books, lecture script, etc.), which are not related to the experiment, may not be used during the experiment. In case of offense against these rules we may exclude you from the experiment and all payments.

## Instructions - General Part

At the beginning of the experiment each participant is assigned to a role, either Competitor A or Competitor B. This assignment remains constant during the whole experiment and each participant is informed individually about his role on the screen.

Competitor A and Competitor B sell arbitrarily divisible goods in a market. Each competitor can produce up to 100 units. The production will create no costs. $\{\mathrm{T} 2+\mathrm{T} 3$ : Competitor A is bound to a price guarantee, which guarantees, that his final price will not exceed the price of Competitor B by more than $\{\mathrm{T} 2: 2 ; \mathrm{T} 3: 33\}$ ECU.\}

The sales prices are determined in the following order:

1. Competitor A sets his $\{\mathrm{T} 2+\mathrm{T} 3$ : posted $\}$ price first.
2. Competitor B sees the $\{\mathrm{T} 2+\mathrm{T} 3$ : posted $\}$ price of Competitor A and sets his price. $\{\mathrm{T} 2+\mathrm{T} 3:$ This price is his final price.
3. The final price of Competitor A is determined:

- If Competitor B sets a price which is at least \{T2: 2; T3: 33\} ECU lower than the posted price of Competitor A , the price guarantee of Competitor A is activated. The final price of Competitor A equals the price of Competitor B plus a markup of \{T2: 2; T3: 33\} ECU.
- If Competitor B sets a higher price than Competitor A , or undercuts his price by less than $\{\mathrm{T} 2: 2 ; \mathrm{T} 3: 33\}$ ECU, the price guarantee of Competitor A is not activated. The final price of Competitor A equals his posted price.\}

The sales volume of each competitor depend on the $\{\mathrm{T} 2+\mathrm{T} 3$ : final $\}$ price of Competitor $\mathrm{A}\left(p_{A}\right)$ and the $\{\mathrm{T} 2+\mathrm{T} 3$ : final $\}$ price of Competitor $\mathrm{B}\left(p_{B}\right)$. In the experiment they are calculated by the computer as follows:

Sales Volume Competitor $\mathrm{A}= \begin{cases}\frac{p_{B}-p_{A}+35}{70} \cdot 100 & \text { if } p_{A}+p_{B}<365 \\ \frac{200-p_{A}}{35} \cdot 100 & \text { else. }\end{cases}$

$$
\text { Sales Volume Competitor } \mathrm{B}= \begin{cases}\frac{p_{A}-p_{B}+35}{70} \cdot 100 & \text { if } p_{A}+p_{B}<365 \\ \frac{200-p_{B}}{35} \cdot 100 & \text { else } .\end{cases}
$$

A sales volume cannot be less than 0 units or greater than 100 units. If the formulas above generate a sales volume smaller than 0 , the sales volume is set to 0 . If the formulas above generate a sales volume higher than 100, the sales volume is set to 100 .

The sales volumes and $\{\mathrm{T} 2+\mathrm{T} 3$ : final $\}$ prices lead to the competitors' profits:

Profit of a Competitor $($ in ECU $)=$ His $\{T 2+$ T3:Final $\}$ Price $\cdot$ His Sales Vol-
ume

## Instructions - Scenario Calculator

At the beginning of the experiment a scenario calculator will be provided. With the help of this calculator you can calculate the sales volumes and profits of both competitors for any price combination. The scenario calculator is available during the whole experiment.

The scenario calculator uses the $\{\mathrm{T} 2+\mathrm{T} 3$ : posted\} price of Competitor A and the $\{T 2+T 3$ : final $\}$ price of Competitor $B$ as inputs:

- You can enter any value between 0 and 200 for each competitor.
- The input is either entered in an input field or by using a slider-bar.
- Inputs via the input field can have any number of decimal places and must be confirmed with the button next to it.

Note: Consider in your simulations that Competitor B makes his decision after Competitor A and therefore knows Competitor A's $\{\mathrm{T} 2+\mathrm{T} 3:$ posted $\}$ price.

After both prices are entered, the scenario calculator displays:

- \{T2+T3: whether the price guarantee is activated:
- This is the case when the final price of Competitor B is at least $\{\mathrm{T} 2: 2 ; \mathrm{T} 3$ : $33\}$ ECU lower than the posted price of Competitor A.
- If the price guarantee is activated, the following applies: Final Price Competitor A $=$ Final Price Competitor $\mathrm{B}+\{\mathrm{T} 2: 2 ; \mathrm{T} 3: 33\}$ ECU $\}$
- the sales amounts of every competitor:
- \{T2+T3: The calculation is based on the final prices. $\}$
- Sales volumes can vary between 0 and 100 units.
- \{T2+T3: the final prices of both competitors:
- The final price of Competitor A equals his posted price, if the price guarantee is not activated, or equals the price of Competitor $B$ plus $\{T 2: 2 ; \mathrm{T} 3: 33\}$ ECU if the price guarantee is activated.
- The final price of Competitor B is always his entered price because Competitor B is not restricted by a price guarantee.\}
- the profits of both competitors:
- The profits of both competitors are the respective $\{\mathrm{T} 2+\mathrm{T} 3$ : final $\}$ price multiplied by the respective sales volume: Profit $=\{T 2+T 3$ : Final $\}$ Price $\cdot$ Sales Volume [in ECU $]$

Familiarize yourself with the calculations and use the scenario calculator as often as you like. Your entries in the scenario calculator will not affect your payoff at the end of the experiment.

## Instructions - Decision Stage

In this stage of the experiment you interact with other competitors which will be matched to you. The interaction takes place in the setting you already know from the scenario calculator.

The decision stage consists of 15 independent periods. The course in each period is identical. However, the competitor matched to you differs from period to period. The matching procedure is as follows:

- Your competitor will be randomly determined each period. However, it is assured that you are never matched with the same competitor in two consecutive periods.
- Your competitor will differ from you in the assigned role, in other words a Competitor A always competes with a Competitor B.

Timing within a period:

1. At the beginning of every period, Competitor A sets his $\{\mathrm{T} 2+\mathrm{T} 3$ : posted $\}$ price. Meanwhile, Competitor B sees a waiting screen.
2. After Competitor A has set his $\{\mathrm{T} 2+\mathrm{T} 3$ : posted $\}$ price, Competitor B sees it and sets his $\{\mathrm{T} 2+\mathrm{T} 3$ : final $\}$ price.
Meanwhile, Competitor A sees a waiting screen.
3. Finally, the computer calculates $\{\mathrm{T} 2+\mathrm{T} 3$ : the final price of Competitor A and $\}$ the sales volumes. These are displayed, in addition to the profits of both competitors, in the period summary.

After completing 15 periods, your profits of all periods will be displayed and summed up in a final summary. The total sum is then converted into Euro to the exact cent and paid to you in cash at the end of the experiment. The payment additionally includes a premium of 4.00 Euro for showing up and another premium of 4.00 Euro for completing the questionnaire. The exchange rate is $€ 1$ per ECU 14,000.

Once the experiment ended, a short questionnaire appears on your screen. Please fill out this questionnaire, while the experimenters prepare your payoff. Afterwards, you will be called by your cabin number for your payment.

## Thank you for your participation!

## D German Instructions (original)

## Instruktionen - Allgemeine Experimentregeln

## Herzlich Willkommen zum Experiment!

In diesem Experiment können Sie Geld verdienen. Wie viel Sie verdienen werden, hängt von Ihren Entscheidungen beziehungsweise den Entscheidungen anderer Experimentteilnehmer ab. Unabhängig von den Entscheidungen während des Experimentes erhalten Sie zusätzlich 4,00 Euro für Ihr Erscheinen sowie weitere 4,00 Euro für das Ausfüllen eines Fragebogens am Ende des Experimentes.

Während des Experimentes wird die Währung ECU (Experimental Currency Units) verwendet. Am Ende des Experimentes werden alle ECU-Beträge, welche Sie im Laufe des Experimentes verdienen, in Euro umgerechnet und Ihnen ausgezahlt. Der Umrechnungskurs beträgt 1 Euro für 14000 ECU.

Alle Entscheidungen, die Sie während des Experimentes treffen, sind anonym. Ihre Auszahlung am Ende des Experimentes wird vertraulich behandelt.

Bitte kommunizieren Sie ab sofort nicht mehr mit den anderen Teilnehmern. Falls Sie jetzt oder während des Experimentes eine Frage haben, heben Sie bitte die Hand. Wir werden dann zu Ihnen kommen und Ihre Frage beantworten. Während des Experimentes bitten wir Sie außerdem, Ihr Mobiltelefon auszuschalten. Unterlagen (Bücher, Vorlesungsskripte, etc.), die nichts mit dem Experiment zu tun haben, dürfen während des Experimentes nicht verwendet werden. Bei Verstößen gegen diese Regeln können wir Sie vom Experiment und allen Auszahlungen ausschließen.

## Instruktionen - Allgemeiner Teil

Zu Beginn des Experimentes wird jedem Experimentteilnehmer eine Rolle zugeteilt, entweder Wettbewerber $A$ oder Wettbewerber B. Diese Zuteilung bleibt das gesamte Experiment über bestehen und wird jedem Experimentteilnehmer individuell auf dem Bildschirm mitgeteilt.

Wettbewerber A und Wettbewerber B verkaufen am Markt beliebig teilbare Güter. Sie können jeweils bis zu 100 Einheiten produzieren. Bei der Produktion fallen keine Kosten an. $\{\mathrm{T} 2+\mathrm{T} 3$ : Beim Verkauf ist Wettbewerber A an eine Preisgarantie gebunden, welche garantiert, dass sein endgültiger Preis den Preis von Wettbewerber B um nicht mehr als \{T2: 2; T3: 33$\}$ ECU überschreitet.\}

Die Festlegung der Verkaufspreise geschieht in folgender Reihenfolge:

1. Wettbewerber A legt zuerst seinen $\{\mathrm{T} 2+\mathrm{T} 3$ : vorläufigen $\}$ Preis fest.
2. Wettbewerber B sieht den $\{\mathrm{T} 2+\mathrm{T} 3$ : vorläufigen $\}$ Preis von Wettbewerber A und legt seinen Preis fest. $\{\mathrm{T} 2+\mathrm{T} 3$ : Dieser Preis ist zugleich sein endgültiger Preis.
3. Der endgültige Preis von Wettbewerber A wird bestimmt:

- Sollte Wettbewerber B den vorläufigen Preis von Wettbewerber A um mindestens \{T2: 2; T3: 33\} ECU unterbieten, so wird dessen Preisgarantie aktiviert. Der endgültige Preis von Wettbewerber A entspricht dann dem Preis von Wettbewerber B zuzüglich \{T2: 2; T3: 33\} ECU.
- Sollte Wettbewerber B einen höheren Preis festlegen als Wettbewerber A oder dessen vorläufigen Preis um weniger als \{T2: 2; T3: 33\} ECU unterbieten, so wird die Preisgarantie von Wettbewerber A nicht aktiviert. Der endgültige Preis von Wettbewerber A entspricht dann seinem vorläufigen Preis.\}

Die Absatzmengen der Wettbewerber hängen von dem $\{\mathrm{T} 2+\mathrm{T} 3$ : endgültigen $\}$ Preis von Wettbewerber $\mathrm{A}\left(p_{A}\right)$ und dem $\{\mathrm{T} 2+\mathrm{T} 3$ : endgültigen $\}$ Preis von Wettbewerber B $\left(p_{B}\right)$ ab. Sie werden im Experiment durch den Computer wie folgt berechnet:

Absatzmenge Wettbewerber $\mathrm{A}= \begin{cases}\frac{p_{B}-p_{A}+35}{70} \cdot 100 & \text { falls } p_{A}+p_{B}<365 \\ \frac{200-p_{A}}{35} \cdot 100 & \text { sonst. }\end{cases}$
Absatzmenge Wettbewerber $\mathrm{B}= \begin{cases}\frac{p_{A}-p_{B}+35}{70} \cdot 100 & \text { falls } p_{A}+p_{B}<365 \\ \frac{200-p_{B}}{35} \cdot 100 & \text { sonst. }\end{cases}$

Eine Absatzmenge kann niemals kleiner als 0 Einheiten oder größer als 100 Einheiten sein. Falls sich aus den obigen Formeln eine kleinere Absatzmenge als 0 Einheiten ergibt, so wird die Absatzmenge auf 0 Einheiten gesetzt. Falls sich aus den obigen Formeln eine Absatzmenge von mehr als 100 Einheiten ergibt, so wird die Absatzmenge auf 100 Einheiten gesetzt.

Aus den Absatzmengen und den $\{T 2+\mathrm{T} 3$ : endgültigen $\}$ Preisen ergeben sich die Gewinne der Wettbewerber:

Gewinn eines Wettbewerbers (in ECU) $=$ Sein $\{T 2+T 3:$ endgültiger $\}$ Preis $\cdot$ seine Absatzmenge

## Instruktionen - Szenario-Rechner

Zu Beginn des Experimentes wird Ihnen ein Szenario-Rechner bereitgestellt. Mit Hilfe dieses Rechners können Sie für beliebige Preiskombinationen die Absatzmengen und Gewinne beider Wettbewerber berechnen. Der Szenario-Rechner steht Ihnen während des gesamten Experimentes zur Verfügung.

Als Eingabe benötigt der Szenario-Rechner den \{T2 + T3: vorläufigen\} Preis von Wettbewerber A und den \{T2 + T3: endgültigen\} Preis von Wettbewerber B:

- Sie können für jeden Wettbewerber beliebige Werte zwischen 0 und 200 eingeben.
- Die Eingabe erfolgt entweder über eine Schiebeleiste oder ein Eingabefeld.
- Eingaben über das Eingabefeld können beliebig viele Nachkommastellen haben und müssen mit dem nebenstehenden Knopf bestätigt werden.

Hinweis: Berücksichtigen Sie bei Ihren Simulationen, dass Wettbewerber B nach Wettbewerber A entscheidet und zum Zeitpunkt seiner Entscheidung dessen $\{T 2+\mathrm{T} 3$ : vorläufigen\} Preis kennt.

Nachdem beide Preise eingegeben sind, wird angezeigt,

- \{T2+T3: ob die Preisgarantie aktiviert wird:
- Dies ist immer der Fall, wenn der endgültige Preis von Wettbewerber B den vorläufigen Preis von Wettbewerber A um mehr als \{T2: 2; T3: 33\} ECU unterschreitet.
- Sofern die Preisgarantie aktiviert wird, gilt: Endgültiger Preis Wettbewerber A = Endgültiger Preis Wettbewerber B

$$
+\{\mathrm{T} 2: 2 ; \mathrm{T} 3: 33\} \mathrm{ECU}\}
$$

- welche Absatzmenge jeder Wettbewerber hat:
- \{T2+T3: Die Berechnung erfolgt auf Basis der endgültigen Preise. $\}$
- Die Absatzmengen können zwischen 0 und 100 Einheiten betragen.
- \{T2+T3: wie die endgültigen Preise beider Wettbewerber lauten:
- Der endgültige Preis von Wettbewerber A ist sein vorläufiger Preis, falls die Preisgarantie nicht aktiviert wird, beziehungsweise der Preis von Wettbewerber B zuzüglich \{T2: 2; T3: 33\} ECU, falls die Preisgarantie aktiviert wird.
- Der endgültige Preis von Wettbewerber B ist immer sein eingegebener Preis, da Wettbewerber B an keine Preisgarantie gebunden ist.\}
- wie hoch die Gewinne beider Wettbewerber sind:
- Die Gewinne beider Wettbewerber ergeben sich aus der Multiplikation des jeweiligen $\{\mathrm{T} 2+\mathrm{T} 3$ : endgültigen $\}$ Preises und der jeweiligen Absatzmenge: Gewinn $=\{\mathrm{T} 2+\mathrm{T} 3:$ Endgültiger $\}$ Preis $\cdot$ Absatzmenge [in ECU]

Machen Sie sich mit den Berechnungen vertraut und nutzen Sie den SzenarioRechner. Sie können ihn beliebig oft verwenden. Ihre Eingaben im SzenarioRechner haben keinen Einfluss auf Ihre Auszahlungen am Ende des Experimentes.

## Instruktionen - Entscheidungsstufe

Nachdem Sie sich nun mit dem Szenario-Rechner vertraut gemacht haben, interagieren Sie in diesem Teil des Experimentes mit Ihnen zugeteilten anderen Wettbewerbern in dem Ihnen aus dem Szenario-Rechner bekannten Setting.

Die Entscheidungsstufe besteht aus insgesamt 15 Runden, wobei jede Runde vom Ablauf identisch ist. Von Runde zu Runde unterschiedlich ist, mit welchem Wettbewerber sie konkurrieren. Hierbei gilt:

- Ihr Wettbewerber wird jede Runde zufällig neu bestimmt. Es wird sichergestellt, dass Sie in zwei aufeinanderfolgenden Runden niemals demselben Wettbewerber zugeordnet sind.
- Ihr Wettbewerber hat immer eine von Ihnen unterschiedliche Rolle, sodass immer ein Wettbewerber A mit einem Wettbewerber B konkurriert.


## Ablauf einer Runde:

1. Zu Beginn jeder Runde setzt Wettbewerber A zuerst seinen $\{\mathrm{T} 2+\mathrm{T} 3$ : vorläufigen $\}$ Preis.
Wettbewerber B sieht währenddessen einen Wartebildschirm.
2. Nachdem Wettbewerber A seinen $\{\mathrm{T} 2+\mathrm{T} 3$ : vorläufigen $\}$ Preis gesetzt hat, wird dieser Wettbewerber B angezeigt, welcher nun seinen $\{T 2+\mathrm{T} 3$ : endgültigen\} Preis setzt.

Wettbewerber A sieht währenddessen einen Wartebildschirm.
3. Anschließend werden vom Computer $\{\mathrm{T} 2+\mathrm{T} 3$ : der endgültige Preis von Wettbewerber A und\} die Absatzmengen bestimmt. Diese werden zusammen mit den Gewinnen beider Wettbewerber in der abschließenden Rundenzusammenfassung angezeigt.

Nach Abschluss der 15 Runden wird eine Auflistung Ihrer sämtlichen Rundengewinne angezeigt und aufsummiert. Die Gesamtsumme wird am Ende des Experimentes zum Umrechnungskurs von $1 €$ pro 14000 ECU auf den Cent genau umgerechnet und zuzüglich zu der Prämie von $4 €$ für das Erscheinen und der $4 €$ - Prämie für das Ausfüllen des Fragebogens Ihnen in bar ausbezahlt.

Nach der Rundenübersicht erscheint der kurze Fragebogen auf dem Bildschirm. Bitte füllen Sie diesen aus, während die Experimentatoren Ihre Auszahlungen vorbereiten. Im Anschluss werden Sie anhand Ihrer Kabinennummer zur Auszahlung aufgerufen und das Experiment ist beendet.

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[^1]:    ${ }^{1}$ For exact condition terms of the guarantee see Shell Deutschland Oil GmbH (2016).
    ${ }^{2}$ See Bundeskartellamt $(2014,2015)$ for details.
    ${ }^{3}$ See Bundeskartellamt (2011).

[^2]:    ${ }^{4}$ Hviid (2010) provides a detailed survey.

[^3]:    ${ }^{5}$ Corts (1996) finds, for a special case where informed customers have the less elastic demand and firms can offer price-beating guarantees, that prices fall for both groups.
    ${ }^{6}$ Mao (2005) comes to a similar conclusion when focusing on the costs of returning of ex ante uninformed customers to stores that provide price-matching.

[^4]:    ${ }^{7}$ A markup smaller than zero would be an exotic form of a price-beating guarantee, which is activated if the competitors price is not sufficiently higher than the price of the guarantee issuing firm. For a discussion of price-beating guarantees see the previous literature section and the references therein.
    ${ }^{8}$ For simplification, the analysis is restricted to cases where $m<t$, since otherwise the market share would be zero for Firm A whenever the price guarantee is active.

[^5]:    ${ }^{9}$ This is technically equivalent to offering a guarantee with an infinitely high markup on the competitor's price, which therefore cannot be activated.

[^6]:    ${ }^{10}$ As proven in Appendix B, prices in equilibrium are identical if additionally Firm B provides a price-matching guarantee with a non-negative markup.

[^7]:    ${ }^{11}$ The instruction in English language can be found in Appendix C. The original German instructions are available upon request.
    ${ }^{12}$ Imposing an upper bound for posted prices was necessary, due to a technical reason: Subjects entered prices via a slider bar, which requires a lower and an upper bound. Thus, we have chosen to set the upper bound to the prohibitive price level of 200 , since all posted prices higher than 200 are at least weakly dominated. Thus, this restriction does not affect the equilibrium point predictions stated in Table 2.

[^8]:    ${ }^{13}$ Perfect collusion is reached when both firms have an effective price of 182.50 . This is the only price combination which extracts the full rent of the indifferent consumer and minimizes overall transportation costs at the same time.
    ${ }^{14}$ The price guarantee in T3 was activated in 17 observations. In none of these cases Firm B undercut by more than 36 .

[^9]:    ${ }^{15}$ The numbers stated refer to the overall treatment specific period effect. For T1 and T3 the effect is the sum of the treatment unspecific period effect and the interaction effect.

[^10]:    ${ }^{16}$ Note that this reasoning holds only for the sequential game with otherwise symmetric firms considered in the theory section, as it relies on Firm B setting the lower price in the equilibrium of the $1 P G$ Game. This reasoning may not hold if firms are asymmetric, for example if Firm B has a disadvantageous cost structure.

