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SEQUENTIAL VERSUS BUNDLE AUCTIONS FOR RECURRING PROCUREMENT

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Sequential versus Bundle Auctions for Recurring Procurement*

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Abstract

We compare sequential and bundle procurement auctions in a framework of successive procurement situations, where current success positively or negatively affects future market opportunities. We find that in bundle auctions procurement cost is lower and less risky than in sequential standard auctions, but still higher than in the optimal sequential auction. Only a sequential second price auction leads to the efficient outcome.

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1 Introduction

Every year governments and private sector firms place numerous orders by running procurement auctions. For instance, governments of Western nations procure about 10 % of national product annually.[1] For the private sector, Burt, Norquist, and Anklesaria (1990) report that in the United States the fraction of purchased inputs increased from 20 % to 56 % of the selling price of finished goods during the last 50 years. A substantial fraction of this procurement takes place by competitive bidding.

Usually, over time firms participate in more than one procurement auction. In this case, a firm's cost of providing one object is quite likely not independent of the allocation of other production rights. Rather, one often observes that the production right for one object guarantees the firm a comparative cost advantage in a follow–up procurement situation. This may be due to scale effects or due to technology improvements from earlier production processes. There may also be situations where current success decreases future opportunities, for example if a firm already produces close to its capacity bound. Rational firms will account for the expected comparative cost (dis)advantage in future auctions in their price offers today.

In particular if several consecutive projects are complementary (as for example, building a facility and the service contract for this facility), a government may consider to procure the contracts as a bundle instead of running a sequence of separate auctions. The reason is that if the contracts are complements bids are presumably more aggressive since firms bidding for the bundle base their offers on much lower expected total production cost.[2] Note, however, that procurement of the bundle has to take place before production of the first object is due. Thus, after the bundle auction firms may observe additional cost relevant information for parts of the contracted issues. Moreover, small firms (that are typically excluded in the bundle auction[3]) may still be in the position to compete for parts of the bundle. All this suggests that aftermarket trade should play an important role after a bundle auction. As a matter of fact, subcontracting is a common phenomenon in many procurement situations where contracts oblige the successful firm to provide a variety of

[1] Cf. McAfee and McMillan (1987).

[2] Also, the literature on optimal auctions suggests that bundling may increase the auctioneer's revenue, even in absence of synergies. Cf. Palfrey (1983), Armstrong (2000), and Avery and Hendershott (2000).

[3] Usually firms have to prove their ability to carry out the project in advance and are only eligible to bid in an auction for a contract if they qualified.

goods and services.[4] In contrast, in separate auctions each object can be procured relatively close to its actual delivery date, when the details of the corresponding contract can be well defined. Moreover, all firms that are qualified to deliver a single object can participate in the respective auction.

The model analyzed in this paper is a stylized version of the procurement setting described above. We have two objects that may be complements or substitutes.[5] Costs for the first object are observed at an earlier point in time (at stage one) than costs for the second object (at stage two). Several multiproduct firms are eligible to bid for both objects, while additional singleproduct firms can only compete for the second one. Delivery of the first (second) object is due at the end of period one (two). Prior to production, each object can be reallocated among the firms (this implies that the allocation of the first object is irreversible at stage two).

In this framework we compare a sequential and a bundle auction. We find that the sequential auction allocates efficiently[6], whereas the bundle auction achieves lower procurement cost. The reason is that the winner of the bundle auction anticipates relatively high profits from subcontracting at stage two which are competed away in the auction at stage one. A comparison with the *optimal sequential auction* reveals that the bundle auction actually implements almost the optimal allocation rule (that minimizes overall procurement cost). In both mechanisms the stage two–allocation rule favors the stage one–winner and discriminates against the losers in order to increase competition at stage one. Moreover we show that the price is less risky in the bundle auction, since here the seller passes the price risk associated with the final allocation of the second object to the bidders. For complementary goods (where bundle auctions are a much more natural choice than for substitutes), our results apply to both, first and second price auctions.

Let us finally discuss the literature related to the topics we analyze. Von der Fehr and Riis (1998), Jeitschko and Wolfstetter (2002), and Jofre-Bonet and Pesendorfer (2005) analyze sequential auctions with the same timing of information revelation. Von der Fehr and Riis study how future market opportunities affect the bidding behavior (and thereby the equilibrium price sequence) in sequential second–price auctions auctions, while Jeitschko

[4]Kamien (1989), as well as Gale, Hausch, and Stegeman (2000) give a variety of examples for horizontal subcontracting.

[5]Although most of our results apply also to substitutes, complementary goods are the more natural scenario to consider bundle auctions.

[6]The efficiency result holds only if the sequential auction is second price.

and Wolfstetter, as well as Jofre-Bonet and Pesendorfer compare sequential first– and second–price auctions in the presence of stochastic economies or diseconomies of scale. They find that second price auctions yield lower (higher) procurement cost if the objects are complements (substitutes). Their results imply that the revenue ranking between bundle and sequential auctions extends to first price auctions in case of complementary goods, but not necessarily if goods are substitutes.

A seminal contribution to the question of bundling versus separate sales is Palfrey (1983). He finds that, even without synergies, a monopolist often prefers bundling as compared to selling the objects in independent auctions. There are also a number of recent theoretical analyses of multi unit auctions. Armstrong (2000) and Avery and Hendershott (2000) derive properties of the optimal multi unit auction when types are multidimensional.[7] Both find that the optimal auction favors bundling in a probabilistic sense: a high bid on one product increases the probability of winning another product. However, additional competition for a product reduces the profitability of all bundles including this product for the auctioneer. Levin (1997) and Branco (1997) characterize the optimal multi unit auction in case of synergies. Both authors make the problem tractable by reducing it to a onedimensional mechanism design problem (i. e. each bidder observes only one private signal that determines his valuation for each single object, and for the bundle). Levin considers a model with only multiproduct bidders who may also submit bids on single objects, whereas Branco also considers singleproduct bidders, however, multiproduct bidders are not allowed to bid for single units.

Frequently used auction rules are analyzed and compared by a variety of papers. Mostly they assume one–dimensional types and model synergies by addition of a positive constant.[8] Menezes and Monteiro (2003) find that in a model with only multiproduct bidders with superadditive valuations a sequential auction and a bundle auction are revenue equivalent.

Only few papers consider the possibility of trade after the auction. Gale, Hausch, and Stegeman (2000) analyze sequential procurement auctions where resale is profitable due to convex cost.[9] Haile (1999) and Gupta and Lebrun (1998) analyze resale which is due to

[7]Armstrong has only multiproduct bidders, whereas Avery and Hendershott consider one multiproduct bidder competing with several singleproduct bidders. Unlike in our model, the authors assume that all valuations are drawn at the same time.

[8]This model goes back to Krishna and Rosenthal (1996) and was employed, e. g. by Branco (1997a) and Albano, Germano, and Lovo (1999).

[9]This is also the reason for subcontracting in Kamien, Li, and Samet (1989).

an inefficient outcome of the initial auction. In Haile this results from noisy signals at the time of the initial auction, in Gupta and Lebrun the initially inefficient allocation is due to asymmetries between bidders.

The paper is organized as follows: In section 2 we state the model. In section 3 we give a description of the bundle and the sequential auction and derive equilibrium bidding strategies and prices. In section 4 we characterize the optimal sequential auction. A comparison of all three mechanisms is given in section 5. Section 6 concludes.

2 The Model

We consider procurement of two contracts that need to be acquired in two successive periods. Bidders privately observe their cost at the beginning of each period, i. e. the second period's draw is not yet known in the first period.[10] The market for the second item may be more competitive. In particular, m multiproduct firms are eligible to acquire both contracts, while $n-m$ additional singleproduct firms are only eligible to bid for the second one ($n \geq m$).

We denote by $Q = (Q_1, \ldots, Q_m)$ the multiproduct firms' costs for the first contract. Cost Q_i, $i = 1, \ldots, m$, is distributed on the interval $[\underline{Q}, \overline{Q}]$ with c.d.f. G_i and density g_i. The winner of the first contract is assumed to have a stochastic comparative cost advantage or disadvantage for the second contract. We shall call this firm "incumbent" (I), while the multiproduct firms that do not provide the first contract are called "contestants" (C).

The random vector $X = (X_1, X_2, \ldots, X_m, \ldots, X_n) \in [0,1]^n$ denotes production costs for the second contract. We order the components of X such that X_1 denotes the incumbent's, $X_2, \ldots, X_m$ the contestants', and $X_{m+1}, \ldots, X_n$ the singleproduct bidders' cost for the second contract. Cost X_i, $i = 1, \ldots, n$, is distributed according to c.d.f. F_i with density f_i. Contestants are assumed to be symmetric with respect to their cost for the second item, i. e. the random variables $X_2, \ldots, X_m$ follow the same distribution as a reference variable X_C that is distributed according to c.d.f. F_C with density f_C. We assume that X_1 and X_C can be ranked by first order stochastic dominance. We say that the contracts are complements (substitutes) if $X_1 \leq_{FSD} X_C$ ($X_1 \geq_{FSD} X_C$).

Finally, we assume that the components of (Q, X) are independent. We denote by

[10]This kind of assumption is suitable if there is a considerable lapse of time between the two periods, or when the exact specification of the contract is not yet communicated by the procurement agency.

$Q_{(j)}$ and $X_{(j)}$ the jth order statistic of the random variables in Q and X, respectively, where we order from lowest to highest cost. Furthermore, $X_{(j)}^{\{-1\}}$ and $X_{(j)}^{\{-C\}}$ denote the jth order statistic of all random variables in X except for X_1 (the incumbent's cost) and one representative contestant's cost, respectively.

In the following, we focus on two different mechanisms: (a) a bundle auction of both contracts in period one, or (b) sequential procurement of one contract each period. We assume that every single transaction is made by a second–price auction where the lowest bidder wins and is paid the second lowest bid.

3 Equilibria

3.1 The Sequential Auction

In a sequential auction every bidder who is eligible to provide a single object can participate in the respective competition. The auction of the first (second) contract takes place in period one (two), after the firms have observed their private cost of providing the contract that is auctioned off.

We analyze the game by backward induction. In the second auction, bidding the true cost of the second contract is a (weakly) dominant strategy for every bidder. Therefore, the expected price in the second auction is equal to the expected value of the second highest cost, that is

$$E[P_2] = E[X_{(2)}]. \tag{1}$$

Now consider the first auction. A multiproduct bidder who wins the first auction has an additional profit, Π_2^I, from the second auction, whenever his cost for the second item is lowest. Thus, in the first period an incumbent's expected profit from the second auction is given by[11]

$$E[\Pi_2^I] = E[X_{(1)}^{\{-1\}} - X_1; X_1 \leq X_{(1)}^{\{-1\}}]. \tag{2}$$

However, a multiproduct bidder who did not win the first auction (a contestant) also faces a positive profit from the second auction, Π_2^C. A contestant's expected profit from the second auction is given by

$$E[\Pi_2^C] = E[X_{(1)}^{\{-C\}} - X_C; X_C \leq X_{(1)}^{\{-C\}}]. \tag{3}$$

[11] In order to simplify notation we define $E[V; A] = E[V|A]Prob[A]$.

In appendix A we show that the *value of incumbency*, $E[\Pi_2^I] - E[\Pi_2^C]$, is positive (negative) if the contracts are complements (substitutes). That is, a bidder i who wins the first auction incurs cost q_i but also "wins" an additional expected profit from the second auction, $E[\Pi_2^I] - E[\Pi_2^C]$, which is negative in case of substitutes. Therefore, perceived cost of winning the first contract, $q_i - (E[\Pi_2^I] - E[\Pi_2^C])$, differs from the real cost of providing the first contract, q_i. Iterated elimination of weakly dominated strategies yields that bidders in the first auction bid their *perceived cost* of winning the first contract, that is $b_i = q_i - (E[\Pi_2^I] - E[\Pi_2^C])$.[12] Therefore, the expected price in the sequential auction is given by

$$\begin{aligned} E\left[P_{SEQ}\right] &= E\left[Q_{(2)}\right] - E\left[\Pi_2^I\right] + E\left[\Pi_2^C\right] + E\left[P_2\right] \\ &= E\left[Q_{(2)}\right] - E\left[X_{(1)}^{\{-1\}} - X_1; X_1 \leq X_{(1)}^{\{-1\}}\right] \\ &\qquad + E\left[X_{(1)}^{\{-C\}} - X_C; X_C \leq X_{(1)}^{\{-C\}}\right] + E\left[X_{(2)}\right]. \end{aligned} \tag{4}$$

We can rearrange the expression to get

Proposition 1 *Expected procurement cost in the sequential auction is*

$$\begin{aligned} E\left[P_{SEQ}\right] = E\left[Q_{(2)}\right] + E\left[X_1\right] - E\left[X_1 - X_{(2)}^{\{-1\}}; X_{(2)}^{\{-1\}} \leq X_1\right] \\ + E\left[X_{(1)}^{\{-C\}} - X_C; X_C \leq X_{(1)}^{\{-C\}}\right]. \end{aligned} \tag{5}$$

3.2 The Bundle Auction

A bundle auction necessarily takes place in period one, before the first contract has to be allocated. At that time bidders have observed their individual cost for the first, but not for the second item. Only firms that are eligible to compete for both contracts can bid in a bundle auction.[13]

The fact that bidders observe their cost for the second contract *after* the bundle auction has taken place implies potentially positive gains from trade of the second contract at stage

[12] Obviously, the first auction bid falls short of the cost of providing the first item if the value of incumbency, $E[\Pi_2^I] - E[\Pi_2^C]$, is positive, which is true in the case of stochastic scale effects. From appendix A it follows immediately that the reverse is true if $X_1 \geq_{FSD} X_C$ (stochastic diseconomies). These findings mirror the findings of von der Fehr and Riis (1998).

[13] This is only relevant if there is a positive number of single unit bidders, i. e. $n > m$. Otherwise, all bidders can participate.

two (if the incumbent's production cost for the second contract turns out to be higher than the cost observed by one or more competitors). We allow for resale and assume that the winner of the bundle auction chooses the resale mechanism. The characterization of the optimal auction rules in this context goes back to Myerson (1981):

Lemma 1 (Optimal Subcontracting Mechanism Γ^*) *Define virtual cost of bidder i by*

$$\gamma_i(x_i) = x_i + \frac{F_i(x_i)}{f_i(x_i)}, \qquad i = 2, \ldots, n, \tag{6}$$

and assume that virtual cost $\gamma_i(x_i)$ is strict monotone increasing.[14] *The incumbent's expected cost of providing the second contract is minimized if he awards the subcontract to the firm with the lowest virtual cost $\gamma_i(x_i)$, provided it is lower than its own cost, x_1.*

The firm that is awarded the contract is paid the highest cost it could have had such that its virtual cost were still lower than the minimum of the lowest virtual cost among its competitors and the incumbent's cost, x_1.

Obviously, any bidder expects the same additional profits from the subcontracting stage as incumbent and contestant, respectively. We denote those (random) profits by $\Pi_S^I(\Gamma^*)$ and $\Pi_S^C(\Gamma^*)$.

The expected profit at stage two affects a bidder's *perceived cost* of providing the bundle of contracts: Bidder i's expected cost of providing both contracts himself is $q_i + E[X_1]$. The expected profit from the subcontracting stage as incumbent, $E[\Pi_S^I(\Gamma^*)]$, lowers the expected perceived cost of bidder i (i. e. makes winning the bundle auction more valuable). However, also the outside option is positive, since in case of loosing the auction a multiproduct bidder still faces positive expected profits $E[\Pi_S^C(\Gamma^*)]$ from the subcontracting stage. This makes him less eager to win the bundle auction. Summing up, perceived cost of providing both items is $q_i + E[X_1] - E[\Pi_S^I(\Gamma^*)] + E[\Pi_S^C(\Gamma^*)]$.[15] At a bid equal to his perceived cost bidder i is just indifferent between winning or loosing the bundle auction. This yields the following

Proposition 2 *Expected procurement cost in the bundle auction is*

$$E[P_{BS}] = E[Q_{(2)}] + E[X_1] - E[\Pi_S^I(\Gamma^*)] + E[\Pi_S^C(\Gamma^*)]. \tag{7}$$

[14] A sufficient condition is that reverse hazard rates $f_i(x_i)/F_i(x_i)$ are strict monotone decreasing.

[15] Note that without a subcontracting stage it holds that $E[\Pi_S^I(\Gamma)] = E[\Pi_S^C(\Gamma)] = 0$.

4 The Optimal Sequential Auction

In order to assess the relative performance of the bundle and the sequential auction, in the following we derive expected total procurement cost depending on the stage one and stage two allocation rules denoted $\phi^1(q)$ and $\phi^2(x)$.[16] We assume that the auctioneer has to commit to a set of rules prior to period one and cannot modify those rules after the first period.[17] Denote by $\pi_i^j(y,z)$ bidder i's expected payoff in period j if he observed cost y but reported cost z. We get

Lemma 2 *Any incentive compatible procurement mechanism where bidders participate voluntarily at each stage where they are eligible to bid, expected procurement cost is given by*

$$\begin{aligned}
&\int_{[\underline{Q},\overline{Q}]^m} \sum_{i=1}^{m} \left(q_i + \frac{G_i(q_i)}{g_i(q_i)} \right) \phi_i^1(q) dG(q) \qquad (8)\\
&+ \sum_{i=1}^{m} \left[\pi_i^1(\overline{Q},\overline{Q}) - \int_0^1 \pi_C^2(x_C,x_C) f_c(x_C) dx_C \right]\\
&+ \int_{[0,1]^n} \left(x_1 \phi_1^2(x_1) + \sum_{C=1}^{m} \left[x_C + \frac{n}{n-1} \frac{F_C(x_C)}{f_C(x_C)} \right] \phi_C^2(x_C) \right.\\
&\quad \left. + \sum_{s=m+1}^{n} \left[x_s + \frac{1}{n-1} \frac{F_C(x_s)}{f_C(x_s)} + \frac{F_s(x_s)}{f_s(x_s)} \right] \phi_s^2(x_s) \right) f(x) dx + \sum_{i=m+1}^{n} \pi_i^2(1,1),
\end{aligned}$$

where $\pi_i^1(\overline{Q},\overline{Q}) \geq \int_0^1 \pi_C^2(x_C,x_C) f_c(x_C) dx_C$.

Proof See appendix B. □

From the above expression, we can easily deduce the *optimal sequential auction* that minimizes expected procurement cost. The first line implies that the first auction is an ordinary first or second price auction. The second line becomes zero since a firm that observes the highest possible cost for the first object is just left with a contestant's expected profit as an outside option. The integral in line three to four is minimized pointwisely by allocation to the bidder with the lowest virtual cost as defined in the following

[16] A standard result of mechanism design theory is that expected payoffs are fully determined by the allocation rule up to a constant.

[17] See also Jofre-Bonet and Pesendorfer (2005) who use the optimal sequential auction rule in the case of only two multiproduct bidders to provide intuition for the comparison between first and second price sequential auctions.

Proposition 3 (Optimal Sequential Auction) *Define virtual costs $\psi_i(x_i)$ of the three types of bidders in the second auction as follows:*

$$\begin{array}{ll} \psi_1(x_1) = x_1 & \textit{for the incumbent,} \\ \psi_C(x_C) = x_C + \frac{n}{n-1}\frac{F_C(x_C)}{f_C(x_C)} & \textit{for a contestant,} \\ \psi_S(x_S) = x_S + \frac{1}{n-1}\frac{F_C(x_S)}{f_C(x_S)} + \frac{F_S(x_S)}{f_S(x_S)} & \textit{for a singleproduct bidder.} \end{array} \tag{9}$$

Procurement cost is minimized by conducting a regular first or second price auction in period one and awarding the second contract to the bidder with the lowest virtual cost for that contract, as defined by (9). This bidder is paid the highest cost he could have had such that his virtual cost were still lowest. The other bidders pay or receive nothing at stage two.

Note that the stage two allocation rule of the optimal sequential auction differs substantially from the the allocation rule the auctioneer would like to implement *after* period one.[18] Independently of the relative strengths of the different bidders (i. e. independently of whether the goods are complements or substitutes), the optimal *sequential* auction always favors the incumbent at the second stage. The reason is that by favoring the incumbent and discriminating against the contestants and the singleproduct bidders at stage two, the auctioneer makes winning the first auction more valuable. In particular, the incumbent's expected stage two profit is relatively high, while a contestant's expected profit (the "loser's option value") is rather low. Note moreover that the participation constraint at stage one requires that discrimination against the contestants and the singleproduct bidders is such that any multiproduct bidder is just indifferent between being treated as singleproduct bidder and contestant at stage two, which reflects in the corresponding virtual costs. We can summarize that in the optimal sequential auction the stage two allocation rule is designed to extract as much of the expected future profits as possible already at stage one.

5 Bundling versus Sequential Procurement

Now we are in the position to compare sequential and bundle sales and relate the two auctions to the optimal sequential auction.

Note that from propositions 1 to 3 it follows that in all three mechanisms the allocation of the first contract is efficient. That is, the mechanisms differ only with respect to the

[18] In a nutshell, the optimal auction at stage two (in ignorance of the first stage) would *discriminate against the strong bidders*, that is, against the incumbent in case of complements and against the contestants in case of substitutes.

stage two–allocation rule: While in the sequential second price auction also the allocation of the second contract is efficient, in the bundle auction and the optimal sequential auction this is not the case. In both auctions distortion is based on two kinds of information about the bidders: (a) their distributions and (b) whether they have won at the first stage or not. Note that both, the bundle and the optimal auction favor the incumbent at stage two independently of his distribution, however, to a different extent. We can prove the following

THEOREM 1 (BUNDLE VERSUS SEQUENTIAL AUCTIONS) *Suppose all bidders bidding for the subcontract are symmetric.*[19] *The following claims hold true:*

(i) In the bundle auction expected procurement cost is lower than in the sequential auction but still higher than in the optimal sequential auction, i. e. $E[P^{OSA}] < E[P^{BA}] < E[P^{SEQ}]$.

(ii) The sequential auction is efficient, whereas the bundle auction is inefficient.

(iii) The price in the bundle auction second order stochastically dominates the overall price in the sequential auction.

PROOF (i) Note that expected procurement cost as given by (8) is minimized by always awarding the second contract to the bidder with the lowest virtual cost as defined by (9). Compared to the optimal rule, in the bundle auction we get a "misallocation" whenever $x_1 \in \left[\min_{i\in\{2,\ldots,n\}}\left\{x_i + \frac{F_C(x_i)}{f_C(x_i)}\right\}, \min_{i\in\{2,\ldots,n\}}\left\{x_i + \frac{n}{n-1}\frac{F_C(x_i)}{f_C(x_i)}\right\}\right]$. Since in those cases the allocation rule of the bundle auction deviates from the allocation rule that pointwisely minimizes (8), expected procurement cost must be higher in the bundle auction than in the optimal auction.

In order to see why procurement cost in the sequential second price auction is higher than in the bundle auction, note that the second price auction produces a misallocation whenever $x_1 \in \left[\min_{i\in\{2,\ldots,n\}} x_i, \min_{i\in\{2,\ldots,n\}}\left\{x_i + \frac{n}{n-1}\frac{F_C(x_i)}{f_C(x_i)}\right\}\right]$. Obviously, the sequential second price auction "misallocates" whenever the bundle auction does, but the reverse is not true. Thus, from (8) it immediately follows that procurement cost must be lower in the bundle auction than in the sequential second price auction.
(ii) Is obvious given the allocation rules.

[19] Theorem 1 can be extended to situations, where the random variables $X_C, X_{m+1}, \ldots X_n$ do not follow the same distribution. In a previous version of this paper (Grimm, 2004) we show that only if the contestants are "weak" compared to the singleproduct bidders we need a very mild condition to establish the result.

(iii) Suppose the stage two–allocation rule after the bundle auction was also efficient (i. e. the incumbent subcontracts by a second price auction with a reserve price equal to his own observed cost x_i). Then, due to the revenue equivalence theorem it must hold that $E[P^{BA}] = E[P^{SEQ}]$.[20] We denote the efficient subcontracting mechanism by Γ^e and define $\Delta^* = E[\Pi_S^I(\Gamma^*) - \Pi_S^C(\Gamma^*)] - E[\Pi_S^I(\Gamma^e) - \Pi_S^C(\Gamma^e)]$. Then, we have

$$\begin{aligned} P_{BS} &= Q_{(2)} + E[X_1] - E[\Pi_S^I(\Gamma^e) - \Pi_S^C(\Gamma^e)] - \Delta^* \qquad (10) \\ &= Q_{(2)} - E[\Pi_2^I - \Pi_2^C] + E[P_2] - \Delta^*, \\ P_{SEQ} &= Q_{(2)} - E[\Pi_2^I - \Pi_2^C] + P_2 \qquad (11) \\ &= P_{BS} + \Delta^* + [P_2 - E[P_2]] . \end{aligned}$$

Note that $Q_{(2)}$ and P_2 are independent random variables. Therefore, P_{SEQ} is a mean preserving spread of $P_{BS}+\Delta^*$, which implies that P_{BS} second order stochastically dominates P_{SEQ},[21] which completes the proof. □

It turns out that from a revenue point of view the government benefits from delegating the allocation of the second contract to the incumbent by running a bundle auction. The reason is that compared to the efficient allocation rule the incumbent expects a higher and a contestant a lower profit from allocation of the second contract. This makes stage one bids more competitive in the bundle auction. Note that the result is independent of whether the goods are complements or substitutes.

We have also shown that, although to a lesser extent, the bundle auction exhibits the same features as the optimal sequential auction. The two allocation rules differ only in one detail: Virtual cost of the contestants and the singleproduct bidders is slightly higher in the optimal auction, i. e. $\psi_i(x_i) = \gamma_i(x_i) + \frac{1}{n-1}\frac{F_C(x_i)}{f_C(x_i)}$ for all $i = 2, \ldots, n$. It is easy to see that for both cases — substitutes and complements — the allocation rule that prevails in the bundle auction is "closer" to the optimal auction than the efficient allocation rule of the sequential second price auction. In other words, the advantage that the incumbent receives by the power to subcontract the second contract at stage two moves the incentives in the right direction and lowers the procurement cost. Note that as the total number of bidders, n, increases, the fraction $\frac{1}{n-1}$ becomes small and the bundle auction comes close to the optimal sequential auction.

The three different allocation rules are visualized in figure 1 for $m = n = 2$ and

[20] A detailed proof of this claim can be found in a previous version of this paper, Grimm (2004).

[21] See e. g. Hadar and Russell (1969), Theorem 4.

$F_C(x) = x$. The incumbent's cost are plotted on the ordinate and the contestant's cost on the horizontal axis. Each line represents the second period allocation rule in one of the mechanisms, as indicated. To the left of the respective line the second period contract is awarded to the contestant, to the right of the line to the incumbent. Note that in none of the mechanisms the allocation depends on the incumbent's distribution. Thus, the picture applies to both cases, substitutes and complements. Obviously, the allocation rule of the

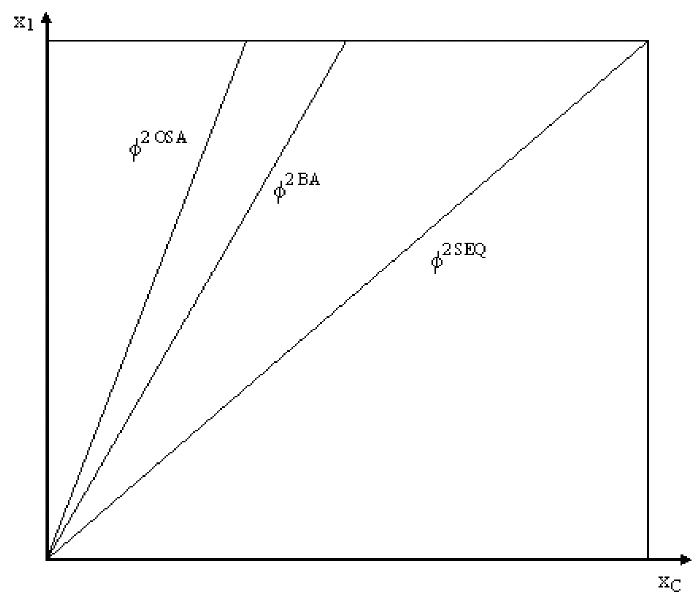

Figure 1: Visualization of the three period–two–allocation rules.

bundle auction is always closer to the optimal auction than the efficient second price rule.

The figure can also be used to illustrate why in the case of complements (which is the case where it is quite natural to think about bundling) our result applies also to first price auctions, but this is not necessarily true in case of substitutes. Observe first that first and second price *bundle* auctions are revenue equivalent. As established by Jeitschko and Wolfstetter (2002) and Jofre-Bonet and Pesendorfer (2005), the sequential first price auction yields higher procurement cost than the sequential second price auction if the contracts are complements. Graphically this means that the line representing the first price auction allocation rule is located to the right of the second price auction line[22], and thus,

[22]This can also be seen in Jofre–Bonet and Pesendorfer (2005) who use the same graph to provide intuition for their ranking of first and second price sequential auctions.

the ranking of bundle and sequential first price auction is unambiguous. For substitutes, the first price allocation rule lies to the left of the second price allocation rule, i. e. in the same direction as the bundle auction. Thus, a comparison is ambiguous and likely depends on the incumbent's distribution (which affects procurement cost in the first price, but not in the bundle auction).

6 Discussion and Concluding Remarks

In this paper, we have compared sequential and bundle procurement auctions of two contracts, where competition for the second contract may be more intense. We have found that a bundle auction yields a lower and less risky procurement cost than the sequential second price auction, which still exceeds procurement cost in the optimal sequential auction. However, while in the bundle auction and in the optimal sequential auction the final allocation is inefficient, the sequential second price auction achieves the efficient allocation. Procurement cost is less risky in the bundle auction than in the sequential second price auction, since here the incumbent is paid the *expected cost* of the second contract at stage one, where provision of the second object is delegated to him. In all auctions we analyzed, the incumbent faces the risk of making an overall loss if the contracts are complements, since he gambles on the value of incumbency in his first stage bid.

Our findings imply that the choice of mechanism clearly depends on the objectives of the auctioneer. If efficiency is the predominant concern (which is plausible if the auctioneer is a public authority), the sequential auction is the appropriate mechanism among the mechanisms considered here. If the auctioneer maximizes revenue (e. g. a private sector firm), a bundle auction is a good choice. As we have shown, the allocation rule of the bundle auction is quite "close" to the optimal allocation rule. Moreover, the bundle auction even has some advantages compared to the optimal sequential auction: First, procurement cost is less risky since the stage two–price risk is borne by the incumbent. Second, whereas in the optimal sequential auction the auctioneer has an incentive to deviate from the rules he announced for stage two after the first period is over, in bundle auction this problem does not occur.[23]

Let us conclude with two considerations beyond the analysis of this paper. First, consider the case that the incumbent's comparative advantage is endogenous, i. e. has to

[23]In fact, the auctioneer does not have an incentive to prohibit resale before or after the bundle auction.

be induced by specific investment. Then, there will be no incentive to incur such cost in the bundle auction (where, by specific investment, the incumbent would only decrease his expected profits from subcontracting), while the sequential auction might give rise to "wasteful" expenditures that only aim at discriminating against potential competitors in the second auction. Second, the choice of mechanism may have an impact on the competition for the second production right. In the bundle and the optimal sequential auction, strong singleproduct bidders are discriminated against even more than the contestants. Therefore, their incentives to enter the game are presumably lower in those auctions than in the sequential second price auction.

A The Value of Incumbency

In order to prove that $E[\Pi_2^I] - E[\Pi_2^C] > (<) 0$ if the objects are complements (substitutes), we define the vector of all bidders' expected cost for the second item except the incumbent's and one (representative) contestant's cost by $\tilde{X} := (X_i)$, $i \neq 1, 2$. We denote by $\tilde{X}_{(1)}$ the lowest cost among those bidders and by $\tilde{F}_{(1)}$ ($\tilde{f}_{(1)}$) the corresponding c.d.f (density function). Now we can decompose as follows:

$$E[\Pi_2^I] = E\left[X_C - X_1; X_1 \leq X_C \leq \tilde{X}_{(1)}\right] + E\left[\tilde{X}_{(1)} - X_1; X_1 \leq \tilde{X}_{(1)} \leq X_C\right] \tag{12}$$

and (13)

$$E[\Pi_2^C] = E\left[X_1 - X_C; X_C \leq X_1 \leq \tilde{X}_{(1)}\right] + E\left[\tilde{X}_{(1)} - X_C; X_C \leq \tilde{X}_{(1)} \leq X_1\right].$$

First, we derive $E[\Pi_2^C]$. We get

$$\begin{aligned} E[\Pi_2^C] = &\int_0^1 \left[(1 - \tilde{F}_{(1)}(v)) \int_0^v F_C(u)du\right] f_1(v)dv \\ &+ \int_0^1 \left[(1 - F_1(v)) \int_0^v F_C(u)du\right] \tilde{f}_{(1)}(v)dv. \end{aligned} \tag{14}$$

Integration by parts of the second term in (14) yields

$$\begin{aligned} &\int_0^1 \left[(1 - F_1(v)) \int_0^v F_C(u)du\right] \tilde{f}_{(1)}(v)dv \\ &= \left|(1 - F_1(v)) \int_0^v F_C(u)du \tilde{F}_{(1)}(v)\right|_0^1 \\ &\quad - \int_0^1 \tilde{F}_{(1)}(v) \left[(1 - F_1(v))F_C(v) - f_1(v) \int_0^v F_C(u)du\right] dv \\ &= - \int_0^1 \tilde{F}_{(1)}(v) \left[(1 - F_1(v))F_C(v) - f_1(v) \int_0^v F_C(u)du\right] dv. \end{aligned} \tag{15}$$

Inserting (15) in (14) gives

$$E[\Pi_2^C] = \int_0^1 \int_0^v F_C(u)dudv - \int_0^1 \tilde{F}_{(1)}(v)(1-F_1(v))F_C(v)dv \tag{16}$$
$$= |F_1(v)\int_0^v F_C(u)du|_0^1 - \int_0^1 F_1(v)F_C(v)dv$$
$$- \int_0^1 \tilde{F}_{(1)}(v)(1-F_1(v))F_C(v)dv,$$

which yields

$$E[\Pi_2^C] = \int_0^1 (1-\tilde{F}_{(1)}(v))(1-F_1(v))F_C(v)dv. \tag{17}$$

Following the same calculations, we get

$$E[\Pi_2^I] = \int_0^1 (1-\tilde{F}_{(1)}(v))F_1(v)(1-F_C(v))dv. \tag{18}$$

Clearly, if X_C is exceeds (falls short of) X_1 in the sense of first order stochastic dominance, it holds that $(1-F_1(v))F_C(v) < (>) F_1(v)(1-F_C(v))$ for every $v \in [0,1]$, which proves the assertion. □

B The Optimal Sequential Auction

In this section we derive the optimal auction rule under the assumption that the auctioneer has to fix it before period one and cannot change it between the periods. We know that we can restrict our attention to direct incentive compatible mechanisms. We denote such a mechanism by the quadruple of vectors $(\phi^1(q), \phi^2(x), t^1(q), t^2(x))$, where $\phi_i^k(\cdot)$ is the probability that firm i is awarded the contract in period k given that reports q (respectively x) have been made and $t_i^k(\cdot)$ denotes the transfer to i in period k given the reports. Let $\Phi_i^1(q_i)$ and $T_i^1(q_i)$ denote the *expected* probability to win and the *expected* transfer in period one if firm i reports q_i and the remaining firms report truthfully, i. e. $\Phi_i^1(q_i) = \int_{[\underline{Q},\overline{Q}]^m} \phi_i^1(q)dG_{-i}(q_{-i})$ and $T_i^1(q_i) = \int_{[\underline{Q},\overline{Q}]^m} t_i^1(q)dG_{-i}(q_{-i})$. Define $\Phi_i^2(x_i)$ and $T_i^2(x_i)$ analogously, accounting for the fact that we potentially have more bidder in period two, i. e. $n \geq m$. Then, the expected profits of firm i from period one and two if its cost are q_i and x_i but it reports $\hat{q}_i$ and $\hat{x}_i$ are given by

$$\pi_i^2(x_i,\hat{x}_i) = T_i^2(\hat{x}_i) - x_i\Phi_i^2(\hat{x}_i) \tag{19}$$

$$\begin{aligned}\pi_i^1(q_i,\hat{q}_i) &= T_i^1(\hat{q}_i) - q_i\Phi_i^1(\hat{q}_i) + \Phi_i^1(\hat{q}_i)\int_0^1 \pi_1^2(s,s)f_1(s)ds \\ &\quad +(1-\Phi_i^1(\hat{q}_i))\int_0^1 \pi_C^2(s,s)f_C(s)ds\end{aligned} \tag{20}$$

The incentive constraints require that

$$\pi_i^1(q_i,q_i) \geq \pi_i^1(q_i,\hat{q}_i) \quad \text{for all } \hat{q}_i \neq q_i, \tag{$IC1$}$$

$$\pi_i^2(x_i,x_i) \geq \pi_i^2(x_i,\hat{x}_i) \quad \text{for all } \hat{x}_i \neq x_i, \tag{$IC2$}$$

and the participation constraints require that

$$\pi_i^2(x_i,x_i) \geq 0, \tag{$PC2$}$$

$$\pi_i^1(q_i,q_i) \geq \int_0^1 \pi_C^2(s,s)f_C(s)ds. \tag{$PC1a$}$$

$$\int_0^1 \pi_C^2(s,s)f_C(s)ds \geq \int_0^1 \pi_S^2(s,s)f_C(s)ds \tag{$PC1b$}$$

From (IC1) and (IC2) it follows that $\Phi_i^k(\cdot)$, $k=1,2$, is monotone decreasing and a bidder's expected equilibrium profit is determined by the probability of winning up to a constant (which determines the bidder's profit if he observes his worst possible type), i. e. $\pi_i^1(q_i,q_i) = \int_{q_i}^{\overline{Q}} \Phi_i^1(s)ds + \pi_i^1(\overline{Q},\overline{Q})$, and analogously for $\pi_i^2(x_i,x_i)$.[24] Note that from the last equation it follows (integration by parts) that

$$\begin{aligned}\int_{\underline{Q}}^{\overline{Q}} \pi_i^1(q_i,q_i)g_i(q_i)dq_i &= \int_{\underline{Q}}^{\overline{Q}} \Phi_i^1(q_i)G_i(q_i)dq_i + \pi_i^1(\overline{Q},\overline{Q}), \\ \int_0^1 \pi_i^2(x_i,x_i)f_i(x_i)dx_i &= \int_0^1 \Phi_i^2(x_i)F_i(x_i)dx_i + \pi_i^2(1,1).\end{aligned}$$

This leads to the following expected transfers of bidder i in period one and two, respectively,

$$\begin{aligned}T_i^2(x_i) &= \pi_i^2(x_i,x_i) + x_i\Phi_i^2(x_i) \\ &= \int_{x_i}^1 \Phi_i^2(s)ds + \pi_i^2(1,1) + x_i\Phi_i^2(x_i)\end{aligned} \tag{21}$$

$$\begin{aligned}T_i^1(q_i) &= \pi_i^1(q_i,q_i) + q_i\Phi_i^1(q_i) - \Phi_i^1(q_i)\int_0^1 \pi_1^2(s,s)f_1(s)ds \\ &\quad -(1-\Phi_i^1(q_i))\int_0^1 \pi_C^2(s,s)f_C(s)ds\end{aligned} \tag{22}$$

[24] See, for example, Krishna (2002), pp. 63 ff.

Now we compute the sum of expected transfer payments to the bidders which the auctioneer aims to minimize:

$$
\begin{aligned}
\sum_{i=1}^{m}\int_{\underline{Q}}^{\overline{Q}} T_i^1(q_i)g_i(q_i)dq_i &= \sum_{i=1}^{m}\left[\int_{\underline{Q}}^{\overline{Q}} \pi_i^1(q_i,q_i)g_i(q_i)dq_i + \int_{\underline{Q}}^{\overline{Q}} q_i\Phi_i^1(q_i)g_i(q_i)dq_i\right] \\
&\quad -\sum_{i=1}^{m}\Phi_i^1(q_i)\int_0^1 \pi_1^2(s,s)f_1(s)ds \\
&\quad -\sum_{i=1}^{m}(1-\Phi_i^1(q_i))\int_0^1 \pi_C^2(s,s)f_C(s)ds \\
&= \sum_{i=1}^{m}\left[\int_{\underline{Q}}^{\overline{Q}} G_i(q_i)\Phi_i^1(q_i)dq_i + \pi_i^1(\overline{Q},\overline{Q}) + \int_{\underline{Q}}^{\overline{Q}} q_i\Phi_i^1(q_i)g_i(q_i)dq_i\right] \\
&\quad -\int_0^1 \pi_1^2(s,s)f_1(s)ds - (m-1)\int_0^1 \pi_C^2(s,s)f_C(s)ds \\
&= \int_{[\underline{Q},\overline{Q}]^m}\sum_{i=1}^{m}\left(q_i+\frac{G_i(q_i)}{g_i(q_i)}\right)\phi_i^1(q)dG(q) + \sum_{i=1}^{m}\pi_i^1(\overline{Q},\overline{Q}) \\
&\quad -\int_0^1 F_1(s)\Phi_1^2(s)ds - \pi_1^2(1,1) - (m-1)\int_0^1 F_C(s)\Phi_C^2(s)ds \\
&\quad -(m-1)\pi_C^2(1,1).
\end{aligned}
$$

$$
\begin{aligned}
\sum_{i=1}^{n}\int_0^1 T_i^2(x_i)f_i(x_i)dx_i &= \sum_{i=1}^{n}\int_0^1 F_i(x_i)\Phi_i^2(x_i)dx_i + \sum_{i=1}^{n}\pi_i^2(1,1) \\
&\quad +\sum_{i=1}^{n}\int_0^1 x_i\Phi_i^2(x_i)f_i(x_i)dx_i.
\end{aligned}
$$

Adding both terms we get the expected transfer to the firms:

$$
\begin{aligned}
E[T] &= \int_{[\underline{Q},\overline{Q}]^m}\sum_{i=1}^{m}\left(q_i+\frac{G_i(q_i)}{g_i(q_i)}\right)\phi_i^1(q)dG(q) + \sum_{i=1}^{m}\pi_i^1(\overline{Q},\overline{Q}) \\
&\quad +\sum_{i=1}^{n}\int_0^1 x_i\Phi_i^2(x_i)f_i(x_i)dx_i + \sum_{i=m+1}^{n}\int_0^1 F_i(x_i)\Phi_i^2(x_i)dx_i + \sum_{i=m+1}^{n}\pi_i^2(1,1).
\end{aligned}
$$

Adding and subtracting the term $m\cdot\int_0^1 \pi_C^2(x_C,x_C)f_c(x_C)dx_C = m\cdot\int_0^1 F_C(x_C)\Phi_C^2(x_C)dx_C$ and subtracting $\frac{n-m}{n-1}\left[\int_0^1 \Phi_C(s)F_C(s)ds - \int_0^1 \Phi_S(s)F_C(s)ds\right]$ (= 0 due to (PC1b)) yields an expression that allows to determine the optimal auction rule:

$$
\begin{aligned}
E[T] &= \int_{[\underline{Q},\overline{Q}]^m}\sum_{i=1}^{m}\left(q_i+\frac{G_i(q_i)}{g_i(q_i)}\right)\phi_i^1(q)dG(q) \\
&\quad +\sum_{i=1}^{m}\left[\pi_i^1(\overline{Q},\overline{Q}) - \int_0^1 \pi_C^2(x_C,x_C)f_c(x_C)dx_C\right] \\
&\quad \int_{[0,1]^n}\left(x_1\phi_1^2(x_1) + \sum_{C=1}^{m}\left[x_C + \frac{n}{n-1}\frac{F_C(x_C)}{f_C(x_C)}\right]\phi_C^2(x_C)\right. \\
&\quad \left. + \sum_{s=m+1}^{n}\left[x_s + \frac{1}{n-1}\frac{F_C(x_s)}{f_C(x_s)} + \frac{F_s(x_s)}{f_s(x_s)}\right]\phi_s^2(x_s)\right) f(x)dx + \sum_{i=m+1}^{n}\pi_i^2(1,1)
\end{aligned}
$$

At the first stage, if the firms are symmetric, a standard first or second price auction is optimal. At the second stage, the optimal auction favors the incumbent and discriminates against the contestants and the single product bidders. The reason is that decreasing a contestant's expected profits from the second stage makes the stage one–bids more competitive. In the above formula, the incumbent is indexed by "1", and the contestants and singleproduct bidders by "C and S, respectively. The optimal mechanism is stated in proposition 3.

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