Equilibrium Investment is Reduced if we Allow for Forward Contracts*

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Abstract

In this paper we analyze incentives to invest in capacity prior to a sequence of Cournot spot markets with varying demand. We compare equilibrium investment in the absence and in presence of the possibility to trade on forward markets. We find that the possibility to trade forwards reduces equilibrium investments.

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1 Introduction

In the course of the liberalization of electricity markets one of the major objectives has been the implementation of a market design that enhances competition. In this context, the role of forward markets in mitigating market power has been discussed extensively. The debate was initiated by Allaz (1992) and Allaz and Vila (1993), who show that the strategic use of forward markets enhances competition in a Cournot oligopoly. The literature on electricity market design adopted this model, arguing that the introduction of forward markets could decrease spot prices and thereby enhance efficiency.[1]

In this debate, incentives to invest in capacity under different market designs have long been ignored. On several recent occasions, however, shortages of transmission and/or generation capacity provoked serious breakdowns of electricity power supply. For example in California, wholesale electricity prices during the Summer of 2000 were nearly 500% higher than they were during the same months in 1998 or 1999. Some customers were required to involuntarily curtail electricity consumption in response to supply shortages.[2] In the United States and Canada, the great blackout of 2003 knocked out power to 50 million people over a 9,300-square-mile area stretching from New England to Michigan.[3] Those events demonstrated that the reliability of energy provision — and thus, the general functioning of energy markets — depends crucially on the existence of sufficient capacities at high levels of demand. Consequently, investment incentives have to be considered when judging the attractiveness of a market design. Moreover, a market design that enhances competition if capacity choices are not taken into account, does not necessarily have the same effect when one accounts for capacity choices upfront.

For both reasons, investment incentives recently have become a mayor

[1]See for example Newberry (1998) and Le Coq and Orzen (2002).

[2]See the discussion in Joskow (2001) and Borenstein (2002).

[3]CBSnews.com, August 15, 2003 at
http://www.cbsnews.com/stories/2003/08/15/national/main568422.shtml

topic in the debate on electricity market design. Von der Fehr and Harbord (1997), as well as Castro-Rodriguez et al. (2001) show that from a social welfare point of view, investments in generation capacity are too low in a perfectly competitive market. Gabszewicz and Poddar (1997) analyze capacity choices prior to Cournot competition, where demand is uncertain at the time where capacity choices are made. They show that in equilibrium firms have excess capacity compared with the capacity of the Cournot certainty equivalent game. Murphy and Smeers (2003) analyze a capacity expansion model where at the production stage players compete on a sequence of Cournot markets with varying demand. They show that capacities are higher and prices are lower if capacity choices are observable prior to production. In a follow–up paper, Murphy and Smeers (2004) show that if firms can choose their capacities prior to a single Cournot spot market, the introduction of forward markets is ineffective. In equilibrium, equal capacities are chosen in the cases with and without forward trading. Hence, equilibrium quantities are the same.

In this paper, we analyze capacity investments prior to a continuum of subsequent Cournot spot markets with different demand realizations. We compare the resulting two stage game to a market design where forwards can be traded prior to each spot market. We find that capacity investments generally decrease upon the introduction of forward markets.

Forward contracts as analyzed by Allaz and Vila have the same impact on the Cournot outcome as the delegation of output decisions to managers or the delegation to retailers, as analyzed among others by Vickers (1985) and Fershtman and Judd (1987). In all those models, the firm uses the incentive scheme to commit to a more aggressive behavior, which in equilibrium leads to higher quantities and lower price. Our result applies to all those models: If capacity investments are an issue in the market under consideration, we have to expect lower capacities if the firms have access to strategic devices.

The paper is organized as follows: In section 2 we state the model. In section 3 we analyze the game without forward contracts. Section 4 analyzes the game in the presence of forward markets and compares the results of the two scenarios. Section 5 concludes.

2 The Model

We analyze a duopoly where firms have to make a capacity choice before they compete on a continuum of successive spot markets. Also prior to production, they have the possibility to trade forward contracts, by which they commit to sell a certain quantity on a specific spot market at a fixed price. The situation we have in mind is captured by the following three stage game:

At stage one each firm i, $i = 1, 2$, invests in capacity $x_i \in \mathbb{R}_+$, $i = 1, 2$, at a unit cost k (firms are assumed to be symmetric with respect to their cost of investment).

At stage two, having observed the capacity choices x, firms have the possibility to sell any quantity up to their capacity constraint on the forward market at a fixed price for each spot market $t \in [0, T]$. Forward contracts $f(t) = (f_i(t), f_{-i}(t))$ are sold in an arbitrage-free market.[4,5]

At stage three firms face the capacity constraints inherited from stage one and hold the forward positions from stage two. They simultaneously choose outputs for each spot market $t \in [0, T]$, denoted by $y(t) = (y_i(t), y_{-i}(t))$, $i = 1, 2$. Demand at time t, $P(Y, t)$ has the functional form[6] $P(Y, t) = at - Y(t)$, where $Y(t) = y_i(t) + y_{-i}(t)$ is the aggregate quantity produced by the two firms at time t, $a \geq 0$, and $t \in [0, T]$.[7] Both firms have the same marginal cost of production which is assumed to be constant. Without loss of generality we normalize marginal cost to zero.

[4]Since we analyze the case of demand certainty we are interested in forward contracts as a strategic device, as introduced by Allaz and Vila (1993).

[5]We denote by $-i$ the firm other than i.

[6]The majority of the contributions to the topic we analyze concentrate on the case of linear demand. Examples are Allaz and Vila (1993), or Murphy and Smeers (2004).

[7]The demand realizations are ordered from low to high for tractability reasons. However, as it is easy to show, the result holds also for any other ordering of demand realizations.

Firm i's profit from operating in the time interval $[0, T]$ if capacities and forwards are given by x and $f(t)$ and firms have chosen feasible[8] load curves $y(t)$, is given by[9]

$$\pi_i(x_i, y) = \int_0^T \left[at - (y_i(t) + y_{-i}(t))\right] y_i(t)dt - kx_i. \tag{1}$$

The game we consider is a three stage game with observability after each stage. We look for subgame perfect Nash equilibria in pure strategies. The assumption that spot market quantities for the entire interval $[0, T]$ have to be chosen simultaneously prior to $t = 0$ is made for expositional simplicity. All results are still true if firms can choose load curves for the subsequent time interval at finitely many points in time.

3 Equilibrium without Forward Contracts

In this section we analyze the game without the possibility to trade forward contracts. This is equivalent to exogenously fix forwards at $f(t) = 0$ for all t. Thus, we have a two stage game where firms invest at stage one and decide upon quantities at stage two. We derive the subgame perfect equilibrium of the game by backward induction, that is, we first solve for the equilibria at stage two and then derive equilibrium capacity choices given that firms anticipate equilibrium play at stage two.

Stage II First note that for given investment levels x we can solve the maximization problem of firm i pointwise. That is, firm i's profit as given by (1) is maximized whenever the integrand is maximized at each $t \in [0, T]$.[10] Thus, an equilibrium $y^*(x, t)$ satisfies simultaneously for both firms and for

[8] That is, $f_i(t) \leq y_i(t) \leq x_i$ for all $t \in [0, T]$, $i = 1, 2$.

[9] At this point we completely disregard the problem of integrability and comment later on it when it becomes an issue.

[10] Any function $\hat{y}(t)$ that differs from $y^*(t)$ at a finite number of points also maximizes π. However, note that this does not affect the optimal investment.

each $t \in [0, T]$

$$y_i^*(x,t) \in \arg\max_{\mathrm{y}} \left\{ \left[at - (\mathrm{y} + y_{-i}^*(t))\right] \mathrm{y} \right\} \qquad \text{s.t.} \quad 0 \leq \mathrm{y} \leq x_i.$$

The above considerations imply that an equilibrium of the game at stage two, $(y_i^*(x,t), y_{-i}^*(x,t))$, is given by the equilibrium outputs of the capacity constrained Cournot games at each $t \in [0, T]$.

It is easy to show that the firms' unconstrained reaction functions at time t have the from $\tilde{y}_i^{BR}(y_{-i}, t) = \frac{at - y_{-i}}{2}$ and that the unconstrained Cournot equilibrium is that both firms produce $\tilde{y}_i^*(t) = \frac{at}{3}$, $i = 1, 2$. Suppose now that firm i's investment is (weakly) lower than firm $-i$'s. Depending on how much the firms have invested at stage one relative to the demand realization at time t, we have to distinguish three cases.

(CN) **No firm is constrained** if $x_i \geq \tilde{y}_i^*(t) = \frac{at}{3}$, $i = 1, 2$, i. e. each firm's unconstrained Cournot quantity is lower than its maximal possible output given the capacity choices. Obviously, this is the case whenever $0 \leq t \leq \frac{3x_i}{a}$, $i = 1, 2$. In this interval the equilibrium of the second stage corresponds to the unconstrained Cournot Nash equilibrium (denoted EQ^{CN}):

$$t \in \left[0, \frac{3x_i}{a}\right) \qquad \Leftrightarrow \qquad y_i^*(x,t) = \frac{at}{3}, \quad i = 1, 2.$$

Equilibrium profits are

$$\pi_i^{CN}(x,t) = \left(\frac{at}{3}\right)^2, \qquad i = 1, 2.$$

(Ci) **Firm i is constrained** if $t > \frac{3x_i}{a}$ and therefore $x_i \leq \frac{at}{3}$. In this case firm i cannot play its unconstrained Cournot output, but will produce at capacity. As long as firm $-i$ is not yet constrained, it will play its best response to firm i producing x_i, that is $\tilde{y}_{-i}^{BR}(x_i, t) = \frac{at - x_i}{2}$. This implies that firm $-i$ is unconstrained for all $t \leq \frac{2x_{-i} + x_i}{a}$. Thus, if $t \in (\frac{3x_i}{a}, \frac{2x_{-i}+x_i}{a}]$, in equilibrium the low-capacity firm i produces at capacity, but firm $-i$ does not (denoted EQ^{Ci}).

$$t \in \left[\frac{3x_i}{a}, \frac{2x_{-i}+x_i}{a}\right) \quad \Leftrightarrow \quad \left[y_i^*(x,t), y_{-i}^*(x,t)\right] = \left[x_i, \frac{at-x_i}{2}\right].$$

Equilibrium profits are

$$\pi_i^{Ci}(x,t) = \left(\frac{at-x_i}{2}\right)x_i, \quad \pi_{-i}^{Ci}(x,t) = \left(\frac{at-x_i}{2}\right)^2.$$

(CB) **Both firms are constrained** for demand realizations higher than $t = \frac{2x_{-i}+x_i}{a}$. In this case in equilibrium both firms produce at capacity (denoted EQ^{CB}).

$$t \in \left[\frac{2x_{-i}+x_i}{a}, T\right] \quad \Leftrightarrow \quad y_i^*(x,t) = x_i, \quad i = 1,2.$$

Equilibrium profits are

$$\pi_i^{CB}(x,t) = (at - x_i - x_{-i})\, x_i, \qquad i = 1,2.$$

As we already mentioned in section 2 the results do not change if we allow the firms to choose load curves at a finite number of points in time. This is obvious since due to uniqueness of the equilibrium at each t, only playing $y_i^*(x,t)$ satisfies subgame perfection. Figure 1 summarizes the results.

Stage I For a given t, figure 1 shows which type of equilibrium exists for each given pair of investment levels, x. Building on these results we can now derive firm i's profit from investing x_i, given that the other firm invests x_{-i} and quantity choices at stage two are given by y^*. A firm's profit from given levels of investments, x, is the integral over equilibrium profits at each t given x on the domain $[0, T]$. For each t, firms anticipate equilibrium play at stage two, which gives rise to one of the three types of equilibria, EQ^{CN}, EQ^{Ci}, or EQ^{CB}. Note that for any possible investment levels $x > 0$ if t is close enough to zero, both firms are unconstrained in equilibrium. Thus, any

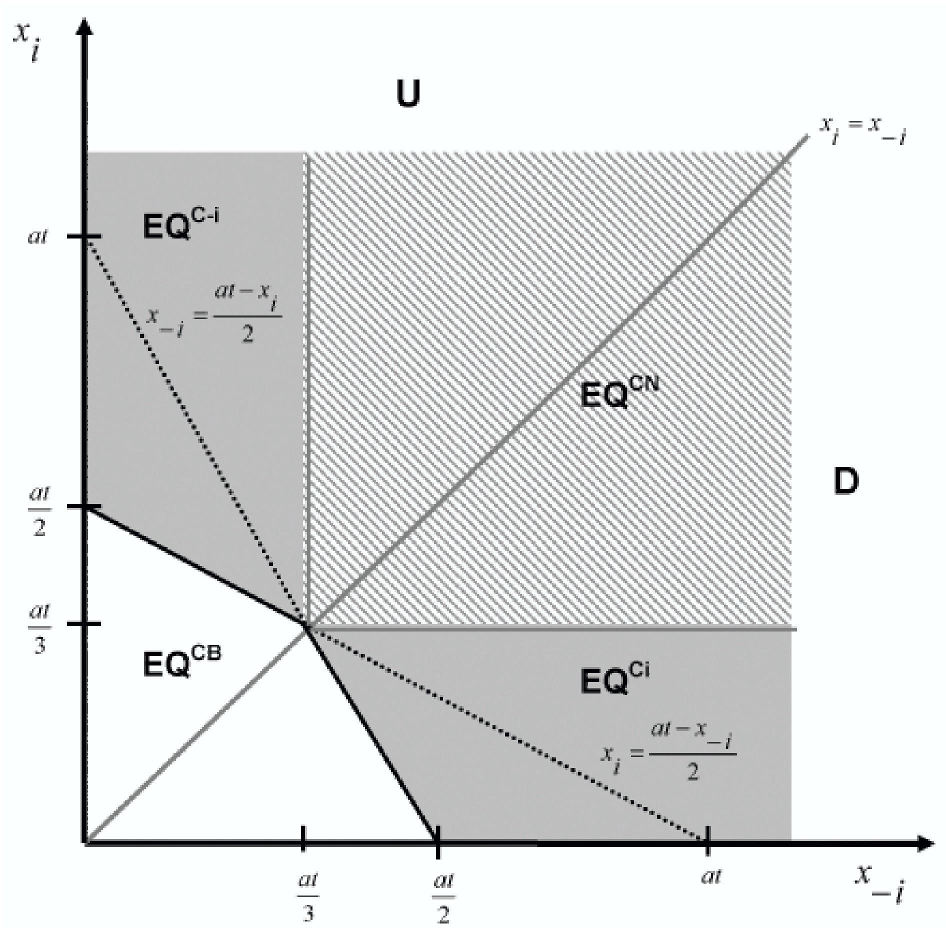

Figure 1: Nash equilibria at stage two of the market game without forward contracts.

$x > 0$ gives rise to the unconstrained equilibrium in this case. An increase of t corresponds to a dilation of all regions outwards with center zero. Thus, a pair of investment levels that initially gave rise to an EQ^{CN} leads to an EQ^{Ci} for a higher t. As t increases even more, x finally is located in the region where both firms are constrained (EQ^{CB}). For investment levels where both firms are constrained in the highest demand scenario the profit function is given by[11]

$$
\begin{aligned}
\pi_i^U(x,y^*) &= \int_0^{\frac{3x_{-i}}{a}} \pi_i^{CN}\,dt + \int_{\frac{3x_{-i}}{a}}^{\frac{2x_i+x_{-i}}{a}} \pi_i^{C-i}\,dt + \int_{\frac{2x_i+x_{-i}}{a}}^{T} \pi_i^{CB}\,dt - kx_i \quad (2)\\
&= \frac{(aT-x_{-i})(aT-x_{-i}-2x_i)x_i}{2a} + \frac{x_{-i}^3+2x_i^3}{3a} - kx_i
\end{aligned}
$$

for $x_i \geq x_{-i}$ and $x_i \leq \frac{aT-x_{-i}}{2}$ (denoted region $\underline{U}$), and

$$
\begin{aligned}
\pi_i^D(x,y^*) &= \int_0^{\frac{3x_i}{a}} \pi_i^{CN}\,dt + \int_{\frac{3x_i}{a}}^{\frac{x_i+2x_{-i}}{a}} \pi_i^{Ci}\,dt + \int_{\frac{x_i+2x_{-i}}{a}}^{T} \pi_i^{CB}\,dt - kx_i \quad (3)\\
&= \frac{(aT-x_i)(aT-2x_{-i}-x_i)x_i}{2a} + \frac{x_i\,x_{-i}^2}{a} - kx_i.
\end{aligned}
$$

for $x_i \leq x_{-i}$ and $x_{-i} \leq \frac{aT-x_i}{2}$ (denoted region $\underline{D}$).

Notice that for $x_i = x_{-i}$ we obtain $\pi_i^U = \pi_i^D$, implying that the profit function $\pi_i(x,y^*)$ is continuous for all x. Given $y^*(x,t)$ we can now derive the equilibrium of stage one which yields the subgame perfect equilibrium of the two stage game.

PROPOSITION 1 *The market game where firms first invest in capacity and then engage in quantity competition in a continuum of spot markets has a unique subgame perfect Nash equilibium. In equilibrium firms invest*

$$x_i^* = \frac{1}{3}\left(aT - \sqrt{2ak}\right), \quad i = 1,2.$$

They produce the unconstrained Cournot best reply quantities at stage two whenever this is possible, and at capacity otherwise.

Proof: see Appendix A.

Since the main objective of the paper is to compare the level of total investment with and without forward markets, we define

$$I^{NF} = \{x \in \mathbb{R}_+^2 : \, x_i + x_{-i} = \frac{2}{3}(aT - \sqrt{2ak})\}. \quad (4)$$

[11]For investment levels where one firm is unconstrained at the highest demand realization the last integral has to be dropped and the upper limit of the second integral has to be substituted by T (regions $\overline{U}^I$ and $\overline{D}^I$ in figure 2). If both firms are unconstrained at the highest demand realization the two last integrals have to be dropped and the upper limit of the first integral has to be substituted by T (regions $\overline{U}^{II}$ and $\overline{D}^{II}$).

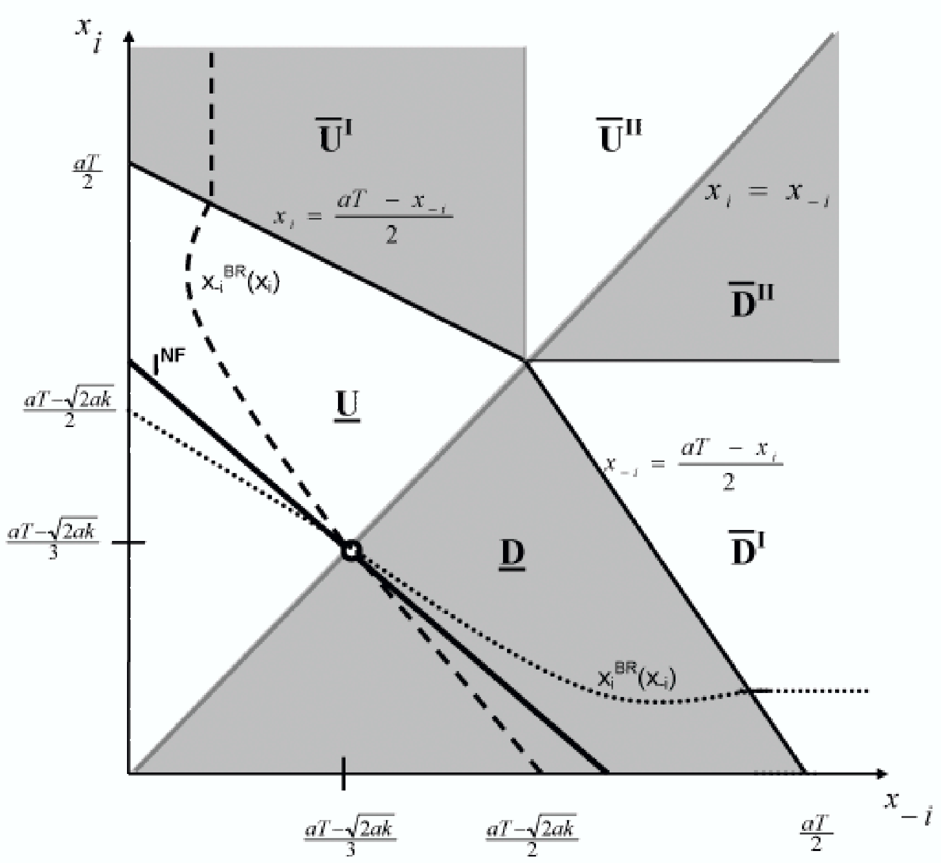

Figure 2: Best replies, equilibrium, and the isoinvestment line I^{NF} for the market game without forward contracts.

The *isoinvestment line* I^{NF} contains all investment levels x_i, x_{-i} leading to the same total investment as the equilibrium of the the market game without forward contracts we analyzed in this section. Best reply functions at stage one and the isoinvestment line are depicted in figure 2.

4 Equilibrium with Forward Contracts

If we include forward markets, we have to analyze the three stage game already described in section 2, where prior to production but after investments have been made, forwards can be traded.

The impact of forward markets on Cournot competition has already been analyzed by Allaz and Vila (1993). In section 4.1 we extend the analysis to the presence of capacity constraints. In section 4.2 we will use the subgame perfect equilibria of the parameterized subgames starting at stage two in order to characterize equilibrium investments at stage one (prior to a continuum of Cournot markets) and compare them to equilibrium investments in the market game without forward markets.

4.1 Forward Trading in the Presence of Capacity Constraints

Stage III In each subgame starting at stage three the firms have observed investment levels $x = (x_i, x_{-i})$ and the quantities traded forward, $f(t) = (f_i(t), f_{-i}(t))$. Again, firm i's profit as given by (1) is maximized whenever the integrand is maximized at each $t \in [0, T]$. Thus, an equilibrium of stage three satisfies simultaneously for both firms and for each $t \in [0, T]$[12]

$$y_i^*(x, f, t) \in \arg\max_{\mathrm{y} \geq 0} \{(at - \mathrm{y} - y_{-i}^*)(\mathrm{y} - f_i(t))\} \quad \text{s.t.} \quad f_i(t) \leq \mathrm{y} \leq x_i. \tag{5}$$

Note that $y_i^*(t)$ only depends on the forwards traded for period t, $f(t)$.

Now we solve for the equilibrium of stage three. As a first step we ignore the capacity constraint and derive the best reply of firm i to a given quantity produced by $-i$,

$$\tilde{y}_i^{BR}(y_{-i}; f, t) = \frac{at + f_i - y_{-i}}{2}, \quad i = 1, 2. \tag{6}$$

Thus, the equilibrium of the unconstrained market game at stage three is

$$\tilde{y}_i^*(f, t) = \frac{at + 2f_i - f_{-i}}{3}, \quad i = 1, 2.$$

[12] With a slight abuse of notation, we use the same symbols as in the case without forward contracts.

From equations (5) and (6) immediately follow the capacity constrained best reply-functions,

$$y_i^{BR}(y_{-i}; x, f, t) = \min\left\{\tilde{y}_i^{BR}(y_{-i}; f, t), x_i\right\}, \quad i = 1, 2.$$

It is straightforward to show that for each (x, f, t) the equilibrium[13] $\{y_1^*(x, f, t), y_2^*(x, f, t)\}$ of stage three is unique. Depending on the values of x, f, and t none of the firms, one of them, or both are capacity constrained in equilibrium. We now become specific on equilibrium quantities and profit functions in each of those cases:

(CN) **No firm is constrained** if for both firms the unconstrained Cournot quantities given f are lower than capacity. This holds true, whenever

$$x_i > \tilde{y}_i^*(f, t), \quad i = 1, 2. \tag{7}$$

We denote by $F^{CN}(x, t)$ the set of all f for which both inequalities in (7) are satisfied at (x, t). For all $f \in F^{CN}(x, t)$, equilibrium quantities at stage three are $y_i^*(x, f, t) = \tilde{y}_i^*(f, t)$, $i = 1, 2$, and equilibrium profits are

$$\pi_i^{CN}(x, f, y^*, t) = \frac{(at - f_i - f_{-i})(at + 2f_i - f_{-i})}{9}. \tag{8}$$

(Ci) **Only firm i is constrained** if firm i's unconstrained Cournot quantity given f exceeds its capacity, but firm $-i$ is not constrained in equilibrium. This holds true, whenever

$$x_i \leq \tilde{y}_i^*(f, t) \quad \text{and} \quad x_{-i} \geq \tilde{y}_{-i}^{BR}(x_i; f, t). \tag{9}$$

We denote by $F^{Ci}(x, t)$ the set of all f for which both inequalities are satisfied at (x, t). For all $f \in F^{Ci}(x, t)$, equilibrium quantities at stage three are $y_i^*(x, f, t) = x_i$, $y_{-i}^*(x, f, t) = \tilde{y}_{-i}^{BR}(x_i; x, f, t) \leq x_{-i}$. Equilibrium profits are

$$\pi_i^{Ci}(x, f, y^*, t) = \frac{x_i(at - f_{-i} - x_i)}{2}. \tag{10}$$

$$\pi_{-i}^{Ci}(x, f, y^*, t) = = \frac{(at - x_i)^2 - f_{-i}^2}{4}. \tag{11}$$

[13] Nash equilibrium in pure strategies.

(CB) **Both firms are constrained** if they cannot play their unconstrained best reply given the other firm produces at capacity. This holds true, whenever

$$x_i \leq \tilde{y}_i^{BR}(x_{-i}; f, t), \quad i = 1, 2.$$

We denote by $F^{CB}(x,t)$ the set of all f for which both inequalities are satisfied at (x,t). For all $f \in F^{CB}(x,t)$, equilibrium quantities at stage three are $y_i^*(x,f,t) = x_i$. Equilibrium profits are

$$\pi_i^{CB}(x, f, y^*, t) = (at - x_i - x_{-i})x_i, \quad i = 1, 2. \tag{12}$$

Stage II Now we derive all subgame perfect equilibria of the parameterized subgames starting at stage two. Again, given investment levels and equilibrium play at stage three, we can solve pointwise for the equilibria at stage two for each $t \in [0,T]$.

It is important to notice that uniqueness of the equilibrium at stage three implies that for each investment level x, the sets $F^{CB}(x,t), F^{Ci}(x,t), F^{C-i}(x,t)$, and $F^{CN}(x,t)$ partition the set $F = [0, x_i] \times [0, x_{-i}]$ of all feasible levels of forward trades given x. For each set, we can now characterize the subgame perfect equilibria (f^*, y^*). Within each set, any equilibrium leads to unique quantities y^* at stage three, that may, however, be supported by various quantities of forward contracts traded at stage two. Lemmas 1 to 3 state the equilibrium quantities, as well as the values of x for which an equilibrium exists in the different regions. The proofs are relegated to appendix B.

Lemma 1 **(No firm is constrained)**

(i) If $f^*(x,t) \in F^{CN}(x,t)$*, then* $y_i^*(f^*(x,t), x, t) = \frac{2at}{5}$*,* $i = 1, 2$ *(denoted* EQ^{CN}*).*[14]

(ii) EQ^{CN} *exists, if and only if* $x_i \geq (1 - \frac{2\sqrt{2}}{5})at =: \frac{at}{c_{2.3}} \approx \frac{at}{2.3}$*,* $i = 1, 2$.

[14] That is, any equilibrium in the unbounded region yields the solution found by Allaz and Vila (1993).

Lemma 2 **(One firm is constrained)**

(i) If $f^*(x,t) \in F^{Ci}(x,t)$*, then* $y_i^*(f^*(x,t),x,t) = x_i$ *and* $y_{-i}^*(f^*(x,t),x,t) = \frac{at-x_i}{2}$ *(denoted* EQ^{Ci}*).*

(ii) EQ^{Ci} *exists if and only if* $x_i < \frac{at}{2}$ *and* $x_{-i} \geq \frac{at-xi}{2}$.

Lemma 3 **(Both firms are constrained)**

(i) If $f^*(x,t) \in F^{CB}(x,t)$*, then* $y_i^*(f^*(x,t),x,t) = x_i$, $i = 1,2$ *(denoted* EQ^{CB}*).*

(ii) EQ^{CB} *exists if and only if* $x_i \leq \frac{at-x_{-i}}{2}$, $i = 1,2$.

Lemmas 1 to 3 enable us to determine which of the four possible equilibria exist for each given investment levels x. Note for example that for high investment levels ($x_i \geq \frac{at}{c_{2.3}}$, $i = 1,2$), the unconstrained equilibrium exists (lemma 1). However, if investments of bidder i are in that region but low enough ($\frac{at}{c_{2.3}} \leq x_i \leq \frac{at}{2}$), also EQ^{Ci} exists (lemma 2). Thus, for all $x_i \in [\frac{at}{2}, \frac{at}{c_{2.3}}]$ both equilibria exist, provided x_{-i} is high enough.

Figure 3 summarizes the results of lemmas 1 to 3. The figure shows for each possible combination of investment levels, which of the four possible types of equilibria exist.

In order to analyze all subgame perfect equilibria of the game it is necessary to determine the profit functions for all different choices of equilibria at stages two and three. This, however, seems to be impossible, since in regions with multiple equilibria for each t another equilibrium of the subgame starting at stage two can be chosen. Moreover, the selection of equilibria of the continuation game may depend on the history of the game, that is, on x. Let us define

Definition 1 (σ-subgame perfect equilibrium, $SPE(\sigma)$) *A σ-subgame perfect equilibrium is a subgame perfect equilibrium of the three stage game where in every small interval* $[t, t+\delta]$, $\delta \to 0$*, the equilibrium preferred by firm i has share σ and the equilibrium preferred by firm $-i$ has share* $1-\sigma$.

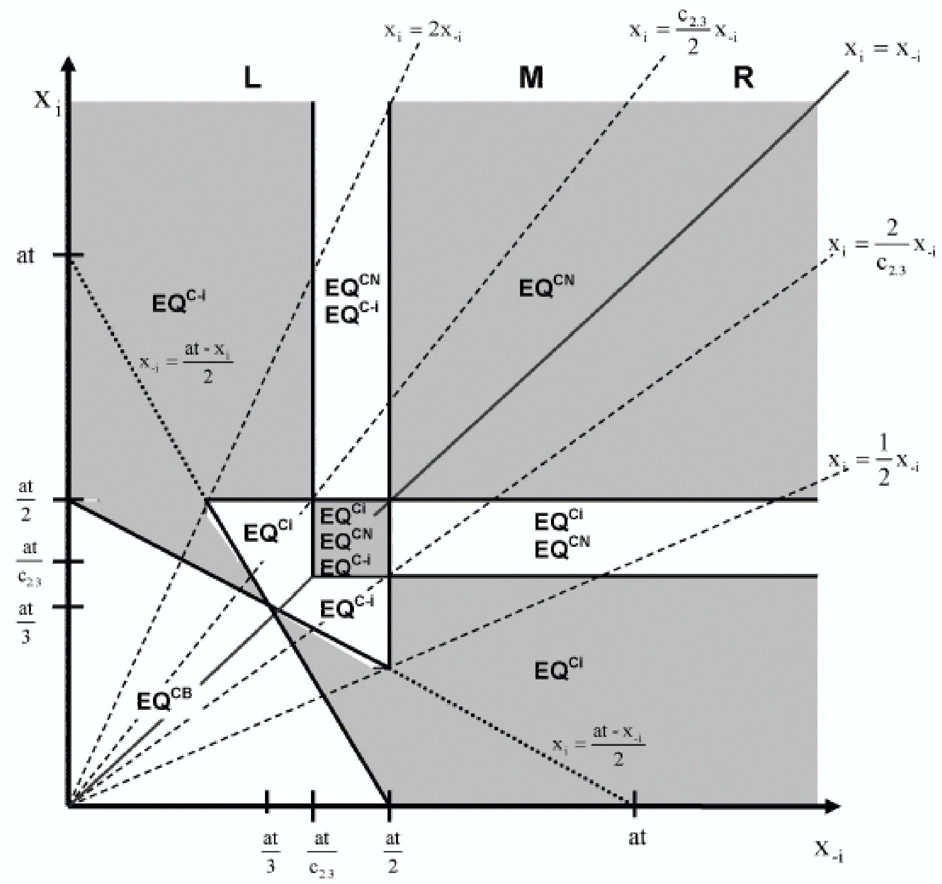

Figure 3: Subgame perfect equilibria of the parameterized subgames starting at stage two.

In the following we consider only the $SPE(\sigma)$ of the market game with forward contracts. This excludes any equilibrium where the choice of equilibria at stages two and three depends on choices of x, or on t. We make this restriction mainly for tractability reasons. However, with the parametrization chosen it should be possible to approximate a huge variety of plausible operational markets. Let us give two examples for equilibria that are covered by this formulation: (1) The equilibrium preferred by one of the firms is always played (e.g. because that firm has the commitment power to enforce this). (2) The equilibrium is chosen randomly with probabilities σ, $1 - \sigma$ at each t where multiple equilibria exist.

As we mentioned in section 2, we do not need the assumption that firms decide on $y(t)$ prior to $t = 0$. We can also allow for the choice of load curves prior to a finite number of time intervals. Note that the spot market equilibrium $y^*(x, f, t)$ is unique for all t and thus, is the only equilibrium play satisfying subgame perfection if load curves are chosen repeatedly (but forwards for all t are chosen prior to $t = 0$). In general this does not hold true for the choice of forward quantities. Here multiplicity of equilibria leaves scope for credible threats that may support outcomes other than f^*, y^* for some $t \in [0, T]$. However, the σ-subgame perfect equilibria we consider in the following do not allow for conditioning on past equilibrium outcomes. Thus, all equilibria covered by this concept are also equilibria of the game where forwards are chosen repeatedly prior to a finite number of time intervals.[15]

4.2 Equilibrium Investments

Stage I Now that we have determined the equilibria of the subgames starting at stage two for all possible capacities, we can turn towards solving the subgame perfect equilibria of the market game with forward contracts. Figure 3 depicts the areas of existence of the different types of equilibria for a given value of t. A firm's profit from given levels of investments, x, is the integral over equilibrium profits at each t given x on the domain $[0, T]$.

Note that for any possible investment levels $x > 0$ if t is close enough to zero, both firms are unconstrained in equilibrium. Thus, any $x > 0$ gives rise to the unconstrained equilibrium in this case. An increase of t corresponds to a dilation of all regions outwards with center zero. Observe furthermore that in the three slices L, M, and R, different types of equilibria exist and that also their sequence is different. Thus, the exact form of the profit function

[15]Finally note that conditioning on past outcomes does not make sense in the present model since demand realizations are ordered. Thus, the evolution of the game over time is meaningless. The model would have to be substantially modified in order to analyze those issues.

depends on the location of the investment levels x.

Suppose for example that we want to determine bidder i's profit $\pi_i(x, f^*, y^*)$ from a given pair of investment levels x, where $x_i > 2x_{-i}$. That is, we have to integrate parameterized equilibrium profits of the subgames starting at stage two from $t = 0$ to $t = T$ given that x is located in the lower left area of figure 3 (region L). In case both firms are contrained at the highest demand realization, the profit function looks as follows:

$$\begin{aligned}\pi_i^{\underline{L}}(x, f^*, y^*, d) = &\int_0^{\frac{2x_{-i}}{a}} \pi^{CN}(x, f^*, y^*, t)dt + \sigma \int_{\frac{2x_{-i}}{a}}^{\frac{c_{2.3}x_{-i}}{a}} \pi^{CN}(x, f^*, y^*, t)dt \\ &+ (1-\sigma) \int_{\frac{2x_{-i}}{a}}^{\frac{c_{2.3}x_{-i}}{a}} \pi^{C-i}(x, f^*, y^*, t)dt + \int_{\frac{c_{2.3}x_{-i}}{a}}^{\frac{2x_i+x_{-i}}{a}} \pi^{C-i}(x, f^*, y^*, t)dt \\ &+ \int_{\frac{2x_i+x_{-i}}{a}}^{T} \pi^{CB}(x, f^*, y^*, t)dt - kx_i.\end{aligned} \tag{13}$$

Starting from $t = 0$, any $x > 0$ lies in the region where only EQ^{CN} exists. Thus, the relevant profit for low values of t is $\pi^{CN}(x, f^*, y^*, t)$ as given by equation (8). That region is left when $x_{-i} = \frac{at}{2}$ (see figure 3), or equivalently, $t = \frac{2x_{-i}}{a}$. This explains the upper limit of the first integral.

As t becomes larger than $\frac{2x_{-i}}{a}$ we enter into a region where multiple equilibria (of type EQ^{CN} and EQ^{C-i}) exist. Obviously, different selections of equilibria of the continuation games played at each t in such a region yield different equilibrium capacity choices at stage one. The parameter σ determines which of the equilibria of the subgame starting at stage two is selected at the operating stages. Firm i prefers EQ^{CN} and thus, receives share σ of the corresponding profit π_i^{CN}. The other firm prefers EQ^{C-i} which is why firm i receives share $1 - \sigma$ of the corresponding profit π_i^{C-i}.

As t increases beyond $\frac{c_{2.3}x_{-i}}{a}$, first only EQ^{C-i} exists and finally, for high values of t, both firms are constrained, i. e. they play EQ^{CB}. This explains the fourth and fifth integral of equation (13).[16]

[16]Capacity choices in region $\underline{L}$ (see figure 4)lead to a situation where both firms are constrained at the highest demand realization. This is the case described here. For investment levels in region $\overline{L}$, x is never inside the region CB, such that the last integral (or the two or four last integrals) have to be dropped. See also footnote 10.

Note that in the remaining regions, M and R the profit function looks different since the sequence of the areas of existence of the different types of equilibria is different (see figure 3). In appendix C we derive the profit functions for all three regions. We obtain a parameterized profit function $\pi_i(x, f^*, y^*, \sigma)$ that is continuous at all x, but not everywhere differentiable. From this profit function we derive a continuous but not everywhere differentiable upper bound for firm i's best reply function $\bar{x}_i^{BR}(x_{-i}, f^*, y^*, \sigma)$.

Now we can compare investment levels in the two market games (with and without forward trading) by comparing $\bar{x}_i^{BR}(x_{-i}, f^*, y^*, \sigma)$ with the isoinvestment line I^{NF} in the market without forward contracts defined by equation (4). If the best reply function lies below the isoinvestment line for all $x_i \geq x_{-i}$, no equilibrium of the game with forward contracts can yield higher total investment than the game without forward contracts. The result is summarized in the following

LEMMA 4 *The best reply function of firm i at stage one,* $x_i^{BR}(x_{-i}, f^*, y^*, \sigma)$, *yields* $x_i^{BR}(x_{-i}) + x_{-i} < x_j + x_{-j}$ *for all* $(x_j, x_{-j}) \in I^{NF}$ *whenever* $x_i^{BR}(x_{-i}, f^*, y^*, \sigma) \geq x_{-i}$.

For a detailed proof see appendix C.

Figure 4 illustrates the lemma. It depicts the isoinvestment line I^{NF} in the case without forward markets, as well as (in the region above the 45-degree line) the upper bound of firm i's best reply in the presence of forward markets, $\bar{x}_i^{BR}(x_{-i}, f^*, y^*, \sigma)$. As the latter always lies below the isoinvestment line in absence of forward trading, we can conclude:

THEOREM 1 *Every* $SPE(\sigma)$ *of the market game with forward contracts gives rise to strictly less total investment than the unique equilibrium of the game without forward contracts.*

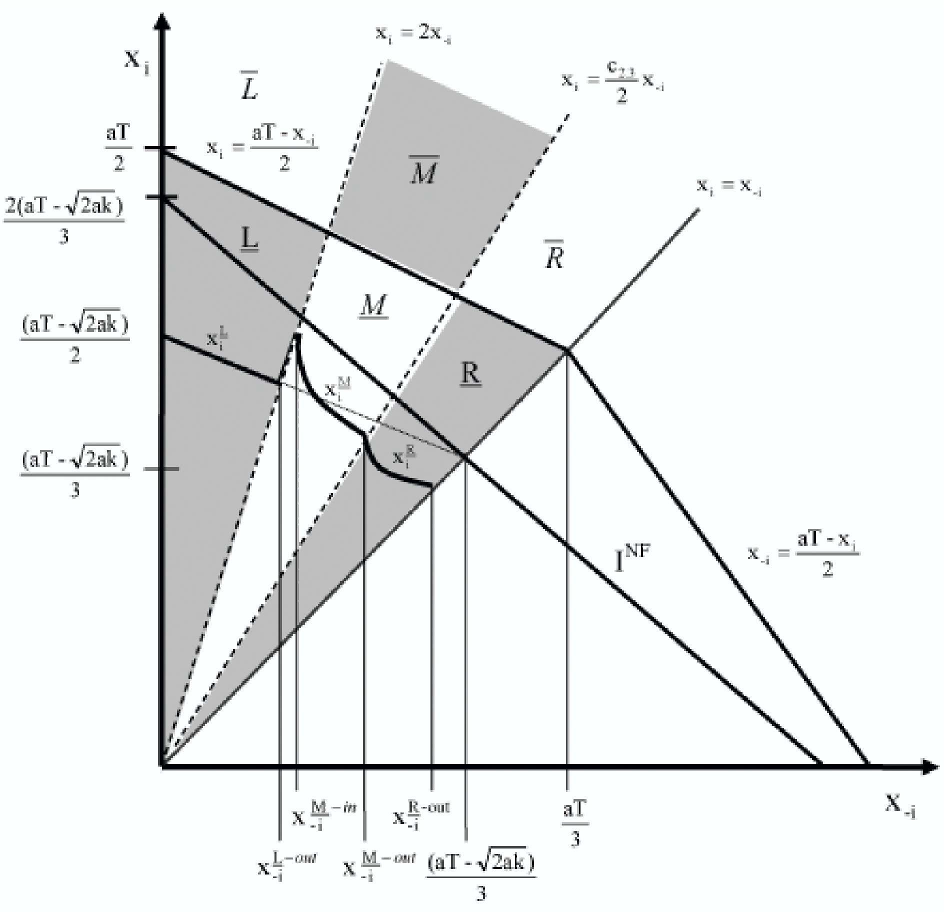

Figure 4: The upper bound of firm i's best reply function, $x_i^{BR}(x_{-i}, f^*, y^*, \sigma)$, and the isoinvestment line I^{NF}.

5 Concluding Remarks

In this paper we analyzed a market game where firms choose capacities prior to a sequence of Cournot markets. We compared the game with and without the possibility to trade on forward markets prior to the production stages. The analysis was considerably complicated by the fact that multiple equilibria exist in the market game with forward contracts. In order to be able to compare equilibrium investments we considered a class of parameterized subgame perfect equilibria of the game, $SPE(\sigma)$, which allowed us to approximate many reasonable operational markets. We found that every $SPE(\sigma)$ yields lower equilibrium investment than the game without forward contracts.

This result contributes to an ongoing policy debate on electricity market design. Low generation (and transmission) capacities put the general functioning of energy markets at risk. Thus, capacity investment incentives under a certain market design are an important issue. In many countries, a further reduction of generation capacity is considered undesirable in the long run.

The model we propose could be reinterpreted in terms of demand uncertainty. In this case the model also covers the case of investment decisions prior to a single Cournot market with uncertain demand at the last stage.[17] In this interpretation, the real state of the world is revealed directly after the investment decision. Forward contracts and quantities are traded under complete information about the demand scenario. An interesting extension of this model is the case where on the spot market firms still face demand uncertainty. This, however, would imply supply function bidding (Klemperer and Meyer (1989)) at the last stage, which presumably further complicates the analysis.

Another issue that could be analyzed in our model is the welfare effect of the introduction of forward markets. In the absence of a capacity stage, forward markets reduce market power and therefore increase welfare. In the presence of capacity choices this might no longer be true. However, the welfare effect is ambiguous. On the one hand, with forward trading production will be higher in low demand scenarios when firms are unconstrained. In high demand scenarios, however, production is lower in the presence of forward markets since capacities are lower. A welfare comparison is a possible extension of this paper but will be complicated by the multiplicity of equilibria of the market game with forward contracts.

[17] The case we analyzed would correspond to a uniform distribution of demand for T=1.

6 References

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A Proof of Proposition 1.

In section 3 we have already analyzed the last stage of the game, where firms decide on production levels. At the first stage, firms choose capacities, anticipating optimal production decisions at the second stage. In the following we first derive the firms' best response functions at stage one (part I), Then (part II) we solve the equilibrium of the game and show uniqueness.

Part I First we determine the best response function of firm i.
(a) Region $\underline{U} = \{x \in \mathbb{R}^2_+ : x_i \geq x_{-i} \,\text{and}\, x_i \leq \frac{aT - x_{-i}}{2}\}$: In this region firm i has the higher capacity and both firms are capacity constrained at the highest possible demand realization. The first order condition of firm i's maximization problem (see equation (2) for firm i's profit function $\pi_i^{\underline{U}}$) is satisfied at

$$x_i^{\max,\min}(x_{-i}) = \frac{aT - x_{-i} \mp \sqrt{2ak}}{2},$$

where $x_i^{\max}(x_{-i}) = \frac{aT - x_{-i} - \sqrt{2ak}}{2}$ is the local maximum and $x_i^{\min}$ the local minimum.

As firm i increases its quantity, the upper bound $\frac{aT - x_{-i}}{2}$ is reached before the profit function attains its local minimum at $x_i^{\min}$. Since the (cubic) function π_i^h increases towards ∞ only for values of x_i above this local minimum, we obtain that $\pi_i^{\underline{U}}$ attains its maximum in region $\underline{U}$ at

$$x_i^{\underline{U}}(x_{-i}) = \frac{aT - x_{-i} - \sqrt{2ak}}{2} \tag{14}$$

for $0 \leq x_{-i} \leq x_{-i}^{\underline{U}-out}$, where $x_{-i}^{\underline{U}-out} = \frac{aT-\sqrt{2ak}}{3}$ is the value of x_{-i} where $x_i^{\underline{U}}(x_{-i})$ hits the righthandside border of region $\underline{U}$ (given by $x_i = x_{-i}$, see figure 2).

Region $\underline{D} = \{x \in \mathbb{R}_+^2 : x_i \leq x_{-i} \,\text{and}\, x_i \leq aT - 2x_{-i}\}$. In this region firm i has the higher capacity and both firms are constrained at the highest demand realization, i. e. $x_{-i} \leq \frac{aT-x_i}{2}$. Firm i's profit function in this case is given by equation (3). By the same reasoning as above we obtain for the maximum of $\pi^{\underline{D}}$ in region $\underline{D}$

$$x_i^{\underline{D}}(x_{-i}) = \max\left\{0, \frac{2aT - 2x_{-i} - \sqrt{6ak + a^2T^2 - 2aTx_{-i} - 2x_{-i}^2}}{3}\right\} \quad (15)$$

for $x_{-i}^{\underline{D}-in} \leq x_{-i} \leq x_{-i}^{\underline{D}-out}$, where $x_{-i}^{\underline{D}-in} = \frac{aT-\sqrt{2ak}}{3}$ and $x_{-i}^{\underline{D}-out} = \min\{\frac{aT}{2}, \frac{aT+\sqrt{12ak+a^2T^2}}{6}\}$. Again, $x_i^{\underline{D}-in}$ ($x_i^{\underline{D}-out}$) is the value of x_{-i} where $x_i^{\underline{D}}(x_{-i})$ hits the lefthandside (righthandside) border of region $\underline{D}$ given by $x_i = x_{-i}$ and $x_{-i} = \frac{aT-x_i}{2}$, respectively (see figure 2).

Region $\overline{D}^I = \{x \in \mathbb{R}_+^2 : x_i \geq aT - 2x_{-i} \,\text{and}\, x_i \leq \frac{aT}{3}\}$: We finally consider the case that firm i has the higher capacity and firm $-i$ always has excess capacity even at the highest demand realization, whereas firm i is constrained at least in the highest demand scenario .

In this region, the profit of firm i is given by equation (3), however, EQ^{CB} cannot occur in this case. Since in region $\overline{D}^I$ it holds that $\frac{2x_{-i}+x_i}{a} > T$, we have to drop the last integral and substitute the upper limit of the second integral by T. We obtain

$$\begin{aligned}
\pi_i^{\overline{D}^I}(x, y^*) &= \int_0^{\frac{3x_i}{a}} \left(\frac{at}{3}\right)^2 dt + \int_{\frac{3x_i}{a}}^{T} \left(\frac{at - x_i}{2}\right) x_i \, dt - kx_i \\
&= \frac{x_i(a^2T^2 + x_i^2 - 2a(2k + Tx_i))}{4a} + \frac{x_i \, x_{-i}^2}{a} - kx_i.
\end{aligned}$$

The function $\pi_i^{\overline{D}^I}$ attains its maximum[18] at

$$x_i^{\overline{D}^I}(x_{-i}) = \max\{0, \frac{2aT - \sqrt{12ak + a^2T^2}}{3}\} \quad (16)$$

[18] Again the the first order condition is satisfied at the local maximum and the local minumum. Since we reach the upper bound of region $\overline{D}^I$ however before the local minimum is reached the solution to the first order condition gives the global maximum in region $\overline{D}^I$.

for $x_{-i}^{\overline{D}^I-in} \leq x_{-i}$, where $x_{-i}^{\overline{D}^I-in} = \min\{\frac{aT}{2}, \frac{aT+\sqrt{12ak+a^2T^2}}{6}\}$ is the intersection point of $x_i^{\overline{D}^I}(x_{-i})$ and the lefthandside border of region $\overline{D}^I$.

Remark 1 *For $k \geq \frac{aT^2}{4}$ it is always optimal for both firms to choose capacities such that at the highest demand realization T we obtain a spot market equilibrium where both firms are constrained. On the contrary for $k \leq \frac{aT^2}{4}$, whenever x_{-i} is big enough, no matter how big the capacity installed by firm $-i$ is, it is always optimal to build up the constant amount $0 < x_i^{\overline{D}^I} < \frac{aT}{3}$.*

(b) It is important to notice that the equations (14), (15) and (16) form a continuous line. Also recall that the overall profit function is continuous. Thus, the continuous function given by equations (14), (15), and (16) determines the profit maximizing capacity choices over all three regions

$$\underline{U} \cup \underline{D} \cup \overline{D}^I := \left\{ x \in \mathbb{R}_+^2 : \begin{array}{ll} x_i \leq \frac{aT-x_{-i}}{2} & \text{for } 0 \leq x_{-i} \leq \frac{aT}{3} \\ x_i \leq \frac{aT}{3} & \text{for } \quad x_{-i} \geq \frac{aT}{3} \end{array} \right\} \tag{17}$$

(c) It remains to show that deviations outside the region $\underline{U} \cup \underline{D} \cup \overline{D}^I$ are not profitable for firm i, i. e. that equations (14), (15), and (16) determine the locus of $\arg\max_{x_i \geq 0} \pi_i(x_i, x_{-i})$.

We have to distinguish three different cases:

(I) Region $\overline{U}^I = \{x \in \mathbb{R}_+^2 : x_{-i} \leq \frac{aT}{3} \text{ and } x_i > \frac{aT-x_{-i}}{2}\}$: The profit of firm i is given by equation (2), dropping its last integral,

$$\pi_i^{\overline{U}^I}(x, y^*) \quad = \quad \int_0^{\frac{3x_{-i}}{a}} \left(\frac{at}{3}\right)^2 dt + \int_{\frac{3x_{-i}}{a}}^T \left(\frac{at - x_{-i}}{2}\right)^2 dt - kx_i \tag{18}$$

$\pi_i^{\overline{U}^I}(x, y^*)$ is a linear function in x_i and attains its maximum at the lowest possible value, making a deviation into this region undesirable.

(II) Region $\overline{U}^{II} = \{x \in \mathbb{R}_+^2 : x_{-i} \geq \frac{aT}{3} \text{ and } x_i > x_{-i}\}$: The profit of firm i is given by equation (2), dropping its last two integrals. This profit depends on x_i only through the term $-kx_i$. Thus, it attains its maximum at the lowest possible value of x_i, making a deviation into this region undesirable.

(III) Region $\overline{D}^{II} = \{x \in \mathbb{R}^2_+ : x_i \geq \frac{aT}{3}$ and $x_i < x_{-i}\}$: The profit of firm i is given by equation (3), dropping its last two integrals. The profit depends on x_i only through the term $-kx_i$. Thus, the function attains its maximum at the lowest possible value of x_i, making a deviation into this region undesirable.

Summing up, the best response function of firm i is given by

$$x_i^{BR}(x_{-i}) = \begin{cases} x_i^U(x_{-i}) & \text{for} \quad 0 \leq x_{-i} \leq \frac{aT-\sqrt{2ak}}{3} \\ x_i^D(x_{-i}) & \text{for} \quad \frac{aT-\sqrt{2ak}}{3} \leq x_{-i} \leq \min\{\frac{aT}{2}, \frac{aT+\sqrt{12ak+a^2T^2}}{6}\} \\ x_i^{\overline{D}^I}(x_{-i}) & \text{for} \quad \min\{\frac{aT}{2}, \frac{aT+\sqrt{12ak+a^2T^2}}{6}\} \leq x_{-i} \end{cases} \quad (19)$$

for the parameter values $a > 0$, $T > 0$, and $k \in [0, \frac{aT^2}{2}]$.[19]

Part II Now we can determine all equilibria (x_i^*, x_{-i}^*) of the market game without forward contracts. We assume without loss of generality that $x_i \geq x_{-i}$. (x_i^*, x_{-i}^*) is an equilibrium if and only if (x_i^*, x_{-i}^*) is a fixed point of the best reply correspondence, i. e. it satisfies the following two equations:

$$x_i = \frac{aT - x_{-i} - \sqrt{2ak}}{2} \qquad \Leftrightarrow \qquad x_{-i} = aT - 2x_i - \sqrt{2ak} =: g(x_i), \quad (20)$$

$$x_{-i} = \max\left\{0, \frac{2aT - 2x_i - \sqrt{6ak + a^2T^2 - 2aTx_i - 2x_i^2}}{3}\right\} =: h(x_i). \quad (21)$$

At $x_i = x_{-i} = \frac{aT-\sqrt{2ak}}{3}$ both equations are satisfied and thus, we have a symmetric equilibrium. For $x_i > x_{-i}$ however, $g(x_i)$ decreases with slope -2, whereas $h(x_i)$ changes at the smaller rate

$$\frac{dh}{dx_i} = -\frac{2}{3} + \frac{aT + 2x_i}{3\sqrt{6ak + a^2T^2 - 2aTx_i - 2x_i^2}} \qquad \left(> -\frac{2}{3} \forall\, a, T, k \right),$$

for all x_i such that $h(x_i) > 0$ and remains constant otherwise. Thus, for $x_i > x_{-i}$ no further equilibrium exists. We conclude that

$$x_i = \frac{aT - \sqrt{2ak}}{3}, \qquad i = 1, 2$$

[19] Investment in the market is profitable only if $k < \frac{aT^2}{2}$. At higher cost it would not even be profitable to invest for a monopolist ($x_{-i} = 0$).

is the unique subgame-perfect equilibrium of the market game without forward contracts. The result is illustrated in figure 2.

B Proofs of lemmas 1 to 3

B.1 Proof of Lemma 1:

Part I We first show that any equilibrium EQ^{CN}, if it exists, is given by $f_i^*(\cdot) = \frac{1}{5}at,\ y_i^*(\cdot) = \frac{2}{5}at,\ i = 1,2.$

Suppose that $(\breve{f}^*, \breve{y}^*)$ is an equilibrium and that $\breve{f}^* \in F^{CN}(x,t)$. Thus, we know from section 4.1 that at the third stage we have the unique solution $\breve{y}_i^*(x, \breve{f}^*, t) = \frac{at+2\breve{f}_i^*-\breve{f}_{-i}^*}{3}$, $i = 1,2$. Since $F^{CN}(x,t)$ is an open set, $\breve{f}_i^*$ is a maximizer of $\pi_i(x, f_i, \breve{f}_{-i}^*, \breve{y}^*, t)$ in some neighborhood of $\breve{f}_i^*$.

Since the profit function of the game without capacity constraints $\pi_i^\infty(f, \breve{y}^*, t) = \pi_i(x_i = \infty, x_{-i} = \infty, f, \breve{y}^*, t)$ is concave in f_i (compare equation (8) and Allaz and Vila (1993)), $\breve{f}_i^*$ is also the global maximizer for all $f_i \geq 0$. Consequently, $(\breve{f}^*, \breve{y}^*)$ is the unique equilibrium of the unrestricted game, which according to Allaz and Vila (1993) has the unique solution $(f_i^* = \frac{1}{5}at,\ y_i^* = \frac{2}{5}at)$.

Part II Conditions for existence of the equilibrium $f_i^*(\cdot) = \frac{1}{5}at$, $y_i^*(\cdot) = \frac{2}{5}at$, $i = 1,2$:

(a) First note that $(f_i^*, f_{-i}^*) = (\frac{1}{5}at, \frac{1}{5}at) \in F^{NC}(x,t)$ if and only if $x_i > \frac{2}{5}at$, $i = 1,2$.

(b) However, depending on the capacity choices at stage one, $f_i = \frac{1}{5}at$ might not be the profit maximizing choice of firm i given that firm $-i$ chooses $f_{-i} = \frac{1}{5}at$. Recall that for $f_i = \frac{1}{5}at$, $i = 1,2$, none of the firms is constrained at the production stage. Now observe that, given that firm $-i$ chooses $f_{-i} = \frac{1}{5}at$, by varying the number of forward contracts traded, firm i can provoke a situation where either of the two firms is constrained. The corresponding profits and forward contracts traded are as follows:

$$\pi_i(f_i, f^*_{-i}, \cdot) = \begin{cases} \pi_i^{C-i}(\cdot) = \frac{(at-x_{-i})^2-f_i^2}{4} & \text{for} \quad 0 \leq f_i \leq \frac{7}{5}at - 3x_{-i} \quad (F^{C-i}) \\ \pi_i^{CN}(\cdot) = \frac{(\frac{4}{5}at-f_i)(\frac{4}{5}at+2f_i)}{9} & \text{for} \quad \frac{7}{5}at - 3x_{-i} \leq f_i \leq \frac{3}{2}x_i - \frac{2}{5}at \quad (F^{CN}) \\ \pi_i^{Ci}(\cdot) = \frac{x_i(\frac{4}{5}at-x_i)}{2} & \text{for} \quad \frac{3}{2}x_i - \frac{2}{5}at \leq f_i \leq x_i \quad (F^{Ci}) \end{cases}$$

Note that the above profits correspond to the profits that have been derived in section 4.1 for the cases CN (no firm is constrained) and Ci, $C-i$ (firm $i/-i$ is constrained). Furthermore note that if condition (a), $x_i \geq \frac{2}{5}at$, $i = 1, 2$, is satisfied, the region where none of the firms is constrained cannot disappear. That is, given that firm $-i$ chooses $f_{-i} = \frac{1}{5}at$, firm i can always sell forwards such that both firms are unconstrained at stage three.

Now observe that the unconstrained equilibrium quantities at stage three, $y^*_i(x, f_i, f^*_{-i}, t) = \frac{at+2f_i-f^*_{-i}}{3}$, $i = 1, 2$, imply that if firm i trades less forwards, its quantity sold at stage three decreases, whereas the quantity sold by firm $-i$ increases. Thus, if firm $-i$'s capacity is sufficiently low, a low quantity of forwards traded by firm i can provoke a situation where firm $-i$ is capacity constrained at stage three. This happens if firm $-i$'s capacity x_{-i} is lower than its unconstrained equilibrium quantity $\tilde{y}^*_{-i}(x, f_i, f^*_{-i}, t) = \frac{at+2f^*_{-i}-f_i}{3}$ (see equation (9)). Solving for the corresponding value of f_i yields $f_i \leq \frac{7}{5}at - 3x_{-i}$. Thus, for $f_i \in [0, \frac{7}{5}at - 3x_{-i}]$, $(f_i, f^*_{-i}) \in F^{C-i}(x, t)$. Obviously, firm i can only provoke this situation if x_{-i} is low enough, i. e. $x_{-i} \in [\frac{2}{5}at, \frac{7}{15}at]$.

A similar reasoning explains the case that $(f_i, f^*_{-i}) \in F^{Ci}(x, t)$. Obviously, this case can only occur if firm i's capacity is low enough, i. e. $x_i \leq \frac{4}{5}at$.

It is easy to check that the above profit function π_i is continuous. Thus, since π_i^{Ci} is a constant, deviation upwards, $f_i > f^*_i$, is never profitable. Furthermore, π_i has two local maxima, one at $f^*_i = \frac{1}{5}at$ and another one at $f^0_i = 0$. Obviously f^* is an equilibrium if and only if f^*_i is the global maximum of $\pi_i(f_i, f^*_{-i})$ which is the case iff

$$\begin{aligned} \pi_i^{CN}(f^*_i, f^*_{-i}) = \frac{2}{25}(at)^2 \quad &\geq \quad \frac{1}{4}(at - x_{-i})^2 = \pi_i^{C-i}(f^0_i, f^*_{-i}) \\ \Leftrightarrow \; x_{-i} \quad &\geq \quad at(1 - \frac{2}{5}\sqrt{2}) =: \frac{at}{c_{2.3}} \left(\approx \frac{at}{2.3}\right) \end{aligned}$$

We conclude that $(f_i^*(\cdot) = \frac{1}{5}at, y_i^*(\cdot) = \frac{2}{5}at)$, $i = 1, 2$, is a SPE of the parameterized subgames starting at stage two if and only if $x_i \geq \frac{at}{c_{2.3}}$, $i = 1, 2$.

B.2 Proof of Lemma 2:

Part I If there exists an equilibrium (f^*, y^*) such that $f^* \in F^{Ci}(x, t)$, then by construction it holds that $y_i^* = x_i$. The profit of firm $-i$ in this case is given by[20]

$$\pi_{-i}^{Ci}(x_i, f_{-i}, y^*, t) = \frac{(at - x_i)^2 - f_{-i}^2}{4},$$

which is maximized at $f_{-i}^* = 0$. Thus, in any equilibrium EQ^{Ci} it holds that $f_{-i}^* = 0$, which implies that firm $-i$'s equilibrium output at stage three is given by $y_{-i}^*(f_{-i}^*) = \frac{at - x_i}{2}$. This proves part (i) of the lemma.

Part II Let $f_i^* = x_i$, $f_{-i}^* = 0$, and $f_i' \in [0, x_i)$. We now show that if (f_i', f_{-i}^*, y^*), $(f_i', f_{-i}^*) \in F^{Ci}(x, t)$, is an equilibrium EQ^{Ci}, then also (f^*, y^*), $f^* \in F^{Ci}(x, t)$, is an equilibrium EQ^{Ci}.

We have already shown in part I that, given firm i produces at capacity, firm $-i$ always chooses $f_{-i}^* = 0$.

Now consider deviations of firm i. Since (f_i', f_{-i}^*, y^*) is an equilibrium, deviations $f_i \neq f_i'$ cannot be profitable. In particular, deviations $f_i \in (f_i', x_i]$ leave firm i's payoff unchanged, since increasing the quantity contracted forward leaves firm i constrained at stage three.

This implies that whenever (f_i', f_{-i}^*, y^*), $(f_i', f_{-i}^*) \in F^{Ci}(x, t)$, is an equilibrium EQ^{Ci}, then so is (f_i^*, f_{-i}^*, y^*).

Part III The findings of part I and II imply that whenever at least one equilibrium EQ^{Ci} of the parameterized subgames starting at stage two exists, $(f_i^*, f_{-i}^*, y^*) = (x_i, 0, y^*)$ is an equilibrium EQ^{Ci} (part II) and that all such equilibria give rise to the same quantities at the production stage (part I). We now establish necessary and sufficient conditions for the existence of at least one equilibrium EQ^{Ci}.

[20] see equation 11.

(a) First, we check whether $(f_i^*, f_{-i}^*) = (x_i, 0) \in F^{Ci}(x,t)$. In order to do so, we substitute $(f_i^*, f_{-i}^*) = (x_i, 0)$ into the inequalities (9). As it turns out, $f^* \in F^{Ci}(x,t)$ whenever it holds that

$$x_i \leq at \quad \text{and} \quad x_{-i} \geq \frac{at - x_i}{2}.$$

In order to establish that (f^*, y^*) is indeed an equilibrium it remains to show that none of the firms wants to deviate from its quantity of forwards sold given the other firm's choice.

(b) Let us first consider deviations of firm $-i$. Since $f_{-i}^* = 0$, only deviation upwards is possible. Note that since $f_i = x_i$ firm i is committed to sell its whole capacity at stage three ($y_i = x_i$) and as we have already shown in part I, the best firm $-i$ can do is to stick to $f_{-i}^* = 0$.

(c) Now we consider deviations of firm i. Since $f_i^* = x_i$, only deviation downwards is possible, which can lead to $(f_i, f_{-i}^*) \in F^{CN}$.[21] Given that $f_{-i}^* = 0$, firm i's profit function is

$$\pi_i(f_i, f_{-i}^*, \cdot) = \begin{cases} \pi_i^{Ci}(\cdot) = \frac{x_i(at-x_i)}{2} & \text{for } \frac{3x_i - at}{2} \leq f_i \leq x_i \quad (F^{Ci}) \\ \pi_i^{CN}(\cdot) = \frac{(at-f_i)(at+2f_i)}{9} & \text{for } 0 \leq f_i \leq \frac{3x_i-at}{2} \quad (F^{CN}) \end{cases}$$

It is easy to check that π_i is continuous at $f_i = \frac{3x_i - at}{2}$. Furthermore note that $\pi_i^{Ci}(f_i, f_{-i}^*)$ is a constant and $\pi_i^{CN}(f_i, f_{-i}^*)$ is a quadratic function reaching its maximum at $f_i = \frac{at}{4}$. This implies that a deviation of firm i such that $(f_i, 0) \in F^{CN}(x,t)$ is profitable if and only if

$$\frac{at}{4} \leq \frac{3x_i - at}{2} \quad \Leftrightarrow \quad x_i \geq \frac{at}{2}.$$

Summing up, we obtain that $(f^*; y_i^*, y_{-i}^*) = (f^*; x_i, \frac{at - x_i}{2})$, $i = 1, 2$, is a SPE of the parameterized subgames starting at stage two if and only if $x_{-i} \geq \frac{at-x_i}{2}$ [from (a)] and $x_i < \frac{at}{2}$ [from (c)].

[21] Note that for $x_1 \leq \frac{1}{3}at$ (which is the unconstrained Cournot quantity) deviation into F^{CN} is impossible.

B.3 Proof of Lemma 3:

Part (i) is satisfied by construction since $f^* \in F^{CB}(x,t)$. In order to prove part (ii), take any $\breve{f}_i > 0$, $\breve{f}_{-i} > 0$ such that $(\breve{f}_i, \breve{f}_{-i}) \in F^{CB}(x,t)$.

Given $\breve{f}_{-i}$, firm i's profit function $\pi_i(f_i, \breve{f}_{-i}, \cdot)$ is[22]

$$\pi_i(f_i, \breve{f}_{-i}, \cdot) = \begin{cases} \pi_i^{C-i}(\cdot) = \frac{(at-x_{-i})^2 - f_i^2}{4} & \text{for} \quad 0 \le f_i \le 2x_i + x_{-i} - at \ \ (F^{C-i}) \\ \pi_i^{CB}(\cdot) \ = (at - x_i - x_{-i})x_i & \text{for} \quad 2x_i + x_{-i} - at \le f_i \le x_i \ \ (F^{CB}) \end{cases}$$

Notice that π_i is continuous at $f_i = 2x_i + x_{-i} - at$ and that π_i^{CB} is constant in f_i. It is easy to see that deviation to $f_i = 0$ is always profitable for firm i whenever it leads to $(f_i = 0, \breve{f}_{-i}) \in F^{C-i}$. Such a deviation is impossible however if $2x_i + x_{-i} - at \le 0$. Accordingly $(\breve{f}_i, \breve{f}_{-i})$ is an equilibrium if and only if

$$2x_i + x_{-i} - at \le 0 \qquad \Leftrightarrow \qquad x_i \le \frac{at - x_{-i}}{2}, \qquad i = 1, 2.$$

C Proof lemma 4

The proof proceeds as follows. In part I we consider the set of investment levels where both firms are constrained at the highest demand realization. Within this set we derive the investment level x_i of firm i that maximizes firm i's profit given an investment level x_{-i} of firm $-i$. In part II we show that the function derived in part I is an upper bound for the best response of firm i to a given investment level of firm $-i$. Finally, in part III we show that the upper bound of firm i's best response always lies below the isoinvestment line (equation(4)) that contains all investment levels that yield the same total capacity as the market game in absence forward markets. Throughout the proof we consider only investment levels such that $x_i \ge x_{-i}$, since this is sufficient to prove the lemma.

[22] Notice if firm i reduces f_i such that $(f_i, \breve{f}_{-i})$ exits F^{CB}, then for all values of f_i firm $-i$ will remain constrained, since firm $-i$ has even stronger incentives to increase it's output at stage three.

Part I As a first step, we consider the region where firm i's investment is higher than firm $-i$'s and both firms are constrained at the highest demand realization, that is $x_i(x_{-i}) \in \underline{U} = \{x \in \mathbb{R}^2_+ : x_i \geq x_{-i} \,\text{and}\, x_i \leq \frac{aT - x_{-i}}{2}\}$. Within this region, we derive the investment level x_i of firm i that maximizes firm i's profit given an investment level x_{-i} of firm $-i$. We have to proceed in three steps, since firm i's profit function looks differently in the three subregions $\underline{L}$, $\underline{M}$, and $\underline{R}$ (see figure 3).

Region $\underline{L} = \{x \in \mathbb{R}^2_+ : x_i \geq 2x_{-i}$ and $x_i \leq \frac{aT - x_{-i}}{2}\}$: Firm i's profit function $\pi_i^{\underline{L}}(x, f^*, y^*, \sigma)$ is given by equation (13). Note that differentiation of $\pi_i^{\underline{L}}(\cdot)$ leads to the same first order condition as differentiation of $\pi_i^{\underline{U}}$ (equation (2)) in the case without forward contracts (see appendix A). This is because all terms depending on x_i coincide for the two profit functions. Thus, $\pi_i^{\underline{L}}(\cdot)$ attains its maximum at

$$x_i^{\underline{L}}(x_{-i}) = \frac{aT - x_{-i} - \sqrt{2ak}}{2} \tag{22}$$

for $0 \leq x_{-i} \leq x_{-i}^{\underline{L}-out}$, where $x_{-i}^{\underline{L}-out} = \frac{1}{5}(aT - \sqrt{2ak})$ is the value of x_{-i} where $x_i^{\underline{L}}(x_{-i})$ intersects with the righthandside border of region $\underline{L}$, given by $x_i = 2x_{-i}$.

For values $x_i > x_i^{\underline{L}}(x_{-i})$, $\pi_i^{\underline{L}}$ is decreasing in x_i since the local minimum is located above the upper bound of region $\underline{L}$ given by $x_i = \frac{aT - x_{-i}}{2}$. Thus, for $x_{-i} > x_{-i}^{\underline{L}-out}$, the maximizer $x_i^{\underline{L}}$ in region $\underline{L}$ is given by its lower bound $x_i^{\underline{L}}(x_{-i}) = 2x_{-i}$.

Region $\underline{M} = \{x \in \mathbb{R}^2_+ : 2x_{-i} \geq x_i \geq \frac{c_{2.3}}{2} x_{-i}$ and $x_i \leq \frac{aT - x_{-i}}{2}\}$: The profit of firm i in region $\underline{M}$ is given by[23]

$$\begin{aligned}\pi_i^{\underline{M}}(x, f^*, y^*, \sigma) = & \int_0^{\frac{2x_{-i}}{a}} \pi^{CN}(\cdot)dt + \sigma \int_{\frac{2x_{-i}}{a}}^{\frac{c_{2.3}x_{-i}}{a}} \pi^{CN}(\cdot)dt + (1-\sigma) \int_{\frac{2x_{-i}}{a}}^{\frac{c_{2.3}x_{-i}}{a}} \pi^{C-i}(\cdot)dt \\ & + \int_{\frac{c_{2.3}x_{-i}}{a}}^{\frac{2x_i}{a}} \pi^{C-i}(\cdot)dt + \sigma \int_{\frac{2x_i}{a}}^{\frac{x_i+2x_{-i}}{a}} \pi^{Ci}(\cdot)dt + (1-\sigma) \int_{\frac{2x_i}{a}}^{\frac{x_i+2x_{-i}}{a}} \pi^{C-i}(\cdot)dt \\ & + \int_{\frac{x_i+2x_{-i}}{a}}^{\frac{2x_i+x_{-i}}{a}} \pi^{C-i}(\cdot)dt + \int_{\frac{2x_i+x_{-i}}{a}}^{T} \pi^{CB}(\cdot)dt - kx_i;\end{aligned}$$

The first order condition of firm i's maximization problem is satisfied at

$$\begin{aligned} x_i^{\underline{M}max}(x_{-i}) &= \frac{1}{2+\sigma}\left(aT - \sqrt{\phi(x_{-i}, \sigma, k)} - \left(1 - \frac{5\sigma}{4}\right)x_{-i}\right), \\ x_i^{\underline{M}min}(x_{-i}) &= \frac{1}{2+\sigma}\left(aT + \sqrt{\phi(x_{-i}, \sigma, k)} - \left(1 - \frac{5\sigma}{4}\right)x_{-i}\right), \end{aligned}$$

where $\phi(x_{-i}, \sigma, k) = 2ak + \frac{1}{2}\sigma\left(2ak - a^2T^2 + 7aTx_{-i} - \left(11 - \frac{5\sigma}{8}\right)x_{-i}^2\right)$.

Starting at $x_i = 0$, for a given x_{-i}, $\pi_i^{\underline{M}}$ increases until $x_i^{\underline{M}max}(x_{-i})$, then decreases until $x_i^{\underline{M}min}(x_{-i})$, and from there on increases to infinity. Thus, $x_i^{\underline{M}max}$ is the maximizer of $\pi_i^{\underline{M}}$ in region $\underline{M}$, whenever $x_i^{\underline{M}max} \neq x_i^{\underline{M}min}$ and $x_i^{\underline{M}max} \in \underline{M}$, whereas $x_i^{\underline{M}min}$ lies outside that region (in this case, $\pi_i^{\underline{M}}$ is quasiconcave in region $\underline{M}$). Unfortunately, this is not always the case. We start the analysis of $\arg\max_{x_i \leq \frac{aT - x_{-i}}{2}} \pi_i^{\underline{M}}$ by characterizing the case where the above holds true.

As a first step note that the locus where $x_i^{\underline{M}max}$ and $x_i^{\underline{M}min}$ coincide can be derived (setting $\phi(x_{-i}, \sigma, k) = 0$) as

$$\left\{x^S \in \mathbb{R}^2_+ : x_i^S = \frac{1}{2+\sigma}\left(aT - \left(1 - \frac{5\sigma}{4}\right)x_{-i}^S\right)\right\}.$$

For values $x_{-i} \leq x_{-i}^S$, $\pi_i^{\underline{M}}$ is monotonically increasing in x_i with a point of inflection at x_i^S. Thus, if $x^S \in \underline{M}$, the maximizer of $\pi_i^{\underline{M}}$ coincides with the upper bound of region $\underline{M}$ for $x_{-i} \leq x_{-i}^S$. Moreover, for all $x_{-i} > x_{-i}^S$, $x_i^{\underline{M}min}$ might be inside region $\underline{M}$ such that $\pi_i^{\underline{M}}$ is not even quasiconcave in region $\underline{M}$. As it turns out, whether x^S is in region $\underline{M}$ depends on the cost of investment,

[23]The profit in region $\underline{M}$ is derived analogously to the profit in region $\underline{L}$, see equation (13).

k. Furthermore, x_i^{Mmin} is outside region $\underline{M}$ whenever x^S is outside region $\underline{M}$.

Let us first determine the value of k for which x^S coincides with the lefthandside border of region two. Calculating the point of intersection of x^S with the lefthandside border given by $x_i = 2x_{-i}$ and inserting the value obtained for x_{-i} into $\phi(x_{-i}, \sigma, k)$ yields that $\phi = 0$ at $k = k^S := \frac{8\sigma+9\sigma^2}{2(20+3\sigma)^2} aT^2$. Since k^S is increasing in σ, x^S lies outside region $\underline{M}$ for all σ if $k \geq \frac{17}{2(23)^2} aT^2 := \frac{aT^2}{c_{62}} \approx \frac{aT^2}{62}$.

We now show that $\pi_i^{\underline{M}}$ is quasiconcave in x_i in region $\underline{M}$ for all $\sigma \in [0, 1]$ if $k \in [\frac{aT^2}{c_{62}}, \frac{aT^2}{2}]$.[24] This is the case if $x_i^{\underline{M}min}(x_{-i})$ is above region $\underline{M}$ for all x_{-i}. In order to verify this, notice that $x_i^{\underline{M}min}(x_{-i})$ crosses the lefthandside border of region $\underline{M}$ given by $x_i = 2x_{-i}$ at

$$x_{-i}^{min\underline{M}-in} = \frac{aT + \sqrt{2ak - \frac{a\sigma}{50}(aT^2 - 2k)}}{5 + \frac{\sigma}{10}} \quad \left(\geq \frac{aT}{5} \;\; \forall\, k, \sigma \right).$$

This is above the upper bound of region $\underline{M}$ given by $x_i = \frac{aT - x_{-i}}{2}$, which intersects the line $x_i = 2x_{-i}$ at $x_{-i} = \frac{aT}{5}$. Since $x_i^{\underline{M}min}$ increases in x_{-i} and since the upper bound of region $\underline{\mathrm{M}}$, $x_i = \frac{aT - x_{-i}}{2}$, decreases in x_{-i}, we obtain that $x_i^{\underline{M}min}$ is always above region $\underline{\mathrm{M}}$ for $\sigma \in [0, 1]$ and for $k \in [\frac{aT^2}{c_{62}}, \frac{aT^2}{2}]$. Thus, for $k \in [\frac{aT^2}{c_{62}}, \frac{aT^2}{2}]$ the maximum of $\pi_i^{\underline{M}}$ in region $\underline{M}$ is given by

$$x_i^{\underline{M}}(x_{-i}) = \frac{1}{2+\sigma} \left(aT - \sqrt{\phi(x_{-i}, \sigma, k)} - \left(1 - \frac{5\sigma}{4}\right) x_{-i} \right) \tag{23}$$

for $x_{-i}^{\underline{M}-in} \leq x_{-i} \leq x_{-i}^{\underline{M}-out}$, where

$$x_{-i}^{\underline{M}-in} = \frac{aT - \sqrt{2ak - \frac{a\sigma}{50}(aT^2 - 2k)}}{5 + \frac{\sigma}{10}},$$

$$x_{-i}^{\underline{M}-out} = \frac{aT - \sqrt{2ak + 0.056a\sigma(aT^2 - 2k)}}{(1 + c_{2.3}) - 0.18\sigma},$$

are the values of x_{-i} where $x_i^{\underline{M}}$ intersects with the lefthandside and righthandside border of region $\underline{\mathrm{M}}$ given by $x_i = 2x_{-i}$ and $x_i = \frac{c_{2.3}}{2} x_{-i}$, respectively.

[24] Recall that at cost $k > \frac{aT^2}{2}$ even a potential monopolist would not enter the market.

Notice that (22) and (23) do not form a continuous line, since $x_{-i}^{\underline{L}-out} < x_{-i}^{\underline{M}-in}$. Since $\pi_i^{\underline{M}}$ is quasiconcave in region $\underline{M}$, the values of x_i that maximize $\pi_i^{\underline{M}}$ for $x_{-i} < x_{-i}^{\underline{M}-in}$ are given by the lefthandside border of region $\underline{M}$.

For $k \in [0, \frac{aT^2}{c_{62}}]$, x^S lies inside region $\underline{M}$. Thus, for $x_{-i} \leq x_{-i}^S$, $x_i^{\underline{M}max}$ coincides with the lefthandside border of region $\underline{M}$. For values $x_{-i} > x_{-i}^S$, $x_i^{\underline{M}min}$ might be inside region $\underline{M}$, which makes corner solutions possible. Lemma 4 can also be shown to hold true for $k \in [0, \frac{aT^2}{c_{62}}]$. However, the proof requires a much heavier mathematical burden and will therefore be abandoned to a supplement that can soon be downloaded at merlin.fae.ua.es/grimm/.

Region $\underline{R} = \{x \in \mathbb{R}_+^2 : \frac{c_{2.3}}{2} x_{-i} \geq x_i \geq x_{-i}$ and $x_i \leq \frac{aT - x_{-i}}{2}\}$: The profit of firm i in region $\underline{R}$ is given by

$$\begin{aligned}
\pi_i^{\underline{R}}(x, f^*, y^*, \sigma) \quad = \quad & \int_0^{\frac{2x_{-i}}{a}} \pi^{CN}(\cdot)dt + \sigma \int_{\frac{2x_{-i}}{a}}^{\frac{2x_i}{a}} \pi^{CN}(\cdot)dt + (1-\sigma) \int_{\frac{2x_{-i}}{a}}^{\frac{2x_i}{a}} \pi^{C-i}(\cdot)dt \\
& + \sigma \int_{\frac{2x_i}{a}}^{\frac{x_i + 2x_{-i}}{a}} \pi^{Ci}(\cdot)dt + (1-\sigma) \int_{\frac{2x_i}{a}}^{\frac{x_i + 2x_{-i}}{a}} \pi^{C-i}(\cdot)dt \\
& + \int_{\frac{x_i + 2x_{-i}}{a}}^{\frac{2x_i + x_{-i}}{a}} \pi^{C-i}(\cdot)dt + \int_{\frac{2x_i + x_{-i}}{a}}^{T} \pi^{CB}(\cdot)dt - kx_i.
\end{aligned}$$

The first order condition of firm i's maximization problem is satisfied at

$$\begin{aligned}
x_i^{\underline{R}max}(x_{-i}) \quad &= \quad \frac{1}{2 - \frac{9}{25}\sigma} \left(aT - \sqrt{\psi(x_{-i}, \sigma, k)} - \left(1 - \frac{\sigma}{4}\right) x_{-i} \right) \\
x_i^{\underline{R}min}(x_{-i}) \quad &= \quad \frac{1}{2 - \frac{9}{25}\sigma} \left(aT + \sqrt{\psi(x_{-i}, \sigma, k)} - \left(1 - \frac{\sigma}{4}\right) x_{-i} \right),
\end{aligned}$$

where $\psi(x_{-i}, \sigma, k) = 2ak + \frac{\sigma}{50}(-18ak + 9a^2T^2 + 7aTx_1 - (91 - \frac{133\sigma}{8})x_1^2)$.

Similar to the analysis of region $\underline{L}$, we can show that we always reach the upper bound of region $\underline{R}$, $x_i = \frac{aT - x_{-i}}{2}$, before the local minimum $x_i^{\underline{R}min}$ of $\pi_i^{\underline{R}}$ is reached.[25]

Thus, in region $\underline{R}$, $\pi_i^{\underline{R}}$ attains its maximum at

$$x_i^{\underline{R}}(x_{-i}) = \frac{1}{2 - \frac{9}{25}\sigma} \left(aT - \sqrt{\psi(x_{-i}, \sigma, k)} - \left(1 - \frac{\sigma}{4}\right) x_1 \right) \tag{24}$$

[25] This can be checked for by evaluating the following inequality for all k, σ: $\frac{aT + \sqrt{2ak + 0.056a\sigma(aT^2 - 2k)}}{3.30 - 0.18\sigma} > \frac{aT}{c_{2.3} + 1}$, where the LHS is the x'_{-i} satisfying $x_i^{\underline{M}min}(x'_{-i}) = 2x'_{-i}$ and the RHS is the x''_{-i} satisfying $\frac{aT - 2x''_{-i}}{2} = 2x''_{-i}$. Furthermore $x_i^{\underline{R}min}(x_{-i})$ is increasing in x_{-i}, whereas the upper limit of Region $\underline{R}$ is decreasing in x_{-i}.

for $x_{-i}^{\underline{R}-in} \leq x_{-i} \leq x_{-i}^{\underline{R}-out}$, where

$$\begin{aligned} x_{-i}^{\underline{R}-in} &= \frac{aT - \sqrt{2ak + 0.056a\sigma(aT^2 - 2k)}}{(c_{2.3}+1) - 0.18\sigma}, \\ x_{-i}^{\underline{R}-out} &= \frac{aT - \sqrt{2ak + \frac{11}{450}a\sigma(aT^2 - 2ak)}}{3 - \frac{11}{150}\sigma} \end{aligned}$$

are the values of x_{-i} where $x_i^{\underline{R}}$ intersects with the lefthandside and righthandside border of region $\underline{\text{R}}$ given by $\frac{c_{2.3}x_{-i}}{2}$ and $x_i = x_{-i}$, respectively. Notice that $x_{-i}^{\underline{M}-out} = x_{-i}^{\underline{R}-in}$.

Summing up we can now state the maximizer over all three regions. Since π_i is continuous at all x, we obtain that the maximizer $x_i^{\underline{L}\cup\underline{M}\cup\underline{R}}(x_{-i})$ of π_i in the Region $\underline{L} \cup \underline{M} \cup \underline{R} \ = \{x \in \mathbb{R}_+^2 : x_i \geq x_{-i} \,\text{and}\, x_i \leq (\frac{aT-x_{-i})}{2}\}$ is given by

$$x_i^{\underline{L}\cup\underline{M}\cup\underline{R}}(x_{-i}) = \begin{cases} x_i^{\underline{L}}(x_{-i}) & \text{for} \quad 0 \leq x_{-i} \leq x_{-i}^{\underline{L}-out} \\ 2x_{-i} & \text{for} \quad x_{-i}^{\underline{L}-out} \leq x_{-i} \leq x_{-i}^{\underline{M}-in} \\ x_i^{\underline{M}}(x_{-i}) & \text{for} \quad x_{-i}^{\underline{M}-in} \leq x_{-i} \leq x_{-i}^{\underline{R}-in} \\ x_i^{\underline{R}}(x_{-i}) & \text{for} \quad x_{-i}^{\underline{R}-in} \leq x_{-i} \leq x_{-i}^{\underline{R}-out} \end{cases} \tag{25}$$

Part II In order to establish that $x_i^{\underline{L}\cup\underline{M}\cup\underline{R}}(x_{-i})$ is an upper bound for the best reply function of firm i it remains to show that deviations outside the region $\underline{L} \cup \underline{M} \cup \underline{R}$ are not profitable.

a) We first analyze deviation upwards, i. e. $x_i \geq \frac{aT-x_{-i}}{2}$.

For $x_i \geq 2x_{-i}$ the profit function is given by (13), adjusting, however, the limits of integration. Analogously to appendix A, part I(c), we have to drop the last integral if $x_i \geq \frac{aT-x_i}{2}$ and $x_{-i} \leq \frac{aT}{c_{2.3}}$, drop the last two integrals if $\frac{aT}{c_{2.3}} \leq x_{-i} \leq \frac{aT}{2}$, and drop the last four integrals if $\frac{aT}{2} \leq x_{-i}$. That is, region $\overline{L}$ divides into three different regions in the case of forward markets.

In all three cases the resulting profit of firm i depends on x_i only through the linear expression $-kx_i$, which makes it optimal for firm i to choose the lowest possible value of x_i in each region. Thus, a deviation into the region where one of the firms is unconstrained at the highest demand realization is undesirable.

Similarly it can be shown for the case $2x_{-i} \geq x_i \geq \frac{c_{2.3}}{2} x_{-i}$ (region M) and for the case $\frac{c_{2.3}}{2} \geq x_{-i} \geq x_{-i}$ (region R), that it is always optimal for firm i to choose the lowest possible value of x_i in each region. Thus, a deviation upwards is undesirable.

b) Finally we consider a deviation downwards, i. e. $x_i \leq x_{-i}$.
If deviation downwards for $0 \leq x_{-i} \leq x_{-i}^{\underline{R}-out}$ should be profitable then the curve given by (25) is an upper bound of firm i's best reply function, which is sufficient to prove the lemma.

Finally, for $x_{-i}^{\underline{R}-out} < x_{-i}$ it can be verified that it is never optimal for firm i to choose $x_i = x_{-i}$. In region IV, which is given by $\{x \in \mathbb{R}_+^2 : x_{-i} \geq x_i \geq \frac{2}{c_{2.3}} x_{-i} \,\text{and}\, x_{-i} \leq \frac{aT - x_i}{2}\}$, the derivative of π_i^{IV} at $x_i = x_{-i}$ is given by $\frac{d\pi_i^{IV}}{dx_i}|_{x_i = x_{-i}} = \frac{450-11d}{100a} x_{-i}^2 - 3Tx_{-i} + \frac{aT^2}{2} - k$, which is negative for $x_{-i} \in [x_i^{\underline{R}out}, \frac{aT}{3}]$.[26] Similarly it can be verified that the same holds true also for $x_{-i} > \frac{aT}{3}$. Thus, we can conclude that for $x_i^{\underline{R}out} < x_{-i}$ it is never optimal for firm i to choose $x_i = x_{-i}$.

Part III Now we can show that the best reply function of firm i, x_i^{BR}, is always below the isoinvestment line I^{NF} for all $x_i \geq x_{-i}$

An upper bound for the best reply function of firm i is $\bar{x}_i^{BR} = x_i^{\underline{L} \cup \underline{M} \cup \underline{R}}(x_{-i})$ as given by (25). Furthermore, we have shown that for $x_{-i} > x_{-i}^{\underline{R}out}$ the best reply has to be below the 45-degree-line.

In order to show that the upper bound of firm i's best reply, $\bar{x}_i^{BR}(x_{-i}, f^*, y^*, d)$, given by (25) lies below I^{NF}, we first show that the (continuous) function $\bar{x}_i^{BR}(x_{-i}, f^*, y^*, d)$ is convex in all differentiable parts.[27] Thus, in order to compare $\bar{x}_i^{BR}$ and I^{NF} it is sufficient to compare the points of intersection of $\bar{x}_i^{BR}$ and I^{NF} with the straight lines that separate the three

[26] Recall that $\frac{aT}{3}$ is the value of the upper bound of the region where both firms are constrained at the highest demand realization given $x_i = x_{-i}$.

[27] We obtain $\frac{d(x_i^{\underline{L}})^2}{d^2 x_{-i}} = 0$, $\frac{d(x_i^{\underline{M}})^2}{d^2 x_{-i}} > 0$, and $\frac{d(x_i^{\underline{R}})^2}{d^2 x_{-i}} > 0$.

regions (see figure 4). We now show that at each intersection point with one of the separating lines, the sum of investments on the best reply function in the presence of forward contracts, $\bar{x}_i^{BR}(x_{-i}) + x_{-i}$ is lower than the sum of investments on the isoinvestment line.

The four separating lines that have to be checked are (1) $x_{-i} = 0$, (2) $x_i = 2x_{-i}$, (3) $x_i = \frac{c_{2.3}x_{-i}}{2}$, and $x_i = x_{-i}$. At $x_i = 0$ it holds that $\bar{x}_i^{BR}(0) = \frac{aT-\sqrt{2ak}}{2} < \frac{aT-\sqrt{2ak}}{3}$, where the last expression is the total investment in the market without forward contracts. Along the remaining separating lines, we now compare the values of x_{-i} where $\bar{x}_i^{BR}$ intersects with each of the three lines and the intersection points of I^{NF} with those lines. We get

(2) along $x_i = 2x_{-i}$: $\quad x_{-i}^{M-in} < \frac{2}{9}(aT - \sqrt{2ak})$,

(3) along $x_i = c_{2.3}/2x_{-i}$: $\quad x_{-i}^{M-out} < \frac{4}{3(c_{2.3}+2)}(aT - \sqrt{2ak})$,

(4) along $x_i = x_{-i}$: $\quad x_{-i}^{R-out} < \frac{1}{3}(aT - \sqrt{2ak})$,

where the last terms are the intersection points of the separating line and I^{NF}. It can be shown[28] that inequalities (2) to (4) above are satisfied for the parameter space $k \in [\frac{aT^2}{c_{62}}, \frac{aT^2}{2}]$, $\sigma \in [0, 1]$, $a > 0$, and $T > 0$, which proves the lemma for $k \in [\frac{aT^2}{c_{62}}, \frac{aT^2}{2}]$. For $k \in [0, \frac{aT^2}{c_{62}}]$ see the supplement than can soon be downloaded at merlin.fae.us.es/grimm/.

[28]Notice that by differentiation it can be verified that $x_{-i}^{M-in}, x_{-i}^{M-out}$, and x_{-i}^{R-out} are monotone in σ. Furthermore each inequality can be divided by aT (replacing $k = aT^2k'$). Then, inserting the maximizing values of σ, verification of conditions (2) to (4) is reduced to a one variable problem.