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ENDOGENOUS GROWTH, GREEN INNOVATION AND GDP DECELERATION IN A WORLD WITH POLLUTING PRODUCTION INPUTS

KERSTIN BURGHAUS
PETER FUNK

Department of Economics
University of Cologne
Albertus-Magnus-Platz
D-50923 Köln
Germany

http://www.wiso.uni-koeln.de

Endogenous Growth, Green Innovation and GDP Deceleration in a World with Polluting Production Inputs*

Kerstin Burghaus† Peter Funk‡

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Abstract

We study economic growth and pollution control in a model with endogenous rate and direction of technical change. Economic growth results from growth in the quantity and productivity of polluting intermediates. Pollution can be controlled by reducing the pollution intensity of a given quantity through costly research (green innovation) and by reducing the share of polluting intermediate quantity in GDP. Without clean substitutes, saving on polluting inputs implies that the rate of GDP growth remains below productivity growth (deceleration). While neither green innovation nor deceleration is chosen under laissez-faire, both contribute to long-run optimal pollution control for reasonable parameter values.

Keywords: Endogenous Growth, Direction of Technical Change, Pollution, Green Innovation, Rebound Effect

JEL Codes: O30, O41, O44, Q55

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†Mercator Research Institute on Global Commons and Climate Change (MCC), Torgauer Str. 12-15, 10829 Berlin, Tel.: +49 (0)30 33 85 537 -248, Email: burghaus@mcc-berlin.net

‡Center for Macroeconomic Research, University of Cologne, Albertus-Magnus-Platz, 50923 Köln, Germany Tel.: +49/221/470-4496, Fax: +49/221/470-5143, Email: funk@wiso.uni-koeln.de

1 Introduction

When it comes to the question of whether and how economic growth and environmental conservation should be reconciled, it is widely argued that technical change will play a crucial role (see for instance the Stern Review, Stern (2007), IPCC AR5 Synthesis Report (2014)). By reducing the pollution intensity of production inputs and processes, by developing clean substitutes or by raising input productivity, technological development can help to decouple economic growth from pollution.

However, first, investment in environment-friendly technology diverts resources from other research activities and the resulting costs in terms of economic growth have to be taken into account when evaluating green R&D options. Second, the ecological benefits of technical progress are not undisputed, due to so-called rebound effects: Consider a modern combine harvester bringing in 100 tons of grain in 5 man-hours using 50 gallons of diesel. A technology innovation allows to harvest the same 100 tons of grain in 4 hours and the same amount of fuel or, alternatively, to reduce operating time by only half an hour and at the same time reduce fuel consumption to 45 gallons. As grain becomes cheaper, instead of producing the same amount in shorter time and using less polluting fuel, farmers may choose to produce more grain and demand the same amount of fuel or even more, such that pollution is *not* reduced. On a macroeconomic scale, productivity gains raise aggregate income and may thereby raise demand for polluting inputs across all sectors.[1] Rebound effects of technical progress are one reason for environmental activists like Greenpeace to believe that the world economy should give up economic growth and converge towards stationary levels of consumption and production.[2]

We address the question of whether and how rebound effects from technical progress should be tamed in a socially optimal solution. Our paper offers a comprehensive analysis of the market equilibrium and the social optimum in a model with fully endogenous direction and rate of technical change. While completely clean substitutes for polluting production inputs do not exist in our model, the pollution intensity of polluting goods can be reduced through green innovation. Further, their productivity can be raised. This, at equilibrium, leads to a rebound effect on input quantity. We show that for empirically plausible parameter values, this rebound effect should be mitigated indeed, but doing so comes with unused growth potential (deceleration).

Consider the above example again: In our model research can be directed to either raise productivity and/or to reduce the polluting impact of intermediate inputs, such as fossil fuels. Technology innovations leads to what we call **productivity growth** in this paper. Given the amount of inputs used, productivity growth always raises input-efficiency if the latter is defined as output (grain yield) per amount of intermediate inputs (fuel) used. However, productivity growth also increases the marginal product of these inputs. This stimulates intermediate demand and may induce a rebound effect. We do therefore not classify productivity growth as 'clean' or 'dirty' a priory. What we understand by **green innovation** in this paper is directing research at decreasing the pollution intensity of inputs. Referring to the above example, such innovations could be improvements in

[1] A formal definition of the rebound effect can be given along the lines of Berkhout et al. (2000): A rebound effect denotes the percentage of input saving that is lost due to increased input use. A rebound effect of more than 100%, which implies a net increase in input use, is sometimes referred to as 'backfire'.

[2] Convergence to a stationary economy as demanded by environmental activists usually goes beyond merely giving up long-run growth. Environmental activists believe the world economy to have surpassed sustainable levels of economic activity so that downsizing -'degrowth'- is unavoidable. This belief is shared by a political movement of the same name, which has its origin in France ('décroissance'), see for example Ariès (2005) and Latouche (2004).

catalytic converters. They could also be advances in Carbon Capture and Storage (CCS) or more generally, any improvement which reduces the emissions of greenhouse gases when burning a given amount of fossil fuel.

Along a balanced growth path the growth rate of (polluting) intermediate input quantity equals the rates of productivity and of output growth. We extend this definition to solutions characterized by growth rates which converge towards constant values only asymptotically. We call such solutions 'asymptotically-balanced growth solutions'.[3] In particular, it may be optimal to keep the growth rate of polluting intermediate inputs persistently below the rate of productivity growth to control the rebound effect of the latter. This leads to what we call deceleration: Output and consumption grow more slowly than productivity as well, although faster than intermediate inputs such that the average product of the latter rises.[4] Given productivity growth, controlling the rebound effect thus reduces the growth rate of pollution at the cost of foregone potential growth in consumption and real GDP.

There are two ways then to persistently decouple output- and pollution growth: Green innovation and deceleration. Our main result shows that for reasonable values of model parameters, optimal growth features deceleration: Growth of real GDP and consumption, driven by productivity growth must be persistently accompanied not only by green innovation[5] but also by a moderation of polluting intermediate input growth below productivity and output growth. This occurs when production is inelastic with respect to intermediate quantity compared to productivity. Deceleration then allows to gain from productivity growth in a relatively clean way without incurring a large loss in consumption growth. Further, the social return to green innovation rises in the production elasticity of the polluting input and is therefore comparatively small in this case. Our results imply that policy may have to stimulate technical development and green innovation in particular while at the same time setting incentives to control the rebound effect which offsets potential efficiency gains in the laissez-faire equilibrium.

If polluting quantity persistently falls in absolute terms and not only per labor efficiency unit we speak of **quantity degrowth.**[6] Even with quantity degrowth, productivity, consumption and GDP may still rise. GDP growth is driven by productivity improvements alone. In the example, quantity degrowth reduces fuel consumption X and emissions in absolute terms (while yield Y may still grow). In our model, persistent GDP degrowth towards stationary GDP and consumption levels in absolute terms is preferable to a path with unconstrained pollution growth as it is chosen in the entirely unregulated economy. However, giving up

[3] Asymptotically-balanced growth paths in environmental economic models have been described, e.g., in an Ak-model by Withagen (1995) and in a general one-sector growth model with non-renewable resources (but without pollution) by Groth and Schou (2002).

[4] Formally this requires that the production function is linear homogeneous in intermediates and the technology variable, as in the standard setting with a constant returns to scale production function and labor-augmenting technical change. The reduced form of our production function will be of the Cobb-Douglas type in which case the distinction between labor-augmenting and intermediate input-augmenting change is irrelevant.

[5] While in the long-run laissez-faire equilibrium which does not internalize pollution externalities, neither green innovation nor deceleration is chosen, we set up the model such as to make sure that long-run economic growth is optimal for a sufficiently patient household and always goes along with persistent green innovation. This result is driven by the existence of fixed costs in each individual research unit. Once a research unit is opened up and the fixed costs are paid, making intermediates at least marginally cleaner while making them more productive generates almost no additional cost.

[6] If we interpret intermediate quantity as material used, quantity degrowth (and, in a weaker sense, deceleration) corresponds to what is sometimes called "dematerialization", since it reduces the quantity in absolute terms (or its share in real GDP).

consumption growth altogether is never optimal (for a sufficiently patient household). Parameterization of the model based on empirical estimates suggests that the option of quantity degrowth may very well be a sensitive long-run aim.

Another interesting result can be obtained when extending the baseline model to include non-renewable resources. Notably, the negative pollution externality of production may reduce optimal resource use in a way that a sufficiently large resource stock is never exhausted.

There are two different strands of literature on the direction of technological change, economic growth and pollution. Closest to our model are Hart (2004) and Ricci (2007) as they also consider the choice between a lower pollution intensity and greater productivity. However, they neglect the possibility to lower pollution growth by reducing the rebound effect of productivity growth so that deceleration is not part of the optimal solution of their models. Ricci (2007) concentrates on the analysis of balanced growth paths along which, by definition, deceleration cannot occur. In Hart (2004), not only the quantity component of output but output itself has a negative effect on the environment. This assumption contradicts our intuition that higher productivity is not polluting in itself and implies that deceleration cannot reduce pollution.

While the understanding of green innovation in Hart (2004) and Ricci (2007) is similar to ours, a different definition of green innovation is given, e.g., in Grimaud and Rouge (2008) and Acemoglu, Aghion, Bursztyn and Hemous (2012). Building on Acemoglu (2002) both papers assume separate research sectors, one for a polluting and one for a non-polluting production input[7]. Green innovation increases the productivity of the clean good while leaving the pollution intensity of the dirty input unchanged. Pollution in these papers is optimally controlled by shifting the composition of GDP towards the clean sector. The present paper assumes a unit elasticity of substitution between factors (Cobb-Douglas specification). Adding a further distinction between raising the productivity of one factor versus raising the productivity of the other factor would not affect the analysis. Deviating from the Cob-Douglas specification would make it natural, also in our framework, to add a further dimension to the direction of research (raising the productivity of non-polluting factors, raising the productivity of polluting factors, reducing the polluting impact of polluting factors). As this would entail a non-trivial increase in model complexity, we do not formally study the non unit-elastic case. Instead we informally discuss the robustness of our main result (deceleration is desirable) with respect to deviations from the Cobb-Douglas specification, in particular toward cases where it is easier to substitute clean inputs for dirty inputs, in section.4[8]

The outline of our paper is as follows: Section 2 presents the model setup. We then determine the laissez-faire equilibrium in section 3. In section 4, we characterize the unique long-run optimum. Our main result, theorem 1, shows that for empirically reasonable parameter constellations, the optimal solution includes both green innovation and deceleration to decouple output- and pollution growth. Section 4 also contains an informal discussion of deviations from the Cobb-Douglas specification. Section 5 extends the baseline model to include

[7] Several authors, among them Smulders and de Nooij (2003) and Hassler, Krusell and Olovsson (2012) use the framework by Acemoglu (2002) to analyze energy-saving technical change. As neither of the models takes pollution into account, we do not refer to these contributions in detail here.

[8] Grimaud, Lafforgue and Magné (2011) consider three forms of R&D, but without a detailed microfoundation of the R&D sectors. Moreover, they are less concerned with the characterization of the optimal growth path than with the optimal policy mix for its implementation.

a polluting non-renewable resource. Section 6 concludes.

2 The model

2.1 Setup

In each period, a representative household receives utility $v(c_t) = \frac{\sigma_c}{\sigma_c - 1} c_t^{\frac{\sigma_c - 1}{\sigma_c}}$ from per-capita-consumption $c_t = \frac{C_t}{L}$ and utility $\phi^E(E_t) = \psi \frac{\sigma_E}{\sigma_E - 1} E_t^{\frac{\sigma_E - 1}{\sigma_E}}$ from environmental quality E_t. We assume as, for example, Stokey (1998) as well as Aghion and Howitt (1998, chapter 5), that utility is additively separable. Discounted intertemporal utility is given by

$$U = \int_0^\infty e^{-\rho t} \left(\frac{\sigma_c}{\sigma_c - 1} c_t^{\frac{\sigma_c - 1}{\sigma_c}} + \psi \frac{\sigma_E}{\sigma_E - 1} E_t^{\frac{\sigma_E - 1}{\sigma_E}} \right) L dt \tag{1}$$

where ρ is the rate of time preference, L total household labor-supply and $\sigma_c, \sigma_E > 0$, $\sigma_c, \sigma_E \neq 1$ are the intertemporal substitution elasticities of consumption and environmental quality respectively. ψ measures the weight of environmental quality in instantaneous utility. Utility is increasing and strictly concave in both arguments.

Environmental quality is inversely related to the stock of pollution originating from the intermediate sector:

$$E_t = \frac{1}{S_t} \tag{2}$$

While utility is concave in E_t, the relation between environmental quality and pollution is convex. Depending on σ_E, the disutility $\phi^S(S_t) = -\phi^E(E_t) = \frac{\sigma_E}{1 - \sigma_E} S_t^{\frac{1 - \sigma_E}{\sigma_E}}$ of pollution can be concave or convex in S_t. We assume that it is convex by restricting σ_E to the interval $(0, 1/2)$, so that the marginal disutility of pollution increases in the pollution stock:

$$\sigma_E < \frac{1}{2} \tag{3}$$

The assumption of convex disutility also rules out parameter constellations for which the utility impact of pollution asymptotically becomes negligible relative to that of consumption in a growing economy. This is not an interesting case for the long-run analysis of questions arising out of the trade-off between economic growth and a clean environment. Not only the long-run laissez-faire equilibrium but also the long-run optimal solution would be similar to those in non-environmental models of growth through creative destruction.

The representative household allocates an amount L_{Yt} of its labor supply L to final-good production, L_{Xt} to intermediate production and an amount L_{Dt} to research:

$$L = L_{Yt} + L_{Xt} + L_{Dt} \tag{4}$$

Final output Y_t is produced from labor L_{Yt} and intermediate goods X_{it} of various productivity levels Q_{it}, $i \in [0, 1]$ with the production function

$$Y_t = L_{Yt}^{1-\alpha} \int_0^1 X_{it}^{\alpha} Q_{it}^{1-\alpha} di \tag{5}$$

where $0 < \alpha < 1$. Y_t is used for consumption only.

$$Y_t = c_t L. \tag{6}$$

Intermediate goods are produced with the production function

$$X_{it} = \varphi L_{Xit} Q_t \tag{7}$$

where $\varphi > 0$ is a parameter and $Q_t = \int_0^1 Q_{it} di$ measures aggregate productivity.[9] X_{it} units of intermediate good i is associated with a flow R_{it} of fossil resources used, say the amount of fossil fuel burned in conjunction with the usage of each intermediate input.[10] In the baseline specification of our model, R_{it} does not have to be accounted for explicitly. Formally, this amounts to the assumption that there is an infinite supply of these resources in each period, such that their price is zero. We show in Section 5 that for parameter constellations well in line with empirical evidence, the alternative assumption of a finite initial resource stock does not affect the long-run social optimum so that our main results still hold.

The pollution flow at time t is given by X_t/B_t, where $X_t = \int_0^1 X_{it} di = \int_0^1 R_{it} di$ is the aggregate quantity of intermediate goods and at the same time the total amount of fossil fuels used in the economy. B_t is aggregate cleanliness of intermediate inputs (in their usage in final good production at t). In the above example, $1/B_t$ measures greenhouse gas emissions per unit of fossil fuels burned in the economy at t. To keep the optimization problem tractable, we assume that aggregate cleanliness simply is the average of individual intermediates' cleanliness at t: $B_t = \int_0^1 B_{it} di$.

If, in addition, a fraction δ of the pollution stock is cleaned up by natural regeneration processes in every period, the pollution stock evolves according to the equation of motion[11]:

$$\dot{S}_t = \frac{X_t}{B_t} - \delta S_t \tag{8}$$

Note that due to natural regeneration, pollution growth eventually ceases if there is no growth in intermediate production and therefore fossil resource use. However, if X_t grows, S_t asymptotically grows at the same rate unless the pollution intensity of intermediates is reduced over time by green innovation. Still, even without green innovation, pollution growth remains below its potential if intermediate quantity grows slower than productivity so that the rebound effect of productivity growth is restricted. With the assumed form of the production function (Cobb-Douglas case), this goes along with deceleration: Growth in output remains below productivity growth.

Definition 1 *There is **deceleration** whenever $\widehat{Y}_t < \widehat{Q}_t$ so that Y_t/Q_t declines.*

[9] The dependence of X_{it} on aggregate productivity Q_t is needed to ensure that the allocation of labor supply and thus growth rates of the aggregate variables in our model are constant in the long run. Our results would not change if we assumed that instead of labor, a fraction of final output had to be spent on the production of intermediates.

[10] For example, the fuel tank of a car has to be filled with gasoline before driving. Fossil resources can of course also be inputs in the production of intermediate goods. While the emissions from these fossil fuels accrue before the goods are actually used, we do not separate them from emissions arising from the use of intermediate goods.

[11] In general, we use a dot above a variable to indicate its derivative with respect to time, while we mark growth rates with a cicumflex.

The two sources of slow pollution accumulation (besides natural regeneration) become apparent when rewriting (8) as $\dot{S}_t = \frac{X_t}{Q_t}\frac{Q_t}{B_t} - \delta S_t$: First, $\dot{S}_t$ is small whenever Q_t/B_t is small, which means a sufficiently large share of research must have been oriented towards green innovation in the past. Second, pollution accumulates more slowly if the rebound effect of productivity growth has been controlled such that X_t/Q_t is smaller.[12] If the rebound effect remains uncontrolled ($\widehat{X}_t = \widehat{Q}_t$), then a constant stock of pollution ($\widehat{S}_t = \frac{d\widehat{S}_t}{dt} = 0$) requires $\widehat{B}_t = \widehat{Q}_t$.[13] This suggests the definition of a natural benchmark for the direction of technical change:

Definition 2 *The* ***direction of technical change*** *is ecologically* ***neutral*** *if and only if* $\widehat{B}_t = \widehat{Q}_t$, ***productivity-oriented*** *if and only if* $\widehat{B}_t < \widehat{Q}_t$, *and* ***green*** *if and only if* $\widehat{B}_t > \widehat{Q}_t$.

Both productivity Q and cleanliness B change over time due to innovations from a continuum of R&D-sectors. Entry to the research sector for any intermediate X_{it} is not restricted. For research unit $j \epsilon [0, \infty]$, improving Q_{it} by a rate q_{ijt} and B_{it} by a rate b_{ijt} requires

$$l_{Dijt}(q_{ijt}, b_{ijt}, Q_{it}, B_{it}, Q_t, B_t) = q_{ijt}^2 \frac{Q_{it}}{Q_t} + b_{ijt}^2 \frac{B_{it}}{B_t} + d\frac{Q_{it}}{Q_t} \tag{9}$$

units of labor. We call q_{ijt} and b_{ijt} the step-size of an innovation with respect to productivity and cleanliness respectively. The wage rate is denoted by w_{Dt}. Then $w_{Dt}d\frac{Q_{it}}{Q_t} > 0$ are fixed entry costs for unit j in sector i. Variable costs for each dimension of technology improvement are quadratic in the step-size.[14] Total costs $w_{Dt}l_{Dijt}$ rise with the level of sectoral relative to aggregate productivity Q_{it}/Q_t and cleanliness B_{it}/B_t respectively. The underlying assumption is that technology improvements in a given sector are increasingly difficult the more advanced the technology in that sector is already while there are positive spillovers from the other sectors.[15] Given l_{Dijt}, a trade-off exists between making an intermediate more productive and making it cleaner. On the other hand, there is also an indirect positive relation between research orientations: Once fixed costs have been paid to innovate in one direction, a comparatively small additional labor-investment is needed to increase the other technology stock as well.

If a researcher j enters into the research sector for intermediate X_i at time t, he hires labor l_{Dijt} and chooses a step-size q_{ijt} and b_{ijt} for the improvement in productivity and cleanliness respectively. The wage rate w_{Dt} is taken as given. Innovations occur at the exogenous, constant Poisson arrival-rate μ per unit of time for the individual researcher j. An innovation changes the sectoral productivity level by $q_{ijt}Q_{it}$ and the cleanliness of production by $b_{ijt}B_{it}$. The innovator obtains a patent for the production of the improved intermediate good. He then receives a profit flow from selling the intermediate which eventually ceases when a new innovation arrives

[12] This could in principle be brought about in two different ways: Note that X_t/Q_t can be rewritten $X_t/Q_t = X_t/Y_t \cdot Y_t/Q_t$. One way to reduce X_t/Q_t is deceleration (reducing Y_t/Q_t). Another way is to keep Y_t/Q_t constant, but reduce the average product of polluting inputs, X_t/Y_t. With the assumed Cobb-Douglas technology, a decrease in the average product always goes along with deceleration in the short to medium term as well as in the long run. With better substitutability of production inputs, deceleration may not be needed in the long run, as we discuss in section 4.5.

[13] $\widehat{S}_t = 0$ if and only if $X_t = \delta S_t B_t$ and $\frac{d\widehat{S}_t}{dt} = 0$ if, in addition, $\widehat{X}_t = \widehat{B}_t$. Since $\widehat{X}_t = \widehat{Q}_t$, this requires $\widehat{B}_t = \widehat{Q}_t$.

[14] While fixed costs are needed to guarantee a finite number of research units, assuming costs quadratic in the step-size ensures the existence of an efficient choice of the latter. As we explain in section 4.3, the presence of fixed costs creates a certain complementarity between pollution-reducing and productivity-enhancing innovation, which we believe to be realistic.

[15] Like the intermediate production function, labor required in R&D (equation (9)) must depend on the aggregate and additionally on the sectoral levels of technology to ensure asymptotically constant growth of the aggregate variables.

and the incumbent is replaced by another firm. If n_{it} units decide to enter research sector i in t, innovations arrive at rate μn_{it} in this sector. The expected development of Q_{it} and B_{it} is given by

$$E[\Delta Q_{it}] = \int_0^{n_{it}} \mu q_{ijt} Q_{it} dj \tag{10}$$

$$E[\Delta B_{it}] = \int_0^{n_{it}} \mu b_{ijt} B_{it} dj. \tag{11}$$

While the sectoral technology level faces discontinuous jumps, aggregate technology evolves continuously, because there is a continuum of sectors carrying out research. According to the law of large numbers, the average rates of change $\dot{Q}_t$ and $\dot{B}_t$ of Q and B approximately equal the respective expected rates of change, which are derived by aggregating over sectors in (10) and (11):

$$\dot{Q}_t = \int_0^1 \int_0^{n_{it}} \mu q_{ijt} Q_{it} dj di \tag{12}$$

$$\dot{B}_t = \int_0^1 \int_0^{n_{it}} \mu b_{ijt} B_{it} dj di. \tag{13}$$

2.2 Balanced and asymptotically-balanced growth

The subsequent analysis of our model in this and the following sections extends beyond balanced growth paths to 'asymptotically-balanced growth paths'. The definition below serves to clarify the terminology, where here and in the following, z_∞ refers to the limit $\lim_{t \to \infty} z_t$ of a variable z:

Definition 3 *Assume that for some initial state (Q_0, B_0, S_0), there exists a (market or planner) solution such that the sequence $\left(\widehat{Q}_t, \widehat{B}_t, \widehat{S}_t\right)_{t=0}^{\infty}$ converges towards the vector $\left(\widehat{Q}_\infty, \widehat{B}_\infty, \widehat{S}_\infty\right)$ for $t \to \infty$. We call such a solution an asymptotically-balanced growth (ABG) solution. We say that the model has an asymptotically unique ABG-solution if all ABG-solutions have the same limit vector $\left(\widehat{Q}_\infty, \widehat{B}_\infty, \widehat{S}_\infty\right)$.*
If there exist initial states (Q_0, B_0, S_0) such that the corresponding solution paths are characterized by $\left(\widehat{Q}_t, \widehat{B}_t, \widehat{S}_t\right) = \left(\widehat{Q}_\infty, \widehat{B}_\infty, \widehat{S}_\infty\right)$ for every t, we call the path defined by $\left(\widehat{Q}_t, \widehat{B}_t, \widehat{S}_t\right)_{t=0}^{\infty} = \left(\widehat{Q}_\infty, \widehat{B}_\infty, \widehat{S}_\infty\right)$ the unique balanced growth (BG)-path.

In abuse of terminology, we sometimes refer to the unique limit of all ABG-solutions for $t \to \infty$, characterized by the unique vector $\left(\widehat{Q}_\infty, \widehat{B}_\infty, \widehat{S}_\infty\right)$, as *the* ABG-solution.
Note that a BG-solution, defined by constant growth rates of Q, B and S for all t, is also an ABG-solution. The reverse is not true because there may not exist an initial state (Q_0, B_0, S_0) such that $\widehat{Q}_t$, $\widehat{B}_t$ and $\widehat{S}_t$ are constant for all t. We will see that while for any set of parameters the economy has a unique BG-equilibrium and a unique ABG-optimum, it need not have a BG-optimum. In particular, a BG-optimum does not exist in theorem 1.

3 The laissez-faire equilibrium

The laissez-faire equilibrium is given by sequences of plans for per-capita consumption $\{c_t\}_0^\infty$, assets $\{A_t\}_0^\infty$, labor supply in production $\{L_{Xit}, L_{Yt}\}_0^\infty$ and research $\{L_{Dt}\}_0^\infty$, demand for intermediates $\left\{X_{it}^d\right\}_0^\infty$, demand for labor in production $\left\{L_{Xit}^d, L_{Yt}^d\right\}_0^\infty$ and research labor demand $\{l_{Dijt}\}_0^\infty$, plans for the step-size $\{q_{ijt}\}_0^\infty$ and $\{b_{ijt}\}_0^\infty$ in productivity and cleanliness, as well as sequences of intermediate prices $\{p_{it}\}_0^\infty$ and wages $\{w_{Xit}, w_{Yt}, w_{Dt}\}_0^\infty$ in intermediate production, final good production and research and a path $\{r_t\}_0^\infty$ for the interest rate such that in every period t, (i) the representative household maximizes utility taking into account the budget constraint and the labor market constraint (4), (ii) profits from final and intermediate goods production as well as research profits are maximized, (iii) aggregate expected profits in each research sector i are zero (iv) the markets for intermediate goods, the three types of labor and assets clear (v) all variables with the possible exception of q_{ij} and b_{ij} are non-negative.

The solution of the model under laissez-faire follows closely that in standard endogenous growth models. Define an upper bound $\overline{\rho}^{\text{LF}}$ for the rate of time preference such that $\widehat{Q}^{\text{LF}} > 0$ if and only if $\rho < \overline{\rho}^{\text{LF}}$. Further, define a lower bound $\underline{\rho}^{\text{LF}}$ such that the transversality condition for assets is satisfied if and only if $\rho > \underline{\rho}^{\text{LF}}$.[16] The following proposition describes the balanced-growth equilibrium:

Proposition 1 ***BG laissez-faire equilibrium***
There exists a $\underline{\rho}^{LF}$ such that the transversality condition for assets is satisfied if and only if $\rho > \underline{\rho}^{LF}$.
For $\rho > \underline{\rho}^{LF}$, the model has a unique BG-laissez-faire equilibrium. Further, an upper bound $\overline{\rho}^{LF}$ for the rate of time preference exists such that economic growth is strictly positive if and only if $\rho < \overline{\rho}^{LF}$. Productivity growth leads to equally fast expansion of polluting quantity ($\widehat{X}_\infty^{LF} = \widehat{Q}_\infty^{LF}$). The rebound effect of productivity growth is not controlled and there is neither deceleration nor green innovation. Pollution grows at the same rate as consumption, production and productivity. Given (3), i.e. $\sigma_E < 1/2$, a solution without long-run growth is socially preferable.

Proof. See appendix A.1. ∎

From the previous section it is obvious that in our model, there are no incentives for producers to self-restrict in polluting intermediate production or invest in cleaner intermediates. The rebound effect of productivity growth is particularly strong under laissez-faire as quantity grows at the same rate as productivity. In a growing economy, there is unconstrained pollution growth. This is clearly suboptimal if the disutility of pollution is convex ($\sigma_E < 1/2$) but utility is concave in consumption: The marginal utility gain from an additional unit of consumption becomes negligible relative to the marginal utility loss generated by a unit increase in the pollution stock as consumption and pollution rise. Utility declines persistently without lower bound. If consumption growth is given up in the long run, the pollution stock and utility converge to constant values. Stationary long-run levels of consumption and production as called for environmental activists are therefore welfare-improving over the laissez-faire equilibrium.

[16] The boundary values are $\overline{\rho}^{\text{LF}} = \frac{1}{2}\mu L \left(\left(\frac{1}{\alpha} + \frac{\alpha}{1-\alpha}\right)\left(\sqrt{1+d} - 1\right)\right)^{-1}$ and $\underline{\rho}^{\text{LF}} = \frac{1}{2}\alpha(1-\alpha)\left(1 - \frac{1}{\sigma_c}\right)(1+d)^{-1/2}\mu L$.

4 The Social Planner's solution

The social planner chooses the time paths of Q, B and S as well as consumption c, production Y, X, X_i, x_{ij}, the allocation of labor L_{Yt}, L_{Xt},L_{Xit},L_{Dt}, l_{Dijt}, the number of research units[17] $n_{it} = n_t$ and a step-size q_{ijt} and b_{ijt} for technology improvements in every period t so as to maximize utility (equation (1)). She takes into account the labor market constraint (4), the aggregate resource constraint (6), the effect of pollution on environmental quality (2), the equation of motion for pollution (8), the expected change in Q_i and B_i as given by (10) and (11) as well as the aggregate equations of motion for Q (12) and B (13).

Because all research units j are ex ante symmetric and research costs are convex in q_{ij} and b_{ij}, the social planner chooses the same q_{ijt}, b_{ijt} and therefore l_{Dijt} for every j in sector i. Further, the planner allocates intermediate production in every sector i to the latest innovator because he is the most productive and cleanest while marginal costs are the same for all j. We therefore omit the index j from now on. In fact, it is optimal to choose the same $q_{it} = q_t$ and $b_{it} = b_t$ in every sector, as we explain in appendix B.1. We also show there that given the allocation of resources over firms and sectors just described, the dynamic social planner's problem involves the sector-independent variables Q, B, S, c, X, L_Y, n, q and b only and we derive the first-order conditions.

The long-run optimal solution differs dependent on the parameter constellation considered. To simplify the analysis, we focus on the empirically most relevant case by making the following assumptions[18]:

$$\alpha/(1-\alpha) < 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1-\sigma_E)/\sigma_E} \tag{14}$$

$$\rho > \rho^{\text{delta}} := \frac{1}{2}\left(1 + \left(\frac{\alpha}{1-\alpha}\right)^2\right)^{1/2} d^{-1/2}\mu L - \kappa \frac{(1-\sigma_E)/\sigma_E}{(1-\sigma_c)/\sigma_c}\delta \text{ for } \sigma_c < 1 \tag{15}$$

$$\text{where } \kappa = \left(\frac{1}{\sigma_c} + \frac{\alpha}{1-\alpha}\left(1 - \frac{(\sigma_c - 1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)\right)$$

While the second restriction excludes a boundary case[19] which does not lead to qualitatively different conclusions, the first restriction is crucial for model outcomes. To see that condition (14) indeed describes the most likely parameter constellation, consider the relevant parameters, α, σ_c and σ_E: While there are little reliable empirical results on σ_E, we believe that disutility is convex in the pollution stock ($\sigma_E < 1/2$) so that the marginal disutility of pollution is the larger, the more polluted the environment is. As for the IES in consumption σ_c, the range $\sigma_c \in (0,1)$ is suggested by a large body of empirical literature (e.g. Hall (1988), Ogaki and Reinhart (1998)). Defining a reasonable range for α is less straightforward. Setting α to the capital share would imply $\alpha \approx 1/3$. Interpreting X_t as energy, α would be substantially smaller than the capital share:

[17] To allow for an analytical solution to the planner problem we consider the constrained maximization problem with $n_{it} = n_t$ for all i.

[18] For a full characterization of all cases, we refer the interested reader to an extended appendix to this paper, available upon request from the authors.

[19] We show in proposition 2 that the pollution stock S decreases whenever $\sigma_c < 1$. S can at most decrease at the rate of natural regeneration ($\widehat{S}_\infty \geq -\delta$). To actually reach this rate of decrease, flow pollution would have to become zero and all economic activity would have to be given up. This can clearly not be optimal in finite time as a positive consumption level has to be maintained. Still, it can be optimal to approach $\widehat{S}_\infty = -\delta$ asymptotically by decreasing the pollution flow particularly fast. This case is more difficult to handle analytically and does not offer new insights. Condition (15) ensures that $\widehat{S}_\infty > -\delta$. Note that no such restriction is needed for $\sigma_c > 1$ as in this case, S increases in the long run (see proposition 2).

Energy expenditures as a share of GDP amounted to 8.9% in the U.S. in 2012 (EIA (2013b)). On the other hand, α is also the inverse of the mark-up in the intermediate production sector. Estimates for the manufacturing sector in the U.S. (Roeger (1995)) suggest values of α of at least 0.3. We consider values which do not exceed 0.5 as plausible. With $\sigma_E < 0.5$, $\sigma_c \in (0,1)$ and $0 < \alpha \leq 0.5$, condition (14) is always satisfied.

If we choose a smaller range for α, so that α does not exceed the capital share of 1/3, condition (14) holds for $\sigma_c < 2$ which covers most empirical estimates of the IES in consumption. Setting α to the energy share in real GDP, even extremely high values of σ_c up to 4.4 as found by Fuse (2004) for Japan do not violate the condition.

Before analyzing optimal pollution control in a growing economy (see section 4.3), we shortly describe the conditions for positive growth and the development of the pollution stock along the optimal path.

4.1 Optimality of persistent economic growth

In standard models of endogenous growth, long-run growth is optimal for sufficiently patient households. This result carries over to our model with negative environmental externalities.

Lemma 1 ***Positive long-run consumption growth***
There exists a $\overline{\rho}$, such that for any rate of time preference $\rho < \overline{\rho}$, long-run optimal consumption growth is positive.

Proof. The upper bound $\overline{\rho} = \frac{1}{2}\left(1 + \left(\frac{\alpha}{1-\alpha}\right)^2\right)^{1/2} d^{-1/2}\mu L$ is derived in the extended appendix. The proof follows from the solution of the model, similar to the proof in standard endogenous growth models. ■

The result is not surprising as pollution accumulation can be restricted without giving up consumption growth altogether. Persistent economic growth must however be accompanied by continuous pollution control.

4.2 The optimal relation between economic growth and pollution accumulation

We show in this subsection, that optimal growth does not automatically require constant or decreasing pollution levels. More precisely, we find that for our assumption of convex disutility of pollution ($\sigma_E < 1/2$) whether the pollution stock de- or increases in the long-run optimum depends on the intertemporal elasticity of substitution in consumption:

Lemma 2 ***Development of the pollution stock***
Long-run growth must be accompanied by a persistent restriction of pollution growth. In a growing economy, the optimal pollution stock S_t increases (decreases) for $\sigma_c > 1$ ($\sigma_c < 1$).

Proof. The first statement follows as a corollary of proposition 1. As to the second, we show in appendix B.2 that in a solution with asymptotically-balanced growth under the restriction (15), the following condition must hold:

$$\frac{\sigma_c - 1}{\sigma_c}\widehat{c}_\infty = \frac{1-\sigma_E}{\sigma_E}\widehat{S}_\infty \tag{16}$$

Given $\widehat{c}_\infty > 0$, the left-hand side of (16) is positive whenever $\sigma_c > 1$ while it is negative for $\sigma_c < 1$. Under assumption (3) that the disutility of pollution is convex, $\frac{1-\sigma_E}{\sigma_E}$ on the right-hand side is positive. Therefore the right-hand side of equation (16) is negative if and only if $\widehat{S}_\infty < 0$ and positive if and only if $\widehat{S}_\infty > 0$. It follows that the pollution stock must increase whenever $\sigma_c > 1$ and decrease whenever $\sigma_c < 1$.[20] ■

Equation (16) is the balanced-growth condition described in Gradus and Smulders (1996) which has become standard in models of the environment and endogenous growth: It requires the ratio of instantaneous marginal utility from consumption to instantaneous marginal disutility from pollution to develop proportionally to S/c so that the elasticity of substitution between c and S is unity.

4.3 Pollution control and the direction of technical change

As shown in lemma 1, long-run growth in the optimal solution must go along with a persistent restriction of pollution growth. It is intuitive that green innovation is always part of optimal pollution control: Once research units are opened up and the fixed costs (e.g., for equipment and fixed labor costs) have been paid, it is almost costless to make intermediates a little cleaner while making them more productive.
Unlike green innovation, deceleration is not always optimal in a growing economy as the costs in terms of foregone consumption growth may be substantial. Under the empirically likely parameter constellation given in condition (14), however, it is socially desirable to decelerate.

In the following theorem, we characterize the long-run optimal solution given conditions (3), (14) and (15). We define a lower bound ρ^{TVC} for the rate of time preference so that the transversality conditions are satisfied if and only if $\rho > \rho^{\mathrm{TVC}}$[21].

Theorem 1 ***ABG optimum***
There exists a lower bound ρ^{TVC} for ρ such that the transversality conditions are satisfied if and only if $\rho > \rho^{TVC}$.

For $\rho^{TVC} < \rho < \overline{\rho}$, there exists an asymptotically unique ABG-optimum with the following properties: Pollution growth $\widehat{S}_\infty$ equals the growth rate of flow pollution, $\widehat{X}_\infty - \widehat{B}_\infty$. $\widehat{S}_\infty$ is reduced below the potential rate $\widehat{Q}_\infty$ both by green innovation ($\widehat{B}_\infty > 0$) and by restricting the rebound effect of productivity growth ($\widehat{X}_\infty < \widehat{Q}_\infty$). The latter goes along with deceleration ($\widehat{Y}_\infty < \widehat{Q}_\infty$). The ratio of green relative to productivity-improving innovation is $\widehat{B}_\infty/\widehat{Q}_\infty = \alpha/(1-\alpha)$. The direction of technical change is green (productivity-oriented), i.e., $\widehat{B}_\infty > \widehat{Q}_\infty$ ($\widehat{B}_\infty < \widehat{Q}_\infty$), if and only if $\alpha > 1/2$ ($\alpha < 1/2$).

Proof. See appendix B.3. ■

Given condition (14), i.e. $\alpha/(1-\alpha) < 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$, reductions in pollution intensity are optimally combined with a restriction of polluting quantity growth to restrict the rebound effect, although it goes along with deceleration.

[20] Note that (16) also suggests that under more general assumptions concerning the utility function, whether the pollution stock de- or increases depends on σ_E being smaller or larger than one as well. For $\sigma_E > 1$, pollution is allowed to rise only if $\sigma_C < 1$ while a falling pollution stock is required for $\sigma_C > 1$.

[21] The formal expression for the critical value ρ^{TVC} is $\rho^{\mathrm{TVC}} = \frac{1}{2}\frac{1-1/\sigma_c}{1+\frac{\alpha}{1-\alpha}\left(1-\frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)}\left(1+\left(\frac{\alpha}{1-\alpha}\right)^2\right)^{1/2} d^{-1/2}\mu L$ (see the extended appendix). Note that the condition $\rho > \rho^{\mathrm{TVC}}$ is satisfied for any non-negative ρ if $\sigma_c < 1$. In this case, a positive lower bound for ρ is given by ρ^{delta} in condition (15).

In this case, the cost of controlling the rebound effect is rather low, as the elasticity α of final good production $Y_t = X_t^\alpha (Q_t L_{Yt})^{1-\alpha}$ with respect to intermediate quantity is small: A small elasticity implies that polluting quantity growth has only a minor effect on output growth compared to productivity growth. Restricting quantity growth does not require giving up much consumption growth, i.e. does not require strong deceleration. Further, with the relative unattractiveness of growth in polluting quantity for small α, it becomes less important to reduce the pollution intensity of intermediate goods. The smaller α, the lower therefore the social return to green as opposed to productivity-improving research.

The expression $1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$ is the ratio of green relative to productivity-improving innovation which yields the pollution growth rate reconcilable with asymptotically-balanced growth (according to equation (16)) when the rebound effect of productivity growth remains uncontrolled and there is no deceleration. If the elasticity α is so small that $\alpha/(1-\alpha) < 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$, the relative return to green research is too low to support this research orientation: It is not optimal to bring about the asymptotically-balanced pollution growth rate by green innovation alone. Research then remains rather productivity-oriented but deceleration lowers the rebound effect of productivity growth and thereby helps to restrict pollution growth.

For larger values of α not reconcilable with (14), a balanced-growth optimum without deceleration exists. Pollution control is achieved through green innovation only. This case is described in the extended appendix.

A solution without deceleration becomes less likely as σ_E increases if and only if $\sigma_c < 1$ and more likely if and only if $\sigma_c > 1$. As σ_E increases from close to zero to 1/2, the intertemporal elasticity of substitution in pollution $(\sigma_E/(1-2\sigma_E))$ increases and the optimal pollution path becomes steeper. If $\sigma_c < 1$, this means that the pollution stock must fall faster, so that stronger pollution control is required. If $\sigma_c > 1$, a larger positive pollution growth rate is accepted by the social planner, so that less pollution control is needed.

We have characterized the social optimum in the long run only. The set of necessary conditions generates a complex dynamic system which does not allow to determine the transition path analytically. Numerical analysis suggests, however, that for any initial state of the economy, there exists a path leading towards the long-run optimal solution.

A very strong restriction of the rebound effect occurs if intermediate quantity falls in absolute terms, not only per labor efficiency unit. There is then degrowth in intermediate quantity (but not in GDP). Because quantity degrowth requires extreme deceleration, it is optimal only if the ratio $\alpha/(1-\alpha)$ of production elasticities is particularly small. This result follows as a corollary from theorem 1:

Corollary 1 ***Quantity degrowth***
X_t converges to zero as Q_t grows in the ABG-solution of the social planner's problem described in theorem 1, i.e. there is quantity degrowth ($\widehat{X}_\infty < 0$), if and only if $\frac{\alpha}{1-\alpha} < (1-\alpha)\frac{(1-\sigma_c)/\sigma_c}{(1-\sigma_E)/\sigma_E}$.

Proof. Proof follows directly from setting $\widehat{X}_\infty < 0$ in equation (B.25) in the appendix. ■

Note that quantity degrowth can only be optimal if the pollution stock is required to decline in the long-run optimum (for $\sigma_c < 1$). Further, α should be substantially below the capital share. Quantity degrowth is likely to be optimal if α is interpreted as the energy share in GDP: Setting $\alpha \approx 0.09$ and $1/3 \leq \sigma_E < 1/2$, the optimal solution is characterized by quantity degrowth for values of σ_c from almost the entire interval $(0, 1)$.

4.4 Environmental care and the pace of economic growth

In our model, a stronger research orientation towards green innovation implies slower productivity growth for given total research effort. Further, deceleration needed to control the rebound effect of productivity growth requires to give up potential consumption growth. Intuitively, one might therefore expect environmental care to slow down economic growth relative to the case where the negative environmental externality of intermediate goods is not taken into account.

Comparing the optimal solution of our baseline model to the optimum in a modified setting where the weight of pollution in utility is zero ($\psi = 0$), we find that the above intuition is not necessarily correct. First, economic growth is positive for larger rates of time preference in our framework. Second, depending on parameters, growth rates of consumption, production and productivity may in fact be higher than in the model without a negative external effect from pollution.[22]

Moreover, the degree of the household's preference for a clean environment and therefore the strength of the negative pollution externality, as reflected in the size of ψ, does not influence long-run growth rates at all (given $\psi > 0$). The reason is that a stronger environmental preference does not alter the social return to productivity-oriented research, which is the driver of economic growth. The long-run relation between productivity growth and growth in intermediate quantity, consumption and output is fixed independently of the environmental preference on an ABG-path.[23]

Corollary 2 ***Environmental care and the pace of economic growth***

In the solution of theorem 1 compared to the optimal solution in a modified setting without negative external effect from pollution on utility ($\psi = 0$), (i) the condition on ρ for long-run growth in per capita consumption to be positive is less strict and (ii) long-run optimal growth in per capita consumption is faster if and only if the rate of time preference is sufficiently large.

Given $\psi > 0$, the strength of the representative household's preference for a clean environment, as reflected in the size of ψ, has no influence on long-run optimal growth rates.

Proof. See appendix B.4. ∎

The driving force behind the result is a positive link between green and productivity-oriented research. Green innovation can lead to an increase in the optimal amount of labor devoted to research. It thereby fosters also productivity growth and therefore consumption growth. A similar effect has before been described by Ricci (2007).

[22] A similar result can be obtained if the optimal solution with $\psi > 0$ is compared not to the optimum with $\psi = 0$ but to the laissez-faire equilibrium. It is, however, not possible to attribute faster growth to the environmental externality in particular in this case because equilibrium growth may be suboptimally slow as a result of several other externalities ('standing-on-shoulders' of previous innovators, firms cannot appropriate the whole consumer surplus).

[23] A similar result was found by Gradus and Smulders (1993) in a Lucas–Uzawa-model. While stronger environmental preference has no influence on long-run growth rates, it can be expected to affect the levels of the model variables along the long-run path. These effects can however not be analyzed without studying transitional dynamics.

4.5 The elasticity of substitution and research spill-overs

The production function for final goods in the present paper is of the very specific Cobb-Douglas (C-D) form, entailing an elasticity of substitution between nonpolluting and polluting factors of exactly one. In this subsection, we informally discuss the robustness of our main result (persistent decelaration is optimal), when allowing for easier substitution than in our specific model without going all the way toward perfect substitutes. Consider the following variation of our model with output production function (in reduced form)

$$Y = \left[(1-\alpha)\left(Q_1 L_Y\right)^{\frac{\xi-1}{\xi}} + \alpha X^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}, \xi > 0, \xi \neq 1 \text{ and } X = Q_2 L_X.$$

Inserting X, this is equivalent to $Y = \left[(1-\alpha)\left(Q_1 L_Y\right)^{\frac{\xi-1}{\xi}} + \alpha\left(Q_2 L_X\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}$.
The C-D specification of our model results as the limit when $\xi \to 1$.[24]

Once we deviate from the C-D specification, the distinction between Q_1 and Q_2 becomes relevant. In particular it is natural then to consider a version where *in addition* to green innovation, research can be directed to raising Q_1 *and/or* Q_2 (so that $\widehat{Q}_1$, $\widehat{Q}_2$, and $\widehat{B}$ are endogenous). Returning to the example of the introduction, assume that currently the combine harvester brings in 100 tons of grain in 5 man-hours using 50 gallons of diesel. Beside the green innovation (e.g. improving the catalytic converter), there are two productivity innovations: Innovation 1 allows to produce the 100 tons with 4 man-hours and 50 gallons of diesel, thus raising Q_1 by 20%. Innovation 2 allows to harvest the same 100 tons with 5 man-hours and 40 gallons of fuel, thus raising Q_2 by 20 %.

We conjecture that, while deceleration may not be needed asymptotically, in the very long run, the importance of deceleration for the short to medium term depends on the nature of innovation spill-overs. We differentiate the following cases:

Case 1: No innovation spill-overs or symmetric spill-overs: Assume that spill-overs from past research affect the productivity of research in raising Q_1 and in raising Q_2 in the same way.[25] The results from the literature on directed technical change suggest that there exists a ratio $(Q_1/Q_2)^*$ such that (Q_{1t}/Q_{2t}) converges to $(Q_1/Q_2)^*$ at any market equilibrium. In particular, with the assumed symmetric spill-overs, the convergence towards balanced growth of (Q_1, Q_2) does not depend on the elasticity of substitution $\xi > 0$.[26] In the absence of environmental concerns ($\psi = 0$) this should also describe the optimal path of (Q_{1t}/Q_{2t}). In the presence of environmental concerns ($\psi > 0$), the planner can fight pollution by green innovation ($\widehat{B} > 0$) and/or controlling the rebound effect of productivity growth ($\widehat{X} = \widehat{Q}_2 + \widehat{L}_X < \widehat{Q}_1$). Should the planner use the latter option besides green innovation? If $\xi > 1$ the polluting intermediate $X = Q_2 L_X$ can be substituted by nonpolluting $Q_1 L_Y$ more easily compared to the C-D case assumed in this paper. This makes controlling the rebound effect ($\widehat{X} = \widehat{Q}_2 + \widehat{L}_X < \widehat{Q}_1$) less costly compared to the CD case, so that we conjecture that it

[24] The distinction between Q_1 and Q_2 is not useful in this case since $(Q_1 L)^{1-\alpha}(Q_2 X)^{\alpha} = (QL)^{1-\alpha} X^{\alpha}$ for $Q = Q_1 (Q_2)^{\frac{\alpha}{1-\alpha}}$.

[25] If research is performed with labor alone (as in the present paper) the possibility of strictly positive constant long-run growth requires spill-overs from past research to current research: The productivity of labor in raising Q must rise with rising Q. The additional assumption is that these spill-overs are non specific. An equally standard setting is the so called lab-equipment specification, where research is performed with final output. In this alternative specification no such spill-overs are required and symmetry or research productivity with respect to Q_1 and Q_2 is automatically achieved.

[26] For a discussion of the role of symmetric spill-overs see in particular Acemoglu (2009, Chapter 15) and Funk and Vogel (2004).

becomes optimal under even weaker conditions. In the short to medium term, restricting input growth goes along with deceleration, $\widehat{Y} < \widehat{Q}_1$. Only asymptotically, deceleration fades out and output grows at the rate of productivity growth while the social planner lets the share of dirty inputs in the final consumption good, X/Y, fall persistently.

Case 2: Strongly asymmetric spill-overs: If we follow the assumption of strongly asymmetric spillovers, made for example in Acemoglu et al. (2012), the ratio (Q_{1t}/Q_{2t}) of technologies will tend either to infinity or zero at market equilibrium in the long run, depending on the initial ratio[27]. There does not exist a stable ratio $(Q_1/Q_2)^*$ with $0 < (Q_1/Q_2)^* < \infty$. In the more relevant and from a policy perspective more interesting case where the dirty sector is initially sufficiently more advanced, the level of the clean technology becomes asymptotically negligible relative to the dirty one along the equilibrium path. As the two inputs are good substitutes, the input share of the dirty input X increases under laissez-faire. We conjecture that the social planner would reverse the trend in technology and have Q_1/Q_2 increase and X grow more slowly than Q_1 to control the rebound effect. This will again come with a persistently falling input share X/Y and with deceleration ($\widehat{Y} < \widehat{Q}_1$) in the short to medium term, which fades out asymptotically. Different from case 1 considered above however, once $Q_1/Q_2 > (Q_1/Q_2)^*$, deceleration would occur out of growth reasons even under laissez-faire, when all environmental concerns are disregarded.

Still, persistent deceleration remains an important ingredient to the optimal mix of pollution control when allowing for better substitutes (compared to the C-D case analyzed in the present paper) except in the very long run. This is particularly true if we assume symmetric spill-overs in the above sense. Once more consider our combine harvester. The distinction between the two kinds of factor-augmentation is an abstract way to parametrize a specific subset of possible innovations. In particular it would be completely mistaken to think that raising Q_i is more closely linked in a physical sense to factor i than to factor $j \neq i$. We don't see any a priori reason in this example, to deviate from the assumed symmetry with respect to research spill-overs. Now assume instead, that besides the conventional combine harvester with an internal combustion diesel engine, there also exists a version with electric engine and batteries charged with electricity produced with solar energy. Although far from imperative, it seems more plausible now, that the current level of knowledge, say about electric motors, has stronger positive externalities on the productivity of future research directed to further improve electric motors than on research directed to improve combustive engines' further productivity. If so, the race between the augmentation of the two types of engines may be best described by the above case with asymmetric spillovers.

In conclusion, we think that persistent deceleration is important for the following reasons: First, even when the internal combustion engine should be gradually replaced by the electric motor, no currently known production technology is completely clean in reality. Transportation and storage of renewable energy, manufacturing and disposal of batteries, solar cells or wind turbines generate emissions and other forms of pollution. Hence, even within relatively clean sectors, technical progress can be directed to productivity gains and/or to towards reductions in pollution intensity. Second, even when considering the race between different sectors with 'conventional' and relatively 'clean' technology, control of rebound effects will be important and require deceleration for the not too distant future. This is true in particular if cross spill-overs are sufficiently strong compared to

[27] See Acemoglu (2009, Chapter 15).

own spill-overs (from 'clean' technology to future research on 'clean' technology) for Case 1 to apply. But even if spillovers are strongly asymmetric as in case 2 and thus enable a transition to clean technology in finite time, the transition is bound to come slowly: More than 80% of today's energy consumption is not produced from renewable energy but from oil, gas and coal and total energy consumption is growing for all forms of energy, particularly in non-OECD countries.[28] Therefore, for quite a while, reducing pollution intensity and rebound effects within the conventional technology should be considered as potential instruments of world-wide pollution control, although it will go along with deceleration in GDP growth.

5 The model with a non-renewable resource

Pollution in the baseline model studied so far arises as a by product of intermediate good usage. As an example, we have suggested that fossil fuels are used in proportion to intermediate goods in the production of the final good and that pollution is due to emissions of greenhouse gases contained in these fossil fuels. So far we have assumed that there is no restriction on the total amount of fossil fuel used over time. In reality, there only exists a limited stock. It is therefore important to account for the exhaustibility of fossil resources along with the pollution externality. This section examines the robustness of the main results from the baseline model with respect to the consideration of a polluting non-renewable (fossil) resource stock. More precisely, intermediate production is assumed to explicitly use a resource drawn from an exhaustible stock as production input. For simplification, other production inputs such as labor are ignored. We prove that the results of our baseline model for the long-run social optimum still apply if the optimal solution of the baseline model is characterized by quantity degrowth and the initial resource stock is large enough.[29] We have argued before that judging by empirical estimates for the model parameters, a solution with quantity degrowth is reasonable. The negative pollution externality of intermediate production then reduces optimal resource use in a way that a sufficiently large resource stock is never exhausted.

5.1 Setup

We denote the resource stock in period t by F_t. Starting from a finite positive initial level F_0, the resource stock is depleted proportionally to resource use:

$$\dot{F}_t = -R_t. \tag{17}$$

We assume that the resource is owned by the representative household and, for simplification, that it can be extracted at zero cost (see also Barbier (1999), Schou (2000) and Groth and Schou (2002)).

The resource stock F_t must be non-negative for any t. Therefore total extraction must not exceed the initial stock F_0, a requirement which is formally represented in the condition

$$\int_0^\infty R_t dt \leq F_0. \tag{18}$$

[28] See for instance the International Energy Outlook (EIA (2013a)). Unexploited supply of fossil fuels is enough to sustain further growth for a long time and China and India alone are projected to substantially raise fossil fuel consumption.

[29] The introduction of a scarce resource slows growth in the laissez-faire equilibrium, as is shown in the extended appendix.

Suppose that one unit of the intermediate good is produced by one unit of the non-renewable resource so that

$$X_{it} = R_{it} \tag{19}$$

is resource input in sector i and $X_t = \int_{i=0}^{1} X_{it} di = R_t$ aggregate resource use in period t. With a finite resource stock, it is obvious that resource use and therefore intermediate production must ultimately decline to zero in the long run, both in the socially optimal solution and the laissez-faire equilibrium. There has to be quantity degrowth.

Lemma 3 *If intermediate goods are produced with a non-renewable resource according to equation (19), the growth rate $\widehat{X}$ of intermediate quantity is negative in the long run. Any solution path is characterized by quantity degrowth for $t \to \infty$.*

Proof. It follows from (19), that aggregate resource use is $R_t = X_t$. Substitution into equation (18) yields $\int_0^\infty X_t dt \leq F_0$. To satisfy the condition, the integral must converge, which requires $\lim_{t\to\infty} \widehat{X}_t = \widehat{X}_\infty < 0$ as a necessary condition. ■

We now consider the optimal outcome in more detail.[30]

5.2 Resource scarcity in the long-run social optimum

We first characterize the long-run social optimum in case of a binding natural resource constraint. This case is commonly studied in related literature (Schou (2000, 2002), Grimaud and Rouge (2008)). The Lagrange-multiplier λ_{Rt} for the natural resource constraint reflects the social costs of producing one unit of intermediates, i.e., the social price of the non-renewable resource. λ_{Rt} increases over time according to the modified Hotelling rule

$$\widehat{\lambda}_R = \rho. \tag{20}$$

While the social price λ_{Rt} of the non-renewable resource increases with progressing resource scarcity, the shadow price v_{St} of pollution moves along with the marginal disutility of pollution on an asymptotically-balanced growth path[31]. The shadow price therefore falls towards $v_{S\infty}^{\mathrm{R}} = 0$ as the stock of the polluting resource gets exhausted and the pollution stock declines. It is shown in the appendix that in this case, green innovation is no longer optimal in the long run, i.e.,

$$b_\infty^{\mathrm{R}} = \widehat{B}_\infty^{\mathrm{R}} = 0.$$

However, we have suggested earlier that the natural resource constraint need not be binding in the social planner's solution. We know from corollary 1 in subsection 4.3, that the social planner may choose to let the quantity of intermediates decrease in the long run even if there is no constraint imposed on intermediate production by resource scarcity. More precisely, this is the case if preferences are such that a declining pollution stock is desired and the factor elasticity of intermediates is particularly small so that quantity degrowth is not too costly in terms of foregone potential consumption growth. Whenever there is quantity degrowth in the long run, the integral $\int_0^\infty X_t dt$ converges to a finite value. In the modified setting where intermediates are produced

[30] As before, we focus on balanced and asymptotically-balanced growth solutions.

[31] Recall the derivation of equation (16) in the appendix.

from a non-renewable resource, the resource constraint is then not binding given that the initial resource stock is not too small. We prove in the appendix that the long-run optimal solution of the resource model is the same as in our baseline model without resources.

Proposition 2 ***ABG-optimum with an exhaustible resource***
Assume that intermediates are produced with a non-renewable resource according to equation (19). Assume further that the path $\{X_t\}_0^\infty$ for intermediate quantity is continuous.

There is always quantity degrowth in the long-run optimal solution. Further, the following holds:
(a) Binding resource constraint *If the resource constraint is binding, all labor in research and development is shifted to productivity improvements asymptotically and green innovation comes to a halt ($\widehat{B}^R_\infty = 0$).*
(b) Non-binding resource constraint *Given that the conditions for quantity degrowth in the baseline model (see corollary 1) are satisfied and given a sufficiently large (but finite) initial resource stock F_0, the natural resource constraint is not binding in the social planner's problem. There exists an asymptotically unique ABG-solution which for $t \to \infty$ is identical to the ABG-solution with quantity degrowth described in section 4.3. More precisely, growth in output and consumption is positive, given a sufficiently small rate of time preference ρ, and entirely driven by productivity growth. The pollution stock S declines both due to quantity degrowth and because the pollution intensity of intermediate goods is reduced by green innovation. The orientation of research and technical change is given by $\widehat{B}^R_\infty/\widehat{Q}^R_\infty = \alpha/(1-\alpha)$.*
Proof. *See appendix C.2.* ■

In case of a binding resource constraint, resource scarcity forces the social planner to save on polluting inputs to such an extent that investing in green innovation to bring about an even faster decline in pollution is not optimal in the long run. On the other hand, the depletion of the non-renewable resource poses an increasing threat to economic growth over time. Therefore, asymptotically, green innovation comes to a halt. All labor in the research sector is shifted towards productivity improvements. Productivity growth raises the productivity of intermediate goods and thereby dampens the adverse effects from resource scarcity on output and consumption growth.

With a binding resource constraint, saving scarce resources solves the pollution problem. Proposition 2, however, also suggests that under realistic conditions it may be vice versa: The preference for a clean environment may make it optimal to restrict resource use in a way that the resource stock is never exhausted. This also means that the inevitable deceleration induced by resource scarcity will not solve the pollution problem. We have pointed out that the parameter constellations for which there is quantity degrowth in the long-run optimal solution are well in line with empirical evidence. In particular, quantity degrowth has been shown to be a likely outcome of the social planner's optimization problem if the intermediate good is interpreted as energy input and its production elasticity α as the energy share in GDP. Further, although fossil resources are effectively bounded, the large stocks particularly of coal still in the ground suggests that the assumption of a finite but large initial resource stock is also realistic. We conclude that without too strong restrictions on the parameter range, the long-run results from the socially optimal solution of the baseline model extend to a model with a non-renewable resource.

6 Conclusion

Pollution accumulation in our endogenous growth model can be controlled by green innovation and by reducing rebound effects from productivity growth on input quantity. The latter goes along with a cost in terms of foregone potential growth in consumption and GDP which we referred to as 'deceleration'. If intermediate quantity falls in absolute terms, we say that there is 'quantity degrowth'.

On a BGP decelaration cannot occur since output, polluting intermediary inputs, consumption and productivity all grow at the same rate. This means that rebound effects are not restricted. A channel of pollution control is thus neglected in otherwise related literature focussing on balanced growth. The first contribution of this paper is to extend the analysis beyond balanced growth paths. This enables us to address the question of whether and when a deliberate reduction of consumption growth below productivity growth to decrease the growth of polluting inputs may be socially desirable.

By construction of our model, no growth would generally be socially preferable to the laissez-faire equilibrium (which exhibits neither green innovation nor deceleration). At the same time, given the possibility of pollution control, long-run economic growth is a desirable aim from a social planner's perspective. The second contribution of this paper is to show that for empirically reasonable parameter values, optimal pollution control involves green innovation and persistent reduction of rebound effects which requires persistent deceleration. Fostering productivity growth while investing in green innovation to decrease the pollution intensity of production does not achieve the optimal balance between consumption and pollution growth. It has to be ensured that productivity growth is used to raise the average product of polluting inputs to reduce their percentage in GDP and does not merely lead to a faster expansion of production: The rebound effect of productivity growth must be restricted. The model also shows that we cannot rely on resource scarcity to induce sufficient deceleration to solve the pollution problem.

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A Appendix to section 3 (Laissez-faire)

The derivation of the laissez-faire equilibrium can be found in an extended appendix to this paper, available upon request.

A.1 Proof of proposition 1

1. **Existence and Uniqueness:** Proof of existence and uniqueness follows the proof in the standard Schumpeterian growth model and is contained in the extended appendix.

2. **Welfare comparison:** To prove that for convex disutility of pollution, a path without long-run growth would be welfare-improving, consider the utility function as function of the pollution stock S which is obtained using (2):
$$U = \int_0^\infty e^{-\rho t}\left(\frac{\sigma_c}{\sigma_c - 1} c_t^{\frac{\sigma_c - 1}{\sigma_c}} - \psi \frac{\sigma_E}{1-\sigma_E} S_t^{\frac{1-\sigma_E}{\sigma_E}}\right) L dt \tag{A.1}$$
For convex disutility of pollution ($\sigma_E < 1/2$), $\frac{1-\sigma_E}{\sigma_E}$ is at least one while $\frac{\sigma_c-1}{\sigma_c}$ is smaller than one. Along the balanced-growth path, $\widehat{S}^{\text{LF}} = \widehat{S}^{\text{LF}}_\infty = \widehat{c}^{\text{LF}}$. Instantaneous utility $u_t = \frac{\sigma_c}{\sigma_c-1} c_t^{\frac{\sigma_c-1}{\sigma_c}} - \psi \frac{\sigma_E}{1-\sigma_E} S_t^{\frac{1-\sigma_E}{\sigma_E}}$ converges to $-\phi^S(S_t) = -\psi \frac{\sigma_E}{1-\sigma_E} S_t^{\frac{1-\sigma_E}{\sigma_E}}$ and declines persistently towards $(-\infty)$. The long-run growth rate is $\frac{1-\sigma_E}{\sigma_E}\widehat{S}^{\text{LF}}_\infty$. Now assume instead that economic growth is given up in a period s: Consumption growth drops to zero instantly while pollution growth converges to zero over time. Initially, there is a loss in per-period-utility compared to the laissez-faire equilibrium. This loss is only transitory: In the long-run, the pollution stock is constant and so is utility, while utility decreases in the laissez-faire equilibrium. Therefore, from a certain time onwards, not growing yields a utility-gain in each period which increases as $t \to \infty$. Because of the concavity of the utility from consumption and convexity of the disutility from pollution, the transitional welfare-loss is smaller, the later in time the regime-switch occurs and converges to zero as $s \to \infty$. Giving up economic growth in the long-run therefore yields an increase in intertemporal welfare.

B Appendix to section 4 (Social Planner)

B.1 Maximization problem

To see that the optimal q_{it} and b_{it} are the same for all sectors i, i.e. $q_{it} = q_t$ and $b_{it} = b_t$, note that the social planner chooses the step-size in every sector i so as to reach a given rate of change $\dot{Q}_t$ and $\dot{B}_t$ in the respective aggregate technology level with a minimum labor investment. From the equations of motion (12) and (13) for Q and B together with the R&D-cost function (9) we can conclude that the marginal gain of an increase in b_i and q_i, in terms of faster technological progress, and the additional amount of labor required increase in the sectorial technology levels Q_{it} and B_{it} in the same way. Therefore sectorial differences are irrelevant for the optimal choice of q_i and b_i.

The dynamic optimization problem then depends on aggregate variables only: From (9), with $\int_0^1 Q_{it} di = Q_t$, $\int_0^1 B_{it} di = B_t$ and $n_{it} = n_t$, the amount of labor allocated to research in period t is $L_{Dt} = n_t(q_t^2 + b_t^2 + d)$. To produce X_t units of intermediates requires $L_{Xt} = \frac{1}{\varphi}\frac{X_t}{Q_t}$ units of labor. The labor market constraint can be written as

$$L = \frac{1}{\varphi}\frac{X_t}{Q_t} + L_{Yt} + n_t(q_t^2 + b_t^2 + d). \tag{B.1}$$

The equations of motion (12) for Q and (13) for B are:

$$\dot{Q}_t = \mu n q_t Q_t \tag{B.2}$$

$$\dot{B}_t = \mu n b_t B_t \tag{B.3}$$

Given aggregate intermediate production X_t the decision over X_{it} is static. The planner optimally allocates a higher share of aggregate intermediate production to the sectors with higher productivity level so as to maximize Y_t. The optimal X_{it} is:

$$X_{it} = X_t \frac{Q_{it}}{Q_t} \tag{B.4}$$

With (B.4), the aggregate resource constraint can be rewritten as:

$$L_{Yt}^{1-\alpha} X_t^{\alpha} Q_t^{1-\alpha} = c_t L \tag{B.5}$$

The dynamic maximization problem is solved by finding the optimal paths for Q, B, S, c, X, L_Y, n, q and b subject to (8), (B.1), (B.2), (B.3) and the resource constraint (B.5). The current-value Hamiltonian is given by:

$$\begin{aligned} H \quad = \quad & \left(\frac{\sigma_c}{\sigma_c - 1} c_t^{\frac{\sigma_c - 1}{\sigma_c}} - \psi \frac{\sigma_E}{1 - \sigma_E} S_t^{\frac{1-\sigma_E}{\sigma_E}}\right) L \\ & + v_{St}\left(\frac{X_t}{B_t} - \delta S_t\right) \\ & + v_{Qt}\mu n_t q_t Q_t \\ & + v_{Bt}\mu n_t b_t B_t \\ & + \lambda_{Yt}\left(X_t^{\alpha} Q_t^{1-\alpha} L_{Yt}^{1-\alpha} - c_t L\right) \\ & + \lambda_{Lt}(L - \frac{1}{\varphi}\frac{X_t}{Q_t} - L_{Yt} - n_t(q_t^2 + b_t^2 + d)) \end{aligned}$$

where v_{St}, v_{Qt} and v_{Bt} are the shadow-prices of S_t, Q_t and B_t respectively and λ_{Yt} and λ_{Lt} are Lagrange-multipliers.

B.2 First-order conditions

The first-order conditions are:

$$\frac{\partial H}{\partial c_t} = 0 \Leftrightarrow \lambda_{Yt} = c_t^{-1/\sigma_c} \tag{B.6}$$

$$\frac{\partial H}{\partial X_t} = 0 \Leftrightarrow \frac{v_{St}}{B_t} + \lambda_{Yt}\alpha X_t^{\alpha-1}L_{Yt}^{1-\alpha}Q_t^{1-\alpha} - \lambda_{Lt}\frac{1}{\varphi Q_t} = 0 \tag{B.7}$$

$$\frac{\partial H}{\partial q_t} = 0 \Leftrightarrow v_{Qt}\mu n_t Q_t = 2\lambda_{Lt}n_t q_t \tag{B.8}$$

$$\frac{\partial H}{\partial b_t} = 0 \Leftrightarrow v_{Bt}\mu n_t B_t = 2\lambda_{Lt}n_t b_t \tag{B.9}$$

$$\frac{\partial H}{\partial n_t} = 0 \Leftrightarrow v_{Qt}\mu q_t Q_t + v_{Bt}\mu b_t B_t = \lambda_{Lt}\left(q_t^2 + b_t^2 + d\right) \tag{B.10}$$

$$\frac{\partial H}{\partial L_{Yt}} = 0 \Leftrightarrow \lambda_{Yt}(1-\alpha)X_t^{\alpha}Q_t^{1-\alpha}L_{Yt}^{-\alpha} = \lambda_{Lt} \tag{B.11}$$

$$\frac{\partial H}{\partial S_t} = \rho v_{St} - \dot{v}_{St} \Leftrightarrow -\psi S_t^{(1-2\sigma_E)/\sigma_E}L - \delta v_{St} = \rho v_{St} - \dot{v}_{St} \tag{B.12}$$

$$\frac{\partial H}{\partial Q_t} = \rho v_{Qt} - \dot{v}_{Qt}$$

$$\Leftrightarrow \quad v_{Qt}\mu n_t q_t + \lambda_{Yt}(1-\alpha)X_t^{\alpha}Q_t^{-\alpha}L_{Yt}^{1-\alpha} + \lambda_{Lt}\frac{X_t}{\varphi}\frac{1}{Q_t^2} = \rho v_{Qt} - \dot{v}_{Qt} \tag{B.13}$$

$$\frac{\partial H}{\partial B_t} = \rho v_{Bt} - \dot{v}_{Bt} \Leftrightarrow -v_{St}\frac{X_t}{B_t^2} + v_{Bt}\mu n_t b_t = \rho v_{Bt} - \dot{v}_{Bt} \tag{B.14}$$

$$\frac{\partial H}{\partial v_{St}} = \dot{S}_t \Leftrightarrow \frac{X_t}{B_t} - \delta S_t = \dot{S}_t \tag{B.15}$$

$$\frac{\partial H}{\partial v_{Qt}} = \dot{Q}_t \Leftrightarrow \mu n_t q_t Q_t = \dot{Q}_t \tag{B.16}$$

$$\frac{\partial H}{\partial v_{Bt}} = \dot{B}_t \Leftrightarrow \mu n_t b_t B_t = \dot{B}_t \tag{B.17}$$

$$\frac{\partial H}{\partial \lambda_{Yt}} = 0 \Leftrightarrow X_t^{\alpha}Q_t^{1-\alpha}L_{Yt}^{1-\alpha} = c_t L \tag{B.18}$$

$$\frac{\partial H}{\partial \lambda_{Lt}} = 0 \Leftrightarrow L = \frac{1}{\varphi}\frac{X_t}{Q_t} + L_{Yt} + n_t(q_t^2 + b_t^2 + d) \tag{B.19}$$

Further, the transversality conditions $\lim_{t\to\infty}\left(e^{-\rho t}v_{Qt}Q_t\right) = \lim_{t\to\infty}\left(e^{-\rho t}v_{Bt}B_t\right) = \lim_{t\to\infty}\left(e^{-\rho t}v_{St}S_t\right) = 0$ as well as the non-negativity constraints $Q_t, B_t, S_t, c_t, X_t, L_{Yt}, n_t \geq 0$, $\forall t$ must hold.

From the first-order conditions, four key equations crucial for the determination of the long-run optimum are derived: The condition (16) for *asymptotically-balanced growth* in the text follows from the first-order conditions for X and S: The first-order condition (B.7) for X yields a relation $\widehat{v}_{S\infty} = (1-1/\sigma_c)\,\widehat{c}_\infty + \widehat{B}_\infty - \widehat{X}_\infty$ between the growth rates of the marginal utility c_t^{-1/σ_c} of consumption and the shadow price v_S of pollution for $t\to\infty$. From the first-order condition (B.12) for the pollution stock, it follows that along an ABG path, the ratio $S_t^{(1-2\sigma_E)/\sigma_E}/v_{St}$ must be constant for v_S to grow at a constant rate. In the long run, v_S must therefore grow at the same rate as the (instantaneous) marginal disutility $\psi S^{(1-2\sigma_E)/\sigma_E}$ of pollution, $\widehat{v}_{S\infty} = \left(\left(1-2\sigma_E\right)/\sigma_E\right)\widehat{S}_\infty$. Setting equal with the expression for $\widehat{v}_{S\infty}$ obtained from (B.7) and rearranging, taking into account that $\widehat{S}_\infty = \widehat{X}_\infty - \widehat{B}_\infty$ under condition (15), yields (16) in the proof of proposition 2.

We are interested in solution candidates with $n_\infty > 0$. Solving (B.8) and (B.9) for v_Q and v_B respectively, substituting in the first-order condition (B.10) for n and taking the limit for $t \to \infty$ yields

$$q_\infty^2 + b_\infty^2 = d \tag{B.20}$$

Condition (B.20) is an *indifference condition.* It guarantees that the social planner is indifferent between all possible values for n.

Dividing by v_{Qt}, setting $t = \infty$ and rearranging, (B.13) can be written as:

$$(1/\sigma_c)\,\widehat{c}_\infty + \rho = \frac{1}{2}\mu q_\infty^{-1}\left(L_{Y\infty} + \frac{1}{\varphi}\left(\frac{X}{Q}\right)_\infty\right) + \alpha\widehat{X}_\infty + (1-\alpha)\mu n_\infty q_\infty \tag{B.21}$$

Equation (B.21) is a version of the *consumption Euler-equation,* where we replaced the shadow-prices and Lagrange-multipliers as well as their growth rates using (B.8), (B.11) and (B.6).
Both research directions, that is, increasing Q and increasing B, must yield the same social net return. We manipulate the first-order condition (B.14) for B similarly to the one for Q, using (B.9) as well as the expression $v_{St} = \left(\lambda_{Lt}\frac{1}{\varphi Q_t} - \lambda_{Yt}\alpha X_t^{\alpha-1}L_{Yt}^{1-\alpha}Q_t^{1-\alpha}\right)B_t$ from (B.7), and equations (B.11) and (B.6). Setting equal the right-hand sides of (B.21) and the modified first-order condition for B, we obtain the *research-arbitrage condition*

$$\frac{1}{2}\mu q_\infty^{-1}\left(L_{Y\infty} + \frac{1}{\varphi}\left(\frac{X}{Q}\right)_\infty\right) = \frac{1}{2}\mu b_\infty^{-1}\left(\frac{\alpha}{1-\alpha}L_{Y\infty} - \frac{1}{\varphi}\left(\frac{X}{Q}\right)_\infty\right). \tag{B.22}$$

B.3 Proof of theorem 1

If growth rates are to be constant asymptotically, equation (B.22) requires intermediate quantity in efficiency units, more precisely the ratio $(X/Q)_\infty$, to be constant in the limit as well.

A balanced growth path, along which productivity and cleanliness grow at constant rates not only asymptotically, must be characterized by a strictly positive $(X/Q)_\infty$[32]. There must therefore be equal growth in intermediate quantity, productivity and (from the resource constraint) also consumption. Equation (16) then yields a ratio $\widehat{B}_\infty/\widehat{Q}_\infty$:

$$\widehat{B}_\infty/\widehat{Q}_\infty = 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1-\sigma_E)/\sigma_E}. \tag{B.23}$$

If $\alpha/(1-\alpha) < 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$ (see (14)), a balanced growth solution to the social planner's problem does not exist, because the ratio $\widehat{B}_\infty/\widehat{Q}_\infty$ in (B.23) is not reconcilable with equation (B.22) for any nonnegative $(X/Q)_\infty$. As $X/Q < 0$ has no sensible interpretation, the optimal solution is to let X/Q converge to zero asymptotically by choosing $\widehat{X}_\infty < \widehat{Q}_\infty$. According to (B.22), the optimal ratio $\widehat{B}_\infty/\widehat{Q}_\infty$ corresponds to

$$\widehat{B}_\infty/\widehat{Q}_\infty = \frac{\alpha}{1-\alpha}. \tag{B.24}$$

With the definition of the direction of technical change, it follows straightforwardly that technical change is green (productivity-oriented) if and only if $\alpha > 1/2$ ($\alpha < 1/2$).

To compute the relation between the growth rates $\widehat{X}_\infty$ and $\widehat{Q}_\infty$, we use (16), substituting $\widehat{X}_\infty - \widehat{B}_\infty = \widehat{X}_\infty - \frac{\alpha}{1-\alpha}\widehat{Q}_\infty$ for $\widehat{S}_\infty$ and $\alpha\widehat{X}_\infty + (1-\alpha)\,\widehat{Q}_\infty$ from the resource constraint for $\widehat{c}_\infty$. After some manipulation,

[32] On a balanced growth path, $(X/Q)_\infty = 0$ implies $X_t/Q_t = 0$ for all t. This is only possible if $X_t = c_t = 0$ for all t which cannot be an optimal path for X because the utility function satisfies the Inada-conditions for c_t.

we obtain:

$$\circ\widehat{X}_{\infty} = \frac{1+\left(\frac{\alpha}{1-\alpha}\right)^2 - \left(1-\frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}-\frac{\alpha}{1-\alpha}\right)}{1+\frac{\alpha}{1-\alpha}\left(1-\frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)}\widehat{Q}_{\infty} \tag{B.25}$$

For $\frac{\alpha}{1-\alpha} < 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$, it is obvious that $\widehat{X}_{\infty} < \widehat{Q}_{\infty}$ given $\widehat{Q}_{\infty} > 0$.

B.4 Proof of corollary 2

See the extended appendix.

C Appendix to section 5.2 (Optimum with a non-renewable resource)

C.1 First-order conditions

Three changes occur in the set of necessary first-order conditions compared to the baseline model: First, the shadow price λ_R of the non-renewable resource contributes to the marginal social cost of intermediate production instead of the marginal labor requirement, so that the first-order condition for X becomes

$$\frac{\partial H}{\partial X_t} = 0 \Leftrightarrow \frac{v_{St}}{B_t} + \lambda_{Yt}\alpha X_t^{\alpha-1} L_{Yt}^{1-\alpha} Q_t^{1-\alpha} - \lambda_{Rt} = 0. \tag{C.1}$$

In the first-order condition (B.13) for Q, the last term on the left-hand side $(\lambda_{Lt}(1/\varphi)\left(X_t/Q_t^2\right)$ drops out because Q no longer affects the production of intermediate goods.

Second, the first-order conditions are complemented by a complementary slackness condition:

$$\frac{\partial H}{\partial \lambda_{Rt}} \le 0 \Leftrightarrow F_0 - \int_0^{\infty} X_t dt \ge 0 \quad \lambda_{Rt} \ge 0 \quad \lambda_{Rt}\left(F_0 - \int_0^{\infty} X_t dt\right) = 0 \tag{C.2}$$

Third, labor is only allocated to research and output production. The first order condition for λ_{Lt} changes to:

$$\frac{\partial H}{\partial \lambda_{Lt}} = 0 \Leftrightarrow L = L_{Yt} + n_t(q_t^2 + b_t^2 + d) \tag{C.3}$$

The set of first-order conditions is otherwise unaffected by the modifications in the model setup.

C.2 Proof of proposition 2

C.2.1 (a) Binding constraint

(i) **Quantity degrowth:** If there is quantity degrowth, $S_{\infty} = 0$ so that $v_{S\infty} = 0$, while λ_R grows persistently. To satisfy the first-order condition (C.1) for X, the social marginal product of X in production must equal λ_R asymptotically:

$$c_{\infty}^{-1/\sigma_c}\alpha X_{\infty}^{\alpha-1} L_{Y\infty}^{1-\alpha} Q_{\infty}^{1-\alpha} = \lambda_{R\infty} \tag{C.4}$$

Note that we already substituted $\lambda_Y = c_{\infty}^{-1/\sigma_c}$ from the first-order condition for c. Condition (C.4) replaces condition (16) for asymptotically-balanced growth from the baseline model. Computing growth rates on both sides of (C.4) yields $(-1/\sigma_c \cdot \widehat{c}_\infty) - (1-\alpha)\left(\widehat{X}_\infty - \widehat{Q}_\infty\right) = \rho$. From this equation, using $\widehat{c}_\infty = \alpha\widehat{X}_\infty + (1-\alpha)\widehat{Q}_\infty$, we derive the growth rate $\widehat{X}^{\mathrm{R}}_\infty$ for any given $\widehat{Q}^{\mathrm{R}}_\infty$:

$$\widehat{X}^{\mathrm{R}}_\infty = \frac{1}{\frac{\alpha}{1-\alpha}\frac{1}{\sigma_c}+1}\left(\left(1-\frac{1}{\sigma_c}\right)\widehat{Q}^{\mathrm{R}}_\infty - \frac{1}{1-\alpha}\rho\right) \tag{C.5}$$

If $\sigma_c < 1$, it can be seen directly that $\widehat{X}^{\mathrm{R}}_\infty < 0$. For $\sigma_c > 1$ the transversality conditions, which require $\rho > \left(1-\frac{1}{\sigma_c}\right)\widehat{Q}^{\mathrm{R}}_\infty$, together with $(1-\alpha) < 1$ guarantee that indeed $\widehat{X}^{\mathrm{R}}_\infty < 0$.

(ii) **Green Innovation:** The research-arbitrage equation is:

$$\frac{\mu}{2q_\infty}L_{Y\infty} = \frac{\mu}{2b_\infty}L_{Y\infty}\left(\frac{\alpha}{1-\alpha} - \frac{1}{1-\alpha}\left(\frac{\lambda_R}{\lambda_Y}\right)_\infty\left(\frac{X}{Q}\right)^{1-\alpha}_\infty L^{\alpha-1}_{Y\infty}\right) \tag{C.6}$$

Substituting (C.4) in (C.6) shows that investing in the cleanliness of technology is not optimal in the long run:

$$\begin{aligned}\frac{\mu}{2b_\infty}L_{Y\infty}\left(-\frac{\alpha}{1-\alpha}+\frac{\alpha}{1-\alpha}\right) &= (\rho - (1-1/\sigma_c)\,\widehat{c}_\infty) \\ \Leftrightarrow b^{\mathrm{R}}_\infty &= 0\end{aligned}$$

From $q^2_\infty + b^2_\infty = d$ it follows that $q^{\mathrm{R}}_\infty = \sqrt{d}$ so that labor in the R&D-sector is entirely used for productivity-oriented innovation.

C.2.2 (b) Unbinding constraint

Given the assumption of continuity of the path for X, the integral $\int_0^\infty X_t dt$ converges if there is quantity degrowth in the long run (see the extended appendix). Therefore $\int_0^\infty X_t dt < F_0$ for a sufficiently large F_0. In this case, the natural resource constraint is not binding and it follows from (C.2) that $\lambda_{Rt} = 0$, $\forall t$. If $\lambda_R = 0$, differences in the first-order conditions compared to the baseline model only arise because labor is no longer used in intermediate production in the model of this section. But for parameter constellations such that there is quantity degrowth in the baseline model, labor use in intermediate production converges to zero in the baseline model as well, so that the first-order conditions and therefore the long-run solutions are identical for $t \to \infty$.