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HOUSEHOLD SPECIALIZATION AND THE LABOR-SUPPLY ELASTICITIES OF WOMEN AND MEN

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Abstract

This paper studies gender differences in the elasticity of labor supply in a model of household specialization. I show that household specialization implies larger Frisch elasticities for the partner that specializes in home production. Quantitatively, empirical time-use ratios alone imply differences in the Frisch elasticity between women and men of more than 50%. Similar results are obtained for long-run elasticities. My results imply that the elasticity of labor supply is not a deep parameter which can, e.g., explain parts of the state-dependent effects of fiscal policy.

Keywords: Labor-supply elasticity, gender, home production
JEL classification: E24, J22, J16, D13

1 Introduction

The elasticity of labor supply is a key concept in many parts of economics including, next to labor economics, macroeconomics and optimal taxation theory. There is empirical consensus that labor supply is not equally elastic across the population (e.g., Francesconi 2002 and Keane 2011). Differences in labor-supply elasticities have implications for the behavior of different population groups over the business cycle, for the effects of fiscal policy, and for the distribution of optimal marginal tax rates.

This paper focusses on gender differences in labor-supply elasticities. There is strong empirical evidence that women's labor supply is in general more elastic than men's (Cogan 1981; Eckstein and Wolpin 1989; Bourguignon and Magnac 1990; van der Klaauw 1996; Francesconi 2002; Dechter 2013). Concerning the Frisch elasticity which governs short-run reactions to wage changes, empirical estimates are about twice as high for women than for men (Keane 2011).

I propose an explanation for gender differences in the Frisch elasticity of labor supply in a model of household specialization. In the model, the partner that specializes in

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home production (traditionally, the wife) has the higher Frisch elasticity of labor supply. Gender differences in the Frisch elasticity are caused by different allocations of working time to home production and market work. For any given level of market hours, marginal disutility of market work increases with the number of hours worked in the household, which causes stronger labor-supply reactions to wage changes. In my model, the Frisch elasticity of *market* labor supply is approximately equal to the elasticity of *total* work multiplied by the ratio of total to market work.[1]

Generally, it is well known that home production increases the elasticity of labor supply as many home-made goods are likely to have close market-purchased substitutes (e.g., Cahuc and Zylberberg 2004 and Rogerson and Wallenius 2009). Cahuc and Zylberberg (2004) have informally argued that this can explain gender differences in labor-supply elasticities simply because home production is more important empirically as a time use for women than for men. While related, the mechanism in my model is different from the one described above for the following reasons.

My explanation does not rely on substitutability between home-made and market-purchased goods. It would also apply if home produced goods were perfect complements to market-purchased consumption goods as long as individuals view market hours and home hours as substitutes.

While this is related, it implies that my explanation also applies to the Frisch elasticity while the above explanation does not. If a household substitutes home production by market consumption, marginal utility of the latter does exactly not stay constant. Reactions to transitory shocks can be magnified by complementarity between the two goods but the Frisch elasticity is unaffected.

I also quantify the role of household specialization on Frisch elasticities. Applying the results for the Frisch elasticity in my model, one can explain about half of the gender differences in empirical Frisch elasticities using time use evidence on household specialization (Ramey and Francis 2009). The model can easily match the stylized empirical observation of women's Frisch elasticities being twice as large as men's if one allows for gender differences in preferences. However, my results suggest that endogenous determinants are quantitatively important for the gender differences in labor-supply elasticities.

My analysis further implies that estimates of preference parameters are biased when home production is omitted from the estimated model. I demonstrate this by estimating a standard model with an artificial data set generated in a model with home production. There, I estimate pronounced gender differences in work preferences although the data-generating model features identical preferences. Also this documents that, neglecting home production, one would mistake the gender differences in labor-supply elasticities as exogenous while they are, in fact, to a substantial part endogenous.

[1] The exact Frisch elasticity also contains the effects of substitution within home production and between home production and leisure. These effects are comparably small but tend to magnify the gender differences in Frisch elasticities, see below.

This has interesting implications. First, it implies that the Frisch elasticity is policy-variant. Policies that affect the distribution of time use within the household such as subsidized child care or direct subsidies for home production change the responsiveness of labor supply to transitory changes in wage rates. Further, my results imply that Frisch elasticities vary over the business cycle. In recessions, the lower share of market work to total work raises the Frisch elasticity. Since this elasticity is one of the major determinants of the transmission of fiscal-policy shocks, this might be one of the reasons behind the finding that fiscal policy appears to be more effective in recessions (Auerbach and Gorodnichenko 2012; Auerbach and Gorodnichenko 2013).

I also analyze the implications of my model for long-run elasticities. Considering gender differences, I find that also long-run labor-supply elasticities tend to be higher for partners that specialize in home production. This has important policy implications as policy makers can expect rather strong effects of policies which change long-run earnings potentials of population groups that initially work much in home production. But, my model also implies that the labor-supply elasticity is not a deep parameter and is thus generally policy-variant. E.g., public provision or subsidizing of child care which reduces the amount of mothers' work at home can reduce the effectiveness of employment subsidies for women.

Next to the relation between home production and the elasticity of labor supply, this paper contributes to the wider literature on the non-preference determinants of labor-supply elasticities. Imai and Keane (2004) show that estimates of the Frisch elasticity are downward-biased when the estimated model omits the effects of on-the-job human capital accumulation. Similarly, Domeij and Floden (2006) demonstrate the importance of borrowing constraints for estimates of the Frisch elasticity. The arising downward bias is less pronounced for women (Bredemeier, Gravert, and Juessen 2015). Concerning gender differences, the higher labor-supply elasticities of women at the macro level has also been related to the higher importance of the extensive margin for this group (Chang and Kim 2006). The literature has also recognized the role of female labor supply as an insurance device (e.g., Ortigueira and Siassi 2013) which implies a different cyclicality of women's hours worked compared to men's.

The paper is also related to the literature on cross-sectional differences in gender-specific labor-supply elasticities. Compatible with my predictions, Kaya (2014) shows that female labor-supply elasticities are particularly high in couples with young children (where home production is arguably important) and in situations where the degree assortative mating is high (which increases the importance of household specialization, see Bredemeier and Juessen 2013).

The remainder of this paper is organized as follows. Section 2 presents the model set-up. Section 3 analyzes Frisch labor-supply elasticities. Section 4 studies long-run labor-supply elasticities. Section 5 concludes.

2 Model

A household consists of two spouses, a wife F and a husband M, who live together forever. There are two commodities c and d. c is a usual consumption good which is produced and purchased on the market while d is a Beckerian home commodity that the household produces itself. Spouses face a joint budget constraint and engage jointly in home production. Each household chooses consumption quantities of the two commodities, hours worked on the market n, and hours worked in home production h for both members.

The household's decision problem is to maximize the Lagrangian

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \left[\begin{array}{c} \sum_{g=M,F} \mu_{g,t+s} \cdot \left(\frac{c_{g,t+s}^{1-\sigma}}{1-\sigma} + \nu_d \frac{d_{t+s}^{1-\kappa}}{1-\kappa} - \nu_l \frac{(n_{g,t+s}+h_{g,t+s})^{1+\eta_g^{-1}}}{1+\eta_g^{-1}} \right) \\ +\lambda_{t+s} \left[\begin{array}{c} w_{F,t+s} n_{F,t+s} + w_{M,t+s} n_{M,t+s} + (1+r_{t+s})\, b_{t+s} \\ +\pi_{t+s} - c_{M,t+s} - c_{F,t+s} - b_{t+s+1} \end{array} \right] \\ +\chi_{t+s} \left[A_{t+s} h_{F,t+s}^{\theta} h_{M,t+s}^{1-\theta} - d_{t+s} \right] \end{array} \right], \quad (1)$$

where w are wage rates, b are bonds, r the rate of return on bonds, and π summarizes dividends, taxes, transfers, and other lump-sum incomes or expenditures. A denotes TFP in home production, σ, ν_d, κ, ν_l, η_g, and θ are parameters, and λ and χ Lagrange multipliers. Note that I allow for gender differences in labor disutility. However, the gender differences in preferences which are needed to rationalize the gender differences in labor-supply elasticities are small. The $\mu_{g,t+s}$ are utility weights which sum up to one. Every point on the Pareto frontier can be reached by the appropriate $\mu_{M,t+s}$ and $\mu_{F,t+s} = 1-\mu_{M,t+s}$. I thus consider cooperative decision making as is standard in collective models of the household (Chiappori 1988; Chiappori 1992).

In order to highlight effects, I consider three variants of the model, one without home production and two variants that include home production.[2] In the *model version without home production (variant 1),* I set the valuation of the home-made good to zero, $\nu_d = 0$. In the first model variant with home production, I assume that home production variables, $h_{M,t}$, $h_{F,t}$, and d_t have to be determined one period in advance. In this *model variant with pre-determined home production (variant 2)*, the household problem is to

$$\max \mathcal{L} \text{ over } \left\{ c_{g,t+s}, n_{g,t+s}, h_{g,t+s+1}, d_{t+s+1}, b_{t+s+1} \right\}_{s=0}^{\infty}.$$

This model variant is supposed to be understood as a means of demonstration. It allows to disentangle the effect of working in home production per se from substitution within home production or between home production and leisure. In the third model variant, I allow households to choose home production simultaneously with market labor supply. In this *model variant with simultaneous choice (variant 3),* the household problem is to

$$\max \mathcal{L} \text{ over } \left\{ c_{g,t+s}, n_{g,t+s}, h_{g,t+s}, d_{t+s}, b_{t+s+1} \right\}_{s=0}^{\infty}.$$

[2]In Section 3.2, I further consider two variations of the model with different utility functions to demonstrate the role of substitutabilities.

The first-order conditions for $c_{M,t}, c_{F,t}, n_{M,t}, n_{F,t}, h_{M,t}, h_{F,t}, d_t, b_{t+1}, \chi_t$, and λ_t, respectively, are

$$\mu_{M,t} c_{M,t}^{-\sigma} = \lambda_t, \tag{2}$$
$$\mu_{F,t} c_{F,t}^{-\sigma} = \lambda_t, \tag{3}$$
$$\mu_{M,t} \nu_l l_{M,t}^{\eta^{-1}} = \lambda_t w_{M,t}, \tag{4}$$
$$\mu_{F,t} \nu_l l_{F,t}^{\eta^{-1}} = \lambda_t w_{F,t}, \tag{5}$$
$$E_t \mu_{M,t(+1)} \nu_l l_{M,t(+1)}^{\eta^{-1}} = E_t \chi_{t(+1)} \left(1-\theta\right) A_{t+s} h_{F,t(+1)}^{\theta} h_{M(+1)}^{-\theta}, \tag{6}$$
$$E_t \mu_{F,t(+1)} \nu_l l_{F,t(+1)}^{\eta^{-1}} = E_t \chi_{t(+1)} \theta A_{t+s} h_{F,t(+1)}^{\theta-1} h_{M(+1)}^{1-\theta}, \tag{7}$$
$$\nu_d d_{t(+1)}^{-\kappa} = E_t \chi_{t(+1)}, \tag{8}$$
$$\lambda_t = \beta E_t \lambda_{t+1} \left(1 + r_t - \delta\right), \tag{9}$$
$$d_t = A h_F^{\theta} t h_M^{1-\theta}, \tag{10}$$
$$c_{M,t} + c_{F,t} + b_{t+1} = w_{M,t} n_{M,t} + w_{F,t} n_{F,t} + \left(1 + r_t\right) b_t + \pi_t, \tag{11}$$

where $l_{g,t} = n_{g,t} + h_{g,t}$ is total work of spouse $g = F, M$ and the $(+1)$ indicates the potential pre-determination of home production. Conditions (6), (7), (8), and (10) are completely irrelevant in the model variant without home production and irrelevant for impact reactions to shocks in the model variant with predetermined home production.

3 Frisch elasticities

3.1 Analytical results

How does the couple react to transitory wage changes? Formally, I consider an unanticipated change in $w_{M,t}$ or $w_{F,t}$, respectively. Considering the Frisch labor-supply elasticity, I hold the valuation of wealth, λ, constant. This implies that, to determine the Frisch elasticity, one only needs to consider the first-order conditions (4) - (8) and (10). In order to calculate the Frisch elasticities, I consider the following log-linearized versions of these conditions where hats "ˆ" indicate percentage changes:

$$\eta_M^{-1} \frac{n_M}{l_M} \widehat{n}_{M,t} + \eta_M^{-1} \frac{h_M}{l_M} \widehat{h}_{M,t} = \widehat{w}_{M,t}, \tag{12}$$
$$\eta_F^{-1} \frac{n_F}{l_F} \widehat{n}_{F,t} + \eta_F^{-1} \frac{h_F}{l_F} \widehat{h}_{F,t} = \widehat{w}_{F,t}, \tag{13}$$
$$\frac{n_M}{\eta_M l_M} E_t \widehat{n}_{M,t(+1)} + \left(\frac{h_M}{\eta_M l_M} + \theta\right) \widehat{h}_{M,t(+1)} - \theta \widehat{h}_{F,t(+1)} - E_t \widehat{\chi}_{t(+1)} = 0, \tag{14}$$
$$\left(\theta - 1\right) \widehat{h}_{M,t(+1)} + \frac{n_F}{\eta_F l_F} E_t \widehat{n}_{F,t(+1)} + \left(\frac{h_F}{\eta_F l_F} + 1 - \theta\right) \widehat{h}_{F,t(+1)} - E_t \widehat{\chi}_{t(+1)} = 0, \tag{15}$$
$$-\kappa \widehat{d}_{t(+1)} - E_t \widehat{\chi}_{t(+1)} = 0, \tag{16}$$
$$-\left(1-\theta\right) \widehat{h}_{M,t} - \theta \widehat{h}_{F,t} + \widehat{d}_t = 0. \tag{17}$$

One of the key drivers of the results can be seen in the first two equations above. When agents work some time in home production, a one-percent increase in market hours, $\widehat{n}_{g,t}$, increases marginal labor disutility only by the inverse of the curvature parameter η_g multiplied by the ratio of market to total work. This follows from the fact that a one-percent increase in market hours increases *total* working time by less than one percent. I now go through the three different model variants and calculate the Frisch elasticity.

In the model without home production, I can focus on the first two conditions (12) and (13) as the other four conditions (14) - (17) relate to home-production variables. Further, as spouses do not work in home production in this variant, $h_{g,t} = 0$, market work is equal to total work, $n_M/l_M = n_F/l_F = 1$. The system (12) and (13) thus simplies to

$$\eta_M^{-1} \cdot \widehat{n}_{M,t} = \widehat{w}_{M,t}, \tag{18}$$
$$\eta_F^{-1} \cdot \widehat{n}_{F,t} = \widehat{w}_{F,t}, \tag{19}$$

and is solved by the following Frisch labor-supply functions in log-linear terms:

$$\widehat{n}_{M,t} = \eta_M \cdot \widehat{w}_{M,t}, \tag{20}$$
$$\widehat{n}_{F,t} = \eta_F \cdot \widehat{w}_{F,t}. \tag{21}$$

Thus, in this model variant, I obtain the well-known standard result that the Frisch elasticity is constant and reflects the utility function's curvature in hours worked,

$$FLSE_g = \eta_g. \tag{22}$$

As a consequence, gender differences in the Frisch elasticity can only be generated by gender differences in preferences in this model variant without home production.

In the model variant with predetermined home production, the household can not react to wage changes with changes in home production on impact. Formally, $E_{t-1}\widehat{\chi}_t = 0$ in conditions (14) - (16). It follows that $\widehat{h}_{M,t} = \widehat{h}_{F,t} = \widehat{d}_t = 0$. So, also in this model variant, one can focus on the first two conditions (12) and (13) when determining the impact reactions to wage changes. By contrast to the model variant without home production, spouses do work positive home hours such that the shares of working time devoted to market work, n/l, are below one. The impact reaction to wage changes in this model variant is determined by the system

$$\eta_M^{-1} \cdot \frac{n_M}{l_M} \cdot \widehat{n}_{M,t} = \widehat{w}_{M,t}, \tag{23}$$
$$\eta_F^{-1} \cdot \frac{n_F}{l_F} \cdot \widehat{n}_{F,t} = \widehat{w}_{F,t}, \tag{24}$$

and the resulting Frisch labor-supply functions in log-linear terms are

$$\widehat{n}_{M,t} = \eta_M \cdot \frac{l_M}{n_M} \cdot \widehat{w}_{M,t}, \tag{25}$$

$$\widehat{n}_{F,t} = \eta_F \cdot \frac{l_F}{n_F} \cdot \widehat{w}_{F,t}. \tag{26}$$

Here, the Frisch elasticity depends on time use,

$$FLSE_g = \eta_g \cdot \frac{l_g}{n_g}. \tag{27}$$

Specifically, the Frisch elasticity decreases in the share of total labor that is devoted to market work. With household specialization, this translates into *higher* Frisch elasticities of the partner that specializes in home production – traditionally, the wife.

In the full model with simultaneous choice, one needs to consider all six conditions (12) - (17). In matrix form, I write them as

$$\begin{pmatrix} \frac{\eta_M^{-1} \cdot n_M}{l_M} & \frac{\eta_M^{-1} \cdot h_M}{l_M} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\eta_F^{-1} \cdot n_F}{l_F} & \frac{\eta_F^{-1} \cdot h_F}{l_F} & 0 & 0 \\ \frac{\eta_M^{-1} \cdot n_M}{l_M} & \frac{\eta_M^{-1} \cdot h_M}{l_M} + \theta & 0 & -\theta & 0 & -1 \\ 0 & \theta - 1 & \frac{\eta_F^{-1} \cdot n_F}{l_F} & \frac{\eta_F^{-1} \cdot h_F}{l_F} + (1-\theta) & 0 & -1 \\ 0 & 0 & 0 & 0 & -\kappa & -1 \\ 0 & \theta - 1 & 0 & -\theta & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \widehat{n}_{M,t} \\ \widehat{h}_{M,t} \\ \widehat{n}_{F,t} \\ \widehat{h}_{F,t} \\ \widehat{d}_t \\ \widehat{\chi}_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \widehat{w}_{M,t} \\ \widehat{w}_{F,t} \end{pmatrix}. \tag{28}$$

The solution of this model variant is

$$\begin{pmatrix} \widehat{n}_{M,t} \\ \widehat{h}_{M,t} \\ \widehat{n}_{F,t} \\ \widehat{h}_{F,t} \\ \widehat{d}_t \\ \widehat{\chi}_t \end{pmatrix} = \begin{pmatrix} \eta_M \frac{l_M}{n_M} + \frac{h_M}{n_M} \cdot \frac{1-\theta+\theta\kappa}{\kappa} & \frac{h_M}{\kappa n_M}(\theta - \theta\kappa) \\ \frac{1}{\kappa}(\theta - \theta\kappa - 1) & \frac{1}{\kappa}(-\theta + \theta\kappa) \\ \frac{h_F}{\kappa n_F}(1 - \theta - \kappa + \theta\kappa) & \eta_F \frac{l_F}{n_F} + \frac{h_F}{n_F} \cdot \frac{\theta+\kappa-\theta\kappa}{\kappa} \\ \frac{1}{\kappa}(\theta + \kappa - \theta\kappa - 1) & \frac{1}{\kappa}(-\kappa - \theta + \theta\kappa) \\ \frac{1}{\kappa}(\theta - 1) & -\frac{\theta}{\kappa} \\ -\theta + 1 & \theta \end{pmatrix} \begin{pmatrix} \widehat{w}_{M,t} \\ \widehat{w}_{F,t} \end{pmatrix}. \tag{29}$$

So, the Frisch elasticities in this full model variant are

$$FLSE_M = \eta_M \cdot \frac{l_M}{n_M} + \frac{h_M}{n_M} \cdot \frac{1 - \theta(1-\kappa)}{\kappa}, \tag{30}$$

$$FLSE_F = \eta_F \cdot \frac{l_F}{n_F} + \frac{h_F}{n_F} \cdot \frac{\theta(1-\kappa) + \kappa}{\kappa}. \tag{31}$$

Also here, the Frisch elasticities depend on time use. The Frisch elasticities in the full model contain the term $\eta_g \cdot \frac{l_g}{n_g}$ which are already known from the variant with predetermined home production. Here, a second term is added to the Frisch elasticity that describes substitution within home production (governed by the parameter θ) as well as substitution between home production and leisure (governed by the parameter κ).

It is insightful to consider the special case where $\kappa \to 1$ (a prerequisite for balanced growth as we will see below). In this special case, the second summands in the Frisch elasticities (30) and (31) simplify to h_g/n_g and, hence, is larger for the partner who specializes

Table 1: Summary of Frisch elasticities in the different model variants.

	Men	**Women**
without home production	η_M	η_F
predetermined home production	$\eta_M \cdot \frac{l_M}{n_M}$	$\eta_F \cdot \frac{l_F}{n_F}$
simultaneous choice	$\eta_M \cdot \frac{l_M}{n_M} + \frac{h_m}{n_M} \cdot \frac{1-\theta(1-\kappa)}{\kappa}$	$\eta_F \cdot \frac{l_F}{n_F} + \frac{h_F}{n_F} \cdot \frac{\theta(1-\kappa)+\kappa}{\kappa}$

in home production. In consequence, for $\kappa \to 1$, the Frisch elasticity is unambiguously larger for the spouse who specializes in home production if preferences do not differ by gender.

Table 1 summarizes the Frisch labor-supply elasticities in the three model variants. The main difference between the variant without home production and the two variants with home production is that in the latter ones, the Frisch elasticities depend on time use. The Frisch elasticity is larger for the spouse who specializes in home production since, at any given level of market hours, marginal disutility of labor is larger for this person. Differences between the two model variants with home production follow from substitution effects within home production between home production and leisure. These effects tend to magnify somewhat the differences in Frisch elasticities within couples.

3.2 The role of substitutability between goods and between time uses

This section demonstrates the role of the substitutability between hours worked at home and in the market in contrast to the substitutability between the different goods. I start with a model with perfect complementarity between the goods c and d such that there is no substitution at all between these two goods. This model variation serves the purpose to demonstrate that the mechanism highlighted in this paper does not rely on substitutability between goods.

Model variant with perfect complementarity between goods. If the two consumption goods are perfect complements, holding constant the marginal utility of the market consumption good, λ, implies that also marginal utility of the home production good, χ, is constant. This gives the following system of log-linearized first-order conditions relevant for the Frisch elasticity:

$$\eta_g^{-1} \cdot \frac{n_g}{l_g} \cdot \widehat{n}_{g,t} + \eta_g^{-1} \cdot \frac{h_g}{l_g} \cdot \widehat{h}_{g,t} = \widehat{w}_{g,t} \quad \forall g = F, M, \tag{32}$$

$$\widehat{h}_{F,t} - \widehat{h}_{M,t} = \widehat{w}_{M,t} - \widehat{w}_{F,t}, \tag{33}$$

$$\theta \cdot \widehat{h}_{F,t} - (1-\theta) \cdot \widehat{h}_{M,t} = 0, \tag{34}$$

which are the counterparts to (12)-(15) and (17) above. The system is solved by the Frisch labor-supply functions in log-linear terms

$$\widehat{n}_{M,t} = \left(\eta_M \cdot \frac{l_M}{n_M} + \theta \cdot \frac{h_M}{n_M}\right) \cdot \widehat{w}_{M,t} - \theta \cdot \frac{h_M}{n_M} \cdot \widehat{w}_{F,t} \quad \text{and} \tag{35}$$

$$\widehat{n}_{F,t} = \left(\eta_F \cdot \frac{l_F}{n_F} + (1-\theta) \cdot \frac{h_F}{n_F}\right) \cdot \widehat{w}_{F,t} - (1-\theta) \cdot \frac{h_F}{n_F} \cdot \widehat{w}_{M,t} \tag{36}$$

for husband and wife, respectively. It follows that the Frisch elasticities are given by

$$FLSE_M = \eta_M \cdot \frac{l_M}{n_M} + \theta \cdot \frac{h_M}{n_M} \quad \text{and} \tag{37}$$

$$FLSE_F = \eta_F \cdot \frac{l_F}{n_F} + (1-\theta) \cdot \frac{h_F}{n_F} \tag{38}$$

which resemble (30) and (31) above. The difference is that the substitution between home production and leisure is shut off such that the parameter κ disappears. Importantly, if preferences and home productivities are identical in this model variation, the Frisch elasticity is unambiguously larger for the spouse who specializes in home productions, for whom both l_g/n_g and h_g/n_g are larger. This effect is not a result of a substitutability between goods which is completely shut off here. Rather, it follows from the substitutability of working types which, at any given level of market hours, implies a larger marginal disutility from market work for the spouse who works more in home production.

Model variant without substitutability between working types. In this model variant, I take the reverse position and shut off substitutability between market hours and home hours and allow instead for potentially strong substitutability between the two consumption goods in the model. I consider individual preferences

$$u_{g,t} = \widetilde{u}\left(c_{g,t}, d_t\right) - \nu_n \cdot \frac{n^{1+\eta_n^{-1}}}{1+\eta_n^{-1}} - \nu_n \cdot \frac{n^{1+\eta_n^{-1}}}{1+\eta_n^{-1}} \tag{39}$$

where $\widetilde{u}$ can contain any degree of substitutability between the goods c and d but the additive separability between working types shuts off the discussed effects on the Frisch elasticity of market hours. This can easily be seen in the log-linearized first-order condition for market hours which is

$$\eta_n^{-1} \cdot \widehat{n}_g = \widehat{\lambda}_t + \widehat{w}_{g,t} \quad \forall g = F, M \tag{40}$$

in this model variant. Hence, the Frisch labor-supply elasticites in this model variant are simply determined by preferences and given by

$$FLSE_g = \eta_n \quad \forall g = F, M. \tag{41}$$

Table 2: Weekly time devoted to work activities of 25-54 years old in 2005, from Ramey and Francis (2009).

	Men	Women	Gender ratio
total labor l	54.1	57.2	.946
market work n	36.8	26.1	1.41
home production h	17.3	31.1	.556
share of market work n/l	.680	.456	1.49
home to market hours h/n	.470	1.19	.395

The Frisch elasticities are not affected by the specific form of $\widetilde{u}$ and thus not by substitutability between the market-purchased goods c and home-made commodities d. In fact, such substitutability can amplify the reactions to transitory shocks for given Frisch elasticities but does not affect the Frisch elasticity itself, see e.g. Gnocchi, Hauser, and Pappa (2014). It can thus not explain the gender differences in its estimates.[3] Of course, substitutability between the two goods impacts on the elasticity of labor supply in the long run, i.e. on Marshall elasticities, as argued by Cahuc and Zylberberg (2004). In the following, I return to analyzing the model as outline in Section 2.

3.3 Quantitative Evaluation

Relation to time-use evidence. I now consider which portion of empirical gender differences in Frisch elasticities can be explained through observed differences in time use. Table 2 shows the weekly working times of individuals in prime working age (25-54) in the United States for 2005. The information stems from Ramey and Francis (2009). One sees that differences in total labor are small, women work some three hours more per week than men. By contrast, there are pronounced differences between genders with respect to the allocation of total labor to market work and home production. Men devote about 68% of their labor to market work while, for women, this number is only about 46%.

Using these empirical numbers in the model-implied Frisch elasticities, one sees that they imply substantial gender differences in the elasticity of labor supply even without gender differences in preferences. Equation (27) shows that women have Frisch elasticities which are about 50% higher than those of men in the model variant with predetermined home production if preferences are identical across genders and the model is calibrated to match the observations in Table 2.[4]

In the model with simultaneous choices on home production and labor supply, gender differences in the model-implied Frisch elasticities under identical preferences are even larger. Quantitatively, the gender ratio in Frisch elasticities also depends on η in the full

[3] To estimate Frisch elasticities, the first-order condition of market hours is written in first log differences and the difference in $\ln \lambda$ replaced by fixed effects and an expectation error from the Euler equation which is the residual in an OLS estimation. Since the wage rate change is correlated with this residual, it is instrumented by its expectation. See Blundell and Macurdy (1999). Applying these steps in this model variant does not include $\widetilde{u}$.

[4] See below for a calibration that achieves this goal.

Table 3: Estimation results for the model variant without home production using artificial data stemming from a simulation of the full model.

		prior		posterior		
variable	distrib.	mean	std. dev.	mean	5% conf.	95% conf
η_M	gamma	0.3333	1.000	0.1007	0.1003	0.1011
η_F	gamma	0.3333	1.000	1.7796	1.7779	1.17814

model. Assuming $\kappa \to 1$ and, as an example, $\eta_M = \eta_F = \eta = 1/3$, the ratio of Frisch elasticities evaluates as $FLSE_F/FLSE_M = 1.54/0.96 = 1.60$ and their average level lies in the ballpark of the elasticities reported in Keane (2011).

Given that empirical estimates of the Frisch labor-supply elasticity are somewhat more than twice as high for women than for men (Keane 2011), one sees that more than half of these gender differences in estimated Frisch elasticities can be explained solely by the empirical gender differences in the fractions of working time devoted to home production and market work, respectively.[5]

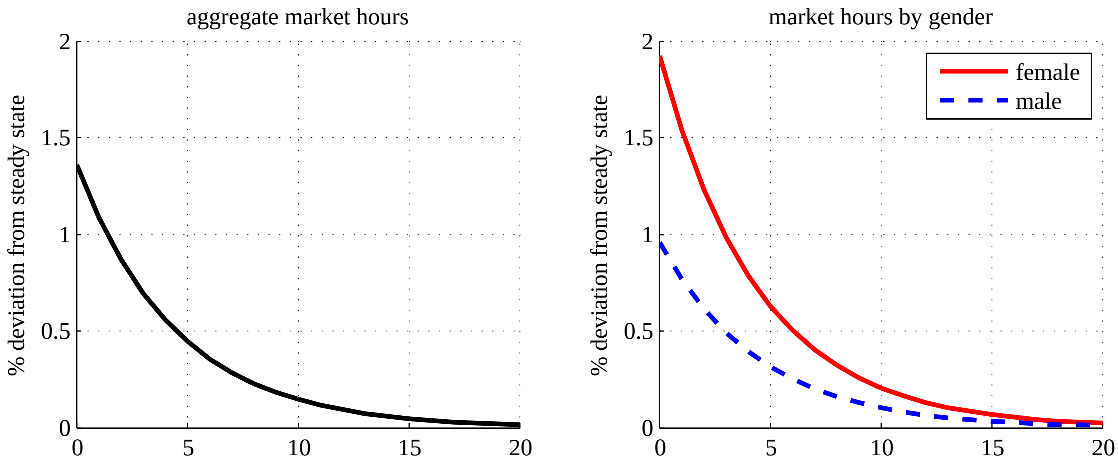
Simulation/estimation exercise. In order to further evaluate the quantitative implications of my results, I perform a simulation/estimation exercise with the model variants 1 and 3. I first simulate the full model with home production and simultaneous choice. Then, I use the artificial data from this simulation to estimate the model variant that does not include home production.

To parameterize the full model, I use a combination of setting certain parameters and calibrating others. Most importantly, I set the parameters that determine the curvature of labor disutility equally to $\eta_M = \eta_F = 1/3$. I set the the values $\sigma = \kappa = 1$ which imply balanced growth, see Section 4 below. I set the time preference rate to $\beta = 0.995$ implying an annual interest rate of 2% as I interpret a period as a quarter. I use two independent AR(1) process for log wages of women and men with a long-run wage gap of $w_F/w_M = 0.8$ but equal persistence of 0.75. I then calibrate the remaining parameters to match the empirical time-use ratios in Table 2. This gives $\theta = 0.5899$ to match the gender ratio of home hours. The utility weights are calibrated to $\mu_M = 0.4239$ and $\mu_F = 0.5761$ to generate the empirical gender ratio of total working times. Finally, to generate the empirical ratios of home production to market work, I obtain $\nu_d = 0.8876$ and gender-specific utility weights on labor, $\nu_{lM} = 11.6263$ and $\nu_{lF} = 6.2954$.

From the simulation of the full model, I save the time series of the cyclical components of male and female wages and market hours ($\widehat{w}_M$, $\widehat{w}_F$, $\widehat{n}_M$, $\widehat{n}_F$). Then, I use this artificial data to estimate the model variant without home production. Specifically, I estimate the labor disutility parameters η_M and η_F and the variances of the wage shocks taking the other parameters as given above (as long as they exist in the model variant without home

[5] The remainder does not necessarily have to be devoted to gender differences in preferences. Bredemeier, Gravert, and Juessen (2015) argue that the gender differences in preferences are over-estimated when usual labor-supply regression techniques are applied.

Figure 1: Effects of tax cuts at baseline steady state.



production). I use a Bayesian estimation technique for dynamic stochastic equilibrium models and use a prior gamma distribution with mean 1/3 and variance 1 for both, η_M and η_F. The estimation results for η_M and η_F are given in Table 3. Although the data-generating model features no gender differences in preferences, the estimation results for the model without home production strongly reject the null hypothesis of equal preferences across genders. This indicates, that researchers may wrongly conclude that there are substantial gender differences in preferences when home production is omitted from the estimated model.

3.4 Policy analysis

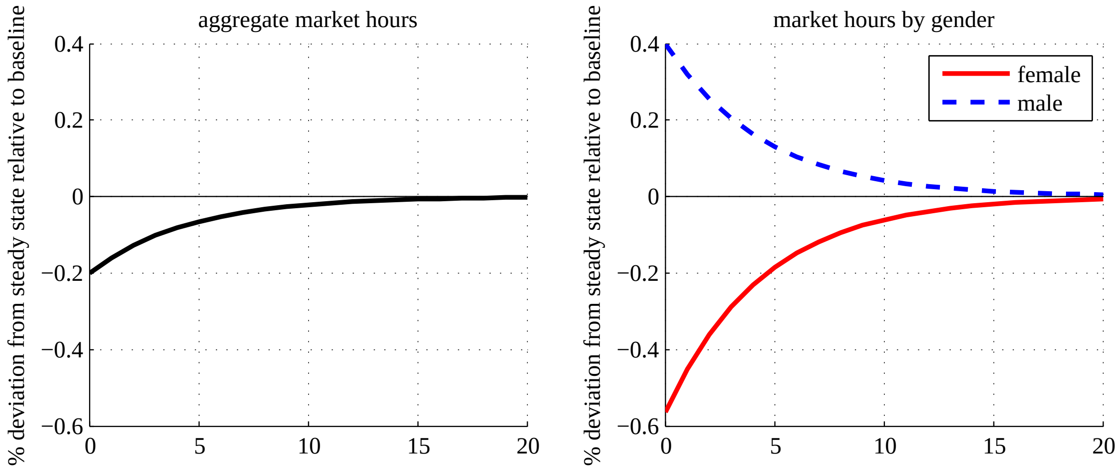
In this section, I add taxes and subsidies to the model. In particular, the household's budget constraint is rendered to

$$\begin{aligned} c_{M,t+s} + c_{F,t+s} + b_{t+s+1} \leq & (1-\tau^w_{M,t+s}) w_{M,t+s} n_{M,t+s} + (1-\tau^w_{F,t+s}) w_{F,t+s} n_{F,t+s} \\ & + \tau^h_M w_{M,t+s} h_{M,t+s} + \tau^h_F w_{F,t+s} h_{F,t+s} \\ & + (1+r_{t+s}) b_{t+s} + \pi_{t+s} - \tau_{t+s}, \end{aligned} \tag{42}$$

where τ^w_g are labor-income taxes, τ^h_g are gender-based home production subsidies which the government pays proportionally to labor earnings foregone during home hours, and τ is a lump-sum tax or transfer. The subsidies τ^h are suited to affect the gender division of home hours and to reduce the importance of household specialization. I assume a small open economy, perfect competition between firms on the labor market, a production technology that is linear in both gender's labor and that the government's budget is balanced in every period.

I perform three policy experiments using the calibration described in Section 3.3 above. In all experiments, I erode the consequences of tax cut that raises $(1-\tau^w)$ by one percent. However, the situation in which this tax cut takes place differs between experiments. I

Figure 2: Effects of tax cuts if policy achieves gender equity in steady-state time use relative to baseline.



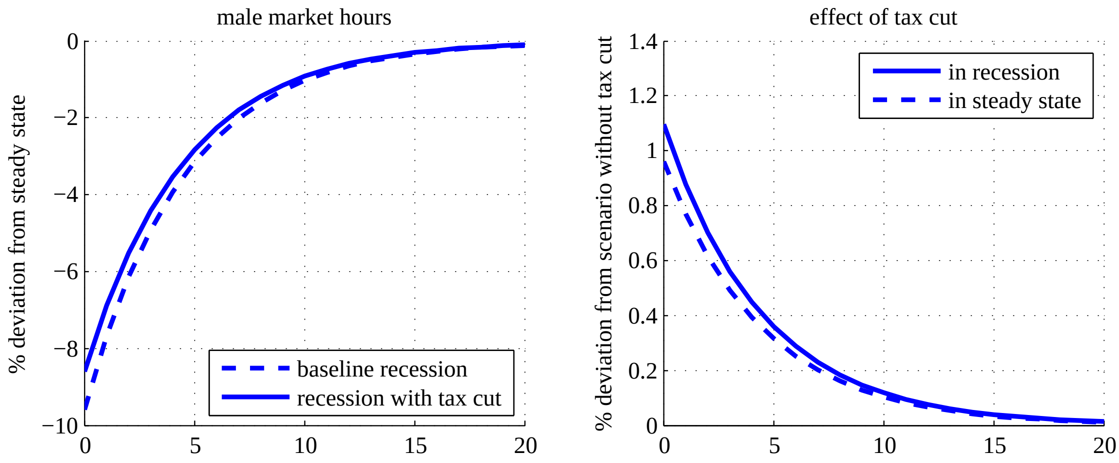
first consider a tax cut in the model's steady state without using the home-production subsidies. Second, to demonstrate policy interdependence, I consider a tax cut in the steady state of a model variant in which the government uses its instruments to achieve gender equity in all time uses. Finally, to demonstrate state dependency of policy effects, I consider a tax cut in a recession.

Baseline. Figure 1 shows the effects of a tax cut that raises both gender's net wages by one percent in a steady state that matches the empirical time uses displayed in Table 2.[6] The left panel shows that total market hours rise by about 1.4% due to the tax cut. The right panel illustrates that there are substantial gender differences in the labor-supply response to the tax cut. Reflecting women's higher Frisch elasticity, their reaction is about two thirds stronger than men's. Remember that there is no gender difference in preferences in this calibration such that this result is solely an effect of household specialization.

Policy interdependence. Figure 2 shows the effects of a tax cut in a scenario where policy offsets the incentives to specialize. In particular, I choose the steady-state values of the tax instruments in a way that achieves that both types of working time are equal across genders and equal to the mean of the time uses reported in Table 2. To facilitate comparison, the figure shows the effects of the tax cut in this scenario relative to the effects in the baseline scenario discussed above. In light of the theoretical results of this paper, it is not surprising that tax cuts have identical effects on both genders if there is no initial specialization. By equalizing time uses, policy has also equalized labor-supply elasticities. Thus, the male response to tax cuts is stronger than in the baseline scenario and the female response is weaker, see right panel of Figure 2. Quantitatively, this translates into a smaller response of total market hours, see left panel.

[6] Here and in the following evaluations, the persistence of the tax rate shock is 0.8.

Figure 3: Effects of tax cuts in different phases of the business cycle.



State dependency of policy effects. Finally, I demonstrate that the effects of fiscal policy are state-dependent in my model. To this end, I consider a recession that is caused by a negative productivity shock that reduces men's wage rates by 10%. The response of men's market hours to this shock is displayed by the dashed line in the left panel of Figure 3. The solid line in the left panel represents the path of men's market hours if a tax cut that raises male wage rates by one percent occurs in this recession scenario. Comparing the two lines, one can see that the tax cut cushions the drop in hours worked in the recession.

Most interesting, however, is to compare the differential effect of the tax cut in the recession with the effects of a tax cut in steady state. I perform this comparison in the right panel of Figure 3. The effect of the tax cut is stronger in the recession. Quantitatively, its effect is raised by about 16%. Economically, the recession is characterized by a reduction in market work both in absolute terms and relative to home hours. This leads to higher Frisch labor-supply elasticities as pointed out in the theoretical analysis above. This makes policy effects state-dependent in the model and, in particular, fiscal policy more effective in recessions.

This result can add to our understanding why fiscal policy effects are found to be larger in downturns (Auerbach and Gorodnichenko 2012; Auerbach and Gorodnichenko 2013). Frequently discussed further explanation for this result build on the absence of counteracting interest rate reactions and the effects of fiscal policy on inflation expectations in liquidity traps (Christiano, Eichenbaum, and Rebelo 2011; Eggertsson 2011; Woodford 2011). My argument is complementary to these arguments.

4 Implications for long-run elasticities

In this section, I consider the implications of my results for the long-run elasticities of labor supply. Empirical evidence suggests that also long-run labor-supply elasticities differ substantially between genders. Marshall (uncompensated) elasticities of labor supply are usually estimated in a range of 0-0.3 for men while estimates for women lie around 0.6 (estimates for both genders in the US are reported by Hausman and Ruud 1984, Triest 1990, Devereux 2004, Eissa and Hoynes 2004, and Dechter 2013).

To determine the long-run elasticities in my model, I consider *permanent* wage changes. I thus consider the model in a steady state where the first-order conditions (2) - (11) simplify to

$$\mu_M c_M^{-\sigma} = \lambda, \tag{43}$$

$$\mu_F c_F^{-\sigma} = \lambda, \tag{44}$$

$$\mu_M \nu_l l_M^{\eta_M^{-1}} = \lambda w_M, \tag{45}$$

$$\mu_F \nu_l l_F^{\eta_M^{-1}} = \lambda w_F, \tag{46}$$

$$\mu_M \nu_l l_M^{\eta_M^{-1}} = \chi \cdot (1-\theta) \cdot A_h h_F^{\theta} h_M^{-\theta}, \tag{47}$$

$$\mu_F \nu_l l_F^{\eta_M^{-1}} = \chi \cdot \theta \cdot A_h h_F^{\theta-1} h_M^{1-\theta}, \tag{48}$$

$$\nu_d d^{-\kappa} = \chi, \tag{49}$$

$$d = A_h h_F^{\theta} h_M^{1-\theta}, \tag{50}$$

$$c_M + c_F = w_M n_M + w_F n_F + T, \tag{51}$$

and $1 = \beta(1+r)$. $T = rb + \pi$ captures all non-labor income.

I consider the following linearized version of the steady-state conditions where x' denotes the change of variable x between two steady states:

$$\mu_M' - \sigma c_M' = \lambda', \tag{52}$$

$$\mu_F' - \sigma c_F' = \lambda', \tag{53}$$

$$\mu_M' + \eta_M^{-1}\frac{n_M}{l_M} n_M' + \eta_M^{-1}\frac{h_M}{l_M} h_M' - \lambda' = w_M', \tag{54}$$

$$\mu_F' + \eta_F^{-1}\frac{n_F}{l_F} n_F' + \eta_F^{-1}\frac{h_F}{l_F} h_F' - \lambda' = w_F', \tag{55}$$

$$\mu_M' + \eta_M^{-1}\frac{n_M}{l_M} n_M' + \eta_M^{-1}\frac{h_M}{l_M} h_M' = A' + \theta h_F' - \theta h_M' + \chi', \tag{56}$$

$$\mu_F' + \eta_F^{-1}\frac{n_F}{l_F} n_F' + \eta_F^{-1}\frac{h_F}{l_F} h_F' = A' - (1-\theta) h_F' + (1-\theta) h_M' + \chi', \tag{57}$$

$$-\kappa d' = \chi', \tag{58}$$

$$d = A' + \theta h_F' + (1-\theta) h_M', \tag{59}$$

$$\frac{c_M}{c} c_M' + \frac{c_F}{c} c_F' - \frac{y_M}{Y} n_M' - \frac{y_F}{Y} n_F' = \frac{y_M}{Y} w_M' + \frac{y_F}{Y} w_F' + \frac{T}{Y} T',. \tag{60}$$

Note that the system of linearized steady-state conditions (52) - (60) contains changes in more exogenous variables than the short-run system (12) - (17). Here, I also consider changes in non-labor income, T', and changes in total factor productivity, A'. These are included for a balanced-growth evaluation in the model with home production. Further, there are also more changes in endogenous variables, as, in the long run, also the marginal valuation of wealth λ may change and with it the consumption levels c of both spouses.

I simplify the system (52) - (60) and express it in matrix form as

$$PX' = QZ' \iff X' = P^{-1}QZ' = SZ' \tag{61}$$

with

$$X = \left(c'_M\, c'_F\, n'_M\, n'_F\, h'_M\, h'_F\, \lambda'\right)^T,\ Z\prime = \left(w'_M\, w'_F\, T'\, A'\right)^T, \tag{62}$$

$$Q = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & n_M w_M \\ 0 & 0 & 1 & 0 & 0 & n_F w_F \\ 0 & 0 & 0 & 1-\kappa & 1-\kappa & 0 \\ 0 & 0 & 0 & 0 & 0 & T \end{pmatrix}^T, \tag{63}$$

and

$$P = \begin{pmatrix} -\sigma & \sigma & 0 & 0 & 0 & 0 \\ \sigma & 0 & \frac{\eta_M n_M}{l_M} & 0 & \frac{\eta_M h_M}{l_M} & 0 \\ 0 & \sigma & 0 & \frac{\eta_F n_F}{l_F} & 0 & \frac{\eta_F h_F}{l_F} \\ 0 & 0 & \frac{\eta_M n_M}{l_M} & 0 & \frac{\eta_M h_M}{l_M} + \theta + \kappa\left(1-\theta\right) & -\left(1-\kappa\right)\theta \\ 0 & 0 & 0 & \frac{\eta_F n_F}{l_F} & -\left(1-\kappa\right)\left(1-\theta\right) & \frac{\eta_F h_F}{l_F} + 1 - \theta + \kappa\theta \\ c_M & c_F & -w_M n_M & -w_F n_F & 0 & 0 \end{pmatrix}. \tag{64}$$

As I consider long-run wage changes, I restrict the analysis to parameter constellations that ensure balanced growth. In the appendix, I show that the model fulfills balanced growth if and only if

$$\sigma = \kappa = 1. \tag{65}$$

Under this condition, I can write the Marshall elasticity of spouse g as

$$MLSE_g = \frac{\Omega_g}{\gamma \cdot n_g} \tag{66}$$

where

$$\gamma = \eta_{-g}^{-1}\eta_g^{-1} + l_{-g}w_{-g}\eta_g^{-1} + l_g w_g \eta_{-g}^{-1} + h_{-g}w_{-g}\eta_{-g}^{-1}\eta_g^{-1} + h_g w_g \eta_{-g}^{-1}\eta_g^{-1}, \tag{67}$$

$$\Omega_g = \left(c\eta_{-g}^{-1} + h_{-g}w_{-g} + h_g w_g + l_{-g}w_{-g} - l_g w_g\right)\left(l_g + h_g \eta_g^{-1}\right), \tag{68}$$

see appendix for a derivation. Now let us consider the role of household specialization for Marshall elasticities. For a given total workload l_g, one can state the following relations between home production h_g and the components of the Marshall elasticity. First, the numerator Ω_g increases in the share of working time devoted to home production, h_g/l_g. Second, the denominator $\gamma \cdot n_g$ decreases in this share (note that γ is independent of gender). Thus, there is a positive relation between the share of working time one spends in home productions and the Marshall elasticity.

Wage rate changes are affecting an individual's labor supply via different effects in the long run. First, there is a standard substitution effect. A higher wage rate induces substitution of leisure and home production against market consumption. This induces the agent to work more after a wage rise. Second, there is also a substitution effect within home production. As in the short run, the household uses the spouse less in home production whose time has become more expensive in terms of foregone labor earnings. Finally, in the long run, wage rate changes also exert an income effect on labor supply. As the household becomes richer, it demands relatively more leisure and reduces labor supply of both spouses. The income effect affects both spouses' total hours worked equally. The two substitution effects are the stronger, the more the agent initially worked in home production. The intuition is the same as in the short run. Putting this together, the Marshall elasticity is larger for the spouse who specializes in home production.

Home production also implies non-zero cross-wage elasticities of labor supply in the long run. As a consequence, the model is compatible with balanced growth even though own-wage Marshall elasticities are positive. This can be seen by setting $(w'_M \, w'_F \, A')^T$ in (61) to $(1 \; 1 \; 1)^T$ and performing the matrix multiplication to obtain $(c'_M \, c'_F \, n'_M \, n'_F \, h'_M \, h'_F)^T = (1 \; 1 \; 0 \; 0 \; 0 \; 0)^T$, i.e. no changes in time use. A model with home production can thus be compatible with balanced growth and still have substantial non-zero long-run labor-supply elasticities. This property differentiates models with home production (e.g., also the one used in Jones, Manuelli, and McGrattan 2003) from models without home production even when the latter can have substantial richness in the short-run labor-supply elasticities.

5 Conclusion

This paper has demonstrated that gender differences in the elasticity of labor supply can be explained as a result of household specialization. While empirical gender differences in Frisch elasticities are estimated to be about factor 2, empirical time-use ratios alone imply differences of more than factor 1.5. The importance of home production for labor-supply elasticities can bring about biased estimates in models without home production. I have further demonstrated that long-run elasticities are also affected by household specialization and that models with home production can have substantial non-zero long-run elasticities and still be compatible with balanced growth. The policy implications of my results include a non-constancy of labor-supply elasticities that policy makers should take into account.

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Appendix

Balanced growth and Marshall elasticities

Calculating $S = P^{-1}Q$ with P and Q defined as in (61), One can determine the Marshall own-wage labor-supply elasticity $MLSE_g$, the cross-wage elasticity $MLSE_g^{cross}$, the labor-supply elasticity to home productivity, LSE_g^A, and the labor-supply elasticity to non-labor income, LSE_g^T, for the husband and the wife as $S_{3,1}$, $S_{3,2}$, $S_{3,3}$, and $S_{3,4}$ and $S_{4,2}$, $S_{4,1}$, $S_{4,3}$, and $S_{4,4}$ respectively. Balanced-growth requires that

$$S_{j,1} + S_{j,2} + S_{j,3} + S_{j,4} = 0 \text{ for } j = 3,4 \tag{69}$$

Define $j(g)$ with $j(M) = 3$ and $j = (F) = 4$ and θ_g with $\theta_F = \theta$ and $\theta_M = 1 - \theta$. Functionally, the elasticities read as

$$\begin{aligned}
MLSE_g &= S_{j(g),j(g)-2} = \Omega_g/\Gamma_g \\
MLSE_g^{cross} &= S_{j(g),j(-g)-2} = \Omega_g^{cross}/\Gamma_g \\
LSE_g^A &= S_{j(g),3} = (1-\kappa)\cdot\Lambda_g/\Gamma_g \\
LSE_g^T &= S_{j(g),4} = T\left(-\kappa\sigma l_g \eta_{-g}^{-1} - \sigma h_g \eta_{-g}^{-1}\eta_g^{-1}\right)/\Gamma_g
\end{aligned}$$

where

$$\begin{aligned}
\Gamma_g =& \kappa c_{-g} n_g \eta_{-g}^{-1}\eta_g^{-1} + \kappa c_g n_g \eta_{-g}^{-1}\eta_g^{-1} + \kappa\sigma l_{-g} n_g w_{-g}\eta_g^{-1} + \kappa\sigma l_g n_g w_g \eta_{-g}^{-1} \\
&+\sigma h_{-g} n_g w_{-g}\eta_{-g}^{-1}\eta_g^{-1} + \sigma h_g n_g w_g \eta_{-g}^{-1}\eta_g^{-1}, \\
\Omega_g =& n_g w_g\left(-\kappa\sigma l_g\eta_{-g}^{-1} - \sigma h_g\eta_{-g}^{-1}\eta_g^{-1}\right) + \kappa c_{-g} l_g \eta_{-g}^{-1} + \kappa c_g l_g \eta_{-g}^{-1} + \kappa\sigma l_{-g} l_g w_{-g} \\
&+c_{-g} h_g \eta_{-g}^{-1}\eta_g^{-1} + c_g h_g \eta_{-g}^{-1}\eta_g^{-1} + \sigma l_{-g} h_g w_{-g}\eta_g^{-1} - \theta_{-g} c_{-g} h_g \eta_{-g}^{-1}\eta_g^{-1} - \theta_{-g} c_g h_g \eta_{-g}^{-1}\eta_g^{-1} \\
&+\theta_{-g}\sigma h_{-g} l_g w_{-g}\eta_{-g}^{-1} + \kappa\sigma h_{-g} l_g w_{-g}\eta_{-g}^{-1} - \theta_{-g}\sigma l_{-g} h_g w_{-g}\eta_g^{-1} + \theta_{-g}\kappa c_{-g} h_g \eta_{-g}^{-1}\eta_g^{-1} \\
&+\theta_{-g}\kappa c_g h_g \eta_{-g}^{-1}\eta_g^{-1} - \theta_{-g}\kappa\sigma h_{-g} l_g w_{-g}\eta_{-g}^{-1} + \theta_{-g}\kappa\sigma l_{-g} h_g w_{-g}\eta_g^{-1} + \sigma h_{-g} h_g w_{-g}\eta_{-g}^{-1}\eta_g^{-1}, \\
\Omega_g^{cross} =& n_{-g} w_{-g}\left(-\kappa\sigma l_g \eta_{-g}^{-1} - \sigma h_g \eta_{-g}^{-1}\eta_g^{-1}\right) - \kappa\sigma l_{-g} l_g w_{-g} - \sigma l_{-g} h_g w_{-g}\eta_g^{-1} \\
&+\theta_{-g} c_{-g} h_g \eta_{-g}^{-1}\eta_g^{-1} + \theta_{-g} c_g h_g \eta_{-g}^{-1}\eta_g^{-1} - \theta_{-g}\sigma h_{-g} l_g w_{-g}\eta_{-g}^{-1} - \kappa\sigma h_{-g} l_g w_{-g}\eta_{-g}^{-1} \\
&+\theta_{-g}\sigma l_{-g} h_g w_{-g}\eta_g^{-1} - \theta_{-g}\kappa c_{-g} h_g \eta_{-g}^{-1}\eta_g^{-1} - \theta_{-g}\kappa c_g h_g \eta_{-g}^{-1}\eta_g^{-1} \\
&+\theta_{-g}\kappa\sigma h_{-g} l_g w_{-g}\eta_{-g}^{-1} - \theta_{-g}\kappa\sigma l_{-g} h_g w_{-g}\eta_g^{-1} - \sigma h_{-g} h_g w_{-g}\eta_{-g}^{-1}\eta_g^{-1}, \\
\Lambda_g =& \sigma h_{-g} l_g w_{-g}\eta_{-g}^{-1} - c_g h_g \eta_{-g}^{-1}\eta_g^{-1} - c_{-g} h_g \eta_{-g}^{-1}\eta_g^{-1} - \sigma l_{-g} h_g w_{-g}\eta_g^{-1}.
\end{aligned}$$

Thus, the balanced-growth condition (69) can be written as

$$\Omega_g + \Omega_g^{cross} + (1-\kappa)\Lambda_g + T\left(-\kappa\sigma l_g\eta_{-g}^{-1} - \sigma h_g\eta_{-g}^{-1}\eta_g^{-1}\right) = 0 \tag{70}$$

As $\Upsilon_g = -\Upsilon_g^{cross}$, see above, one can thus simplify the balanced-growth condition (70) to

$$\Omega_g + \Omega_g^{cross} + (1-\kappa)\Lambda_g + T\left(-\kappa\sigma l_g\eta_{-g}^{-1} - \sigma h_g\eta_{-g}^{-1}\eta_g^{-1}\right) = 0.$$

Using the functional forms of Ω_g and Ω_g^{cross}, one obtains

$$
\begin{aligned}
&\Omega_g + \Omega_g^{cross} + (1-\kappa)\Lambda_g + T\left(-\kappa\sigma l_g \eta_{-g}^{-1} - \sigma h_g \eta_{-g}^{-1}\eta_g^{-1}\right) \\
=\,& n_g w_g \left(-\kappa\sigma l_g \eta_{-g}^{-1} - \sigma h_g \eta_{-g}^{-1}\eta_g^{-1}\right) + \kappa c_{-g} l_g \eta_{-g}^{-1} + \kappa c_g l_g \eta_{-g}^{-1} + \kappa\sigma l_{-g} l_g w_{-g} + c_{-g} h_g \eta_{-g}^{-1}\eta_g^{-1} \\
&+ c_g h_g \eta_{-g}^{-1}\eta_g^{-1} + \sigma l_{-g} h_g w_{-g} \eta_g^{-1} - \theta_{-g} c_{-g} h_g \eta_{-g}^{-1}\eta_g^{-1} - \theta_{-g} c_g h_g \eta_{-g}^{-1}\eta_g^{-1} + \theta_{-g}\sigma h_{-g} l_g w_{-g} \eta_{-g}^{-1} \\
&+ \kappa\sigma h_{-g} l_g w_{-g} \eta_{-g}^{-1} - \theta_{-g}\sigma l_{-g} h_g w_{-g} \eta_g^{-1} + \theta_{-g}\kappa c_{-g} h_g \eta_{-g}^{-1}\eta_g^{-1} + \theta_{-g}\kappa c_g h_g \eta_{-g}^{-1}\eta_g^{-1} \\
&- \theta_{-g}\kappa\sigma h_{-g} l_g w_{-g} \eta_{-g}^{-1} + \theta_{-g}\kappa\sigma l_{-g} h_g w_{-g} \eta_g^{-1} + \sigma h_{-g} h_g w_{-g} \eta_{-g}^{-1}\eta_g^{-1} \\
&+ n_{-g} w_{-g}\left(-\kappa\sigma l_g \eta_{-g}^{-1} - \sigma h_g \eta_{-g}^{-1}\eta_g^{-1}\right) - \kappa\sigma l_{-g} l_g w_{-g} - \sigma l_{-g} h_g w_{-g} \eta_g^{-1} + \theta_{-g} c_{-g} h_g \eta_{-g}^{-1}\eta_g^{-1} \\
&+ \theta_{-g} c_g h_g \eta_{-g}^{-1}\eta_g^{-1} - \theta_{-g}\sigma h_{-g} l_g w_{-g} \eta_{-g}^{-1} - \kappa\sigma h_{-g} l_g w_{-g} \eta_{-g}^{-1} + \theta_{-g}\sigma l_{-g} h_g w_{-g} \eta_g^{-1} \\
&- \theta_{-g}\kappa c_{-g} h_g \eta_{-g}^{-1}\eta_g^{-1} - \theta_{-g}\kappa c_g h_g \eta_{-g}^{-1}\eta_g^{-1} + \theta_{-g}\kappa\sigma h_{-g} l_g w_{-g} \eta_{-g}^{-1} - \theta_{-g}\kappa\sigma l_{-g} h_g w_{-g} \eta_g^{-1} \\
&- \sigma h_{-g} h_g w_{-g} \eta_{-g}^{-1}\eta_g^{-1} + (1-\kappa)\Lambda_g + T\left(-\kappa\sigma l_g \eta_{-g}^{-1} - \sigma h_g \eta_{-g}^{-1}\eta_g^{-1}\right) \\
=\,& (n_g w_g + n_{-g} w_{-g})\left(-\kappa\sigma l_g \eta_{-g}^{-1} - \sigma h_g \eta_{-g}^{-1}\eta_g^{-1}\right) + (1-\kappa)\Lambda_g + T\left(-\kappa\sigma l_g \eta_{-g}^{-1} - \sigma h_g \eta_{-g}^{-1}\eta_g^{-1}\right) \\
&+ \kappa c_{-g} l_g \eta_{-g}^{-1} + \kappa c_g l_g \eta_{-g}^{-1} + c_{-g} h_g \eta_{-g}^{-1}\eta_g^{-1} + c_g h_g \eta_{-g}^{-1}\eta_g^{-1} \\
=\,& \kappa c l_g \eta_{-g}^{-1} + c h_g \eta_{-g}^{-1}\eta_g^{-1} - \kappa\sigma n_{-g} l_g w_{-g} \eta_{-g}^{-1} - \kappa\sigma l_g n_g w_g \eta_{-g}^{-1} - \sigma n_{-g} h_g w_{-g} \eta_{-g}^{-1}\eta_g^{-1} \\
&- \sigma h_g n_g w_g \eta_{-g}^{-1}\eta_g^{-1} + (1-\kappa)\Lambda_g + T\left(-\kappa\sigma l_g \eta_{-g}^{-1} - \sigma h_g \eta_{-g}^{-1}\eta_g^{-1}\right) \\
=\,& \kappa\left(w_g n_g + w_{-g} n_{-g}\right)\left(h_g + n_g\right)\eta_{-g}^{-1} + \left(w_g n_g + w_{-g} n_{-g}\right) h_g \eta_{-g}^{-1}\eta_g^{-1} - \kappa\sigma n_{-g}\left(h_g + n_g\right) w_{-g} \eta_{-g}^{-1} \\
&- \kappa\sigma\left(h_g + n_g\right) n_g w_g \eta_{-g}^{-1} - \sigma n_{-g} h_g w_{-g} \eta_{-g}^{-1}\eta_g^{-1} - \sigma h_g n_g w_g \eta_{-g}^{-1}\eta_g^{-1} \\
&+ (1-\kappa)\Lambda_g + T\left(\left(h_g + n_g\right)\eta_{-g}^{-1} + h_g \eta_{-g}^{-1}\eta_g^{-1} - \kappa\sigma l_g \eta_{-g}^{-1} - \sigma h_g \eta_{-g}^{-1}\eta_g^{-1}\right),
\end{aligned}
$$

where I used $c_g + c_{-g} = c = w_g n_g + w_{-g} n_{-g} + T$ and $l_g = n_g + h_g$. For this to be zero independent of T, it needs to hold that

$$
l_g \eta_{-g}^{-1} + h_g \eta_{-g}^{-1}\eta_g^{-1} - \kappa\sigma l_g \eta_{-g}^{-1} - \sigma h_g \eta_{-g}^{-1}\eta_g^{-1} = (1-\kappa\sigma)\, l_g \eta_{-g}^{-1} + (1-\sigma)\, h_g \eta_{-g}^{-1}\eta_g^{-1} = 0
$$

Which is, for general $l_g, h_g, \eta_{-g}^{-1}, \eta_g^{-1}$, only fulfilled if $\kappa = \sigma = 1$. Is this necessary condition also sufficient? I set $\kappa = \sigma = 1$ and obtain

$$
\begin{aligned}
&\Omega_g + \Omega_g^{cross} + (1-\kappa)\Lambda_g + T\left(-\kappa\sigma l_g \eta_{-g}^{-1} - \sigma h_g \eta_{-g}^{-1}\eta_g^{-1}\right) \\
=\,& \left(w_g n_g + w_{-g} n_{-g}\right)\left(h_g + n_g\right)\eta_{-g}^{-1} + \left(w_g n_g + w_{-g} n_{-g}\right) h_g \eta_{-g}^{-1}\eta_g^{-1} - n_{-g}\left(h_g + n_g\right) w_{-g} \eta_{-g}^{-1} \\
&- \left(h_g + n_g\right) n_g w_g \eta_{-g}^{-1} - n_{-g} h_g w_{-g} \eta_{-g}^{-1}\eta_g^{-1} - h_g n_g w_g \eta_{-g}^{-1}\eta_g^{-1} = 0.
\end{aligned}
$$

This implies that

$$
\kappa = \sigma = 1
$$

is a necessary and sufficient condition for balanced growth. Under this condition, Γ_g, Ω_g, and Υ_g simplify to the terms expressed in Section 4 of the main text.