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WELFARE EFFECTS OF SHORT-TERM COMPENSATION

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Welfare Effects of Short-Time Compensation*

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Abstract

We study welfare effects of public short-time compensation (STC) in a model in which firms respond to idiosyncratic profitability shocks by adjusting employment and hours per worker. Introducing STC substantially improves welfare by mitigating distortions caused by public UI, but only if firms have access to private insurance. Otherwise firms respond to low profitability by combining layoffs with long hours for remaining workers, rather than by taking up STC. Optimal STC is substantially less generous than UI even when firms have access to private insurance, and equally generous STC is worse than not offering STC at all.

Keywords: Short-Time Compensation, Unemployment Insurance, Welfare
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1 Introduction

Virtually all developed countries have public unemployment insurance (UI) systems. In addition, many countries run public short-time compensation (STC) schemes, which pay benefits to workers that have not lost their job entirely but are working reduced hours. In contrast to UI, however, STC has historically not been a universal component of the social insurance systems of developed countries. Before the 2008-2009 crisis, STC schemes existed in 18 out of 33 OECD countries. Such schemes increased in popularity during the crisis, with many countries expanding existing schemes and others introducing new schemes on a temporary basis.[1]

This increase in the popularity of STC has also revived academic interest in this policy instrument. Recent research has primarily focussed on employment effects of STC during the crisis.[2] What has received little attention, both in recent and earlier work, are effects of STC on social welfare. This contrasts with UI, which has been studied extensively from a welfare perspective. In this paper we study welfare effects of STC in a setting in which UI is socially optimal, consistent with the observation that UI is a universal feature of social insurance systems in developed countries. We ask if introducing STC can improve welfare in a situation in which the instrument of UI is already used optimally.

We study this question in a static model of implicit contracts, building on existing theoretical work on STC. Workers are risk averse and ex ante heterogeneous in that they are either attached to a firm or unattached. Both attached and unattached workers can be unemployed ex post. We follow existing work in not separating the role of workers and employers: workers attached to a firm are both the suppliers of its labor input as well as its owners. Firms are subject to idiosyncratic profitability shocks, and can adjust through a combination of layoffs and work sharing in the sense of adjusting hours per worker. Profitability shocks are interpreted as temporary, and layoffs are interpreted as temporary layoffs that do not break attachment to the firm. The government has access to two policy instruments, UI and STC. UI is modeled as a payment to each unemployment worker, where a worker is considered unemployed if it works zero hours.[3]

[1] Arpaia et al. (2010) and Hijzen and Venn (2011) survey STC schemes.

[2] See for example Arpaia et al. (2010), Hijzen and Venn (2011), Boeri and Bruecker (2011), Cahuc and Carcillo (2011), Hijzen and Martin (2013), and Balleer et al. (2014).

[3] Since the model is static, search activity is not modeled explicitly and thus does not enter the definition of unemployment. Unattached workers are eligible for unemployment insurance payments in the model. For this to be consistent with the typical UI system, the status of being unattached should be interpreted as including having worked in the recent past.

Thus workers are unemployed either because they are unattached or on temporary layoff. UI is the only source of income for unattached workers. STC is modeled as a payment for each hour by which working time is reduced below some threshold of normal hours. We allow for the possibility that eligibility for STC may require a minimum reduction in hours per worker, a common feature of existing STC schemes. The government balances the budget through a linear tax on total hours. When studying public insurance, it is important to take into account agents' access to private insurance (PI). Here we consider two polar scenarios: either firms have access to perfect PI, or they have no access to PI.

Welfare effects of UI in this setting are well understood. It has a positive effect on utilitarian welfare via redistribution towards unattached workers. If firms lack access to perfect PI, UI also provides insurance to attached workers. The cost of UI is a distortion of labor inputs, as firms do not internalize the impact of layoffs on the government budget.

Starting from a situation in which the level of UI is chosen to maximize social welfare, the introduction of STC can affect welfare through two channels. First, since private labor input decisions are distorted by UI, STC affects welfare through its impact on these decisions. This is the only welfare effect of STC when firms have access to perfect PI. If firms lack such access, then STC also has a direct insurance effect, since it reallocates resources across firms with different realizations of profitability.

Our analysis proceeds in two main steps. First, we analyze firms' decisions for given values of the policy instruments. In particular, we characterize how firms adjust labor inputs in response to profitability shocks, conditional on the decision to take up STC. This is well known for the case of perfect PI: when profitability is sufficiently low for layoffs to be optimal, a further reduction in profitability causes lower employment, while hours per worker remain constant. For the case of no PI and for our specification of preferences, which is a standard specification in macroeconomics, we establish a new comparative statics property: the availability of UI induces firms to respond to a decline in profitability by reducing employment and *increasing* hours per worker. This occurs because lower profitability raises the marginal utility of consumption relative to the marginal disutility of working longer hours for workers with positive hours. This property turns out to be the key factor in determining the welfare effects of STC when firms lack access to PI.

In the second step, we study welfare-maximizing choices of UI and STC. We rely on computational experiments, calibrating the model by targeting features of the US labor market. We obtain two main results. First, introducing STC substantially improves welfare, but only if firm have access to PI. If firms have such access, then STC can mitigate labor input distortions in form of excessive temporary layoffs caused by UI. This mechanism fails if firms lack access to PI, due to the comparative statics property discussed above. In the absence of STC, unprofitable firms would choose temporary layoffs combined with *high* hours per worker, and this makes adopting STC unappealing. For the same reason, STC has a direct negative insurance effect, but quantitatively this is relatively unimportant. Our second main result is that optimal STC is substantially less generous than UI even when firms have access to PI. In our model there is no reason to expect that equal generosity is optimal, since the optimal levels of STC and UI are governed by different trade-offs. According to our computational experiments, STC should be about one third as generous as UI. Furthermore, equally generous STC is worse than not offering STC at all. This is important, given that equal generosity of STC and UI is a common feature of existing schemes.[4]

We contribute both to the literature using implicit contract models to study STC, and the broader literature using such models to study the response of layoffs and hours per worker to shocks. Our analysis of STC builds heavily on Burdett and Wright (1989, henceforth BW) and Wright and Hotchkiss (1988, henceforth WH). BW use an implicit contract model to study effects of UI and STC on layoffs, hours per worker, and wages. A key feature of their model is that laissez faire is socially optimal. Their analysis is focussed on the distortions induced by UI and STC. They find that while UI distorts the level of employment, STC distorts hours per worker. WH extend the analysis of BW in several directions, two of which are important for our purposes. While BW consider a model in which workers and employers are distinct agents, WH also consider a simplified model which abstracts from this heterogeneity. We adopt this simplification. Second, WH use this simplified model to analyze social welfare. As in BW, having neither UI nor STC is socially optimal. Alternatively, UI and STC can be neutralized through full experience rating. Our main contribution to this literature is an analysis of the welfare effects of STC in a setting in which there is a reason for the existence of public UI. This is an important setting to consider, given that UI is a universal across developed economies. The existence of UI gives rise to a nontrivial trade-off for STC, since STC can

[4]More precisely, a common feature is that the replacement rates received by workers are often the same across UI and STC. What matters in our model is the generosity of the program from the joint perspective of workers and employers. Some programs have equal replacement rates for workers, but impose additional costs of utilizing STC on employers, and thus are effectively less generous than UI.

mitigate distortions induced by UI. In our model we generate a reason for the existence of UI through the presence of unattached workers.[5] But our results concerning STC may also be relevant if the reason for the existence of UI is different, for example if socially inefficient UI persists for political economy reasons.

Since the work of BW and WH on STC, there has been tremendous progress in the development of dynamic models of the labor market. Nonetheless, we think that static implicit contract models remain a natural starting point for studying the welfare effects of STC. What has made this class of models attractive for analyzing STC is the combination of three features: (i) specificity of employment relationships, captured by the attachment of workers to firms, (ii) multi-worker firms, adjusting at both the extensive and the intensive margin, (iii) private insurance arrangements among the agents attached to a firm, in a setting with incomplete markets. While immense progress has been made in developing dynamic models capturing these features individually, models capturing them jointly have not yet been developed.[6] Of course, the static nature of our model prevents us from evaluating some potential effects of STC, such as the common concern that STC reduces the reallocation of workers to more productive firms.[7]

Our contribution to the broader implicit contracts literature is the comparative statics property discussed above, which applies when firms lack access to PI and public UI is available: if profitability is sufficiently low for layoffs to be optimal, then a firm responds to a further reduction in profitability by reducing employment and *increasing* hours per worker. Rosen (1985) and FitzRoy and Hart (1985) study the corresponding comparative statics for the case of perfect PI, and show that hours are constant across profitability levels for which layoffs are optimal. The analysis closest to ours is Miyazaki and Neary (1985), who study the comparative statics of employment of hours for a firm without access to PI. They find that an increase in profitability can reduce both employment and hours per worker if firms have to cover fixed costs that are independent of employment, or

[5]When firms lack access to perfect PI, an additional source of welfare gains from UI and potentially STC in our model is insurance provision against idiosyncratic profitability shocks. In contrast, in both BW and WH shocks are aggregate and thus undiversifiable, whether through public or private insurance.

[6]The paper that comes closest is Cooper, Haltiwanger, and Willis (2007), who construct a model with search frictions with the aim of matching the comovement of labor market variables in the aggregate and at the establishment level. Their model exhibits the first two features. It has two types of agents, risk neutral employers and risk averse workers. To maintain tractability, they assume that employers have all the bargaining power. In equilibrium all workers, whether employed or unemployed, obtain the same level of utility. Workers have GHH preferences, ensuring marginal utilities of consumption are also equalized. Thus it does not matter whether markets are complete or incomplete. They do not analyze a version of their model in which employers are risk averse.

[7]See OECD (2010) for a discussion of this concern.

if income effects are sufficiently strong. Our finding differs in that a change in profitability induces an opposite response of employment and hours, and that this pattern is induced by the presence of UI, which acts like a fixed cost per worker.

The remainder of the paper is organized as follows. We introduce the model in Section 2. In Section 3, we characterize the allocation for a given system of UI and STC. Section 4 contains the computational experiments. Section 5 considers an alternative specification of technology, and Section 6 concludes.

2 Model

There is a continuum of firms. Each firm has a mass N of workers attached to it. We normalize $N = 1$. The firm is jointly owned and operated by these workers. A fraction υ of the total population of workers is not attached to a firm.

Technology. Each firm has the production function

$$xf(nh) \tag{1}$$

where n denotes the mass of workers working strictly positive hours, and h denotes the number of hours worked by each of these workers. Here x parametrizes the profitability of the firm. The function $f : [0, +\infty) \to [0, +\infty)$ is twice continuously differentiable with $f' > 0$ and $f'' < 0$ on $(0, +\infty)$, and satisfies the Inada conditions $\lim_{l\to 0} f'(l) = +\infty$ and $\lim_{l\to\infty} f'(l) = 0$. Profitability x is subject to stochastic shocks that can be of technological or other origin, with probability density $p(x)$ and support $(0, +\infty)$.

Hours per worker and employment enter equation (1) multiplicatively. Thus hours of different workers are perfect substitutes. This specification is used by WH, and dubbed the *standard case* by BW. BW also study a more general specification with imperfect substitutability. We maintain the standard case for most of our analysis. In Section 5 we consider the case in which hours of different workers are perfect complements.

Preferences. The utility function of a worker is

$$\mathbb{E}\left[u(c, h)\right]$$

where c denotes consumption and h denotes hours worked. The function u takes the form proposed by King, Plosser, and Rebelo (1988, KPR):

$$u(c,h) = \frac{[cv(h)]^{1-\sigma} - 1}{1-\sigma} \tag{2}$$

with $\sigma > 1$. The function $v : [0, h_{\max}) \to (0, 1]$ satisfies $v(0) = 1$. Here $h_{\max} \in (0, +\infty]$ is a physical upper limit on hours. The function v incorporates a fixed utility loss from working strictly positive hours: $\lim_{h\to 0} v(h) = v_0$ with $v_0 \in (0, 1)$. The function v is twice continuously differentiable and satisfies $v' < 0$ on $(0, h_{\max})$. We assume that $-\frac{v'}{v}$ is strictly increasing on $(0, h_{\max})$ to ensure that consumption is a normal good. Let $V(h) \equiv -v(h)^{\frac{1-2\sigma}{\sigma}} v'(h)$. We assume $V'(h) > 0$ to ensure that $u(c, h)$ is strictly concave, and we impose the Inada condition $\lim_{h\to h_{\max}} V(h) = +\infty$.

Our specification is more general than BW in that we allow for a fixed utility loss from working strictly positive hours. It is less general than BW in that the KPR functional form restricts the relative strength of income and substitution effects. The KPR functional form is standard in macroeconomic models, since it is necessary for balanced growth. We see this paper as a step towards incorporating STC in a dynamic macroeconomic model, making this functional form a natural choice.

Private Insurance. We consider two polar cases, parametrized by $\chi \in \{0, 1\}$. If $\chi = 0$, then firms have access to perfect PI. If $\chi = 1$, then firms have no access to PI. We assume perfect risk sharing within firms.

Policy Instruments. UI takes the form of a payment $g_{UI} > 0$ to workers with zero hours worked. STC takes the form of a payment $g_{STC} \geq 0$ to employed workers for every hour that hours worked fall short of some normal level $\bar{h}$. We impose the restriction $\bar{h} g_{STC} \leq g_{UI}$, so the maximal amount of STC, obtained by working marginally positive hours, cannot exceed the level of UI. The normal level of hours is taken as given by firms. In equilibrium, it is given by the average level of hours. Most countries with STC schemes require a minimum hours reduction (MHR). To capture this feature, firms are eligible for STC if hours are below $g_{MHR}\bar{h}$, where $g_{MHR} \leq 1$. The government balances the budget through a proportional tax $\tau > 0$ on total hours nh. Thus a firm with employment n

and hours h receives the net subsidy

$$(1-n)g_{UI} + n\mathcal{I}\left[h \leq g_{MHR}\bar{h}\right] \cdot \left(\bar{h} - h\right) \cdot g_{STC} - \tau n h \tag{3}$$

where $\mathcal{I}$ denotes the indicator function. Unattached workers have no opportunity to work positive hours, thus receive the unemployment benefit g_{UI}. Notice that this system of UI and STC is uniform: it does not differentially treat workers based on the profitability of their firm, nor does it distinguish between attached and unattached workers. We do not model the reasons why the government does not use differential benefits.

This specification of policy is based on BW and WH, and generalizes theirs in three directions. First, they restrict attention to the case in which the normal level of hours $\bar{h}$ coincides with the physical upper limit $h_{\max}$. This implies that in their model firms always receive STC. This allows them to ignore the decision of firms whether to take up STC. We allow $\bar{h}$ and $h_{\max}$ to differ. To pin down $\bar{h}$ we require that in equilibrium it is equal to the average level of hours across states of the world. This is as close as we can come to capturing the notion of normal hours in a static setting.

Second, BW restrict attention to two regimes: an American regime with $g_{STC} = 0$, and a European regime in which UI and STC are equally generous, that is, $\bar{h}g_{STC} = g_{UI}$. We allow any value of g_{STC} between 0 and $\bar{h}g_{STC}$. While many countries have equal replacement rates for UI and STC, in some countries STC is effectively less generous. For example, in Germany firms are required to pay social security contributions for STC hours. In our computational experiments it turns out that equal generosity is not optimal.

Third, in their specification firms receive STC whenever hours are below the normal level $\bar{h}$ which, as discussed above, coincides with the physical upper limit $h_{\max}$ in their model. We introduce the parameter g_{MHR} to investigate whether a minimum hours reduction is a desirable feature of STC schemes.

BW assume that the government balances the budget through a lump sum tax. In their setup without a relevant eligibility threshold, a lump sum tax is isomorphic to our specification with a proportional tax on total hours.[8] This is no longer true in our setup with an eligibility threshold. Given this, we prefer the specification with a proportional

[8]Without the eligibility threshold, net subsidy schedule (3) reduces to $(1-n)g_{UI}+n(\bar{h}-h)\cdot g_{STC}-\tau n h$. A system with unemployment benefit $\hat{g}_{UI}$ and short-time compensation $\hat{g}_{STC}$ financed through a lump sum tax $\hat{\tau}$ has the net subsidy schedule $(1-n)\hat{g}_{UI} + n(\bar{h}-h)\cdot\hat{g}_{STC} - \hat{\tau}$. The isomorphism is defined by setting $\hat{g}_{UI} = g_{UI} + \bar{h}\tau$, $\hat{g}_{STC} = g_{STC} + \tau$, and $\hat{\tau} = \bar{h}\tau$.

tax, since it mimics more closely the observed financing of UI through payroll taxes.[9]

Our specification does not include so-called experience rating, which requires that a firm reimburses the government for part of the UI and STC benefits received by its workers. Exactly as in the models of BW and WH, experience rating is redundant in our model: in Appendix C we show that a system with experience rating is equivalent to a system without experience rating and lower benefits. Thus we can omit experience rating without loss of generality. When mapping the model to the data, g_{UI} and g_{STC} should be interpreted as subsidies net of any experience rating. Furthermore, the restriction that UI and STC are uniform should be understood as a restriction on net subsidies.[10]

Firm Optimization Problem. Let $T(x) \in \{0, 1\}$ indicate the decision of the firm to take up STC in state x. Let $\iota(x)$ denote the net-transfer received from PI in state x. The firm chooses $c_w(x)$, $c_b(x)$, $n(x)$, $h(x)$, $\iota(x)$, and $T(x)$ for all $x \in (0, +\infty)$ to maximize

$$\int_0^{\infty} \{n(x)u(c_w(x), h(x)) + (1 - n(x))u(c_b(x), 0)\} \, p(x)dx \tag{4}$$

subject to

$$\begin{aligned} n(x)c_w(x) + (1 - n(x))c_b(x) &= xf(n(x)h(x)) + \iota(x) - \tau n(x)h(x) && \text{(BC)} \\ &+(1 - n(x))g_{UI} + n(x)\left(\bar{h} - h(x)\right) T(x)g_{STC}, \\ n(x) &\leq 1, && \text{(N)} \\ T(x) \cdot \left(h(x) - g_{MHR}\bar{h}\right) &\leq 0, && \text{(MHR)} \\ \chi\iota(x) &= 0 && \text{(NI)} \end{aligned}$$

for all $x \in (0, +\infty)$ and

$$\int_0^{\infty} \iota(x)p(x)dx = 0. \tag{PI}$$

Constraint (PI) requires that PI is actuarially fair. If $\chi = 1$, then (NI) enforces that the firm has no access to PI by requiring $\iota(x) = 0$ in every state.

[9]Of course a payroll tax would be based on wages. Thus it would not only depend on total hours, but also on profitability. Here we exclude policy instruments that condition of profitability.

[10]If only gross benefits are restricted to be uniform, and if experience rating is allowed to differentiate between workers based on profitability or attached status, then the restriction has no content, since any desired differentiation can be implemented through experience rating.

Government Optimization Problem. We collect the policy instruments in a vector $g = \left\{g_{UI}, g_{STC}, g_{MHR}, \bar{h}, \tau\right\}$. We restrict the government to choose g from a set $\mathcal{G}$. By varying $\mathcal{G}$, we can restrict the set of policy instruments available to the government. In our computational experiments, we consider a sequence of expanding sets $\mathcal{G}$, to examine the added value of introducing the policy instrument STC with and without minimum hours requirements. The objective function of the government is utilitarian welfare, giving weight υ to unattached workers.[11] Let $U(g)$ denote the maximized value of the firm optimization problem as a function of the policy vector, and let $c_w(x,g)$, $c_b(x,g)$, $n(x,g)$, $h(x,g)$, $\iota(x,g)$, and $T(x,g)$ denote corresponding maximizers. Given these functions, the optimization problem of the government is to choose $g \in \mathcal{G}$ to maximize

$$(1-\upsilon)U(g) + \upsilon u\left(g_{UI}, 0\right) \tag{5}$$

subject to the government budget constraint

$$\begin{aligned}\int_0^\infty &\Big\{(1-n(x,g))g_{UI} + n(x,g)\left(\bar{h} - h(x,g)\right)T(x,g)g_{STC} \\ &\quad - \tau n(x,g)h(x,g)\Big\}p(x)dx = 0\end{aligned} \tag{6}$$

and the constraint that normal hours coincide with average hours per worker

$$\bar{h} = \frac{\int_0^\infty n(x,g)h(x,g)p(x)dx}{\int_0^\infty n(x,g)p(x)dx}. \tag{7}$$

First-Best Optimization Problem. A useful reference point for the allocations chosen by the government is the first-best allocation. It is obtained by choosing $c_w(x)$, $c_b(x)$, $n(x)$, $h(x)$, and unattached workers' consumption c_ν to maximize utilitarian welfare

$$(1-\nu)\int_0^\infty \left\{n(x)u(c_w(x),h(x)) + (1-n(x))u(c_b(x),0)\right\}p(x)dx + \nu u(c_\nu, 0)$$

subject to the resource constraint

$$(1-\nu)\int_0^\infty \left\{n(x)c_w(x) + (1-n(x))c_b(x) - xf(n(x)h(x))\right\}p(x)dx + \nu c_\nu = 0$$

and constraint (N).

[11] One can also interpret attachment as an initial uninsurable shock. In this interpretation all workers are ex ante identical, and the government simply maximizes expected utility.

When firms have access to perfect PI, the only reason why the government cannot achieve the first-best is that attached workers on layoff are not excluded from UI. If firms do not have access to PI, then an additional reason is that the government's policy instruments do not enable it to condition transfers directly on profitability x.

3 Optimal Firm Behavior

In this section we analyze the firm optimization problem, proceeding in three steps. In Section 3.1 we derive first-order conditions and obtain comparative statics properties of optimal hours. In Section 3.2, we analyze how optimal labor inputs vary with profitability conditional on the decision to take up STC. That is, we fix the take-up decision, and study optimal labor input profiles separately for the cases of take-up and no take-up of STC. In Section 3.3 we combine these results to discuss the take-up decision.

3.1 First-Order Conditions

Let $\lambda(x)p(x)$, $\nu(x)p(x)$, $\zeta(x)p(x)$, $\rho(x)p(x)$, and μ denote the multipliers associated with constraints (BC), (N), (MHR), (NI), and (PI), respectively. The first-order conditions for $c_w(x)$, $c_b(x)$, $n(x)$, $h(x)$, and $\iota(x)$ are

$$
\begin{aligned}
u_c(c_w(x),h(x)) &= \lambda(x), && (8)\\
u_c(c_b(x),0) &= \lambda(x), && (9)\\
u(c_b(x),0)-u(c_w(x),h(x)) &= \lambda(x)\big[xf'(n(x)h(x))h(x)-c_w(x)+c_b(x) && (10)\\
&\qquad -g_{UI}+\big(\bar{h}-h(x)\big)T(x)g_{STC}-\tau h(x)\big]-\nu(x),\\
-n(x)u_h(c_w(x),h(x)) &= \lambda(x)\big[xf'(n(x)h(x))n(x) && (11)\\
&\qquad -n(x)T(x)g_{STC}-\tau n(x)\big]-T(x)\zeta(x),\\
\lambda(x) &= \mu+\rho(x)\chi. && (12)
\end{aligned}
$$

Conditions (8)–(9) imply that consumption levels of employed and unemployed workers are $c_w(x) = c_w^*(\lambda(x),h(x))$ and $c_b(x) = c_b^*(\lambda(x))$, respectively, with $c_w^*(\lambda,h) \equiv \lambda^{-1/\sigma}v(h)^{(1-\sigma)/\sigma}$ and $c_b^*(\lambda) \equiv \lambda^{-1/\sigma}$.

Next, we analyze the first-order conditions that determine the optimal level of hours per worker. We first consider the case in which the employment constraint (N) is slack, and then turn to the case in which it binds. In both cases we focus on the case in which constraint (MHR) is slack, since its impact on optimal hours is straightforward. If the constraints (N) and (MHR) are slack, that is, if $\nu(x) = 0$ and $\zeta(x) = 0$, then combining first-order conditions (10) and (11) yields

$$\begin{aligned} &u(c_b(x), 0) - u(c_w(x), h(x)) + u_h(c_w(x), h(x))h(x) \\ &= \lambda(x)\left[c_b(x) - c_w(x) - g_{UI} + \bar{h} \cdot T(x) g_{STC}\right]. \end{aligned} \tag{13}$$

This is the first-order condition for a variation that reduces employment while increasing hours per worker h to keep total hours nh constant. The left-hand side gives the utility gain from this variation. Each worker now has a larger chance of being on layoff, which yields the utility gain $u(c_b(x), 0) - u(c_w(x), h(x))$. To keep total hours constant, the additional layoff must be compensated by redistributing $h(x)$ hours across the remaining employees, which yields a utility loss of $-u_h(c_w(x), h(x))h(x)$. The right-hand side gives the impact of this variation on the budget constraint. The additional worker on layoff is switched from consumption $c_w(x)$ to consumption $c_b(x)$ and collects the UI benefit g_{UI}. The firm loses $(\bar{h} - h(x)) \cdot T(x) g_{STC}$ in STC for the worker on layoff, and and additional $h(x) \cdot T(x) g_{STC}$ due to higher hours for remaining workers, for a total of $\bar{h} \cdot T(x) g_{STC}$.

Substituting the functions c_w^* and c_b^*, we obtain a condition linking hours and the multiplier λ which does not directly involve profitability x:

$$\begin{aligned} &u(c_b^*(\lambda), 0) - u(c_w^*(\lambda, h), h) + u_h(c_w^*(\lambda, h), h)h \\ &+\lambda\left[c_w^*(\lambda, h) - c_b^*(\lambda) + g_{UI} - \bar{h}T \cdot g_{STC}\right] = 0. \end{aligned} \tag{14}$$

The following proposition establishes that this equation has a unique solution for hours, and characterizes the comparative statics of hours with respect to λ and T.

Proposition 1 *Equation (14) has a unique solution for h given any $\lambda > 0$ and $T \in \{0, 1\}$. If $g_{STC} > 0$, then this solution is strictly decreasing in T. If $g_{UI} - \bar{h}T \cdot g_{STC} > 0$, then it is strictly increasing in λ. If $g_{UI} - \bar{h}T \cdot g_{STC} = 0$, then it is independent of λ.*

Hours are decreasing in T if $g_{STC} > 0$, since g_{STC} subsidizes low hours. The relationship between the multiplier λ and hours is less obvious. In the absence of a net payment from the government ($g_{UI} - \bar{h}T \cdot g_{STC} = 0$), hours are determined by the trade-off between the fixed disutility of working positive hours and the increasing marginal disutility of working long hours. Higher fixed costs favor longer hours, while convex disutility favors spreading hours across many workers. With KPR utility, the optimal level of hours determined by this trade-off is not affected by the multiplier λ. UI benefits introduce an additional fixed cost of working positive hours, incurred in terms of the consumption good. A higher multiplier λ indicates that consumption is more valuable. This shifts the trade-off in favor of higher hours. Thus UI distorts the composition of labor inputs in the direction of higher hours and lower employment. If taken up, STC counteracts this distortion and eliminates it entirely if UI and STC are equally generous, that is, if $\bar{h}g_{STC} = g_{UI}$.

The property that hours are strictly increasing in λ if $g_{UI} - \bar{h}T \cdot g_{STC} > 0$ and independent of λ if $g_{UI} - \bar{h}T \cdot g_{STC} = 0$ also holds for other common specifications of utility. In particular, it also holds when utility is additively separable in consumption and hours. With GHH preferences, hours are independent of λ even if $g_{UI} - \bar{h}T \cdot g_{STC} > 0$.[12]

The key implication of equation (14) is that profitability x does not enter this trade-off directly. Hours are affected by profitability only through the multiplier $\lambda(x)$, which is the marginal utility of consumption. With perfect PI, $\lambda(x)$ does not vary with profitability, hence hours are constant. As we discuss below, without PI $\lambda(x)$ is decreasing in x, hence hours are declining in x. Thus firms experiencing an uninsured decline in profitability and engaging in layoffs have relatively high hours for those workers that remain at work.

Next, consider the case in which the employment constraint is binding. Substituting $n(x) = 1$ along with the function c_w^* into first-order condition (11) yields

$$-u_h(c_w^*(\lambda, h), h) = \lambda \left[x f'(h) - \tau - T \cdot g_{STC} \right].$$

Substituting the functional forms of u_h and c_w^* yields

$$V(h) = \lambda^{\frac{1}{\sigma}} \left[x f'(h) - \tau - T \cdot g_{STC} \right]. \tag{15}$$

The following proposition establishes that this equation has a unique solution for hours, and characterizes the comparative statics of hours with respect to x, λ, and T.

Proposition 2 *Equation (15) has a unique solution for h given any $x > 0$ and $T \in \{0, 1\}$. This solution is strictly increasing in x and λ, and converges to $h_{\max}$ as x converges to infinity. If $g_{STC} > 0$, then it is strictly decreasing in T.*

UI does not directly affect the choice of hours when the firm does not engage in layoffs.

[12] These claims are established at the end of the proof of Proposition 1.

3.2 Labor Input Profiles Conditional on STC Take-Up

In this section we analyze how optimal labor inputs vary with profitability conditional on STC take-up, separately for the cases of perfect PI and no PI. Let $h^0(x)$ and $n^0(x)$ denote the levels of hours and employment that would be optimal if STC is not taken up. Here the superscript indicates that $T = 0$. Analogously, let $h^1(x)$ and $n^1(x)$ denote the corresponding levels if STC is taken up, that is, if $T = 1$.

3.2.1 Perfect Private Insurance

Proposition 3 *If $\chi = 0$, then the functions $h^0(x)$, $n^0(x)$, $h^1(x)$, and $n^1(x)$ are continuous and have the following properties.*

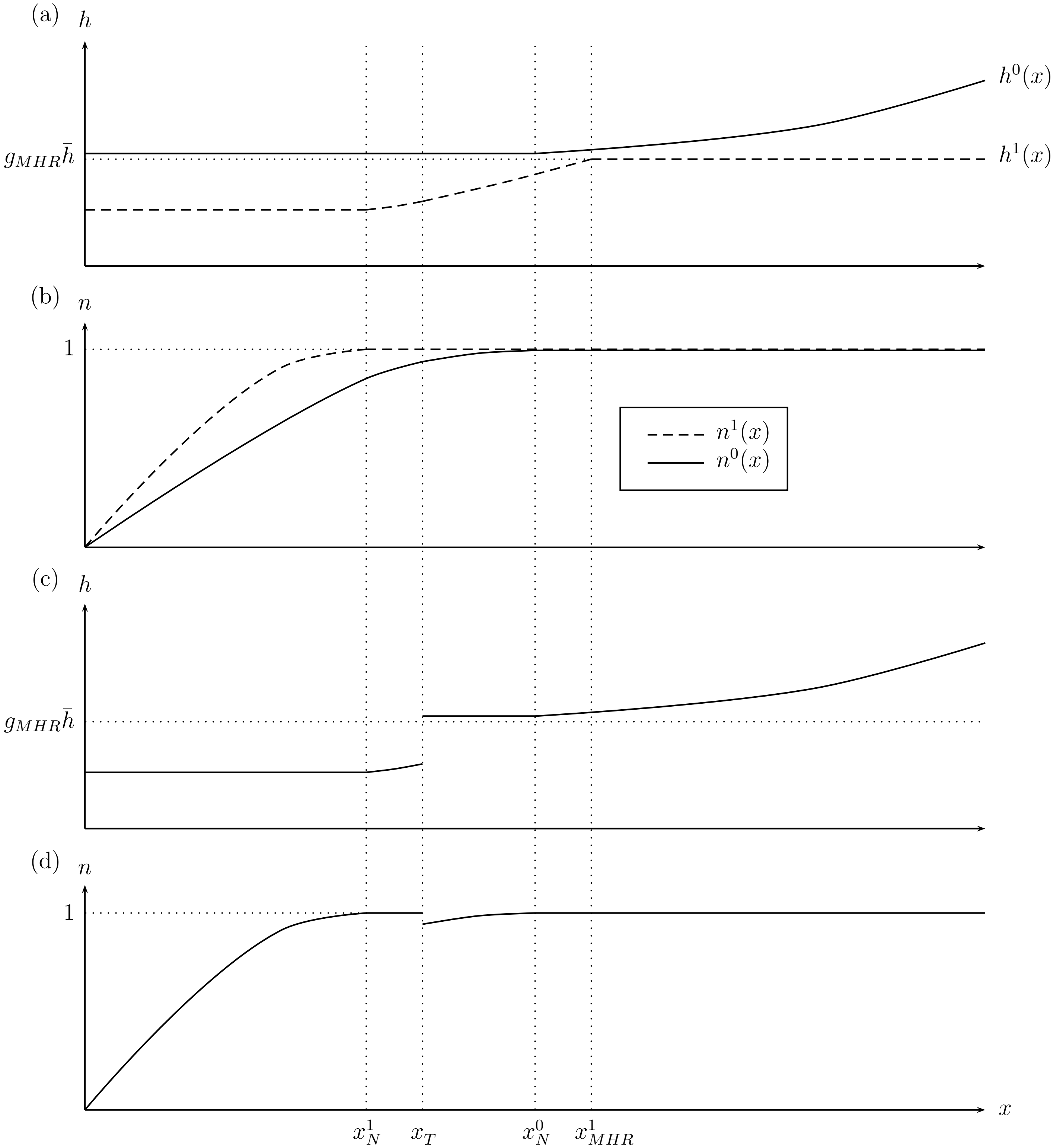
1. *There exists a threshold $x_N^0 \in (0, +\infty)$ such that $h^0(x)$ is constant on $(0, x_N^0)$ and strictly increasing on $(x_N^0, +\infty)$, while $n^0(x)$ is strictly increasing on $(0, x_N^0)$ and equal to one on $(x_N^0, +\infty)$.*

2. *There exist thresholds $x_N^1 \in (0, +\infty)$ and $x_{MHR}^1 \in [x_1^N, +\infty]$ such that $h^1(x)$ is constant on $(0, x_N^1)$, strictly increasing on (x_N^1, x_{MHR}^1), and constant at $g_{MHR}\bar{h}$ on $(x_{MHR}^1, +\infty)$, while $n^1(x)$ is strictly increasing on $(0, x_N^1)$ and equal to one on $\left(x_1^N, +\infty\right)$.*

3. *If $g_{STC} > 0$, then $h_1(x) < h_0(x)$ for all $x \in (0, +\infty)$.*

This proposition is illustrated in Panels (a) and (b) of Figure 1. Part 1 characterizes $h^0(x)$ and $n^0(x)$. There are two profitability regions across which the qualitative behavior of labor inputs differs, divided by a threshold x_N^0 at which the employment constraint becomes binding. Below this threshold the firm engages in layoffs, and hours per workers are constant. The latter follows directly from Proposition 1, which states that hours do not vary with profitability x conditional on the multiplier $\lambda(x)$. Perfect PI implies that $\lambda(x)$ is independent of x, hence hours are constant. Employment is strictly increasing over this region. Above x_N^0 the behavior of hours is governed by Proposition 2. Hours are now strictly increasing in profitability as it is no longer possible to take advantage of higher profitability by raising employment. This characterization of labor input profiles in the case of perfect PI is well-known, and can be found in Rosen (1985), FitzRoy and Hart (1985), and Burdett and Wright (1989), among others.

Figure 1: Labor Input Profiles and STC Take-Up with Perfect PI



Part 2 of the proposition describes $h^1(x)$ and $n^1(x)$. Again there is a threshold x_1^N at which the employment constraint becomes binding, and the qualitative behavior of labor inputs above and below this threshold is very similar to the case of no take-up. The only difference stems from the MHR constraint. Above x_1^N, hours are strictly increasing in profitability until the MHR constraint is binding. It is also possible that the MHR constraint is already binding below x_1^N, in which case hours do not vary with profitability over the entire profitability range $(0, +\infty)$.

Part 3 of the proposition shows that hours under take-up are always below hours under under no take-up. In essence, this follows directly from the comparative statics for hours with respect to take-up T established in Propositions 1 and 2.[13] Notice that Part 3 is silent on the relative position of the employment schedules $n^0(x)$ and $n^1(x)$. First-order condition (10) shows that taking up STC provides an employment subsidy of $(\bar{h} - h)g_{STC}$ per worker, which by itself increase optimal employment. However, the reduction in hours induced by taking up STC reduces the marginal product from employing an additional worker. Thus the total effect of take-up on employment is ambiguous.[14] This implies that the relative position of the thresholds x_0^N and x_1^N is also ambiguous. In our computational experiments the case $n^1(x) < n^0(x)$ always prevails, and in this case $x_1^N < x_0^N$. This is the case illustrated in Panel (b) of Figure 1.

3.2.2 No Private Insurance

Proposition 4 *If $\chi = 1$, then the functions $h^0(x)$, $n^0(x)$, $h^1(x)$, and $n^1(x)$ are continuous and have the following properties.*

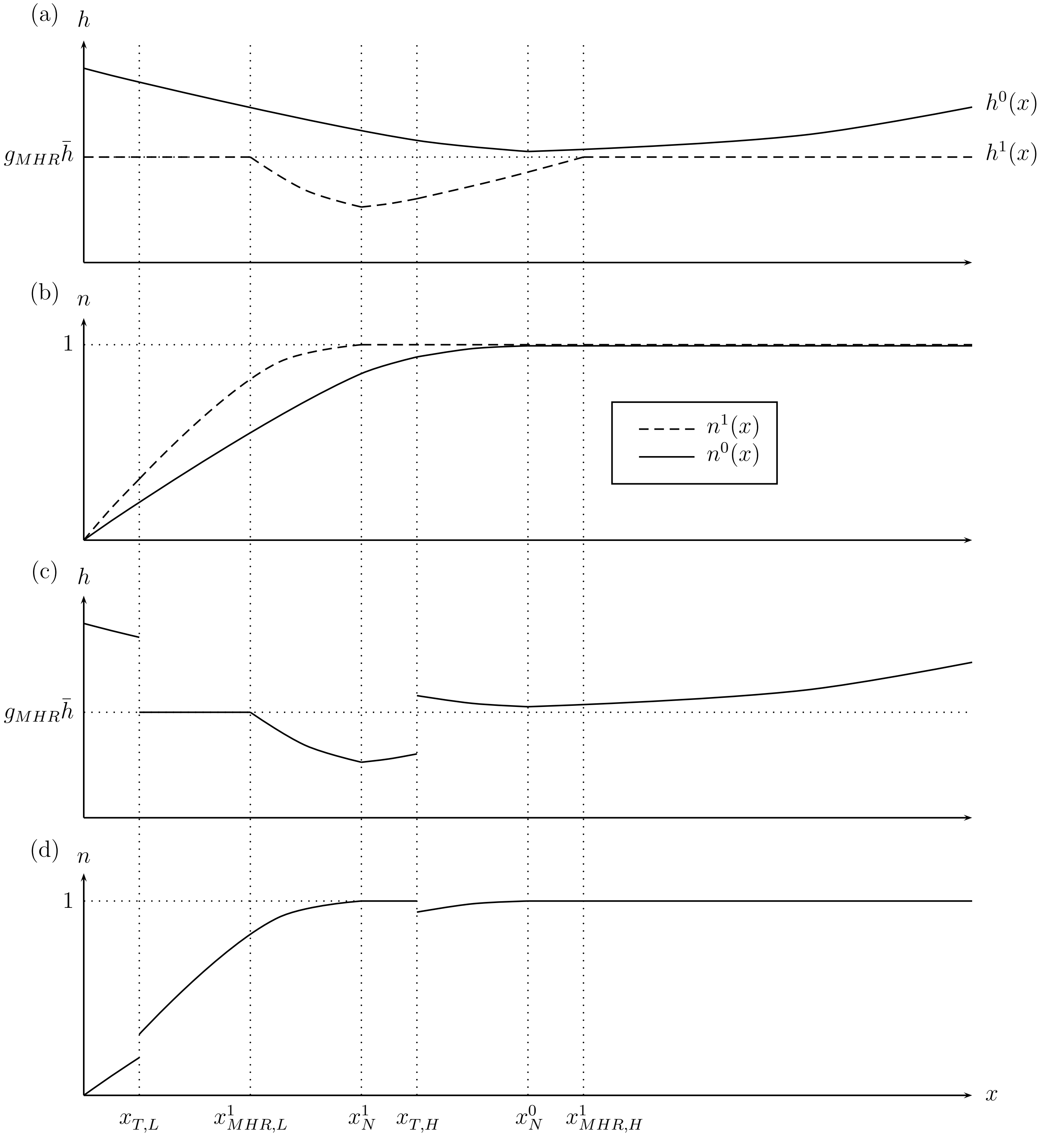
1. *There exists a threshold $x_N^0 \in [0, +\infty]$ such that $h^0(x)$ is strictly decreasing on $(0, x_N^0)$ and strictly increasing on $(x_N^0, +\infty)$, while $n^0(x)$ is strictly increasing on $(0, x_N^0)$ and equal to one on $(x_N^0, +\infty)$.*

2. *There exist thresholds $x_N^1 \in [0, +\infty]$, $x_{MHR,L}^1 \in [0, x_1^N]$, and $x_{MHR,H}^1 \in [x_1^N, +\infty]$ such that $h^1(x)$ is constant at $g_{MHR}\bar{h}$ on $\left(0, x_{MHR,L}^1\right)$, weakly decreasing on $\left(x_{MHR,L}^1, x_N^1\right)$, strictly increasing on $\left(x_N^1, x_{MHR,H}^1\right)$, and constant at $g_{MHR}\bar{h}$ on $(x_{MHR,H}^1, +\infty)$. It is strictly decreasing on $\left(x_{MHR,L}^1, x_N^1\right)$ if $g_{UI} - \bar{h}g_{STC} > 0$. Employment $n^1(x)$ is strictly increasing on $(0, x_N^1)$ and equal to one on $\left(x_1^N, +\infty\right)$.*

[13] Proposition 1 immediately implies this result for levels of profitability below $\min[x_0^N, x_1^N]$, and Proposition 2 does so for the region above $\max[x_0^N, x_1^N]$. The only extra work in the proof of Part 3 of Proposition 3 is to show that this result also holds between x_0^N and x_1^N.

[14] Van Audenrode (1994, p. 84) notes this ambiguity in a similar model.

Figure 2: Labor Input Profiles and STC Take-Up without PI



3. *If $g_{STC} > 0$, then $h_1(x) < h_0(x)$ for all $x \in (0, +\infty)$.*

This proposition is illustrated in Panels (a) and (b) of Figure 2. Employment schedules behave qualitatively as in the case of perfect PI. In contrast, the behavior of hours is qualitatively different. Consider first the case of no take up. The profile $h_0(x)$ is strictly *decreasing* in profitability below the threshold x_0^N. This is explained by Proposition 1, according to which hours are strictly increasing in the multiplier λ if $g_{UI} - \bar{h}T \cdot g_{STC} > 0$, which holds if STC is not taken up. The multiplier λ coincides with marginal utility of consumption. In the absence of PI it is strictly decreasing in profitability.[15] This carries over to hours. As explained in the discussion of Proposition 1, the multiplier affects optimal hours through its interaction which the UI benefit, which acts like a fixed cost of employment in terms of the consumption good. Consumption is scarce after an uninsured decline in profitability. The optimal response of the firm is to send more workers to collect UI benefits, which is one way of obtaining consumption, and to implement longer hours for workers that remain on the job.[16] This comparative statics result is new to the implicit contracts literature. In Section 4 we show that it has important implications for the welfare effects of STC.

Above the full-employment threshold x_N^0, hours are strictly increasing in profitability. Qualitatively, this is as in the case of perfect PI. But the economic forces underlying this result are somewhat different. With perfect PI, the increase in hours is purely driven by a substitution effect, thus our assumption of KPR preferences is not important for this result. In contrast, here the marginal utility of consumption is decreasing in profitability, hence the response of hours depends on the relative strength of the income effect and the substitution effect. KPR preferences imply that these effects would cancel exactly in the absence of policy, that is, if $\tau = 0$. The presence of a positive tax τ makes the income effect relatively weaker. Thus hours remain strictly increasing in profitability.

The hours profile conditional on take-up of STC $h_1(x)$ is qualitatively similar to $h_0(x)$. As in the case of perfect PI, its shape only differs due to the MHR constraint. However, the hours profile would be V-shaped in the absence of the MHR constraint. This implies that in general there are two profitability intervals over which the MHR constraint binds. First, below a threshold $x^1_{MHR,L}$, which lies in the profitability range

[15]This is established in the course of the proof of Proposition 4.

[16]Notice that we have assumed that all fixed costs of employment accrue in terms of utility, so that UI is the only fixed cost in terms of consumption. If other fixed costs also accrue in terms of consumption, then this strengthens the result.

with a slack employment constraint. Second, above a threshold $x^1_{MHR,H}$, which lies in the profitability range over which the employment constraint binds.

Part 3 of the proposition establishes that, as in the case of perfect PI, hours under take-up are always below hours under no take-up.

3.3 STC Take-Up

Having analyzed labor input profiles conditional on take-up, we now discuss optimal take-up. Consider first the case of perfect PI. The next proposition gives sufficient conditions such that take-up is monotone in profitability, occurring at low levels of productivity.

Proposition 5 *Suppose that $\chi = 0$ and $g_{STC} > 0$. If $f(nh) = (nh)^\alpha$ for some $\alpha \in (0, 1)$, and if $x^1_N < x^0_N$, then there exists a threshold $x_T \in [0, +\infty]$ such that the take-up function*

$$T^*(x) = \begin{cases} 1 & for \quad x \in (0, x_T] \\ 0 & for \quad x \in (x_{T,\infty}) \end{cases}$$

is optimal.

The first condition is that the technology is Cobb-Douglas, which is the functional form we employ in our computational experiments. The second condition is that the employment constraint starts to bind at a lower level of productivity in the case of take-up, that is, $x^1_N < x^N_0$. As discussed in the context of Proposition 3, the case $x^N_1 < x^N_0$ prevails in all our computational experiments, although the reverse is a theoretical possibility.

The monotonicity of optimal take-up in Proposition 5 is driven by the complementarity between total hours $n^T(x)h^T(x)$ and profitability. Adopting STC is associated with a reduction in hours. Everything else equal, this leads to lower total hours. This can be countered by an increase in employment, but only if the employment constraint is slack. Once profitability is sufficiently high, firms adopting STC run into the employment constraint. This makes adopting STC more costly, the more so the higher is profitability.

The take-up threshold x_T can lie anywhere in $[0, +\infty]$. Panels (c) and (d) of Figure 1 illustrate the optimal labor input profiles for the case in which x_T lies between the two employment thresholds x^1_N and x^0_N. They are generated from Panels (a) and (b) by selecting the take-up schedules $h^1(x)$ and $n^1(x)$ to the left of x_T, and the no take-up schedules $h^0(x)$ and $n^0(x)$ to the right of x_T. As profitability increases, hours are first flat while employment is increasing. Hours start to increase as the employment constraint becomes binding under take-up at x_1^N. Next, hours jump up and employment jumps down as the take-up threshold x_T is reached. After that, hours are once again flat while employment is increasing until the employment constraint becomes binding under no take-up at x_0^N. Beyond this point, hours are once again increasing.

Next, consider the case of no PI. Here we do not have any theoretical results concerning take-up, as the analysis is substantially complicated by the income effects that arise in this case. As in the case of perfect PI, one economic force that remains at work is that adopting STC is more costly if it would be optimal to choose high hours in the absence of STC. In the case of perfect PI, this gave rise to the following property: take-up is monotone in the hours that the firm would choose conditional on no-take up. Since the latter are monotone in profitability, so is take-up. For the sake of illustration, suppose that this force remains dominant in shaping take-up in the case of no PI. The key difference to the case of perfect PI is that hours are not monotone in profitability, but V-shaped. Given this, one would expect no take-up to occur in two separate regions of profitability, both at very low levels of profitability and at very high levels of profitability. Panels (c) and (d) of Figure 2 illustrate such a case with two take-up thresholds, denoted $x_{T,L}$ and $x_{T,H}$. The lower take-up threshold $x_{T,L}$ is located in the profitability region over which hours conditional on the take-up are strictly declining in profitability, both for $T = 1$ and $T = 0$. In the case illustrated here, the second take-up threshold is located between x_N^1 and x_N^0, when hours are still strictly decreasing in profitability conditional on no take-up, but are already strictly increasing in profitability conditional on take-up due to a binding employment constraint. The labor input schedules in Panels (c) and (d) are generated from Panels (a) and (b) by selecting the no-take schedules $h^0(x)$ and $n^0(x)$ to the left and to the right of $x_{T,L}$ and $x_{T,H}$, respectively, and the take-up schedules $h^1(x)$ and $n^1(x)$ in between. Hours jump down and employment jumps up at $x_{T,L}$, the reverse happens at $x_{T,H}$.

4 Computational Experiments

In this section we carry out computational experiments to examine whether introducing STC can improve on a system restricted to UI in our model. We obtain two main results. First, the ability of STC to improve on UI critically depends on firms' access to PI. STC substantially improves welfare if firms have access to perfect PI, but yields only a negligible improvement when firms lack access to PI. Under perfect PI, STC improves welfare by mitigating labor input distortions induced by UI. This mechanism is greatly diminished if firms lack access to insurance, because the most distressed firms prefer long hours over taking up STC. Second, we find that even with perfect PI, the optimal generosity of STC is substantially below that of UI, and that introducing STC with equal generosity results in a large welfare loss in comparison to having no STC at all.

4.1 Calibration

We calibrate the model to match features of the US labor market. The functional form of the production function is

$$f(nh) = (nh)^{\alpha}.$$

We set $\alpha = \frac{2}{3}$, implicitly assuming that capital cannot be adjusted in response to profitability shocks. For the utility function given in equation (2) above, we specify

$$v(h) = \exp\left(-\eta \frac{h^{1+\psi}}{1+\psi} + \log\left(v_0\right) \mathcal{I}\left[h > 0\right]\right),$$

where η and ψ are strictly positive. The parameter η only affects the level of hours, so we can use it to normalize employment-weighted average hours to one. We set the coefficient of relative risk aversion to $\sigma = 2$, within the "plausible" range 1–5 indicated by micro estimates, see Heathcote, Storesletten, and Violante (2009). The parameter ψ governs the Frisch elasticity of labor supply. Based on the recent survey of the microeconomic evidence in Hall (2009), we target a Frisch elasticity of 0.7.[17] We set $\upsilon = 0.045$, so that 4.5% of workers are not attached to a firm. Together with the level of temporary layoffs targeted below, this matches the average unemployment rate in the US of about 6%.

The density $p(x)$ is log-normal. We normalize the mean of $\log(x)$ to zero. As the standard deviation of $\log(x)$ we choose $\sigma_x = 0.1$, which is a reasonable order of magnitude for firm-level idiosyncratic shocks for a time horizon between six month and one year, see for example Comin and Philippon (2006) and Davis et al. (2007).

We calibrate an economy that has UI but no STC. Thus two parameters remain to be calibrated: the parameter v_0 that governs the fixed utility loss from working strictly positive hours, and the UI benefit g_{UI}. They are jointly calibrated to match two targets.

[17] The Frisch elasticity is $\left(\psi + \frac{\sigma-1}{\sigma}\left(\eta h^{1+\psi}\right)\right)^{-1}$. At average hours, this reduces to $\left(\psi + \frac{\sigma-1}{\sigma}\eta\right)^{-1}$.

First, we target that 1.5% of all workers are unemployed while attached to a firm. Thus 25% of all the unemployed are attached. We choose this target based on the empirical prevalence of temporary layoffs, defined as unemployment in spells that end with being rehired by the previous employer. In the US Current Population Survey, on average 14% of the unemployed are classified as on temporary layoff.[18] Based on the Survey of Income and Program Participation, Fujita and Moscarini (2012) report that a group of unemployed workers of about equal size is not classified as on temporary layoff, but ultimately returns to the previous employer.[19] Second, we target that the replacement rate of UI is 25%, where we define the replacement rate in the model as g_{UI} divided by the average consumption of workers. Recall that experience rating is neutral in our model and g_{UI} corresponds to the UI subsidy net of experience rating. Topel (1983) reports that on average the net subsidy is 31% of earnings. In our model workers jointly own and operate firms, hence implicitly their average consumption reflects income from both wages and profits. This leads us to adopt the somewhat lower target of 25%.

These targets pin down g_{UI} and v_0 as follows. Both g_{UI} and v_0 act as as fixed cost of working positive hours. The fraction of workers on temporary layoff is increasing in fixed costs, so the corresponding target pins down v_0 for given g_{UI}. We then vary g_{UI} to match the target for the replacement rate.

The calibration for both cases, perfect PI and no PI, is summarized in Table 1. The policy parameter g_{UI} is pinned down quite directly by the replacement rate target. Only the utility fixed cost v_0 differs substantially between the two calibrations. With perfect PI it is equal to 0.934, which corresponds to 6.63% in terms of consumption and 9.76% in terms of hours.[20] Its value is higher in the case of no PI, corresponding to 11% in terms of consumption. Lack of insurance makes firms more reluctant to carry out layoffs, thus the fixed cost must be higher to match the targeted level of temporary layoffs. In Appendix B we show that the main results obtained in the remainder of this section are insensitive to changes in parameter and targets over a wide range of values.

We use the calibrated model to carry out the following sequence of policy experiments,

[18] The average is taken over the years 1967-2012.

[19] Compared with other countries for which evidence is available, the incidence of temporary layoffs in the US is about average. In a survey of the available evidence, OECD (2002) reports that temporary layoffs account for almost 40% of unemployment in Canada, 20% of unemployment in Austria and Denmark, and fractions closer to 10% in other European countries such as Germany, Norway and Sweden.

[20] The cost associated with v_0 is expressed in terms of consumption (hours) by considering a compensating proportional decrease in consumption (increase in hours) that leaves workers with $v_0 = 0$ as well off as under the calibrated value.

Table 1: Calibration: Perfect PI and No PI

Value	Perfect PI	No PI	Target
σ	2	2	
α	0.67	0.67	
σ_x	0.1	0.1	
ψ	1.1	1.1	Frisch elasticity 0.7
η	0.664	0.667	$\overline{h} = 1$ (Normalization)
υ	0.045	0.045	Unattached workers 0.045
υ_0 (% c) (% h)	0.934 (6.6)(9.8)	0.89 (11)(16)	Temporary layoffs 0.015
g_{UI}	0.247	0.247	Replacement rate 25

summarized in Table 2. Each experiment is defined by restrictions on the set of policy instruments $\mathcal{G}$ in the government optimization problem of Section 2. First, we restrict the set of policy instruments to UI and determine the welfare-maximizing level of g_{UI}. We denote this level as g^*_{UI}, and also use g^*_{UI} to label this experiment. We use g^*_{UI} rather than the calibrated level of g_{UI} as the starting point for experiments that introduce STC. Otherwise welfare gains from STC could merely reflect a suboptimal level of g_{UI}, rather than a genuine added value of g_{STC} as a policy instrument. The next three experiments introduce short-time compensation, but without a minimum hours requirement, hence $g_{MHR} = 1$. In the first, we determine the optimal level of g_{STC} holding constant g_{UI} at g^*_{UI}. By construction, introducing g_{STC} in this way does not affect the level of consumption of unattached workers. Therefore, to the extent that STC does improve the allocation, it can only do so by mitigating the distortion of labor inputs induced by UI. We refer to the corresponding level of STC and also the entire experiment as $g^*_{STC}|g^*_{UI}$ to indicate that g^*_{STC} is optimal conditional on fixing the level of UI at g^*_{UI}. In the second experiment, we introduce a level of g_{STC} that is as generous as g^*_{UI}. This level satisfies $g_{STC}\bar{h} = g^*_{UI}$, and the corresponding experiment is labeled $g^{\max}_{STC}|g^*_{UI}$. In the next step, we determine the welfare-maximizing combination of g_{STC} and g_{UI} denoting this experiment as $(g_{UI}, g_{STC})^*$. The next two experiments introduce an MHR by allowing g_{MHR} do differ from one. First, in the experiment $(g_{STC}, g_{MHR})^*|g^*_{UI}$ we once again fix the level of UI at g^*_{UI} while jointly choosing g_{STC} and g_{MHR} optimally. Finally, in the experiment $(g_{UI}, g_{STC}, g_{MHR})^*$ we choose all three policy instruments optimally.

Table 2: Policy Experiments

Policy Experiment	Restrictions on the Set of Policy Instruments $\mathcal{G}$	Remarks
g^*_{UI}	$g_{STC} = 0$, $g_{MHR} = 1$	
$g^*_{STC}\|g^*_{UI}$	$g_{UI} = g^*_{UI}$, $g_{MHR} = 1$	
$g^{max}_{STC}\|g^*_{UI}$	$g_{UI} = g^*_{UI}$, $g_{STC}\overline{h} = g^*_{UI}$, $g_{MHR} = 1$	Perfect PI only
$(g_{UI}, g_{STC})^*$	$g_{MHR} = 1$	
$(g_{STC}, g_{MHR})^* \|g^*_{UI}$	$g_{UI} = g^*_{UI}$	
$(g_{UI}, g_{STC}, g_{MHR})^*$	None	
$\overline{g}_{STC}\|g^*_{UI}$	$g_{UI} = g^*_{UI}$, $g_{STC}\overline{h}/g_{UI}$ takes same value as in experiment $g^*_{STC}\|g^*_{UI}$ under perfect PI, $g_{MHR} = 1$	No PI only

4.2 Perfect Private Insurance

Results for the case of perfect PI are displayed in Table 3. The calibration and the first best are shown as points of reference.[21] For each experiment, the first six rows show the values of the policy instruments g_{UI}, g_{STC} and g_{MHR}, and the budget clearing tax τ, along with the replacement rates implied by the values of g_{UI} and g_{STC}, labeled $REPR_{UI}$ and $REPR_{STC}$, respectively.[22] The next row reports the take-up rate for STC, that is, the average fraction of attached workers receiving STC in percent. The next four rows show the average of employment and average (employment weighted) hours for attached workers, denoted $\bar{n}$ and $\bar{h}$, respectively, along with average output $\bar{y}$ and consumption $\bar{c}$ across attached workers. The final row shows, for each allocation, the gain in welfare vis-à-vis the experiment g^*_{UI}. Both in this table and in the remainder of the paper, all welfare gains are expressed in percentage consumption-equivalent terms. Figure 3 compares labor input profiles for the three experiments g^*_{UI}, $g^*_{STC}|g^*_{UI}$, and $(g_{UI}, g_{STC})^*$ and the first best. These correspond to the theoretical labor input profiles of Figure 1, showing hours and employment as a function of profitability x.[23] Thick gray segments indicate the region of STC take-up.

[21] The first-best allocation maximizes utilitarian welfare subject to only the resource constraint of the economy.

[22] $REPR_{UI}$ is defined as the ratio between g_{UI} and average consumption, expressed in percentage terms. Analogously, $REPR_{STC}$ is defined as the ratio between the maximal STC benefit $g_{STC}\bar{h}$ and average consumption. Thus the two replacement rates coincide if $g_{STC}\bar{h} = g_{UI}$.

[23] The x-axis is scaled to the distribution of profitability shocks.

Table 3: Policy Experiments: Perfect PI

	Calibr.	g_{UI}^*	$g_{STC}^*\|g_{UI}^*$	$g_{STC}^{max}\|g_{UI}^*$	$(g_{UI}, g_{STC})^*$	$(g_{STC}, g_{MHR})^*\|g_{UI}^*$	$(g_{UI}, g_{STC}, g_{MHR})^*$	FB
g_{UI}	0.247	0.262	0.262	0.262	0.284	0.262	0.28	
g_{STC}			0.08	0.308	0.13	0.0643	0.0933	
g_{MHR}			1	1	1	0.81	0.82	
τ	0.0158	0.0214	0.0196	0.0501	0.0278	0.017	0.0237	
$REPR_{UI}$ (%)	25	26.7	27.1	29.2	29.9	26.8	29.1	
$REPR_{STC}$ (%)			7.97	29.2	13	6.43	9.32	
STC Take-Up (%)			51.6	49	52.1	16.2	28.1	
$\bar{n}$	0.984	0.968	0.988	1	0.98	0.991	0.982	1
$\bar{h}$	1	1	0.965	0.853	0.943	0.98	0.963	1.02
$\bar{y}$	0.999	0.991	0.979	0.911	0.96	0.991	0.975	1.02
$\bar{c}$	0.987	0.978	0.967	0.898	0.947	0.978	0.961	0.982
Welf. Rel. to g_{UI}^* (%c)	−0.1886		0.30107	−1.8582	0.52629	0.4027	0.60792	6.0165

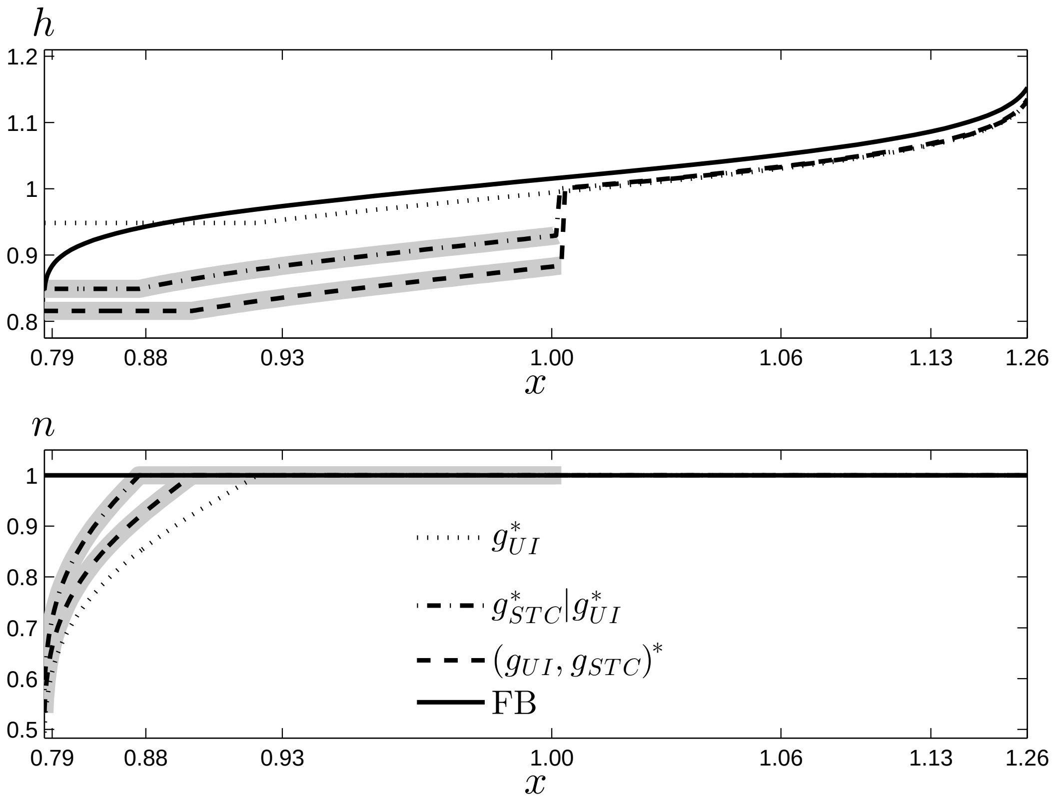
Experiment g^*_{UI} shows that optimal UI is somewhat above the calibrated level. The corresponding level of $\bar{n}$ is 0.984, compared to 0.968 in the calibration. Thus the number of workers on layoff doubles. Hence layoffs respond quite strongly to g_{UI}, a point we return to below. Employment is below one at sufficiently low levels of profitability and increasing. Hours are constant over the profitability range with positive layoffs and increasing otherwise, as established by Part 1 of Proposition 3. In contrast, first-best employment is one irrespective of profitability, and first-best hours are increasing throughout.

Experiment $g^*_{STC}|g^*_{UI}$ shows that introducing STC is optimal when UI is fixed at g^*_{UI}, and it establishes half of our first main result: under perfect PI, STC can substantially improve welfare, here by 0.3%. The optimal level of g_{STC} is modest: the implied replacement rate for STC is 7.97%, compared to 27.1% for UI. Nevertheless, this level of STC is quite effective, reducing layoffs by more than half. As discussed in Section 3.2.1, an increase in employment is not implied by our theoretical analysis, but occurs in all of our computational experiments. Hours per worker drop substantially, so that output is lower than in experiment g^*_{UI}, despite the increase in employment. The reduction in spending on UI outweighs the spending on STC, and government outlays as a percentage of output are reduced from 2.19% under experiment g^*_{UI} to 2%. The labor input profiles for this experiment conform to Propositions 3 and 5. The take-up threshold x_T lies above the threshold x^N_0 at which the employment constraint becomes binding under no take-up.[24] Thus some firms taking up STC would have retained all workers even in the absence of STC. For these firms, STC does not have the benefit of reducing layoffs, but it distorts hours. Employment is then continuous at the take-up threshold, while hours jump up. Throughout the take-up region, hours are strictly lower than in the experiment g^*_{UI} and employment is uniformly higher.

In experiment $g^{\max}_{STC}|g^*_{UI}$, STC eliminates layoffs completely, but induces a very large decline in hours. Overall, this leads to a large welfare loss of 1.85% vis-à-vis experiment g^*_{UI}. Together with the preceding experiment, this establishes our second main result: Optimal STC is substantially less generous than UI, and introducing STC with equal generosity results in a large welfare loss in comparison to having no STC at all. The left panel of Figure 4 illustrates this result by plotting the welfare gain as a function of g_{STC}, with g_{UI} fixed at g^*_{UI} and with g_{STC} varying up to g^{max}_{STC}. In our model there is no natural reason for UI and STC to be equally generous. The optimal levels of UI and STC are

[24] Thresholds are not labeled in the figure, as it contains multiple experiments.

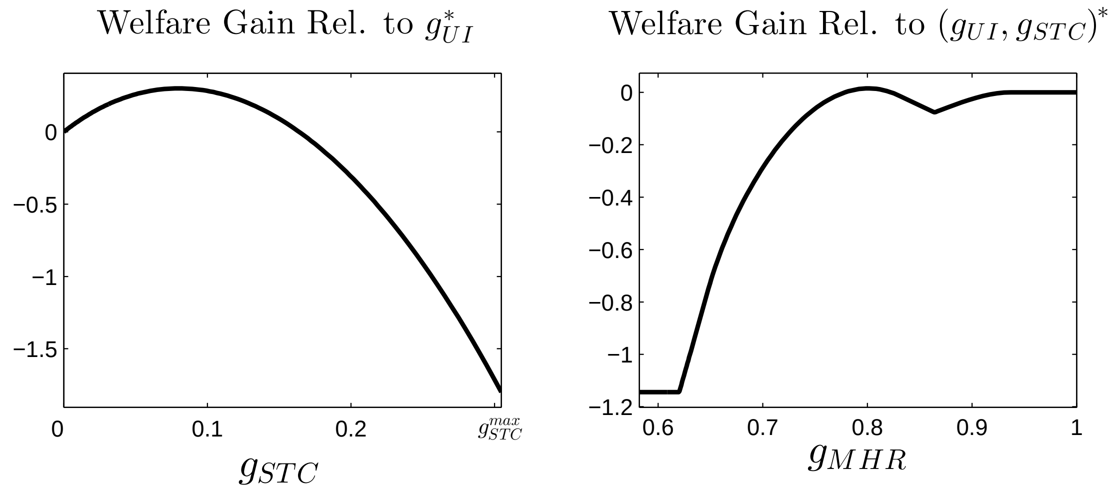
Figure 3: Hours and Employment, Perfect PI



determined by different trade-offs. Optimal UI balances the benefit of making transfers to unattached workers against the cost of distorting the layoff decision of firms. Optimal STC balances mitigation of this distortion against the cost of distorting hours in firms that would abstain from layoffs even in the absence of STC.

Experiment $(g_{UI}, g_{STC})^*$ shows that the optimal combination of UI and STC involves substantially more generous UI than what is optimal if STC is not available: the benefit level g_{UI} increases by more than 8% (from 0.262 to 0.284), which corresponds to an increase in the replacement rate from 27.1% to 29.9%. The mechanism underlying this result is that STC counteracts the distortion of the composition of labor input associated with UI. As a consequence, the availability of STC makes it optimal to offer more generous UI, an indirect insurance effect of STC. As in experiment $g^*_{STC}|g^*_{UI}$, STC is substantially less generous that UI in this experiment. The overall welfare gain of moving from g^*_{UI} to $(g_{UI}, g_{STC})^*$ amounts to 0.53%. About half of this gain can be obtained by moving to $g^*_{STC}|g^*_{UI}$, indicating that adjusting the level of UI is equally important in order to reap the full benefit of the availability of STC as an additional instrument. Qualitatively the pattern of labor inputs across profitability in Figure 3 is very similar to the experiment

Figure 4: Welfare Gains, Perfect PI

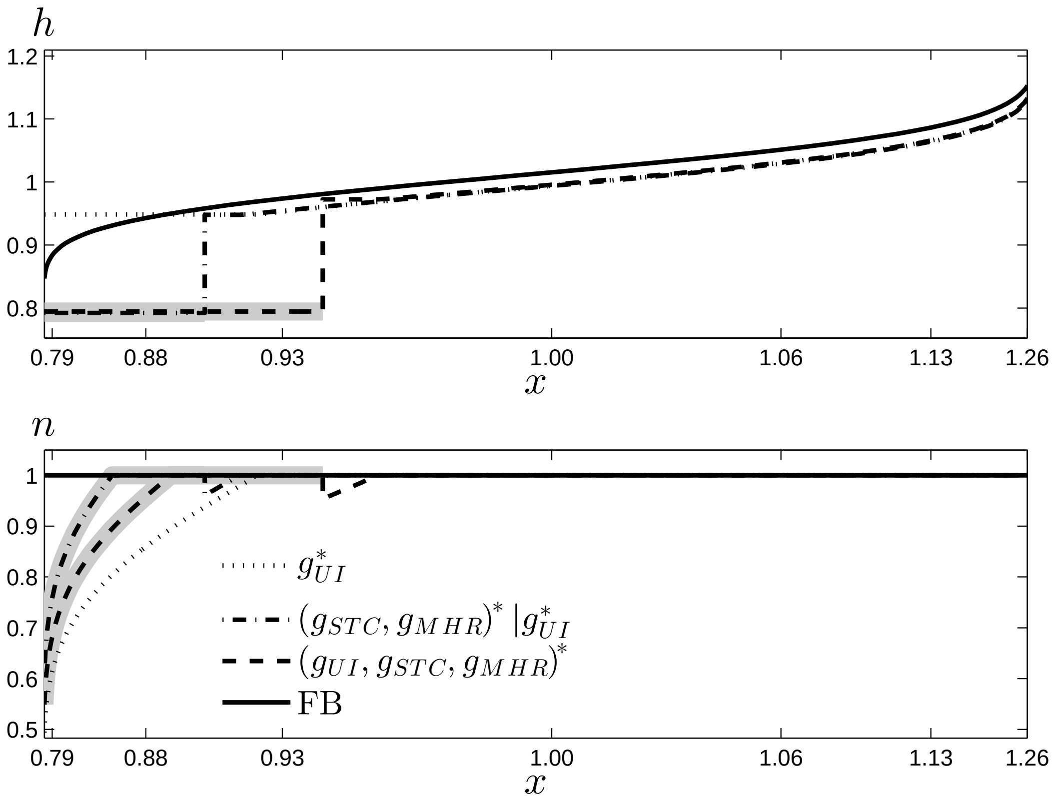


$g^*_{STC}|g^*_{UI}$. However, both hours and employment are lower since both UI and STC are more generous.

In the next two experiments we examine whether requiring a minimum hours reduction (MHR) can further improve welfare. The results for experiments $(g_{STC}, g_{MHR})^*|g^*_{UI}$ and $(g_{UI}, g_{STC}, g_{MHR})^*$ displayed in the Table 3 show that imposing an MHR is indeed optimal, leading to further, albeit small welfare gains. The optimal level of g_{MHR} is similar in the two experiments, at 0.81 and 0.82, respectively. As in the experiments without MHR, optimal STC is substantially less generous than UI. In fact, it becomes slightly less generous, as the MHR makes it possible to achieve the same reduction in layoffs with a lower level of g_{STC}. Figure 5 shows the labor input profiles for these policy constellations. For both experiments, hours are constant at $g_{MHR}\overline{h}$ in the take-up region. The take-up threshold is lower than in the experiments without an MHR, which is reflected in lower take-up rates in Table 3. Because government expenditure on STC is reduced, taxes are lower. Average hours and employment are higher.

To illustrate how the MHR affects welfare, the right panel of Figure 4 plots welfare gains as a function of g_{MHR}, holding g_{STC} and g_{UI} fixed at $(g_{UI}, g_{STC})^*$. Lowering g_{MHR} from a value of one first leaves welfare unaffected because all firms taking up STC have hours strictly below one. As the constraint becomes binding with further reductions in g_{MHR}, some firms with $n = 1$ choose to forgo STC. This effect is desirable since for these firms take-up of STC only distorts hours without any beneficial effect on employment. However, other firms with $n = 1$ reduce hours even further to meet the MHR. Quantitatively, this negative effect dominates and consequently welfare decreases.

Figure 5: Hours and Employment, Perfect PI: Minimum Hours Reduction



For further reductions in g_{MHR}, the MHR constraint also binds for firms with $n < 1$. In contrast to firms with $n = 1$, for these firms imposing the MHR has the additional positive effect of reducing layoffs. Due to this effect, welfare now increases as g_{MHR} is reduced further. The welfare maximizing level of g_{MHR} is reached in this region. Further reductions in g_{MHR} lead to large negative effects due to inefficient hours reductions and because there are now more and more firms with $n < 1$ that do not take up STC. Finally, there is a second flat region at very low levels of g_{MHR} for which take-up is zero.

One noteworthy feature of the calibrated model is that employment and thus unemployment is very sensitive to policy. To put this into perspective, we will now compare this sensitivity with empirical evidence. For this purpose, it is useful to express the sensitivity of unemployment with respect to the replacement rate as a semielasticity. The local semielasticities are 15.6 and 10.4 at g^*_{UI} and $g^*_{STC}|g^*_{UI}$, respectively. Costain and Reiter (2008) estimate a semielasticity of 3.09. This is substantially smaller than the semielasticity implied by our calibration. This suggests that the model may be missing features that reduce the semielasticity. Interestingly, the model implies that STC can play an important role in reducing the semielasticity. Specifically, the impact of an increase in the replacement rate on unemployment is much weaker when STC is adjusted optimally. At $g^*_{STC}|g^*_{UI}$, for example, the associated local elasticity is 4.52.

4.3 No Private Insurance

We now turn to the second scenario in which firms have no access to PI. This opens up an additional channel through which STC can affect welfare, via a direct insurance effect. At first sight, our analysis in Section 4.2 may suggest that this will add an additional positive welfare effect of STC, on top of the mitigation of labor input distortions. In Figure 3 hours per worker are increasing in profitability (weakly so in the region with positive layoffs). Thus total hours are rising faster in profitability than employment. Taking as given this pattern of labor inputs, from an insurance perspective it is then better to make transfers proportional to total hours rather than employment. However, the labor input profiles in Figure 3 are optimal when firms have access to perfect PI. As we have seen in Section 3, labor input profiles are different if firms lack this access. In particular, the hours profile is declining over the profitability region in which firms engage in layoffs. This has two important implications for the welfare effects of STC. First, the positive effect from the mitigation of labor input distortions is greatly diminished. This is because the most distressed firms, which account for most layoffs, prefer high hours per worker. Second, a direct positive insurance effect fails to materialize, and the effect is even slightly negative, because STC fails to transfer resources to the most distressed firms. We find that overall, STC fails to substantially improve welfare.

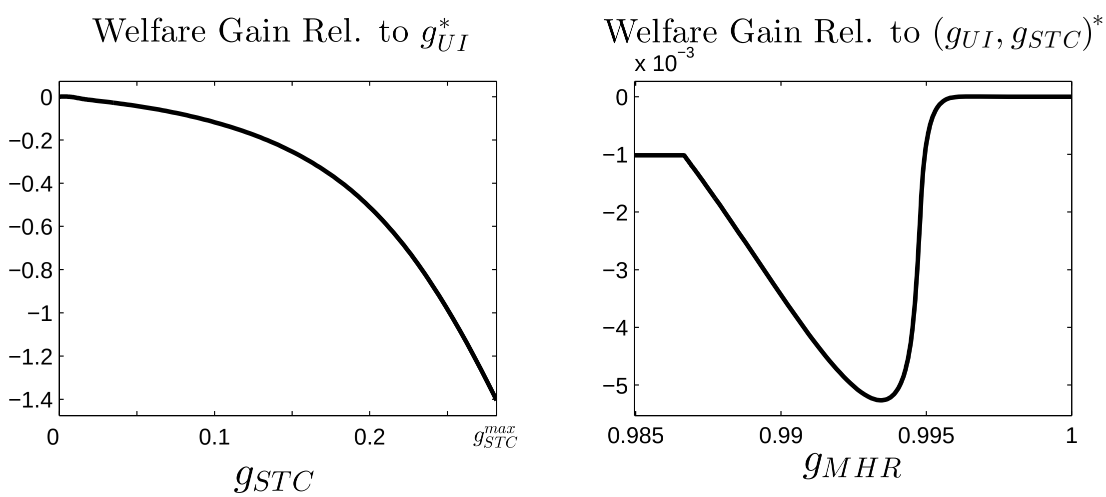
The results of the policy experiments are reported in Table 4. The left panel of Figure 6 plots welfare gains associated with variations in g_{STC} up to $g^{max}_{STC} = g^*_{UI}/h$, given g^*_{UI}. The optimal level of g_{STC} in experiment $g^*_{STC}|g^*_{UI}$ is strictly positive at 0.003, but very close to zero and yields a negligible welfare gain of 0.93 per million that is imperceptible in the plot. This is the second half of our main first main result: STC delivers substantial welfare gains if firms have access to perfect PI, but fails to do so if firms have no access to PI.

Since the optimal level of STC in experiment $g^*_{STC}|g^*_{UI}$ is so small, it comes as no surprise that the availability of STC has a negligible impact on the optimal level of UI when both are optimized jointly in experiment $(g_{UI}, g_{STC})^*$. Under perfect PI, a modest level of STC is optimal. Why is STC of a similar magnitude not optimal here? To shed light on the underlying mechanisms, we include an additional experiment labeled

Table 4: Policy Experiments: No PI

	Calibr.	g_{UI}^*	$g_{STC}^* \vert g_{UI}^*$	$\overline{g}_{STC} \vert g_{UI}^*$	$(g_{UI}, g_{STC})^*$	FB
g_{UI}	0.247	0.25	0.25	0.25	0.25	
g_{STC}			0.00316	0.0758	0.00331	
g_{MHR}			1	1	1	
τ	0.0157	0.017	0.017	0.018	0.0171	
$REPR_{UI}$ (%)	25	25.3	25.4	25.8	25.4	
$REPR_{STC}$ (%)			0.321	7.6	0.336	
STC Take-Up (%)			78.9	51.3	78.8	
$\bar{n}$	0.984	0.98	0.98	0.984	0.98	1
$\bar{h}$	1	1	0.999	0.969	0.999	1.02
$\bar{y}$	0.996	0.994	0.992	0.975	0.992	1.02
$\bar{c}$	0.984	0.982	0.981	0.963	0.981	0.982
Welf. Rel. to g_{UI}^* (%c)	−0.012572		0.00092858	−0.076012	0.0009721	7.0041

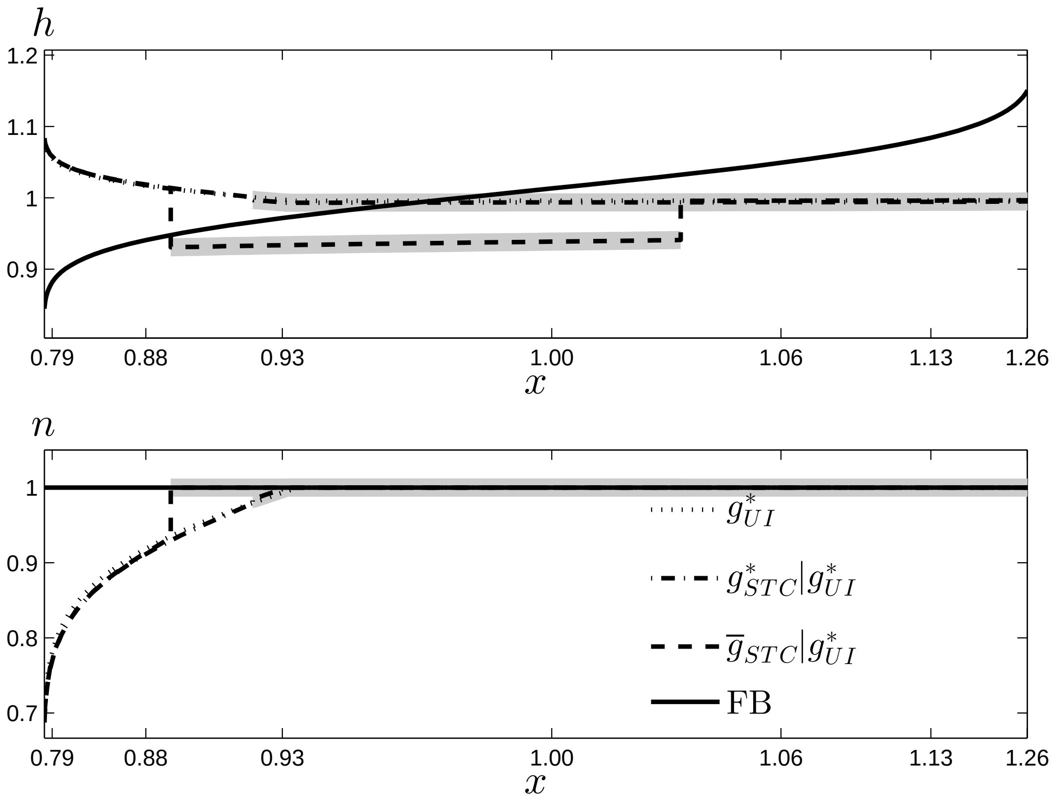
Figure 6: Welfare Gains, No PI



$\overline{g}_{STC}|g_{UI}^*$. Here $\overline{g}_{STC}$ is chosen such that the ratio of STC to UI is the same as in the experiment $g_{STC}^*|g_{UI}^*$ under perfect PI.

Figure 7 is the counterpart of Figure 3, showing labor input profiles for the familiar experiments g_{UI}^* and $g_{STC}^*|g_{UI}^*$, together with the additional experiment $\overline{g}_{STC}|g_{UI}^*$. Starting with the experiment g_{UI}^*, the key difference in the pattern of labor inputs in comparison to Figure 3 is that hours per worker are now strictly decreasing rather than increasing over the profitability range with positive layoffs, in accordance with Proposition 4. Hours are strictly increasing over the region with $n = 1$, in line with Proposition 4, yet quan-

Figure 7: Hours and Employment, No PI



titatively they are virtually flat. Recall from the discussion in Section 3.2.2 that with perfect PI the strictly increasing pattern of hours is due to a substitution effect. For the case of no PI, this substitution effect is counteracted by an income effect. Given KPR preferences, the income effect does not fully offset the substitution effect only because it is weakened by the presence of the tax. Quantitatively this impact of the tax is small.

Since the optimal level of STC in the experiment $g^*_{STC}|g^*_{UI}$ is very small, labor input profiles lie virtually on top of profiles from experiment g^*_{UI}. The thick gray segments indicate that take-up of STC occurs at high levels of profitability, distorting hours without any benefit in terms of reduced layoffs. The effects of STC on the labor input profiles are easier to discern for the experiment $\overline{g}_{STC}|g^*_{UI}$. Here only firms with intermediate profitability take up STC, and for most of these firms STC merely distorts hours, without any reduction in layoffs. Firms with very low profitability, which account for most layoffs, forego STC in favor of high hours. This shows that STC's ability to mitigate the labor input distortions caused by UI is greatly diminished here. Correspondingly, Table 4 shows little positive impact on employment, whereas hours and welfare decline significantly in this experiment.

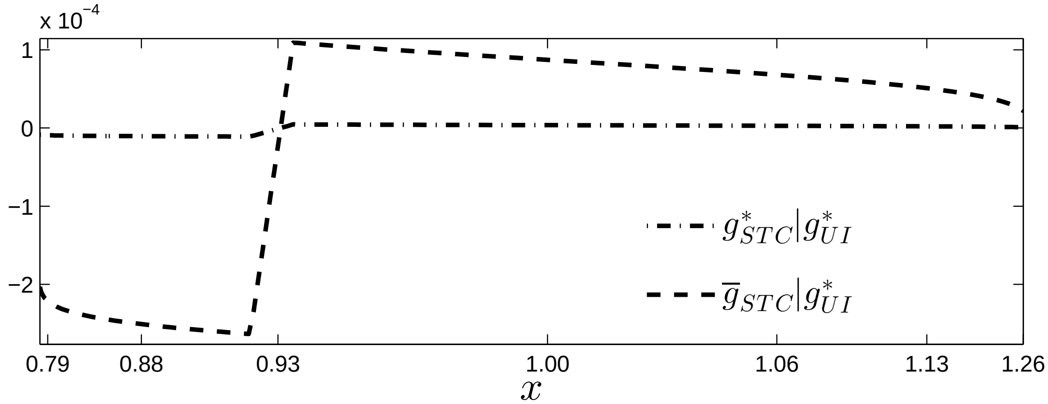
The preceding analysis explains at least part of why STC fails to substantially improve welfare when firms lack access to PI. Yet the analysis is not complete, because STC now also has a direct insurance effect. But the V-shaped pattern of hours induced by the lack of insurance implies that STC is poorly targeted when it comes to providing insurance. The most distressed firms choose to forego STC. STC is collected by more profitable firms. Hence STC redistributes in the wrong direction between these groups. STC shifts consumption in the right direction within the group of firms with the highest levels of profitability, those that employ all workers and take up STC, since hours are strictly increasing over this profitability region. Because hours are virtually flat however, one would expect this positive effect to be very small.

To quantify the overall direct insurance impact of STC, we hold labor input decisions constant at those corresponding to the experiment g^*_{UI} and calculate net government transfers induced by the levels of STC from experiment $g^*_{STC}|g^*_{UI}$ in conjunction with a corresponding budget clearing tax τ. The dash-dotted line in Figure 8 plots the difference in this resulting net transfer schedule and net transfer schedule under g^*_{UI}. Clearly, for the group of low profitability firms who do no take-up STC, net transfers worsen. Within the group of firms that take up STC, low profitability firms' net tranfers improve by more than high profitability firms'. Next, we evaluate the level of welfare associated with the consumption profiles that are induced by these net transfers. Quantitatively, the direct insurance effect of STC on welfare is a negative, but very small at 0.79 per million. Of course this reflects in part that $g^*_{STC}|g^*_{UI}$ is very small. The direct insurance effect in the experiment $\overline{g}_{STC}|g^*_{UI}$ is -0.002%. This is negligible in comparison to the welfare gain of 0.3% induced by a similar STC level in the case of perfect PI. Thus it is the greatly diminished ability to mitigate labor input distortions which explains the failure of STC to substantially improve welfare.

We would like to stress that the finding of small direct insurance effects of STC should not lead to the conclusion that STC can be evaluated in a model with risk neutral firms, when studying firms that do lack access to PI. The degree of access to financial markets shapes how firms adjust labor inputs in response to shocks, and this matters for the ability of STC to affect labor input decisions. Thus taking into account firms' access to financial markets is important, independent of whether STC has small or large direct insurance effects.

The option to combine STC with an MHR does not yield any additional welfare

Figure 8: Change in Net-Transfers Due to STC, No PI



gains. Thus the results from the experiments $(g_{STC}, g_{MHR})^*|g^*_{UI}$ and $(g_{UI}, g_{STC}, g_{MHR})^*$ are omitted from the table, since they are identical to the results from experiments $g^*_{STC}|g^*_{UI}$ and $(g_{UI}, g_{STC})^*$, respectively. The right panel of Figure 6 is the counterpart of the corresponding panel of Figure 4, and illustrates that the introduction of an MHR does not improve welfare in the experiment $(g_{STC}, g_{UI})^*$. As in Figure 4 the impact of reducing g_{MHR} is non-monotone. Once the MHR is low enough to bind, welfare first decreases and then increases, but never exceeds the level obtained for $g_{MHR} = 1$.[25]

5 Intensive-Margin Technology

Up to now, we have focused on a specification of technology in which hours of different workers are perfectly substitutable. It appears likely that the welfare effects of STC vary with the features of technology, such as the substitutability of hours. Thus it would be valuable to carry out a comprehensive investigation of the role of technology for the optimality of STC. In this section we take a first step in this direction by considering a specification that, in terms of the substitutability of hours of different workers, lies at the opposite end of the spectrum in that there is no substitutability at all. A firm then either produces with $n = 1$ or shuts down entirely, and all adjustment of labor inputs occurs along the intensive margin. We refer to this as the *intensive-margin case* in short.

We find that our two main results carry over to this specification. First, STC yields substantial welfare gains only in the case with perfect PI. In fact, optimal STC is zero under no PI. Second, optimal STC is always substantially less generous that optimal UI,

[25] The kink in Figure 6 is not present here, as all firms adopting STC have employment $n = 1$.

and making STC equally generous results in a large welfare loss.

Two main differences emerge in comparison to the standard technology. First, STC has a positive direct insurance effect when firms lack access to private insurance. This does not happen under the standard technology because the most distressed firms adjusted by choosing layoffs in conjunction with high hours. In contrast, here the technology is such that changes in hours per worker are the only way to adjust, short of shutting down. Nevertheless, we find that the direct insurance effect is too small to make STC worthwhile in the case of no PI. Second, we find that welfare gains of STC under perfect PI are smaller. This is because STC can mitigate the distortions caused by UI only via the shutdown margin, which is not very responsive to STC.

To put both the standard technology and the intensive-margin technology into perspective, we start with the more general specification used by BW. In this specification $l(n, h)$ is a function that combines employment and hours into a labor-input index, and output is given by $xf(l(n, h))$. The standard case is then $l(n, h) = nh$. The technology we consider in this section can be written as

$$l(n, h) = \begin{cases} h & \text{for } n = 1, \\ 0 & \text{for } n < 1. \end{cases} \tag{16}$$

The index of labor input is zero whenever employment falls short of one, hence reducing hours per worker is the only possible response to a profitability shock, apart from shutting down production. This case and the standard case are at the two ends of the spectrum of specifications exhibiting a property which BW refer to as Assumption L. This assumption requires that technology is not biased against work sharing, in the sense that reducing hours per worker while keeping total hours constant does not result in a loss of output.[26] Work sharing is neutral under the standard technology, in the sense that reducing hours per worker given constant total hours has no effect on output. The intensive-margin case is most favorable to work sharing: reducing employment for given total hours would result it a complete loss of output.

There is no need to revisit the theoretical analysis of Section 3 for this specification. The only change is that there is no longer a region in which employment lies strictly between zero and one. The analysis of the behavior of hours per worker when the

[26] Formally, the function $l(n, h)$ satisfies Assumption L if $n_2 h_2 = n_1 h_1$ and $n_2 > n_1$ imply $l(n_2, h_2) \geq l(n_1, h_1)$.

employment constraint binds carries over directly. Thus we immediately carry out the sequence of computational experiments described in Section 4, again for the scenarios of perfect and no private insurance.

Table 5: Calibration: Intensive-Margin Case

Value	Perfect PI	No PI	Target
σ	2	2	
α	0.67	0.67	
σ_x	0.1	0.1	
ψ	1.1	1.1	Frisch elasticity 0.7
η	0.663	0.661	$\overline{h} = 1$ (Normalization)
υ	0.045	0.045	Unattached workers 0.045
v_0 (% c) (% h)	0.712 (29)(41)	0.425 (57)(87)	Temporary layoffs 0.015
g_{UI}	0.246	0.245	Replacement rate 25

The calibration for both cases under the intensive-margin technology is shown in Table 5. Calibrated parameters remain essentially unchanged, with exception of the fixed-cost parameter v_0. Temporary layoffs are much less attractive in the intensive-margin case than in the standard case, because they result in a complete loss of output. Matching the targeted rate of temporary layoffs then requires a substantially higher fixed cost of working positive hours. For the case of perfect PI, these amount to a consumption-equivalent value of 28.8% as opposed to 6.63% percent for the standard case. Similarly, for the case of no PI, this cost increases from 11% to 57.5% in consumption equivalents.[27]

5.1 Perfect Private Insurance

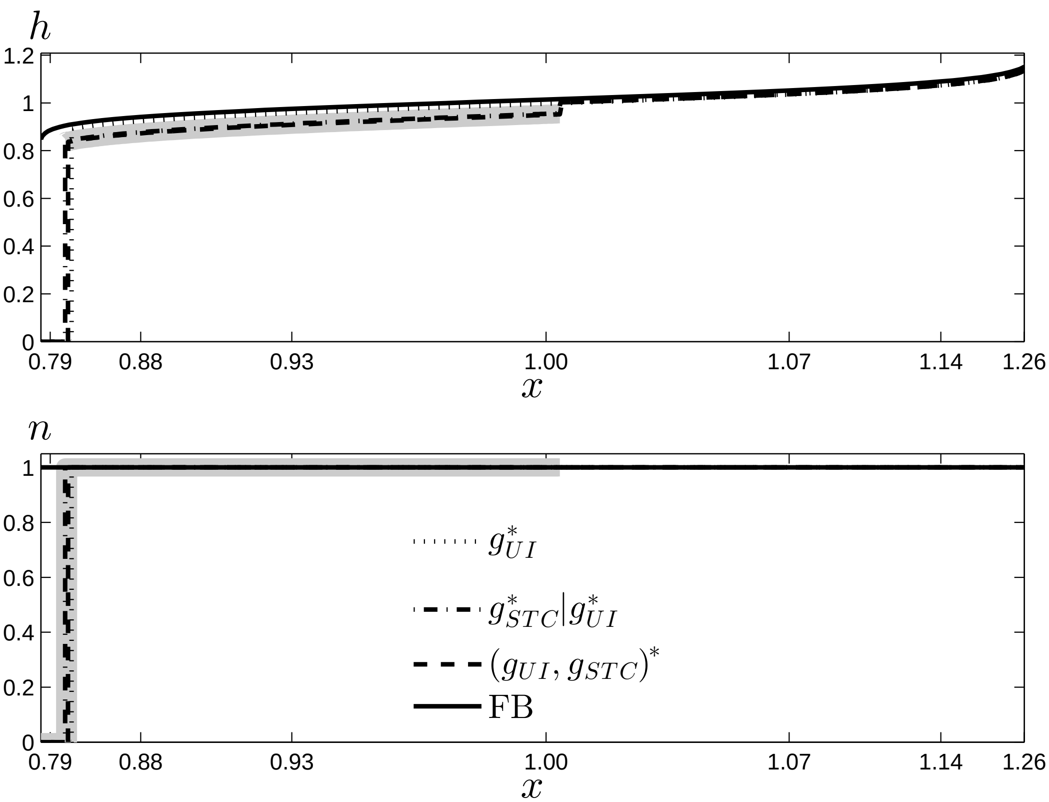
Table 6 and Figure 9 show the results of the policy experiments for the case of perfect PI. Once again it is optimal to introduce STC for a given level of g^*_{UI}. The left panel of Figure 10 shows welfare as a function of g_{STC} with UI fixed at g^*_{UI}. Comparison with the left panel of Figure 4 shows that here the ability of STC to improve welfare is more

[27] By targeting the aggregate rate of temporary layoffs, we implicitly assume that all firms in the economy operate the intensive-margin technology. Similarly, in the benchmark calibration we implicitly assume that all firms in the economy operate the standard technology. It is likely that the technology in some sectors is better described by the standard case, while in other sectors the intensive-margin case is more appropriate. If the fixed cost of employment is similar across sectors, then, everything else equal, one would expect that sectors operating the standard technology have a higher rate of temporary layoffs. In future research, it would be interesting to consider sector-specific calibrations of the model.

Table 6: Intensive-Margin Case, Perfect PI

	Calibr.	g^*_{UI}	$g^*_{STC} \vert g^*_{UI}$	$g^{max}_{STC} \vert g^*_{UI}$	$(g_{UI}, g_{STC})^*$	FB
g_{UI}	0.246	0.268	0.268	0.268	0.272	
g_{STC}			0.0516	0.315	0.0566	
g_{MHR}			1	1	1	
τ	0.0157	0.0218	0.0221	0.0542	0.0236	
$REPR_{UI}$ (%)	25	27.6	27.8	30	28.3	
$REPR_{STC}$ (%)			5.25	30	5.76	
STC Take-Up (%)			49.6	48.8	49.5	
$\bar{n}$	0.984	0.968	0.975	0.994	0.972	1
$\bar{h}$	1	1	0.979	0.85	0.977	1.02
$\bar{y}$	0.996	0.984	0.975	0.904	0.972	1.02
$\bar{c}$	0.984	0.971	0.963	0.892	0.959	0.986
Welf. Rel. to g^*_{UI} (%c)	-0.21582		0.074817	-2.2137	0.082714	4.138

Figure 9: Intensive-Margin Case, Perfect PI



limited. Welfare is maximized at a level that is lower relative to g_{UI}, and the welfare gain of 0.075% is relatively small. The reason is that here STC is less effective in mitigating the distortion of employment levels caused by UI. The employment profiles in Figure 9 exhibit a shutdown region with $n = 0$ for low profitability, immediately followed by an operating region with $n = 1$ for higher profitability levels. In contrast to the standard technology, there is no intermediate region with employment strictly between zero and one. As a consequence, STC cannot raise employment at the margin for a given level of profitability x. As is evident in the figure, it can only affect employment by decreasing the shutdown region. In proportion to $g^*_{STC} = 0.052$, the increase in employment from 0.968 to 0.975 is small in comparison to the response observed under the standard technology. At the same time, STC once again distorts hours over a range of profitability levels for which firms would have chosen $n = 1$ even in the absence of STC. This adverse effect is about as strong as under the standard technology. Taken together, this explains the lower level of optimal STC. The results for experiment $g^{\max}_{STC}|g^*_{UI}$ show that as in the standard case, making STC as generous as UI results in a large welfare loss. This is also visible in the left panel of Figure 10. Policy experiment $(g_{UI}, g_{STC})^*$ involves an increase in UI. Thus, as in the standard case, STC improves insurance indirectly by allowing for more generous UI. The associated welfare gains are negligible, however. This contrasts with the standard case, where jointly optimizing STC and UI accounts for about half of the overall welfare gain of introducing STC.

Figure 10: Welfare Gains, Intensive-Margin Case, Perfect PI

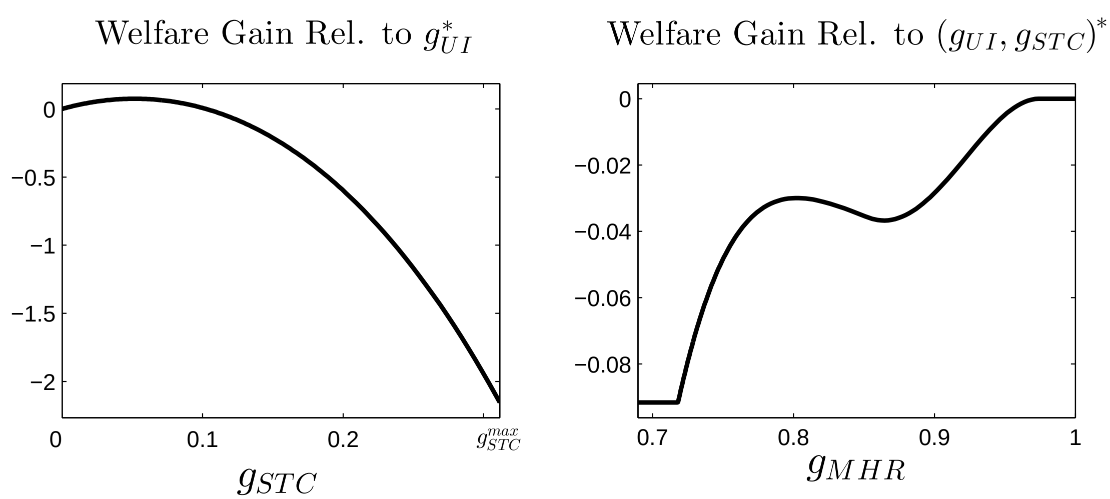


Table 6 does not contain any results for experiments involving g_{MHR}, because it turns out that introducing an MHR is not optimal here. This is illustrated in the right panel of Figure 10, which shows that an MHR cannot increase welfare further in experiment $(g_{UI}, g_{STC})^*$. As in Figure 4, at first there is a flat segment as g_{MHR} is reduced below one because the MHR does not yet bind. As g_{MHR} is reduced further, the usual trade-off arises as some firms forego STC and other firms reduce hours further to meet the MHR. The effect on welfare is non-monotone. In contrast to Figure 4, here welfare always remains lower than in the constellation without MHR.

5.2 No Private Insurance

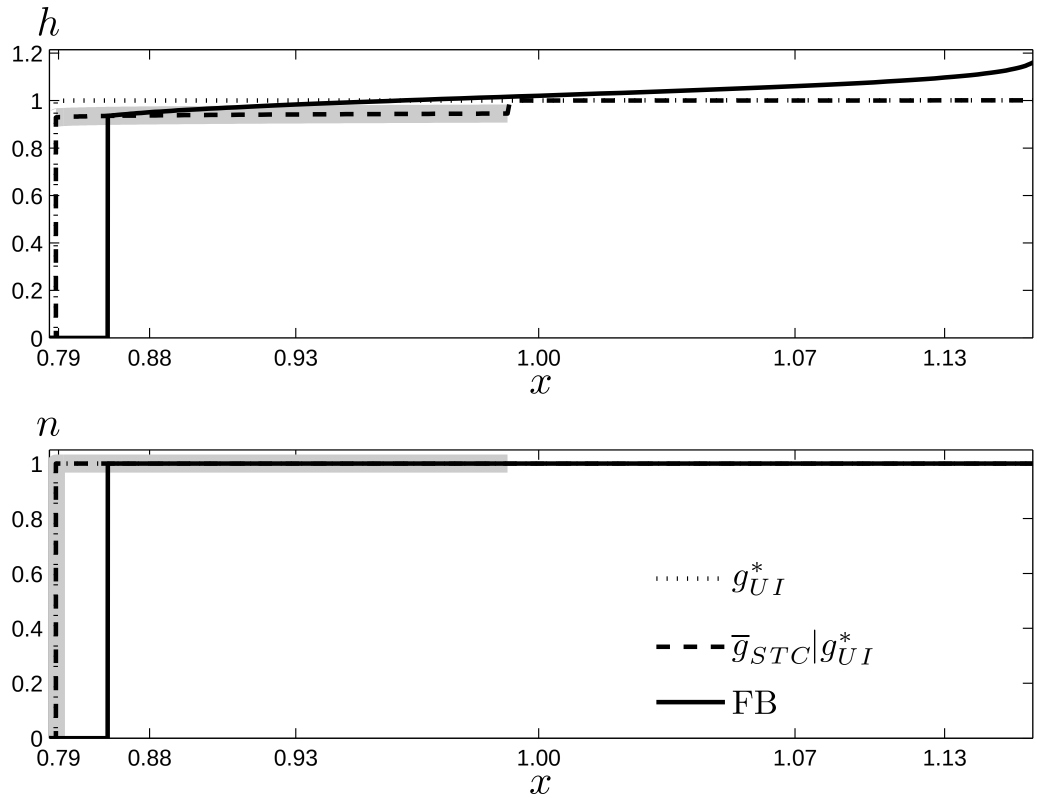
In the case of no PI, STC has a direct insurance effect. For the standard technology we obtained a negative direct insurance effect. This is driven by the declining profile of hours across the region of profitability levels at which firms continue to produce while engaging in some layoffs. This region is absent in the intensive-margin case. All operating firms have $n = 1$, and thus Proposition 4 implies a strictly increasing hours profile, which ensures a positive direct insurance effect of STC.

Table 7: Intensive-Margin Case, No PI

	Calibr.	g^*_{UI}	$\bar{g}_{STC}\vert g^*_{UI}$	FB
g_{UI}	0.245	0.239	0.239	
g_{STC}			0.0723	
g_{MHR}			1	
τ	0.0157	0.0131	0.0146	
$REPR_{UI}$ (%)	25	24.2	24.6	
$REPR_{STC}$ (%)			7.24	
STC Take-Up (%)			45.4	
$\bar{n}$	0.984	0.993	0.992	0.94
$\bar{h}$	1	1	0.973	1.03
$\bar{y}$	0.993	1	0.983	0.978
$\bar{c}$	0.981	0.989	0.972	0.953
Welf. Rel. to g^*_{UI} (%c)	−0.063892		−0.13709	3.4684

Nevertheless, we find that introducing STC is not optimal here. This echoes the finding that the optimal level of STC is very small under the standard technology for the case of no PI. Here STC is even less desirable. The reason is a relatively low optimal level of UI. In all the configurations we have considered so far, the optimal level of UI exceeds the level obtained in the calibration, which in turn implies that the associated level of temporary layoffs exceeds the level targeted in the calibration. In contrast, here g^*_{UI} falls short of the level of UI in the calibration, and at 0.7% the associated level of temporary

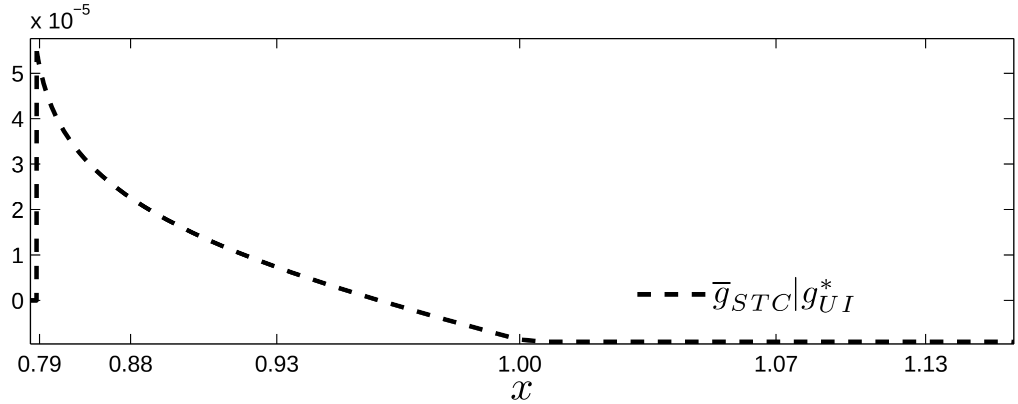
Figure 11: Intensive-Margin Case, No PI



layoffs is much lower that the calibration target. Consequently, once the level of UI is chosen optimally, there is simply not much scope for STC to reduce temporary layoffs. To illustrate what this means for the effects of STC, we again conduct experiment $\overline{g}_{STC}|g_{UI}^*$, introducing a level of STC that is as generous as the optimal level under perfect PI. The results are displayed along with the experiment g_{UI}^* and the first best in Table 7. Not only is there very little scope for STC to increase employment, the introduction of STC even reduces employment slightly. This is because STC still leads to a strong reduction of hours, as is evident from the dashed line in the top panel of Figure 11. Without a strong positive response of employment, there cannot be a sufficient reduction in the transfers to UI recipients to offset the costs of STC. Thus the tax rate τ must increase, which then induces the perverse effect on employment. This result once again puts emphasis on the point that the magnitude of temporary layoffs is a key determinant of the welfare benefits of STC.

The low level of g_{UI}^*, in turn, can be traced to a high marginal welfare loss from increasing the tax rate τ, which is generated by the interaction between the intensive-margin technology and the lack of insurance. This interaction puts firms with adverse profitabil-

Figure 12: Intensive-Margin Case, Change in Net-Transfers Due to STC, No PI



ity shocks in an especially adverse position. Under the standard technology, firms can smoothly adjust to profitability shocks through temporarily layoffs. The intensive-margin technology removes this adjustment mechanism. As a consequence, the average marginal utility from consumption across firms is substantially higher than in the standard case, making it very costly to raise revenue for financing UI.

This discussion is not yet complete, because we still need to consider the positive direct insurance effect of STC. Figure 12 is constructed in the same way as Figure 8, and shows that the net transfers induced by STC in experiment $\overline{g}_{STC}|g^*_{UI}$ shift consumption towards low profitability states. The magnitude of these transfers is very small, however, because the hours profile is virtually flat for the reason discussed in Section 4.3. Consequently, the direct insurance effect on welfare, computed as in Section 4.3, is very small at 0.00023%.

Overall, then, STC does not directly improve insurance in the settings we have studied in this paper. It only has the potential to do so in the absence of perfect PI. Yet the shape of the hours profile precludes substantial positive effects when private insurance is lacking entirely.

6 Conclusion

We have studied the welfare effects of short-time compensation (STC), departing from previous work by considering a setting in which unemployment insurance (UI) is socially optimal, and obtained two main results. First, STC can substantially improve welfare compared to a system that only relies on UI, but only when firms have access to private insurance. Second, optimal STC is substantially less generous than UI even when firms do have access to private insurance, and equally generous STC is worse than not offering

STC at all.

In the course of this analysis, the paper also contributes a new comparative statics result to the implicit contracts literature: lacking access to private insurance, firms engaging in layoffs respond to a further decline in profitability by reducing employment and *increasing* hours per worker. This property is a key determinant of the welfare effects of STC and a testable implication of the model. Thus a natural next step is to investigate this implication empirically.

We see our analysis as groundwork for studying the welfare effects of STC in dynamic models of the labor market. As discussed in the introduction, dynamic models capturing all the features which make implicit contract models a natural choice for studying STC have not yet been developed. Given this, a potentially fruitful next step is to consider a variety of dynamic models, each retaining some of the features of the static model. The findings of the present paper can help to identify which models are likely to be interesting, as well has indicate potential pitfalls.

A relatively straightforward dynamic extension is a model in which firms face credit constraints and self-insure against fluctuations in profitability. Here one could maintain the simplification that attached workers are homogeneous, and assume that attachment is permanent. This setting would be especially interesting for revisiting the direct insurance effect of STC. In our static setting, the lack of private insurance affects low and high profitability levels symmetrically. This generates a flat hours profile across high profitability states without layoffs, as firms cannot save. The option to save would make STC less attractive for highly profitable firms and hours would rise more strongly in response to a temporary increase in profitability, potentially increasing the direct insurance effect of STC.

This extension still does not permit an evaluation potential adverse effects of STC on worker reallocation. Introducing mobility of workers while maintaining incomplete markets may be intractable, however. Proceeding with complete markets and assuming exogenous UI would yield an interesting setting for studying the trade-off between STC's ability to mitigate distortions caused by UI, and potential reallocation effects. When interpreting results from this exercise, however, one should keep in mind that such a model may not capture well the STC take-up behavior of firms facing credit constraints.

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A Proofs

Proposition 1

Substituting the functions c_w^* and c_b^* into equation (14) and using the functional form of the utility function (2) yields

$$\frac{\lambda^{-\frac{1-\sigma}{\sigma}}}{1-\sigma} - \frac{\lambda^{-\frac{1-\sigma}{\sigma}} v(h)^{\frac{1-\sigma}{\sigma}}}{1-\sigma} + \lambda^{-\frac{1-\sigma}{\sigma}} v(h)^{\frac{1-\sigma}{\sigma}} \frac{v'(h)h}{v(h)}$$
$$= \lambda^{-\frac{1-\sigma}{\sigma}} \left[1 - v(h)^{\frac{1-\sigma}{\sigma}}\right] - \lambda \left[g_{UI} - \bar{h}T \cdot g_{STC}\right].$$

Dividing both sides by $\lambda^{-\frac{1-\sigma}{\sigma}}$ and rearranging terms, we obtain

$$\frac{\sigma}{1-\sigma} + v(h)^{\frac{1-2\sigma}{\sigma}} \left[v'(h)h - \frac{\sigma}{1-\sigma} v(h)\right] + \lambda^{\frac{1}{\sigma}} \left[g_{UI} - \bar{h}T \cdot g_{STC}\right] = 0. \tag{17}$$

Evaluating the left-hand side at $h = 0$ gives

$$\frac{\sigma}{1-\sigma}\left(1 - v_0^{\frac{1-\sigma}{\sigma}}\right) + \lambda^{\frac{1}{\sigma}} \left[g_{UI} - \bar{h}T \cdot g_{STC}\right] > 0.$$

This term captures the fixed cost of employing an additional worker. The first summand reflects the utility fixed cost, and is strictly positive since $v_0 \in (0, 1)$. The second term reflects the fixed cost in terms of the consumption good, induced by policy. It is nonnegative since $\bar{h} g_{STC} \leq g_{UI}$.

Let

$$\tilde{V}(h) \equiv v(h)^{\frac{1-2\sigma}{\sigma}} \left[v'(h)h - \frac{\sigma}{1-\sigma} v(h)\right].$$

Straightforward differentiation yields that

$$\tilde{V}'(h) = -hV'(h), \tag{18}$$

where the function $V(h)$ was introduced during the discussion of preferences in Section 2. Since $V'(h) > 0$, it follows that the left-hand side of equation (17) is strictly decreasing in h. Since $\lim_{h \to h_{\max}} V(h) = \infty$, equation (18) implies that $\lim_{h \to h_{\max}} \tilde{V}'(h) = -\infty$. Thus the left-hand side of equation (17) converges to minus infinity as h converges to $h_{\max}$. Consequently, there exists a value of hours in $(0, h_{\max})$ which solves equation (17), and the solution is unique. If $g_{STC} > 0$, then the left-hand side of equation (17) is strictly decreasing in T, and thus the solution for hours is strictly decreasing in T. If $g_{UI} - \bar{h}T \cdot g_{STC} > 0$, then the left-hand side of equation (17) is strictly increasing in λ, hence the solution for hours is strictly increasing in λ. If $g_{UI} - \bar{h}T \cdot g_{STC} = 0$, then the solution for hours is independent of λ.

As discussed in the text, the result that hours are increasing in λ if $g_{UI} - \bar{h}T \cdot g_{STC} > 0$ and independent of λ if $g_{UI} - \bar{h}T \cdot g_{STC} = 0$ also holds for other common specifications of utility. We supplement the proof for the KPR specification with a discussion of additively separable and GHH preferences. If preferences are additively separable, that is, $u(c,h) = \frac{c^{1-\sigma}}{1-\sigma} + v(h)$, then equation (14) becomes

$$v'(h)h - [v(h) - v(0)] + \lambda \left[g_{UI} - \bar{h}T \cdot g_{STC}\right] = 0.$$

Once again hours are strictly increasing in λ if $g_{UI} - \bar{h}T \cdot g_{STC} > 0$ and independent of λ otherwise. If utility takes the GHH form $u(c,h) = \frac{(c+v(h))^{1-\sigma}-1}{1-\sigma}$, then equation (14) becomes

$$\lambda v'(h)h - \lambda [v(h) - v(0)] + \lambda \left[g_{UI} - \bar{h}T \cdot g_{STC}\right] = 0.$$

In this case hours are independent of λ for any value of $g_{UI} - \bar{h}T \cdot g_{STC}$.

Proposition 2

Since $V(h)$ is strictly increasing on $(0, h_{\max})$, it follows that $\lim_{h\to 0} V(h)$ is finite. Since $\lim_{h\to 0} f'(h) = +\infty$, the right-hand side strictly exceeds the left-hand side as h converges to 0. Since $\lim_{h\to h_{max}} V(h) = +\infty$ while $f'(h_{\max})$ is finite, the left-hand side of equation (15) strictly exceeds the right-hand side as h converges to $h_{\max}$. Since V is strictly increasing while f' is strictly decreasing, equation (15) has a unique solution in $(0, h_{\max})$. The right-hand side is strictly increasing in x, hence the solution is strictly increasing in x. Suppose that the solution does not converge to $h_{\max}$ as x converges to infinity. Then the left-hand side converges to a finite value while the right-hand side converges to infinity as x converges to infinity, a contradiction. If $g_{STC} > 0$, then the right-hand side is strictly decreasing in T, hence the solution is strictly decreasing in T. Finally, note that at the solution the term in square brackets on the right-hand side is strictly positive. Thus the right-hand side is strictly increasing in λ, which implies that the solution for hours is strictly increasing in λ.

Proposition 3

We start with preliminary steps that apply to both $T = 0$ and $T = 1$. Let $\tilde{h}^T(\lambda)$ denote the level of hours that solves equation (14), which is well defined according to Proposition 1. Let $\tilde{h}^T_N(x, \lambda)$ denote the level of hours that solves equation (15), which is well defined according to Proposition 2.

With $\chi = 0$, first-order condition (12) implies that the multiplier $\lambda(x)$ does not vary with x at the solution. Let λ^* denote the corresponding constant value of the multiplier. Substituting the functions c^*_w and c^*_b into equation (10), we can solve explicitly for the value that employment would have to take if the employment constraint is slack:

$$\begin{aligned}\hat{n}^T(x, h) \equiv (f')^{-1}\Bigg(\frac{1}{xh}\Bigg\{\frac{1}{\lambda^*}\left[u(c^*_b(\lambda^*), 0) - u(c^*_w(\lambda^*, h), h)\right] \\ + g_{UI} - \bar{h}T \cdot g_{STC} + (\tau + T \cdot g_{STC})h + c^*_w(\lambda^*, h) - c^*_b(\lambda^*)\Bigg\}\Bigg) \cdot \frac{1}{h}.\end{aligned} \tag{19}$$

Consider $\hat{n}^T(x, h)$ as a function of x for a given constant level of hours h. This function is strictly increasing, and the Inada conditions on f' imply that $\hat{n}^T(x, h)$ converges to infinity as x converges to infinity, and converges to zero as x converges to zero. It follows that there exists a unique threshold $\hat{x}^T_N(h)$ such that $\hat{n}^T\left(\hat{x}^T_N(h), h\right) = 1$.

With these preliminaries in hand, we are ready to prove the two parts of the proposition.

1. Across profitability levels with a slack employment constraint, hours are constant at $h^0(x) = \tilde{h}^0(\lambda^*)$, while employment is given by $n^0(x) = \hat{n}^0\left(x, \tilde{h}^0(\lambda^*)\right)$ and thus strictly increasing in x. This implies that the employment constraint becomes binding at $x^0_N \equiv \hat{x}^0_N\left(\tilde{h}^0(\lambda^*)\right)$. Thus the employment constraint is slack on $(0, x^0_N)$ and binding on $(x^0_N, +\infty)$. On the latter interval, hours are given by $h^0(x) = \tilde{h}^0_N(x, \lambda^*)$ and thus strictly increasing.

2. The proof for the case $T = 1$ is similar to the one for $T = 0$ in part 1. The only difference is that constraint (MHR) may be binding. Let $\tilde{h}^1_{MHR} \equiv \min\left[\tilde{h}^1(\lambda^*), g_{MHR}\bar{h}\right]$. Then, across profitability levels with a slack employment constraint, hours are constant at $h^1(x) = \tilde{h}^1_{MHR}$, while employment is given by $n^1(x) = \hat{n}^1\left(x, \tilde{h}^1_{MHR}\right)$ and thus is strictly increasing in x. This implies that the employment constraint becomes binding at $x^1_N \equiv \hat{x}^1_N\left(\tilde{h}^1_{MHR}\right)$. Thus the employment constraint is slack on $(0, x^1_N)$ and binding on $(x^1_N, +\infty)$. On the latter interval, hours are given

by $h^1(x) = \min\left[\tilde{h}_N^1(x,\lambda^*), g_{MHR}\bar{h}\right]$. If $\tilde{h}_N^1(x_N^1,\lambda^*) \geq g_{MHR}\bar{h}$, set $x_{MHR}^1 = x_N^1$. If $\lim_{x\to\infty}\tilde{h}_N^1(x,\lambda^*) \leq g_{MHR}\bar{h}$, set $x_{MHR}^1 = +\infty$. Otherwise, set x_{MHR}^1 to the unique value of x that satisfies $\tilde{h}_N^1(x,\lambda^*) = g_{MHR}\bar{h}$. With this definition of x_{MHR}^1, hours $h^1(x)$ are strictly increasing on (x_N^1, x_{MHR}^1), and constant at $g_{MHR}\bar{h}$ on $(x_{MHR}^1, +\infty)$.

3. There are four cases to consider. First, suppose $x \leq \min[x_N^0, x_N^1]$. Then $h^0(x) = \tilde{h}^0(\lambda^*)$ and $h^1(x) = \min\left[\tilde{h}^1(\lambda^*), g_{MHR}\bar{h}\right]$. The desired result follows immediately from Proposition 1, which implies $\tilde{h}^0(\lambda^*) < \tilde{h}^1(\lambda^*)$. Second, consider $x \geq \max[x_N^0, x_N^1]$. Then $h^0(x) = \tilde{h}_N^0(x,\lambda^*)$ and $h^1(x) = \min\left[\tilde{h}_N^1(x,\lambda^*), g_{MHR}\bar{h}\right]$. The desired result follows immediately from Proposition 2, which implies $\tilde{h}_N^1(x,\lambda^*) < \tilde{h}_N^0(x,\lambda^*)$. Third, suppose that $x_N^1 < x_N^0$ and consider $x \in [x_N^1, x_N^0]$. Now $h^0(x) = \tilde{h}^0(\lambda^*)$ and $h^1(x) = \min\left[\tilde{h}_N^1(x,\lambda^*), g_{MHR}\bar{h}\right]$. The desired result follows from

$$\tilde{h}_N^1(x,\lambda^*) \leq \tilde{h}_N^1(x_N^0,\lambda^*) < \tilde{h}_N^0(x_N^0,\lambda^*) = \tilde{h}^0(\lambda^*)$$

where the first inequality uses that $\tilde{h}_N^1(x,\lambda^*)$ is increasing in x, the second inequality uses Proposition 2, and the final equality uses the definition of x_N^0. Fourth, suppose that $x_N^0 < x_N^1$ and consider $x \in [x_N^0, x_N^1]$. Here $h^0(x) = \tilde{h}_N^0(x,\lambda^*)$ and $h^1(x) = \min\left[\tilde{h}^1(\lambda^*), g_{MHR}\bar{h}\right]$. The desired result follows from

$$\tilde{h}_N^0(x,\lambda^*) \geq \tilde{h}_N^0(x_N^0,\lambda^*) = \tilde{h}^0(\lambda^*) > \tilde{h}^1(\lambda^*)$$

where the first inequality uses that $\tilde{h}_N^0(x,\lambda^*)$ is strictly increasing in x, the equality uses the definition of x_N^0, and the second inequality uses Proposition 1.

Proposition 4

We start with preliminary steps that apply to both $T = 0$ and $T = 1$. As a first step, we analyze the comparative statics of hours and employment with respect to profitability in a relaxed problem without constraints (N) and (MHR). Let $\tilde{n}^T(x)$ and $\tilde{h}^T(x)$ denote the optimal levels of employment and hours associated with this problem. Since we are in the case $\chi = 0$, constraint (PI) is irrelevant, so only the constraints (BC) and (NI) remain. In particular, since there is no insurance, there is no interdependence of the optimization problem across across profitability levels, so we can solve the problem separately for each level of x. We can reduce the problem for a given level of x to an unconstrained problem of choosing hours and employment. To do so, substitute the functions c_w^* and c_b^* along with $\iota = 0$ into the budget constraint. Solving the resulting equation for $\lambda^{-\frac{1-\sigma}{\sigma}}$ yields

$$\lambda^{-\frac{1-\sigma}{\sigma}} = \left[\frac{xf(nh) + (1-n)g_{UI} + n\left(\bar{h} - h\right)T \cdot g_{STC} - \tau nh}{nv(h)^{\frac{1-\sigma}{\sigma}} + (1-n)}\right]^{1-\sigma}. \tag{20}$$

Substituting the functions c_w^* and c_b^* along with $\iota = 0$ into the objective yields

$$nu(c_w, h) + (1-n)\,u(c_b, 0) = \frac{1}{1-\sigma}\lambda^{-\frac{1-\sigma}{\sigma}}\left[(nv(h)^{\frac{1-\sigma}{\sigma}} + (1-n)\right] - \frac{1}{1-\sigma}.$$

Using equation (20) to replace $\lambda^{-\frac{1-\sigma}{\sigma}}$ and dropping the constant $-\frac{1}{1-\sigma}$, the objective can be written as

$$\frac{1}{1-\sigma}\left[xf(nh) + (1-n)g_{UI} + n\left(\bar{h} - h\right)T \cdot g_{STC} - \tau nh\right]^{1-\sigma}\left[nv(h)^{\frac{1-\sigma}{\sigma}} + (1-n)\right]^{\sigma}.$$

The optimal labor input levels $\tilde{n}^T(x)$ and $\tilde{h}^T(x)$ must maximize this objective. Since $\sigma > 1$, this is equivalent to maximizing

$$G(n, h, x) \equiv \log\left[\Omega(n, h, x)\right] - \frac{1}{\psi}\log\left[\Gamma(n, h)\right] \tag{21}$$

where $\psi \equiv \frac{\sigma-1}{\sigma} \in (0, 1)$ and

$$\begin{aligned}\Omega(n, h, x) &\equiv xf(nh) + (1-n)g_{UI} + n\left(\bar{h} - h\right)T \cdot g_{STC} - \tau nh,\\ \Gamma(n, h) &\equiv nv(h)^{-\psi} + (1-n).\end{aligned}$$

We will determine the signs of the derivatives of $\tilde{n}^T(x)$ and $\tilde{h}^T(x)$ by studying this formulation of the problem. To economize on notation, we suppress arguments of functions in this analysis. The functions Ω and Γ have the following properties which are useful in what follows.

$$\Omega_{nh} = -x|f''|nh + \frac{1}{n}\Omega_h, \tag{22}$$

$$\Gamma_{nh} = \frac{1}{n}\Gamma_h, \tag{23}$$

$$\Gamma_{nn} = 0, \tag{24}$$

$$\Gamma_{hh} = -n\psi v^{-(2+\psi)} \left[vv'' - (1+\psi)(v')^2 \right] > 0. \tag{25}$$

The first-order conditions are

$$G_n = \frac{\Omega_n}{\Omega} - \frac{1}{\psi}\frac{\Gamma_n}{\Gamma} = 0, \tag{26}$$

$$G_h = \frac{\Omega_h}{\Omega} - \frac{1}{\psi}\frac{\Gamma_h}{\Gamma} = 0. \tag{27}$$

From now on, all second derivatives are evaluated at the solution to these first-order conditions. Using that $\Omega_x = f$, $\Omega_{nx} = f'h$, and $\Omega_{hx} = f'n$, the second derivatives involving profitability are given by

$$G_{nx} = \frac{f'h\Omega - \Omega_n f}{\Omega^2},$$

$$G_{hx} = \frac{f'n\Omega - \Omega_h f}{\Omega^2}.$$

Using that $\Omega_{nn} = -x|f''|h^2$, $\Omega_{hh} = -x|f''|n^2$, and equation (22), second derivatives for labor inputs are

$$G_{nn} = -\frac{x|f''|h^2\Omega + (\Omega_n)^2}{\Omega^2} - \frac{1}{\psi}\frac{\Gamma_{nn}\Gamma - \Gamma_n^2}{\Gamma^2},$$

$$G_{hh} = -\frac{x|f''|n^2\Omega + (\Omega_h)^2}{\Omega^2} - \frac{1}{\psi}\frac{\Gamma_{hh}\Gamma - \Gamma_h^2}{\Gamma^2},$$

$$G_{nh} = -\frac{x|f''|nh\Omega - \frac{1}{n}\Omega_h\Omega + \Omega_n\Omega_h}{\Omega^2} - \frac{1}{\psi}\frac{\Gamma_{nh}\Gamma - \Gamma_n\Gamma_h}{\Gamma^2}.$$

Using equations (22)–(25) and the first-order conditions, these second derivatives can be written as

$$G_{nn} = -\frac{x|f''|h^2\Omega + (1-\psi)(\Omega_n)^2}{\Omega^2},$$

$$G_{hh} = -\frac{x|f''|n^2\Omega + (1-\psi)(\Omega_h)^2}{\Omega^2} - \frac{1}{\psi}\frac{\Gamma_{hh}}{\Gamma},$$

$$G_{nh} = -\frac{x|f''|nh\Omega + (1-\psi)\Omega_n\Omega_h}{\Omega^2}.$$

The sign of $\frac{d}{dx}\tilde{h}^T(x)$ coincides with the sign of

$$-G_{nn}G_{hx} + G_{nh}G_{nx}.$$

Dropping the denominator Ω^2, which appears in all four second derivatives involved in this condition, we see that the left-hand side has the same sign as

$$\begin{aligned}&\left(x|f''|h^2\Omega+(1-\psi)(\Omega_n)^2\right)(f'n\Omega-\Omega_h f)\\&-(x|f''|nh\Omega+(1-\psi)\Omega_n\Omega_h)(f'h\Omega-\Omega_n f)\,.\end{aligned}\tag{28}$$

Exploiting cancellations, this reduces to

$$-\left[x|f''|h\Omega f+(1-\psi)\Omega\Omega_n f'\right]\cdot\left[h\Omega_h-n\Omega_n\right].\tag{29}$$

Since, from the definition of Ω,

$$h\Omega_h-n\Omega_n=n\left[g_{UI}-\bar{h}T\cdot g_{STC}\right],\tag{30}$$

the expression in equation (29) is strictly negative if $g_{UI}-\bar{h}T\cdot g_{STC}>0$ and zero if $g_{UI}-\bar{h}T\cdot g_{STC}=0$.

The sign of $\frac{d}{dx}\tilde{n}^T(x)$ coincides with the sign of

$$-G_{hh}G_{nx}+G_{nh}G_{hx}.$$

This expression has the same sign as

$$\begin{aligned}&\left(x|f''|n^2\Omega+(1-\psi)(\Omega_h)^2\right)(f'h\Omega-\Omega_n f)\\&-(x|f''|nh\Omega+(1-\psi)\Omega_n\Omega_h)(f'n\Omega-\Omega_h f)\\&+\frac{\Omega^4}{\psi}\frac{\Gamma_{hh}}{\Gamma}G_{nx}.\end{aligned}$$

The first two rows are symmetric, with switched roles of n and h, to equation (28). Consequently, this expression simplifies to

$$\left[x|f''|n\Omega f+(1-\psi)\Omega\Omega_h f'\right]\cdot\left[h\Omega_h-n\Omega_n\right]+\frac{\Omega^4}{\psi}\frac{\Gamma_{hh}}{\Gamma}G_{nx}.$$

Equation (30) implies that the first summand is nonnegative. Next, we show that G_{nx} is strictly positive, which allows to conclude that $\tilde{n}^T(x)$ is strictly increasing in x. Using the definition of Ω, we have

$$G_{nx} = \frac{f'hg_{UI} + (f - f'nh) \cdot \left[g_{UI} - \bar{h}T \cdot g_{STC} + (T \cdot g_{STC} + \tau)h\right]}{\Omega^2}.$$

This expression is strictly positive because strict concavity of f ensures $f - f'nh > 0$, and since $g_{UI} - \bar{h}T \cdot g_{STC} \geq 0$.

To summarize, so far we have shown that $\tilde{n}^T(x)$ is strictly increasing in x, and that $\tilde{h}^T(x)$ is weakly decreasing in x, strictly so if $g_{UI} - \bar{h}T \cdot g_{STC} > 0$. As mentioned in Footnote 15, a by-product of this proof is that marginal utility of consumption, which coincides with the Lagrange multiplier associated with the budget constraint, is a strictly decreasing function of x in the relaxed problem. Let $\tilde{\lambda}^T(x)$ denote the value of this Lagrange multiplier at the optimal solution. For the case $g_{UI} - \bar{h}T \cdot g_{STC} > 0$, Proposition 1 establishes a strictly increasing relationship between $\tilde{\lambda}^T(x)$ and $\tilde{h}^T(x)$. Since we know that $\tilde{h}^T(x)$ is strictly decreasing, this property carries over to $\tilde{\lambda}^T(x)$. For the case $g_{UI} - \bar{h}T \cdot g_{STC} = 0$, Proposition 1 establishes that hours are unrelated to λ. Consequently, here we need to use a different approach to show that $\tilde{\lambda}^T(x)$ is strictly decreasing. Equation (20) implies $\log(\lambda) = \sigma[\log(\Gamma) - \log(\Omega)]$. Since hours are independent of x in this case, we have

$$\frac{d}{dx}\log\left(\tilde{\lambda}^T(x)\right) = \sigma\left[\frac{\Gamma_n}{\Gamma} - \frac{\Omega_n}{\Omega}\right]\frac{d}{dx}\tilde{n}^T(x) = -\sigma\frac{1-\psi}{\psi}\frac{\Gamma_n}{\Gamma}\frac{d}{dx}\tilde{n}^T(x),$$

where the second equality uses first-order condition (26). This expression is strictly negative, since $\psi \in (0,1)$, $\Gamma_n > 0$, and $\frac{d}{dx}\tilde{n}^T(x) > 0$.

Since $\tilde{n}^T(x)$ is strictly increasing, we can determine the unique threshold profitability level x_N^T at which the employment constraint (N) becomes binding. If $\lim_{x\to 0} n^T(x) \geq 1$, let $x_N^T = 0$. If $\lim_{x\to\infty} n^T(x) \leq 1$, let $x_N^T = +\infty$. Otherwise, let x_N^T be the unique level of x that satisfies $\tilde{n}^T(x) = 1$.

So far we have studied the relaxed problem without constraints (N) and (MHR). Next, we modify this problem by imposing the employment constraint (N) with equality. Let $\tilde{h}_N^T(x)$ denote the level of hours that maximizes $G(1, h, x)$. We will show that $\tilde{h}_N^T(x)$ is strictly increasing. This is the case if $G_{hx} > 0$. Evaluated at $n = 1$, the sign of G_{hx} coincides with the sign of

$$f'\Omega - \Omega_h f = f'\left[xf - \tau h + (\bar{h} - h)g_{STC}\right] - \left[xf' - \tau - g_{STC}\right]f$$

$$= f'\bar{h}g_{STC} + [f - f'h] \cdot [\tau + g_{STC}],$$

where the first equality substitutes Ω and Ω_h, and the second equality exploits cancellations. The resulting expression is strictly positive, so $\tilde{h}_N^T(x)$ is indeed strictly increasing in x.

As the final preliminary step, we need to determine how employment varies when constraint (MHR) is binding. Let $\hat{n}^T(x,h)$ denote the level of employment that maximizes $G(n,h,x)$ for a given level of hours h. Above we established that $G_{nx} > 0$, hence $\hat{n}^T(x,h)$ is strictly increasing in x.

Using these preliminary results, we are ready prove the first two parts of the proposition.

1. Across profitability levels with a slack employment constraint, hours and employment are given by $h^0(x) = \tilde{h}^0(x)$ and $n^0(x) = \tilde{n}^0(x)$, respectively. The employment constraint becomes binding at x_N^0. Hours $h^0(x)$ are strictly decreasing since the condition $g_{UI} - \bar{h}T \cdot g_{STC} > 0$ is satisfied for $T = 0$, as $g_{UI} > 0$. Employment $n^0(x)$ is strictly increasing on $(0, x_N^0)$. On $(x_N^0, +\infty)$, employment $n^0(x)$ equals one. Hours are given by $h^0(x) = \tilde{h}_N^0(x)$ on this interval and thus strictly increasing.

2. First, we determine the threshold $x_{MHR,L}^1$. Consider the decreasing function $\tilde{h}^1(x)$. If $\lim_{x \to x_N^1} \tilde{h}^1(x) \geq g_{MHR}\bar{h}$, set $x_{MHR,L}^1 = x_N^1$. If $\lim_{x \to 0} \tilde{h}^1(x) \leq g_{MHR}\bar{h}$, set $x_{MHR,L}^1 = 0$. If neither of these two conditions is satisfied, then set $x_{MHR,L}^1$ to the unique level of $x \in (0, x_N^1)$ that satisfies $\tilde{h}^1(x) = g_{MHR}\bar{h}$. Next, we determine the threshold $x_{MHR,H}^1$. Consider the strictly increasing function $\tilde{h}_N^1(x)$. If $\lim_{x \to x_N^1} \tilde{h}_N^1(x) \geq g_{MHR}\bar{h}$, let $x_{MHR,H}^1 = x_N^1$. If $\lim_{x \to \infty} \tilde{h}_N^1(x) \leq g_{MHR}\bar{h}$, let $x_{MHR,H}^1 = \infty$. Otherwise, let $x_{MHR,H}^1$ be the unique level of $x \in \left(x_1^N, \infty\right)$ that satisfies $\tilde{h}_N^1(x) = g_{MHR}\bar{h}$. Having constructed these thresholds, we have

$$\left(h^1(x), n^1(x)\right) = \begin{cases} \left(g_{MHR}\bar{h}, \hat{n}^1\left(x, g_{MHR}\bar{h}\right)\right) & \text{for} \quad x \in \left(0, x_{MHR,L}^1\right), \\ \left(\tilde{h}^1(x), \tilde{n}^1(x)\right) & \text{for} \quad x \in \left(x_{MHR,L}^1, x_N^1\right), \\ \left(\tilde{h}_N^1(x), 1\right) & \text{for} \quad x \in \left(x_1^N, x_{MHR,H}^1\right), \\ \left(g_{MHR}\bar{h}, 1\right) & \text{for} \quad x \in \left(x_{MHR,H}^1, +\infty\right). \end{cases}$$

The properties of the functions $\hat{n}^1\left(x, g_{MHR}\bar{h}\right)$, $\tilde{h}^1(x)$, $\tilde{n}^1(x)$, and $\tilde{h}_N^1(x)$ imply that $h^1(x)$ and $n^1(x)$ vary with profitability as stated in the proposition.

For the third part of the proposition, we begin by establishing some preliminary results, once again starting with the solution to the relaxed problem without the constraints (N) and (MHR), that is, the functions $\tilde{h}^T(x)$ and $\tilde{n}^T(x)$. So far, we have only defined these functions for $T \in \{0, 1\}$. For the argument that follows, it is convenient to extend the definition to $T \in [0, 1]$, letting take-up vary continuously. We will show that if $\tilde{h}^0(x) \leq \bar{h}$, then $\tilde{h}^T(x)$ is strictly decreasing in T for $T \in [0, 1]$, which in turn implies $\tilde{h}^1(x) < \tilde{h}^0(x)$. Using that $\Omega_T = n(\bar{h}-h)g_{STC}$, $\Omega_{nT} = (\bar{h}-h)g_{STC}$, and $\Omega_{hT} = -ng_{STC}$, second derivatives involving take-up are

$$G_{nT} = \frac{\Omega - \Omega_n n}{\Omega^2}(\bar{h} - h)g_{STC},$$
$$G_{hT} = -\frac{\Omega + \Omega_h(\bar{h} - h)}{\Omega^2} n g_{STC}.$$

To show that $\tilde{h}^T(x)$ is strictly decreasing in T, we need to show that

$$-G_{nn}G_{hT} + G_{nh}G_{nT}$$

is strictly negative. Substituting the four second derivatives into this expression and dropping the denominators Ω^2, we obtain

$$-\left(x|f''|h^2\Omega + (1-\psi)(\Omega_n)^2\right)\left(\Omega + \Omega_h(\bar{h} - h)\right) n g_{STC}$$
$$-\left(x|f''|nh\Omega + (1-\psi)\Omega_n\Omega_h\right)\left(\Omega - \Omega_n n\right)(\bar{h} - h)g_{STC}.$$

Exploiting cancellations, this reduces to

$$-g_{STC}\left\{x|f''|hn\Omega\left[(\bar{h} - h)(\Omega_h h - \Omega_n n) + \Omega\bar{h}\right] + (1-\psi)\Omega\Omega_n\left[\Omega_n n + \Omega_h(\bar{h} - h)\right]\right\}.$$

Equation (30) implies that $h\Omega_h - n\Omega_n$ is nonnegative. Consequently, if $h \leq \bar{h}$, then the preceding expression is strictly negative. Now suppose that $\tilde{h}^0(x) \leq \bar{h}$. It follows that $\tilde{h}^T(x)$ is strictly decreasing in T at $T = 0$, and this in turn ensures that $\tilde{h}^T(x)$ remains below $\bar{h}$ as T increases towards one. It follows that $\tilde{h}^1(x) < \tilde{h}^0(x)$.

Next, we modify the preceding problem by imposing the employment constraint (N) with equality. The optimal level of hours in this problem is $\tilde{h}^T_N(x)$, now defined for $T \in [0, 1]$. Hours $\tilde{h}^T_N(x)$ are strictly decreasing in T if $G_{hT} < 0$. Now suppose that $\tilde{h}^0_N(x) \leq \bar{h}$. Then the condition $G_{hT} < 0$ is satisfied. Thus $\tilde{h}^T_N(x)$ remains below $\bar{h}$ as T

increases towards one. Consequently, $\tilde{h}_N^1(x) < \tilde{h}_N^0(x)$.

Using this preliminary results, we can prove part 3 of the proposition:

3. If $h^0(x) > g_{MHR}\bar{h}$, then the result follows immediately from the MHR constraint, which implies that $h^1(x) \leq g_{MHR}\bar{h}$. So we only need to examine the case $h^0(x) \leq g_{MHR}\bar{h}$. Here there are four cases to consider. First, suppose $x \leq \min[x_N^0, x_N^1]$. Then $h^0(x) = \tilde{h}^0(x)$. Since $\tilde{h}^0(x) \leq g_{MHR}\bar{h} \leq \bar{h}$, the preliminary result established above implies $\tilde{h}^1(x) < \tilde{h}^0(x)$. Thus the MHR constraint is not binding, and $h^1(x) = \tilde{h}^1(x) < h^0(x)$. Second, consider $x \geq \max[x_N^0, x_N^1]$. Then $h^0(x) = \tilde{h}_N^0(x)$. Since $h_N^0(x) \leq g_{MHR}\bar{h} \leq \bar{h}$, the preliminary result established above implies $\tilde{h}_N^1(x) < \tilde{h}_N^0(x)$. Thus the MHR constraint is not binding, and $h^1(x) = \tilde{h}_N^1(x) < h^0(x)$. Third, suppose that $x_N^1 < x_N^0$ and consider $x \in [x_N^1, x_N^0]$. Then $h^0(x) = \tilde{h}^0(x)$. Since $\tilde{h}^0$ is decreasing, we have $\tilde{h}^0(x) \geq \tilde{h}^0(x_N^0) = \tilde{h}_N^0(x_N^0)$. This means that $\tilde{h}_N^0(x_N^0) \leq g_{MHR}\bar{h} \leq \bar{h}$, so we can appeal to the preliminary result established above to conclude that $\tilde{h}_N^0(x_N^0) > \tilde{h}_N^1(x_N^0)$. Since $\tilde{h}_N^1$ is increasing, it follows that $\tilde{h}_N^1(x_N^0) \geq \tilde{h}_N^1(x)$. Thus the MHR constraint is not binding at x, and $h^1(x) = \tilde{h}_N^1(x) < h^0(x)$. Fourth, suppose that $x_N^0 < x_N^1$ and consider $x \in [x_N^0, x_N^1]$. Here $h^0(x) = \tilde{h}_N^0(x)$. Since the function $\tilde{h}_N^0$ is strictly increasing, it follows that $\tilde{h}_N^0(x) \geq \tilde{h}_N^0(x_0^N) = \tilde{h}^0(x_0^N)$. This implies that $\tilde{h}^0(x_0^N) \leq g_{MHR}\bar{h} \leq \bar{h}$, hence the preliminary result established above implies that $\tilde{h}^0(x_0^N) > \tilde{h}^1(x_0^N)$. Since the function $\tilde{h}^1$ is decreasing, it follows that $\tilde{h}^1(x_0^N) \geq \tilde{h}^1(x)$. Thus the MHR constraint is not binding at x, and $h^1(x) = \tilde{h}^1(x) < h^0(x)$.

Proposition 5

As a first step, we examine how the maximized value of the objective conditional on take-up varies with profitability x. For given x and T, the optimal value of the tuple $(c_w(x), c_b(x), n(x), h(x))$ must maximize

$$\begin{aligned} & n(x)u(c_w(x), h(x)) + (1 - n(x))u(c_b(x), 0) \\ & + \lambda^* \big\{ xf(n(x)h(x)) - \tau n(x)h(x) + (1 - n(x))g_{UI} + n(x)\left(\bar{h} - h(x)\right) T \cdot g_{STC} \\ & \quad - n(x)c_w(x) - (1 - n(x))c_b(x) \big\} \end{aligned} \tag{31}$$

subject to the constraints (N) and (MHR). The corresponding optimal values of labor inputs are $h^T(x)$ and $n^T(x)$. Let $U^T(x)$ denote the associated maximized value of objective (31). The optimal take-up decision $T^*(x)$ must maximize $U^T(x)$. By the envelope theorem, we have

$$\frac{dU^T}{dx}(x) = \lambda^* f\left(n^T(x)h^T(x)\right). \tag{32}$$

Having established this preliminary result, we are ready to prove the proposition. The proof proceeds by sequentially analyzing the three regions $(0, x_N^1]$, $[x_N^1, x_N^0]$, and $[x_N^0, +\infty)$, thereby determining the location of the threshold x_T.

First, consider the interval $(0, x_N^1]$. According to Proposition 3, both $h^0(x)$ and $h^1(x)$ are constant over this interval. Furthermore, for both $T = 0$ and $T = 1$, employment is obtained by substituting hours into equation (19). Using the latter relationship to solve for $n^T(x)h^T(x)$ yields

$$n^T(x)h^T(x) = (f')^{-1}\left(\frac{A^T}{x}\right) = \left(\alpha\frac{x}{A^T}\right)^{\frac{1}{1-\alpha}} \tag{33}$$

where A^T is a constant that depends on take-up T, and the second equality uses the assumption $f(nh) = (nh)^\alpha$, which implies $(f')^{-1}(y) = (\alpha/y)^{1/(1-\alpha)}$. Consequently, using equation (32) we obtain

$$\frac{d}{dx}\left[U^1(x) - U^0(x)\right] = \lambda^*\left(\alpha\frac{x}{A^1}\right)^{\frac{\alpha}{1-\alpha}}\left[1 - \left(\frac{A^1}{A^0}\right)^{\frac{\alpha}{1-\alpha}}\right].$$

The sign of the right-hand side does not depend on x. Furthermore, $U^T(0) = u(c_b(\lambda^*), 0) + \lambda^*\left[g_{UI} - c_b(\lambda^*)\right]$ is independent of T. Consequently, if $A^1 > A^0$, then $U^1(x) < U^0(x)$ for all $x \in (0, x_N^1]$. We set $x_T = 0$ in this case. Below we show that no take-up is optimal at all levels of profitability in this case. If $A^1 < A^0$, then $U^1(x) > U^0(x)$ on $(0, x_N^1]$, and the threshold x_T will lie to the right of this interval.

The case $A^1 = A^0$ can also arise as part of the optimal solution. In this case, given λ^*, the firm is indifferent between take-up and no take-up at any given level of $x \in (0, x_N^1]$. At the same time, the budget constraint requires that, across profitability levels, a specific expected value of employment must be allocated to take-up. One optimal choice is to choose a threshold $x_T \in [0, x_N^1]$ and set $T^*(x) = 1$ on $(0, x^T]$ and $T^*(x) = 0$ on $(x^T, x_N^1]$. We proceed with this choice, and the precise value of x_T is then determined by the budget constraint.

Next, consider the interval $[x_N^1, x_N^0]$. Here we need to consider two cases, depending on the results for the interval $(0, x_N^1]$. First, if $U^1(x) \leq U^0(x)$ on the interval $(0, x_N^1]$, then it is clear that take-up remains inferior on $[x_N^1, x_N^0]$: if the employment constraint were not binding for take-up, then the analysis for the interval $(0, x_N^1]$ would directly extend to $[x_N^1, x_N^0]$, and the binding employment constraint makes take-up even less attractive. Second, if $U^1(x) > U^0(x)$, then there are two possibilities. First, take-up may remain optimal on the entire interval $[x_N^1, x_N^0]$. Second, no take-up may be optimal on part of this interval. Next, we show that if there is a switch to no take-up somewhere within this interval, then no take-up remains optimal after this switch. So suppose there is a profitability level $\tilde{x}_T \in [x_N^1, x_N^0]$ such that $U^1(\tilde{x}_T) = U^0(\tilde{x}_T)$. Using equation (32) and the fact that $n^1(x) = 1$ on $[x_N^1, x_N^0]$, we have

$$\frac{d}{dx}\left[U^1(x) - U^0(x)\right] = \lambda^* \left[f\left(h^1(x)\right) - f\left(n^0(x)h^0(x)\right)\right]. \tag{34}$$

Since we are in the case in which $U^1(x) > U^0(x)$ on $\left(0, x_1^N\right]$, we know that this derivative is strictly positive at $x = x_N^1$. To permit $U^1(\tilde{x}_T) = U^0(\tilde{x}_T)$, this derivative must turn strictly negative somewhere between x_N^1 and $\tilde{x}_T$. Consequently, there must exist a profitability level $x_D \in [x_N^1, \tilde{x}_T]$ such that

$$h^1(x_D) < n^0(x_D)h^0(x_D). \tag{35}$$

Now consider $x \in [x_D, x_N^0]$. Equation (33) implies

$$\left(\frac{x}{x_D}\right)^{\frac{1}{1-\alpha}} n^0(x_D)h^0(x_D) = n^0(x)h^0(x). \tag{36}$$

Using the notation from the proof of Proposition 3, let $\tilde{h}_N^T(x, \lambda^*)$ denote the solution to equation (15). Using the assumption $f(h) = h^\alpha$, equation (15) can be written as

$$h = (\alpha x)^{\frac{1}{1-\alpha}} \left[(\lambda^*)^{-\frac{1}{\sigma}} V(h) + \tau + T \cdot g_{STC}\right]^{-\frac{1}{1-\alpha}}.$$

Thus

$$\begin{aligned}
\tilde{h}_N^1(x, \lambda^*) &= \left(\frac{x}{x_D}\right)^{\frac{1}{1-\alpha}} \left[\frac{(\lambda^*)^{-\frac{1}{\sigma}} V\left(\tilde{h}_N^1(x_D, \lambda^*)\right) + (\tau + g_{STC})}{(\lambda^*)^{-\frac{1}{\sigma}} V\left(\tilde{h}_N^1(x, \lambda^*)\right) + (\tau + g_{STC})}\right]^{\frac{1}{1-\alpha}} \tilde{h}_N^1(x_D, \lambda^*) \\
&< \left(\frac{x}{x_D}\right)^{\frac{1}{1-\alpha}} \tilde{h}_N^1(x_D, \lambda^*)
\end{aligned}$$

where the strict inequality follows from the fact that $h_N^1(x,\lambda^*)$ is strictly increasing on $[x_N^1, x_N^0]$, together with the fact that $V(h)$ is strictly increasing in h. Since $h^1(x) = \min\left[\tilde{h}_N^1\left(x,\lambda^*\right), g_{MHR}\bar{h}\right]$ and $h_N^1(x,\lambda^*)$ is strictly increasing, this inequality also holds for $h^1(x)$:

$$h^1(x) < \left(\frac{x}{x_D}\right)^{\frac{1}{1-\alpha}} h^1\left(x_D\right). \tag{37}$$

Combining the two inequalities (35) and (37) with equation (36) yields

$$h^1(x) < n^0(x)h^0(x)$$

for all $x \in [x_D, x_N^0]$. Since $x_D \leq \tilde{x}_T$ and $U^1(\tilde{x}_T) = U^0(\tilde{x}_T)$, equation (34) implies $U^1(x) < U^0(x)$ for all $x \in [\tilde{x}_T, x_N^0]$. Thus there can only be one value $\tilde{x}_T$ in $[x_N^1, x_N^0]$ such that $U^1(\tilde{x}_T) = U^0(\tilde{x}_T)$. If such a value exists, we set the threshold x_T to this value $\tilde{x}_T$. If such a value does not exist, we are in the case in which take-up remains optimal throughout the interval $[x_N^1, x_N^0]$.

Finally, consider the interval $[x_N^0, +\infty)$. Over this range

$$\frac{d}{dx}\left[U^1(x) - U^0(x)\right] = \lambda^*\left[f\left(h^1(x)\right) - f\left(h^0(x)\right)\right]. \tag{38}$$

This expression is strictly negative, since $h^1(x) < h^0(x)$ according to Part 3 of Proposition 3. Again we need to consider two cases, depending on the results obtained for the intervals $(0, x_N^1]$ and $[x_N^1, x_N^0]$. In the first case, we have already set a threshold $x_T \in [0, x_N^0]$ such that $T^*(x) = 0$ on $(x_T, x_N^0]$. In this case inequality (38) implies that no take-up $T^*(x) = 0$ is also optimal on $[x_N^0, +\infty)$. In the second case, we have not yet determined a threshold x_T, and $T^*(x) = 1$ is optimal on $(0, x_N^0]$. In the latter case, inequality (38) implies that there is at most one profitability level $\tilde{x}_T$ in $[x_N^0, +\infty)$ such that $U^1\left(\tilde{x}_T\right) = U^0\left(\tilde{x}_T\right)$. If such a level exists, we set $x_T = \tilde{x}_T$. If such a level does not exist, we set $x_T = +\infty$. With this definition of x_T, $T^*(x) = 1$ is optimal on $[x_N^0, x_T]$, and $T^*(x) = 0$ is optimal on $(x_T, +\infty)$.

Summarizing the results from analyzing the three intervals, we have determined a threshold $x_T \in [0, +\infty]$ such that $T^*(x) = 1$ is optimal on $(0, x_T]$ and $T^*(x) = 0$ is optimal on $x_T \in (x_T, +\infty)$.

B Sensitivity Analysis

In this appendix we show that the the main conclusions obtained in the computational experiments of Section 4 are not sensitive with respect to changes in parameters and targets.

Recall that the parameters σ, α, υ and σ_x were chosen independently, while υ_0, g_{UI}, and ψ were pinned down by targets for temporary layoffs, the replacement rate of UI and the Frisch elasticity. For each parameter in the first group we choose a low and a high value. Similarly, for each of the three targets we choose a low and a high value. We vary one parameter or target at a time, and for each deviation from the benchmark we recalibrate the model and repeat the welfare analysis.

Table 8 and Table 9 display the results for the case of perfect PI and no PI respectively. We omit the results for the production function parameter α, since changing this parameter has very little impact on the welfare effects associated with STC. We also do not report results for the fraction of unattached workers, since it plays a role very similar to that of risk aversion σ.[28] For each change in one of the remaining parameters and targets, we present the results in a pair of rows. The first of these rows, labeled 'Policy', displays the values of the policy instruments for the respective policy experiment. The second row, labeled 'Welfare', displays welfare relative to the experiment g^*_{UI} in consumption equivalents. The first pair of rows provides this information for the benchmark calibration.

B.1 Perfect Private Insurance

Our main results of Section 4.2 are robust with respect to these changes in parameters and targets.

First, introducing STC can always improve on UI, with sizable welfare gains varying between 0.1% and 1.2% for experiment $g^*_{STC}|g^*_{UI}$ and between 0.1% and 2.2% for experiment $(g_{STC}, g_{UI})^*$. When we exclude the experiments involving a change in the degree of risk aversion, these gains vary between 0.2% and 0.5% and 0.5% and 0.8% respectively. Naturally, the degree of risk aversion is a key determinant of the magnitude of welfare gains. With high risk aversion, the motivation to insure unattached workers is stronger,

[28] The benefit of UI is to insure this group of workers, and the magnitude of this benefit is determined by risk aversion in conjunction with the size of this group.

leading to a higher optimal level of g^*_{UI}. Then, the composition of labor inputs is more distorted, leaving more room for STC to mitigate these distortions.

Second, optimal levels of STC are markedly less generous than UI, and the results for the experiment $\bar{g}^{\max}_{STC}|g^*_{UI}$ show that introducing STC with the same generosity as g_{UI} results in large welfare losses.

Another robust result is that the welfare gains from STC are about equally distributed between the direct gain of introducing the STC for a given level of UI and the additional gain from jointly optimizing the levels of UI and STC.

Finally, introduction of a minimum hours reduction does not generally lead to sizable welfare improvements. In most experiments, the gains are less than a third of welfare gains achievable by STC alone and negligible for some experiments.

B.2 No Private Insurance

As in the benchmark without PI, welfare gains from the introduction of STC are generally negligible. Small but non-negligible gains arise for high risk aversion and a low targeted replacement rate. However, in both cases the welfare gains remain an order of magnitude below the gains in the corresponding sensitivity analysis under perfect PI.

Table 8: Sensitivity Analysis, Perfect PI

		g_{UI}^*	$g_{STC}^*\|g_{UI}^*$	$g_{STC}^{max}\|g_{UI}^*$	$(g_{UI}, g_{STC})^*$	$(g_{STC}, g_{MHR})^*\|g_{UI}^*$	$(g_{UI}, g_{STC}, g_{MHR})^*$
Benchmark	Policy	0.262	0.08|0.262	0.308|0.262	(0.284, 0.13)	(0.0643, 0.809)|0.262	(0.28, 0.0933, 0.825)
	Welfare	0	0.3011	−1.858	0.5263	0.4027	0.6079
σ=4	Policy	0.289	0.146|0.289	0.355|0.289	(0.302, 0.185)	(0.113, 0.834)|0.289	(0.302, 0.185, 0.905)
	Welfare	0	1.22	−0.8446	2.204	1.256	2.204
σ=1	Policy	0.24	0.0436|0.24	0.27|0.24	(0.255, 0.0679)	(0.0442, 0.797)|0.24	(0.256, 0.0589, 0.807)
	Welfare	0	0.07122	−1.641	0.1153	0.1326	0.1983
σ_x=0.15	Policy	0.275	0.0882|0.275	0.33|0.275	(0.295, 0.13)	(0.0717, 0.735)|0.275	(0.295, 0.0966, 0.752)
	Welfare	0	0.3385	−2.234	0.5255	0.4707	0.6686
σ_x=0.05	Policy	0.249	0.0629|0.249	0.286|0.249	(0.27, 0.123)	(0.0499, 0.888)|0.249	(0.27, 0.123, 0.941)
	Welfare	0	0.234	−1.644	0.5531	0.2881	0.5531
Frisch Elasticity 1	Policy	0.26	0.0687|0.26	0.319|0.26	(0.287, 0.127)	(0.0523, 0.776)|0.26	(0.281, 0.084, 0.801)
	Welfare	0	0.3293	−2.805	0.6634	0.4171	0.6836
Frisch Elasticity 0.4	Policy	0.266	0.0983|0.266	0.293|0.266	(0.279, 0.135)	(0.0872, 0.86)|0.266	(0.279, 0.11, 0.867)
	Welfare	0	0.2336	−0.7572	0.3365	0.3422	0.4554
Temp. Layoffs 0.025	Policy	0.253	0.084|0.253	0.295|0.253	(0.275, 0.136)	(0.0663, 0.814)|0.253	(0.271, 0.0968, 0.83)
	Welfare	0	0.3336	−1.55	0.5789	0.4307	0.644
Temp. Layoffs 0.005	Policy	0.279	0.0734|0.279	0.331|0.279	(0.299, 0.121)	(0.0612, 0.8)|0.279	(0.297, 0.0879, 0.816)
	Welfare	0	0.2508	−2.446	0.4422	0.3582	0.5492
Rep. Rate 30	Policy	0.3	0.0654|0.3	0.367|0.3	(0.319, 0.109)	(0.0579, 0.79)|0.3	(0.319, 0.0821, 0.805)
	Welfare	0	0.1955	−3.429	0.3466	0.307	0.4797
Rep. Rate 20	Policy	0.227	0.0986|0.227	0.259|0.227	(0.25, 0.154)	(0.0749, 0.833)|0.227	(0.245, 0.111, 0.851)
	Welfare	0	0.4621	−0.677	0.7715	0.5367	0.773

Table 9: Sensitivity Analysis: No PI

		g_{UI}^*	$g_{STC}^*\|g_{UI}^*$	$(g_{UI}, g_{STC})^*$	$(g_{STC}, g_{MHR})^*\|g_{UI}^*$	$(g_{UI}, g_{STC}, g_{MHR})^*$
Benchmark	Policy	0.25	0.00316|0.25	(0.25, 0.00331)	(0.00314, 0.996)|0.25	(0.25, 0.00329, 0.996)
	Welfare	0	0.0009286	0.0009721	0.0009316	0.0009749
σ=4	Policy	0.267	0.0348|0.267	(0.27, 0.0465)	(0.0348, 0.989)|0.267	(0.27, 0.0465, 0.986)
	Welfare	0	0.112	0.1554	0.112	0.1554
σ=1	Policy	0.236	0|0.236	(0.236, 0)	(0, 1)|0.236	(0.236, 0, 1)
	Welfare	0	0	$-1.249e-12$	0	$-1.222e-12$
σ_x=0.15	Policy	0.258	0.000398|0.258	(0.258, 0.000406)	(0.000439, 0.996)|0.258	(0.258, 0.000439, 0.996)
	Welfare	0	$1.072e-05$	$1.096e-05$	$1.562e-05$	$1.59e-05$
σ_x=0.05	Policy	0.243	0.00338|0.243	(0.244, 0.00371)	(0.00338, 0.998)|0.243	(0.244, 0.00371, 0.998)
	Welfare	0	0.001878	0.002057	0.001878	0.002061
Frisch Elasticity 1	Policy	0.249	0.00246|0.249	(0.249, 0.00258)	(0.00244, 0.996)|0.249	(0.249, 0.00256, 0.996)
	Welfare	0	0.0007345	0.0007715	0.0007384	0.0007751
Frisch Elasticity 0.4	Policy	0.25	0.00414|0.25	(0.25, 0.00428)	(0.00412, 0.998)|0.25	(0.25, 0.00426, 0.998)
	Welfare	0	0.000995	0.001033	0.0009969	0.001034
Temp. Layoffs 0.025	Policy	0.243	0.0043|0.243	(0.243, 0.00455)	(0.00427, 0.996)|0.243	(0.243, 0.00452, 0.996)
	Welfare	0	0.001713	0.001812	0.001714	0.001813
Temp. Layoffs 0.005	Policy	0.262	0.00133|0.262	(0.262, 0.00137)	(0.00131, 0.997)|0.262	(0.262, 0.00136, 0.997)
	Welfare	0	0.0001646	0.0001694	0.0001699	0.0001747
Rep. Rate 30	Policy	0.293	0|0.293	(0.293, 0)	(0, 1)|0.293	(0.293, 0, 1)
	Welfare	0	0	$-6.2e-11$	0	$-7.841e-11$
Rep. Rate 20	Policy	0.206	0.0153|0.206	(0.208, 0.0189)	(0.0153, 0.995)|0.206	(0.208, 0.0189, 0.994)
	Welfare	0	0.02069	0.02529	0.02069	0.02529

C Redundancy of Experience Rating

The redundancy of experience rating is a feature our model shares with those of BW and WH, and it applies even to their models in which attached agents are heterogeneous in that some are workers while others are employers. The reason for this redundancy is the absence of imperfections in the risk sharing contract among agents attached to a firm.

First, we establish redundancy for a version of our model in which both UI benefits and experience rating are allowed to differentiate between attached and unattached workers. Following BW and WH, experience rating imposes a tax on a firm which amounts to fraction $e \in [0,1]$ of the total benefits received by the workers attached to that firm. We also allow for experience rating of unattached workers with factor $e^{\nu} \in [0,1]$. [29] With experience rating, the net benefit schedule for firms (3) becomes

$$(1-e)\left[(1-n)g_{UI} + n\mathcal{I}\left[h \leq g_{MHR}\bar{h}\right] \cdot \left(\bar{h}-h\right) \cdot g_{STC}\right] - \tau nh \tag{39}$$

and the benefit received by unattached workers is $(1-e^{\nu})g^{\nu}_{UI}$ where g^{ν}_{UI} is the UI benefit level for unattached worker, which for now is allowed to differ from g_{UI}. It is immediately clear that this system is equivalent to an alternative system (distinguished by a check) given by $\check{g}_{UI} = (1-e)g_{UI}$, $\check{g}_{STC} = (1-e)g_{STC}$, $\check{e} = 0$, $\check{g}^{\nu}_{UI} = (1-e^{\nu})g^{\nu}_{UI}$, and $\check{e}^{\nu}$.

In our model without experience rating, we restrict STC and UI to be uniform. As discussed in the text, this should be understood as a restriction on effective subsidies. Otherwise this restriction has no content, as any differentiation can be implemented through experience rating. Next, we show that experience rating is redundant under the assumption that the effective subsidy cannot differentiate between attached and unattached workers:

$$(1-e)g_{UI} = (1-e^{\nu})g^{\nu}_{UI}.$$

The system consisting of $\check{g}_{UI}$, $\check{g}_{STC}$, and $\check{g}^{\nu}_{UI}$ and no experience rating continues to be equivalent. Moreover, it satisfies $\check{g}_{UI} = \check{g}^{\nu}_{UI}$, so the UI benefit does not differentiate between attached and unattached workers.

[29] A natural level for this is zero, given that these workers are not attached to a firm. However, one can also assume that these workers were previously attached to some firm, and that the government imposes the experience-rating tax on the owners of that firm. In our model of owner-operators, it is internally consistent to assume that unattached workers own the firm from which they became unattached. In their role as owners, they are liable for the experience-rating tax induced by the benefits they receive in their role as workers.