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MONETARY AND FISCAL POLICY WITH SOVEREIGN DEFAULT

JOOST RÖTTGER

Department of Economics
University of Cologne
Albertus-Magnus-Platz
D-50923 Köln
Germany

http://www.wiso.uni-koeln.de

Monetary and Fiscal Policy with Sovereign Default*

Joost Röttger†

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Abstract

How does the option to default on debt payments affect the conduct of public policy? To answer this question, this paper studies optimal monetary and fiscal policy without commitment in a model with nominal debt and endogenous sovereign default. When the government can default on its debt, public policy changes in the short and the long run relative to a setting without default option. The risk of default increases the volatility of interest rates, impeding the government's ability to smooth tax distortions across states. It also limits public debt accumulation and reduces the government's incentive to implement high inflation in the long run. The welfare costs associated with the short-run effects of sovereign default are found to be outweighed by the welfare gains due to lower average debt and inflation.

Keywords: Monetary and Fiscal Policy, Lack of Commitment, Sovereign Default, Domestic Debt, Markov-Perfect Equilibrium

JEL Classification: E31, E63, H63

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†University of Cologne, TU Dortmund University, and RGS Econ. Address: University of Cologne, Center for Macroeconomic Research, Albertus-Magnus-Platz, 50923 Cologne, Germany. Email: `joost.roettger@udo.edu`.

1 Introduction

Suppose that a government faces high nominal debt payments that can only be refinanced at high interest rates. If it is not willing (or able) to raise primary surpluses to pay bond holders, there are essentially two options left: inflation and sovereign default. While default and inflation both can lower the real debt burden, there are several differences between these two policy options which make them imperfect substitutes. For example, a government can collect seigniorage when engineering inflation by issuing currency while a default does not generate additional tax revenues. Another difference is that default directly affects the return on government bonds while inflation impacts on the return on all nominal assets. Being a continuous variable, inflation can also be easily adjusted while the default decision is a discrete choice that does not offer the same degree of flexibility. Given the distinct role of money and government bonds for the private sector, default and inflation may also distort economic activity through different channels.

The contribution of this paper is to study the implications of allowing a policy maker not only to use standard instruments of monetary and fiscal policy but also to choose outright sovereign default. In particular, it extends previous studies on optimal monetary and fiscal policy with nominal debt that focus on the case of lack of commitment but still assume that the policy maker is always committed to service debt (see e.g. Diaz-Gimenez et al., 2008, Martin, 2009, Niemann et al., 2013). In the model studied here, a benevolent government jointly chooses monetary and fiscal policy under discretion to finance exogenous government spending in a representative agent cash-credit economy that is subject to productivity shocks.[1] More specifically, it sets a labor income tax rate, chooses the money growth rate, issues nominal non-state contingent bonds and decides on whether to repay its outstanding debt or not. The default decision is modeled as a binary choice (see Eaton and Gersovitz, 1981). Following the quantitative sovereign default literature,[2] a default is costly because it leads to a deadweight loss of resources that takes the form of a reduction in aggregate productivity and exclusion from financial markets for a random number of periods.

As is common in the literature on optimal monetary and fiscal policy, I consider a closed economy. This paper thus contributes to the study of domestic debt default which, despite being a historically recurring phenomenon with severe economic consequences, has not received a lot of attention in the sovereign default literature (see Reinhart and Rogoff, 2011). In a closed economy, a default does not redistribute resources from foreign lenders to domestic citizens. The government may still choose not to repay its debt to relax its budget constraint and reduce distortionary taxes. The model is calibrated to the Mexican economy which has experienced periods of high inflation and sovereign risk in the recent past. In addition, domestic nominal debt matters for the Mexican government. Du and

[1] I assume that there is only one policy maker, referred to as the government, who is in charge of both, monetary and fiscal policy. However, one can also interpret this setting as one in which a central bank chooses monetary policy and a fiscal authority controls fiscal policy, assuming that they are benevolent and coordinate their policies.

[2] See e.g. Hamann (2004), Aguiar and Gopinath (2006) and Arellano (2008)).

Schreger (2014) document that the ratio of Mexican nominal government debt held by domestic residents to GDP has roughly increased from 10 to 20 percent in the last decade, while the ratio of foreign currency debt to GDP has strongly declined.[3]

I study the Markov-perfect equilibrium of the public policy problem (see Klein et al., 2008). The government's decisions hence only depend on the payoff-relevant state which consists of aggregate productivity and the beginning-of-period public debt position. Since the government optimizes sequentially, it cannot commit do future policies and does not internalize how its current decisions affect household expectations in previous periods. However, the government is aware that (expected) future policy will depend on its borrowing decision because it will affect the incentive to reduce the real debt burden via default or inflation in the next period. With lack of commitment, the option to default thus matters for the government's response to an adverse shock by allowing it to adjust the real debt burden as well as by affecting the cost of borrowing and thus the attractiveness of debt as a shock absorber.

Compared to an otherwise identical economy without default option (or equivalently an economy with prohibitively high costs of default) the availability of sovereign default results in lower average inflation. Since inflation does not reduce the real debt burden when a default takes place, it is lower when default is chosen instead of repayment. However, this direct effect of default on inflation is negligible at a plausible default frequency. The key mechanism that leads inflation to be lower when the default option is available is an indirect one. The attractiveness and hence the probability of default increases with public debt and decreases with aggregate productivity. With default risk, bond prices become more debt elastic in recessions than with only inflation risk and the marginal revenue from debt issuance decreases faster.[4] Consequently, the government borrows less which reduces its incentive to implement high inflation rates. Since lower average debt is associated with less inflation, less money is issued and seigniorage revenues decline. The government then has to adjust the primary surplus, leading to a higher labor tax rate in the long run. In the short run, the increased sensitivity of bond prices to productivity shocks and bond issuance that is induced by sovereign risk impedes the government's ability to smooth tax distortions across states. Relative to an economy without default option, tax and inflation rates are thus more volatile, amplifying the impact of productivity shocks on the economy. In times of high sovereign risk, debt issuance is costly and the government tries to avoid a default by inflating the debt burden. A sovereign debt crisis thus is highly inflationary which is consistent with empirical evidence (see Reinhart and Rogoff, 2009).

From a welfare perspective, it is not obvious whether it is desirable to endow the government with the option to default when it cannot commit to future actions.[5] As discussed

[3] Between 2000 and 2009, the amount of Mexican nominal public debt held by foreign residents increased as well but only accounted for up to about 20 percent of total nominal debt.

[4] Even without sovereign risk, higher debt issuance leads to higher interest rates by increasing expected inflation.

[5] The same is true in the context of consumer default where there exists a trade-off when reducing the costs of filing for bankruptcy. On the one hand, indebted consumers receive the ability to make debt payments state contingent. On the other hand, this flexibility comes at the cost of higher borrowing costs

above, the risk of default affects public policy in the short and the long run. With productivity shocks, the government would like to smooth tax distortions by running a budget deficit (surplus) during bad (good) times, following the logic of Barro (1979). Default risk makes debt issuance more expensive in recessions which leads to welfare losses due to more volatile public policy. The long-run implications of sovereign default might however outweigh these costs. As in Martin (2009) and Diaz-Gimenez et al. (2008), the government chooses positive average debt positions because of its lack of commitment and the presence of a liquidity constraint. By increasing the cost of borrowing in recessions, risk of default renders public debt accumulation less attractive, thus avoiding high debt levels and the implementation of high average inflation. A welfare exercise reveals that the counterfactual elimination of sovereign default leads to a small welfare loss, i.e. Mexico would be worse off without the option to default.

This paper builds on the literature on optimal Markov-perfect monetary and fiscal policy with nominal government debt. Martin (2009, 2011, 2013) extensively studies the short- and long-run properties of public debt and inflation when the government lacks commitment. I will discuss how his findings relate to mine in Section 3. Diaz-Gimenez et al. (2008) show how public policy and welfare depend on whether debt is indexed to inflation or not. Among other things, they find that without commitment welfare can be lower when debt is indexed. In a model with nominal rigidities, Niemann et al. (2013) show that the presence of lack of commitment and nominal government debt leads to persistent inflation. Despite highlighting the role of lack of commitment for public policy, all these studies maintain the assumption of perfect commitment to debt service and thus abstract from sovereign default. This work is also related to recent papers that study domestic debt default. In a model with incomplete markets and idiosyncratic income risk, D'Erasmo and Mendoza (2013) show that a sovereign default can occur in equilibrium as an optimal distributive policy. Pouzo (2013) extends the incomplete markets model of Aiyagari et al. (2002) by allowing the Ramsey planner to default on debt payments. Sosa-Padilla (2013) studies Markov-perfect fiscal policy in a model where a sovereign default triggers a banking crisis. All these papers feature real economies and hence do not discuss monetary policy. This paper is also related to the quantitative sovereign default literature that studies how risk of default affects business cycles in emerging economies.[6] With this literature, it shares the assumption of the government's lack of commitment and the way sovereign default is modeled.

The rest of the paper is organized as follows. Section 2 presents the model that is analyzed in Section 3. The welfare implications of sovereign default are discussed in Section 4. Section 5 concludes.

that compensate lenders for the increased risk of default (see e.g. Livshits et al., 2007).

[6]This literature focuses on small open economies and only considers external real debt. A recent summary of this literature can be found in Aguiar and Amador (2014). Within this literature, the study that is closest to this paper is Cuadra et al. (2010) which discusses fiscal policy but abstracts from monetary policy.

2 Model

The model extends the cash-credit economy studied in Martin (2009) by allowing for sovereign default and productivity shocks. Time is discrete, starts in $t = 0$ and goes on forever. In each period t, the timing is as follows. First, aggregate firm productivity a_t is realized and the public policy choices are made by the government in a discretionary way, i.e. taking future policies as given. To finance exogenous government spending $\bar{g}$ and outstanding nominal debt payments $\tilde{B}_t$, the government can choose from a set of policies that includes the money growth rate μ_t, a linear labor income tax rate τ_t, the binary default decision $d_t \in \{0, 1\}$, and issuance of nominal non-state contingent one-period bonds $\tilde{B}_{t+1}$ at unit price q_t. A default occurs when $d_t = 1$ is chosen, while the government fully repays its obligations for $d_t = 0$. In the default case, the government is excluded from financial markets for a random number of periods (see Aguiar and Gopinath, 2006 or Arellano, 2008). It can thus neither borrow from nor lend to households during this time. Given the policy choices, households and firms then simultaneously optimize in a competitive fashion. Households supply labor l_t at the real wage rate w_t, choose consumption of a cash good c_{1t} and a credit good c_{2t}, and decide on money ($\tilde{m}_{t+1}$) and nominal government bond ($\tilde{b}_{t+1}$) holdings. While both assets are subject to inflation risk, only government bonds are subject to default risk. A role for money is introduced by tying consumption of c_{1t} to beginning-of-period money holdings via a cash-in-advance constraint (see Lucas and Stokey, 1983)

$$\tilde{m}_t \geq \tilde{p}_t c_{1t},$$

with $\tilde{p}_t$ denoting the price of consumption in terms of $\tilde{m}_t$. Using labor n_t as the only input, profit maximizing competitive firms produce a homogeneous good y_t. They are owned by households who receive real profits Π_t.

Following Martin (2009), nominal variables are normalized by the beginning-of-period aggregate money stock $\tilde{M}_t$: $x_t \equiv \tilde{x}_t / \tilde{M}_t$ with $x \in \{B, b, m, p\}$. This normalization renders the model stationary and reduces the number of endogenous aggregate states to B_t since the normalized aggregate money stock $M_t = 1$ is constant over time.

2.1 Private Sector

Given the assumed intra-period timing, the optimal decisions of the private sector - which consists of households and firms - need to be derived before formulating the government's decision problem.

2.1.1 Households

The economy is populated by a unit mass continuum of homogeneous infinitely-lived households who maximize their expected lifetime utility $\mathbb{E}[\sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, l_t)]$ with $\mathbb{E}[\cdot]$ denoting the expectation operator, $\beta \in (0, 1)$ the discount factor and $u : \mathbb{R}^3_+ \to \mathbb{R}$ the period utility function. The utility function is additively separable in all its arguments and satisfies $u_1, u_2, -u_l > 0$ and $u_{11}, u_{22}, -u_{ll} < 0$ with u_x (u_{xx}) denoting the first (second) derivative of $u(\cdot)$ with respect to $x \in \{c_1, c_2, l\}$. A household's individual state variables are its beginning-of-period bond and money holdings (b_t, m_t). Households take as given the aggregate laws of motion for government debt B_{t+1} and productivity a_{t+1} which follows a stationary first-order Markov process with continuous support $\mathcal{A}$ and transition function $f_a(a_{t+1}|a_t)$. I focus on Markov-perfect public policy (see Klein et al., 2008) such that the government's policy choices will be given as functions of the payoff-relevant aggregate state $S_t = (B_t, a_t)$. This equilibrium concept ensures that the chosen policies are time consistent.

Turning to recursive notation and letting $v(\cdot)$ denote a household's value function, its optimal decisions solve the following Bellman equation

$$v(b, m; S) = \max_{c_1, c_2, l, b', m'} u\left(c_1, c_2, l\right) + \beta \mathbb{E}\left[v(b', m'; S')\right]$$

subject to the period budget constraint

$$(1-\tau) w l + \frac{m + (1-d)\, b}{p} + \Pi = c_1 + c_2 + (1+\mu)\, \frac{m' + q b'}{p},$$

and the cash-in-advance constraint

$$\frac{m}{p} \geq c_1.$$

Combining the necessary first-order and envelope conditions associated with the problem above yields the following household optimality conditions:

$$-\frac{u_l(l)}{u_2(c_2)} = (1-\tau) w, \tag{1}$$

$$u_2(c_2) q = \beta \mathbb{E}\left[\left(1-d'\right) u_2(c_2') \frac{p}{p'} \frac{1}{1+\mu}\right], \tag{2}$$

$$u_2(c_2) = \beta \mathbb{E}\left[u_1(c_1') \frac{p}{p'} \frac{1}{1+\mu}\right], \tag{3}$$

and the complementary slackness conditions

$$\lambda = u_1(c_1) - u_2(c_2) \geq 0, m/p - c_1 \geq 0, \lambda\left(m/p - c_1\right) = 0,$$

with λ denoting the Kuhn-Tucker multiplier on the cash-in-advance constraint. Intuitively, the constraint is binding whenever the marginal utility of cash-good consumption exceeds the marginal utility of credit-good consumption. As in Martin (2009), the inequality

$$u_1(c_1) - u_2(c_2) \geq 0, \tag{4}$$

needs to hold in equilibrium to satisfy $\lambda \geq 0$.

Equation (1) characterizes optimal labor supply which is distorted for $\tau \neq 0$. Conditions (2) and (3) are the Euler equations for bonds and money. The normalization of nominal variables implies that the inflation rate in period t is given as

$$\pi_t \equiv \frac{p_t\left(1+\mu_{t-1}\right)}{p_{t-1}} - 1,$$

such that inflation equals money growth in the long run.

2.1.2 Firms

Production of the homogeneous good y is carried out by a unit mass continuum of identical competitive firms. Labor services provided by households are transformed into output according to the linear technology $y = \psi(a,d)n$.[7] Firms take as given their effective productivity $\psi : \mathbb{R}_+ \times \{0,1\} \to \mathbb{R}_+$ which depends on exogenous stochastic productivity a and the government's default decision d. Effective productivity increases with exogenous productivity ($\partial\psi(a,d)/\partial a \geq 0$) and is negatively affected by a default ($\psi(a,0) \geq \psi(a,1)$). Profit maximizing firm behavior leads to the static optimal labor demand condition

$$w = \psi(a,d). \tag{5}$$

When a government lacks commitment to repayment, costs of default are necessary to sustain positive levels of debt. As is common in the quantitative sovereign default literature (see e.g. Arellano, 2008 and Cuadra et al., 2010), there are two types of costs. First, the government is excluded from the bond market for a random number of periods such that it can neither borrow nor lend during this time. Reentry to the bond market takes place with probability θ. Second, the economy experiences a direct resource loss. These costs capture in reduced form output losses that occur in periods of default (and financial autarky). Despite being arguably ad hoc, such a specification allows me not to take a stand on how exactly a sovereign default is propagated through the economy. While there is evidence for domestic output costs, there is still no consensus on which mechanism is the most relevant one (see Panizza et al., 2009). In addition, two recent papers show that models with endogenous default costs that arise due to private credit disruptions (Mendoza and Yue, 2012) or banking crises (Sosa-Padilla, 2013) deliver similar qualitative and quantitative results as those with exogenous default costs.

2.1.3 Market Clearing

In equilibrium, goods, labor and asset markets clear,

$$\begin{aligned} \psi(a,d)n &= c_1 + c_2 + \bar{g}, && (6) \\ n &= l, && (7) \\ b' &= B', \\ m' &= 1. \end{aligned}$$

[7] This production function implies zero firm profits in equilibrium ($\Pi = 0$).

If real balances are high enough, households equalize marginal utility across cash and credit goods. If not, households are cash constrained and the allocation of consumption is distorted. As in Martin (2009), in a monetary equilibrium, i.e. an equilibrium in which money is valued,

$$c_1 = 1/p, \tag{8}$$

holds. Note that this still allows for an unconstrained consumption allocation when the cash-in-advance constraint is just binding such that $\lambda = 0$ and $1/p = u_1^{-1}(u_2(c_2))$.

2.2 Government Problem

When the government defaults, the economy suffers a productivity loss governed by $\psi(\cdot)$ and it cannot access the bond market for a random number of periods. For the default and autarky case, I assume that $q = 0$ holds. This assumption guarantees an equilibrium bond price function that is defined over the whole state space and not just conditional on repayment. Since no debt is issued in this case, this assumption does not affect the results.

Conditional on the default decision, the government budget constraint is

$$g - \tau w l = \begin{cases} (1+\mu)\frac{1+qB'}{p} - \frac{1+B}{p}, & \text{if} \quad d = 0 \\ \frac{\mu}{p}, & \text{if} \quad d = 1 \end{cases}$$

In the default (and autarky) case, the government faces the static decision problem of how to finance public spending $\bar{g}$ with income tax revenues $\tau w l$ and seigniorage $\tau^m \equiv \mu/p$.

Using conditions (1)-(3) and (5)-(8), the government budget constraint can be rewritten as the following (period) implementability constraint:

$$0 = \begin{cases} \beta\mathbb{E}\left[u_2(c_2')\frac{1-d'}{p'}\right]B' + \beta\mathbb{E}\left[u_1(c_1')\frac{1}{p'}\right] \\ +u_l(l)l + u_2(c_2)\left(c_2 - \frac{B}{p}\right), & \text{if} \quad d = 0 \\ \\ \beta\mathbb{E}\left[u_1(c_1')\frac{1}{p'}\right] + u_l(l)l + u_2(c_2)c_2, & \text{if} \quad d = 1 \end{cases} \tag{9}$$

I focus on stationary Markov-perfect equilibria (see Klein et al., 2008). Policy decisions are thus restricted to be functions of the payoff-relevant aggregate state S. In each period, the government then takes as given future variables

$$z', \; z \in \{c_1, c_2, d, p\}, \tag{10}$$

as functions of the future state $S' = (B', a')$.[8] These variables enter the household optimality conditions (2) and (3).

Despite lacking the ability to commit to future policies, the government fully recognizes today that it affects (expected) future policies via its choice of B'. A stationary

[8]The exact definition of the policy functions is given in the definition of the Markov-perfect equilibrium below.

Markov-perfect equilibrium then requires the policy functions that govern future decisions to coincide with the policy functions that determine current public policy for all states S.

The decision problem of the government is formulated recursively and given by the following functional equation:

$$V(B,a) = \max_{d\in\{0,1\}} \left\{(1-d)W^0(B,a) + dW^1(a)\right\} \tag{11}$$

with the value of repayment given as

$$W^0(B,a) = \max_{c_1,c_2,l,p,B'} u\left(c_1,c_2,l\right) + \beta\mathbb{E}\left[V(B',a')\right] \;\; s.t.\; (4),(6)-(10),$$

and the value of default as

$$W^1(a) = \max_{c_1,c_2,l,p} u\left(c_1,c_2,l\right) + \beta\mathbb{E}\left[\theta V(0,a') + (1-\theta)\, W^1(a')\right] \;\; s.t.\; (4),(6)-(10).$$

Whenever the government is indifferent between default and repayment, it is assumed to honor its obligations.[9] For the definition of the equilibrium it is useful to introduce the indicator variable $h \in \{0,1\}$ which denotes whether the government is in financial autarky ($h=1$) or not ($h=0$).

2.3 Equilibrium

The Markov-perfect equilibrium is defined as follows:

Definition 1 *A stationary Markov-perfect equilibrium is given by a set of value functions* $V(B,a)$, $W^0(B,a)$, $W^1(a)$, *and policy functions* $\Gamma(B,a)$, $\Psi^0_B(B,a)$, $\Psi^0_x(B,a)$, $\Psi^1_x(a)$ *with* $x \in \{c_1,c_2,l,p\}$, *such that for all* (B,a):

$$\Gamma(B,a) = \underset{d\in\{0,1\}}{\arg\max}\; \left\{(1-d)W^0(B,a) + dW^1(a)\right\},$$

$$\begin{aligned}
\left\{\Psi^0_x(B,a)\right\}_{x\in\{c_1,c_2,l,p,B\}} &= \underset{c_1,c_2,l,p,B'}{\arg\max}\, u\left(c_1,c_2,l\right) + \beta\mathbb{E}\left[V(B',a')\right] \\
\text{s.t.} &: (4),(6)-(9), \\
d' &= \Gamma\left(B',a'\right), \\
z' &= \left\{\begin{array}{c} (1-\Gamma(B',a')) \times \Psi^0_z(B',a') \\ +\Gamma(B',a') \times \Psi^1_z(a') \end{array}\right\}, \\
z &\in \{c_1,c_2,p\},
\end{aligned}$$

[9] This tie-breaking rule is solely introduced to formally ensure that the default decision is a function of the aggregate state and not a correspondence. Assuming the government to default when it is indifferent doesn't change the results since productivity is drawn from a continuous distribution such that the probability of indifference is zero.

and

$$\begin{aligned}
\left\{\Psi_x^1(a)\right\}_{x\in\{c_1,c_2,l,p\}} &= \underset{c_1,c_2,l,p}{\arg\max}\; u(c_1,c_2,l)+\beta\mathbb{E}\left[\theta V(0,a')+(1-\theta)W^1(a')\right] \\
\text{s.t.} &: (4),(6)-(9), \\
z' &= \left\{\begin{array}{c} h'\times\Psi_z^1(a') \\ +(1-h')\times\left\{\begin{array}{c}(1-\Gamma(0,a'))\times\Psi_z^0(0,a') \\ +\Gamma(0,a')\times\Psi_z^1(a')\end{array}\right\}\end{array}\right\}, \\
z &\in \{c_1,c_2,p\},
\end{aligned}$$

as well as

$$V(B,a)=(1-\Gamma(B,a))\times W^0(B,a)+\Gamma(B,a)\times W^1(a),$$

$$W^0(B,a)=u\left(\Psi_{c_1}^0(B,a),\Psi_{c_2}^0(B,a),\Psi_l^0(B,a)\right)+\beta\mathbb{E}\left[V(\Psi_B^0(B,a),a')\right],$$

and

$$W^1(a)=u\left(\Psi_{c_1}^1(a),\Psi_{c_2}^1(a),\Psi_l^1(a)\right)+\beta\mathbb{E}\left[\theta V(0,a')+(1-\theta)W^1(a')\right].$$

3 Model Analysis

In this section, the role of sovereign default for public policy is investigated. Because the model cannot be evaluated analytically due to the discrete default option, numerical methods are applied. Appendix A.2 contains details regarding the numerical computation of the equilibrium. The next section presents the model specification. A discussion of the public policy choices can be found in Section 3.2. Simulation results are presented in Section 3.3.

3.1 Model Specification

To explore the model properties by computational means, functional forms and parameters need to be chosen.

3.1.1 Functional Forms

Productivity follows a log-normal AR(1)-process,

$$a_t=a_{t-1}^{\rho}\exp(\sigma\varepsilon_t),\ \varepsilon_t\overset{i.i.d.}{\sim}\mathcal{N}(0,1).$$

The household utility function is specified as

$$u(c_1,c_2,l)=\gamma_1\frac{c_1^{1-\sigma_1}-1}{1-\sigma_1}+\gamma_2\frac{c_2^{1-\sigma_2}-1}{1-\sigma_2}+(1-\gamma_1-\gamma_2)\frac{(1-l)^{1-\sigma_l}-1}{1-\sigma_l},$$

with $\gamma_1,\gamma_2,\sigma_i>0$, $i\in\{1,2,l\}$ and $\gamma_1+\gamma_2<1$.[10]

[10]For $\sigma_i=1$, $i\in\{1,2,l\}$, household utility is logarithmic for the respective variable.

Parameter	Description	Value
β	Discount factor	0.9900
$\bar{g}$	Government spending	0.0379
γ_1	Cash-good weight	0.0030
γ_2	Credit-good weight	0.3370
σ_1	Cash-good curvature	2.4200
σ_2	Credit-good curvature	1.0000
σ_l	Leisure curvature	2.0000
$\bar{a}$	Default cost parameter	0.9910
θ	Probability of reentry	0.2000
σ	Std. dev. productivity shock	0.0169
ρ	Persistence of productivity	0.9000

Table 1: Parameter values

The resource costs of default are modeled as in Arellano (2008) and Cuadra et al. (2010):

$$\psi(a, d) = a - d \times \max\{0, a - \bar{a}\}.$$

If a default takes place, effective productivity equals $\bar{a}$ when a exceeds $\bar{a}$ while there are no costs of default when productivity a is below the threshold $\bar{a}$. This default cost specification implies that a default is more costly in booms than in recessions. In the quantitative sovereign default literature, it is well known that this feature is crucial for default to mostly take place in bad states and hence for countercyclical sovereign risk to emerge (see e.g. Aguiar and Amador, 2014). This property is consistent with empirical evidence (see Tomz and Wright, 2007) and also present in models with endogenous costs of default (see Mendoza and Yue, 2012, Sosa-Padilla, 2013).

3.1.2 Parameters

A model period corresponds to one quarter. The selected model parameters are listed in Table 1. They are either set to standard values or chosen to replicate certain short- or long-run properties of the Mexican economy.[11] The productivity parameter ρ is set to 0.9 while σ is chosen to match the standard deviation of HP-filtered Mexican log real GDP. As is common in business cycle models, a discount factor of $\beta = 0.99$ is selected, implying an annual real risk-free rate of 4 percent. Based on World Bank data for 1980-2008, parameter $\bar{g}$ is set to 0.0379 to match an average ratio of public spending to GDP of around 11 percent. The credit-good preference parameter σ_2 is normalized to 1. Targeting a cash-credit good ratio and an average working time of one third each, γ_1 is set to 0.003 and γ_2 to 0.337. For the inverse of the elasticity of leisure σ_l, a rather standard value of 2 is selected. The probability of reentry θ is set to 0.2, implying that financial autarky lasts for 5 quarters on average. This parameter value is in line with values considered in the quantitative sovereign default literature which range from 0.0385 (Chatterjee and Eyigungor, 2012) to

[11] The time series for real GDP and the GDP deflator are taken from Cuadra et al. (2010) and cover the time period from 1980:I to 2007:I. They are seasonally adjusted via EViews' multiplicative X-12 routine.

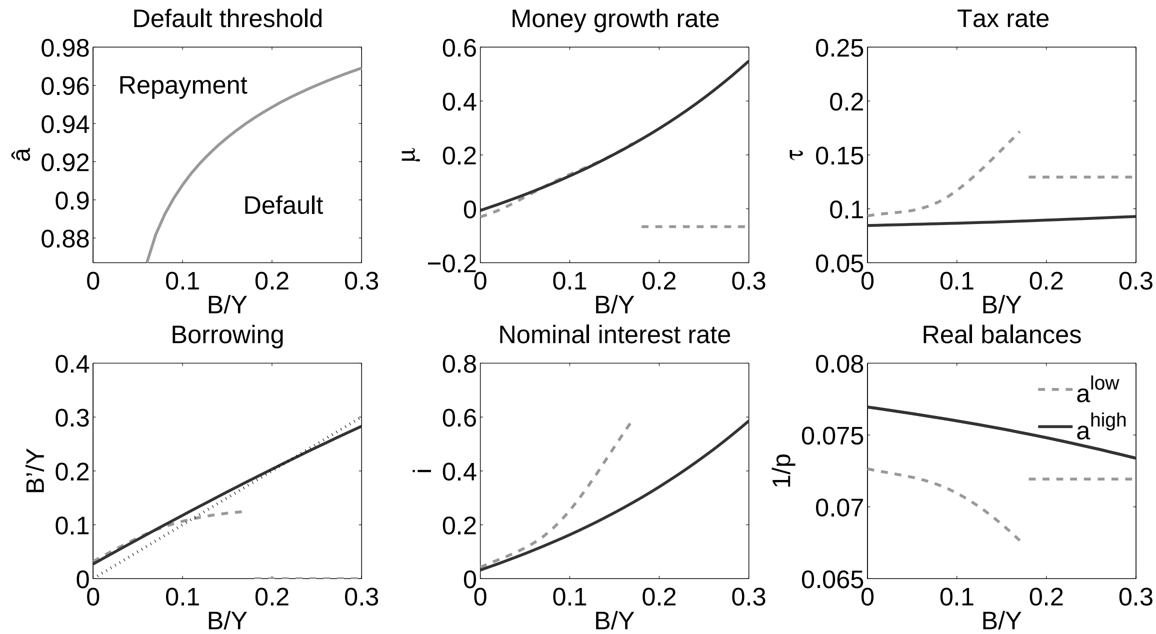

Figure 1: Default threshold and selected policy functions for productivity values 1.5 standard deviations below (a^{low}) and above (a^{high}) the unconditional mean of productivity (baseline economy). The money growth rate and the nominal interest rate are expressed in annual terms.

0.282 (Arellano, 2008).

The size of σ_1 is crucial for long-run debt and inflation as will be explained in the next section. To match the Mexican average annual inflation rate of 29.69 percent, σ_1 is set to 2.42.[12] The incentive to default critically depends on $\bar{a}$. For Mexico, Reinhart (2010) documents that there have been domestic defaults in 1982 and between 1929 and 1938. Based on this observation, I set the default cost parameter to match an annual default frequency of 2 percent. The model is also solved and simulated with prohibitively high productivity costs of default which rule out equilibrium default. This benchmark economy yields the same results as a model without default option and will be referred to as "no-default economy". The model with default option will be referred to as "baseline economy".

3.2 Public Policy Decisions

The optimal policies for the economy with default can be seen in Figure 1. Debt is normalized by nominal output $Y \equiv py$, evaluated at the unconditional mean of productivity. The default decision is visualized using the default threshold $\hat{a}(B)$ which is the lowest productivity level that leads to repayment for given debt B: $W^0(B, \hat{a}(B)) = W^1(\hat{a}(B))$. It separates the state space (B, a) into two areas: the default region ($a < \hat{a}(B)$), i.e. below the line) and the repayment region ($a \geq \hat{a}(B)$, i.e. on and above the line). As in the quantitative

[12]The inflation rate is calculated based on the quarterly GDP deflator time series for Mexico provided by Cuadra et al. (2010). Using alternative measures such as the CPI also yields average inflation rates of around 30 percent.

sovereign default literature, default becomes more attractive with higher debt and lower productivity (see Arellano, 2008).

The remaining policies are presented for productivity levels 1.5 standard deviations below (dashed line) and above (solid line) the unconditional mean of productivity. The nominal interest rate is defined as $i = 1/q - 1$. Since the continuous policy decisions depend on the default decision, the objects displayed in Figure 1 exhibit kinks at states where default is optimally chosen. In the default case, the policies also do not change with B anymore.

The optimal labor and inflation tax distortions reflect the government's financing needs. By relaxing the government's budget, a sovereign default allows to reduce labor taxation and increase real balances relative to full debt repayment. The income tax rate and the price index p both increase with B. An inflationary monetary policy becomes particularly more attractive with higher debt because it lowers the real debt burden. This implies that default and inflation are substitutes since inflation as "partial default" becomes useless for $d = 1$. However, they are only imperfect substitutes due to the discrete nature of default.

The intertemporal policy trade-off can be illustrated via the generalized Euler equation

$$\beta \int_{\hat{a}(B')}^{\infty} \left(\xi' - \xi\right) \frac{u_2\left(c_2'\right)}{p'} f_a(a'|a)da' = \xi \left(\frac{\partial R^b}{\partial B'} B' + \frac{\partial R^m}{\partial B'} \right), \tag{12}$$

where ξ denotes the multiplier on the implementability constraint, $R^b = \left(\left(1+\mu\right) q u_2\right)/p$ average revenues from bond issuance and $R^m = \left(\left(1+\mu\right) u_2\right)/p$ (gross) revenues from money creation.[13] With the model specification of Section 3.1.1 and condition (8), these revenues are given by the following functions of productivity a and borrowing B':

$$\begin{aligned} R^b(B', a) &= \beta \int_{\hat{a}(B')}^{\infty} \frac{\gamma_2 \left(\Psi_{c_2}^0(B', a')\right)^{-\sigma_2}}{\Psi_p^0(B', a')} f_a(a'|a)da', \\ R^m(B', a) &= \beta \times \left\{ \begin{array}{c} \int_0^{\hat{a}(B')} \gamma_1 \left(\Psi_p^1(a')\right)^{\sigma_1 - 1} f_a(a'|a)da' \\ + \int_{\hat{a}(B')}^{\infty} \gamma_1 \left(\Psi_p^0(B', a')\right)^{\sigma_1 - 1} f_a(a'|a)da' \end{array} \right\}. \end{aligned}$$

Households dislike volatile consumption and leisure. In the presence of productivity shocks, the government can issue debt to accommodate these preferences and smooth tax distortions as measured by ξ across states (see the LHS of (12)). Its ability to do so is constrained by financial market incompleteness and lack of commitment. Since only nominal non-state contingent bonds are available, the government has an incentive to make debt payments state contingent via inflation or default. However, because it cannot commit to a state-contingent repayment plan for the next period, public financing conditions will depend on the chosen debt position B' since it affects the risk of inflation and default. The derivatives on the RHS of (12) reflect this channel. The optimal debt policy then trades off the tax smoothing motive against the time-consistency problem, i.e. how current debt issuance affects revenues $R^b(B', a) \times B'$ and $R^m(B', a)$ by changing household expectations of inflation

[13] The derivation of the Euler condition can be found in Appendix A.1. Note that seigniorage does not equal money revenues: $\tau^m = R^m/u_2 - 1/p$.

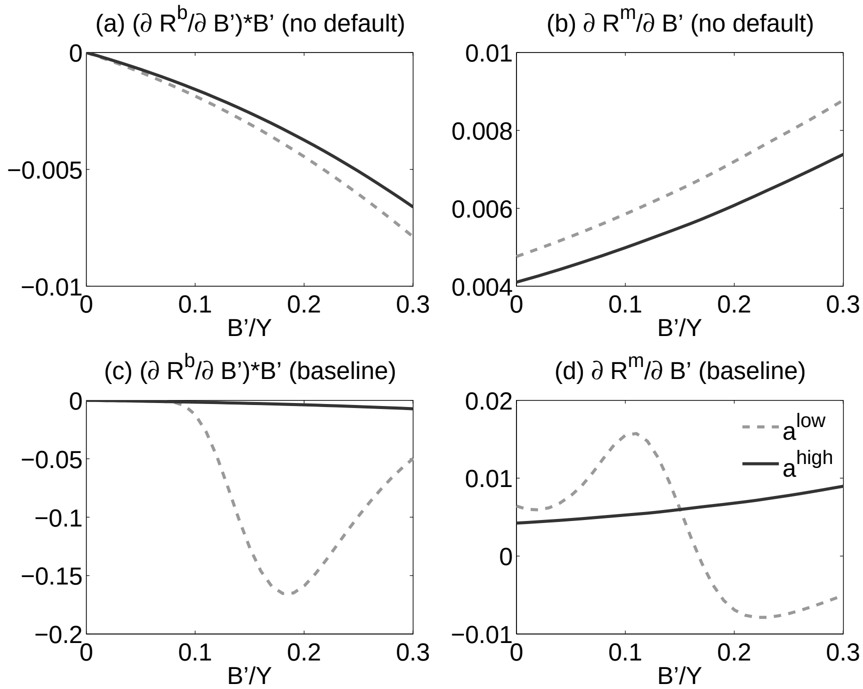

Figure 2: Marginal revenues $\left(\partial R^b/\partial B'\right) \times B'$ and $\partial R^m/\partial B'$ for the no-default economy and the baseline economy.

and default.

The impact of debt issuance on public revenues is visualized in Figure 2. It depicts $\left(\partial R^b/\partial B'\right) \times B'$ and $\partial R^m/\partial B'$ for the no-default economy (first row) and the baseline economy with equilibrium default (second row). The no-default case has previously been discussed by Martin (2009, 2011, 2013) and Diaz-Gimenez et al. (2008).

It is useful to first look at the case without default to understand how sovereign risk affects the debt policy.[14] The sign of $\left(\partial R^b/\partial B'\right) \times B'$ depends on the relation between inflation and beginning-of-period debt (see panel (a)). Due to its lack of commitment, the government does not internalize how its current policy choices affect outcomes in previous periods and decides to erode the real value of beginning-of-period debt via inflation to relax its budget. As a result, expected inflation becomes an increasing function of end-of-period debt and the price of nominal government bonds responds to B' in a negative way, causing borrowing to become more expensive when more debt is issued ($\left(\partial R^b/\partial B'\right) \times B' < 0$). The behavior of $\partial R^m/\partial B'$ depends on the way inflation and household money demand are related (see panel (b)). Given that real balances are a decreasing function of B, the real payoff of money is expected to decrease with borrowing B', reducing household money demand today. However, lower real balances $1/p$ also increase the marginal value of money when the cash-in-advance constraint is binding such that the demand for money increases with B' since households expect to be more cash-constrained. Whether higher borrowing increases net household money demand depends on the size of σ_1. For $\sigma_1 > 1$, it does and

[14] In this case, the default threshold is given as $\hat{a}(B) = 0$, i.e. default is prohibitively costly.

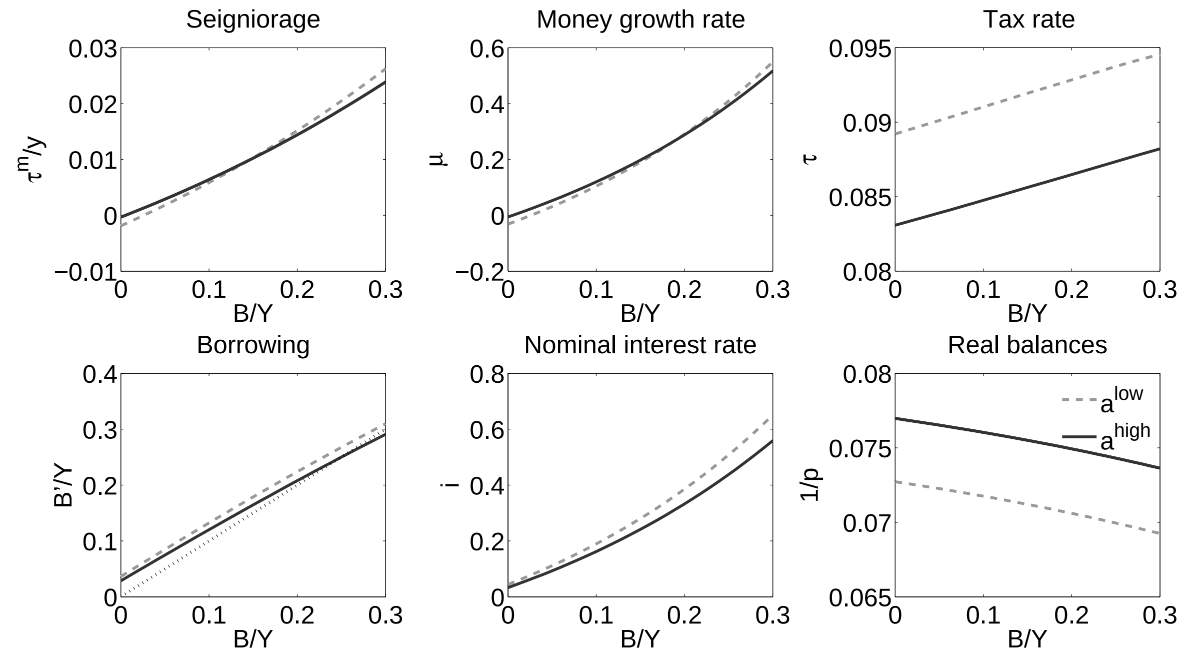

Figure 3: Selected policy functions for productivity values 1.5 standard deviations below (a^{low}) and above (a^{high}) the unconditional mean of productivity (no-default economy). The money growth rate and the nominal interest rate are expressed in annual terms.

an increase in B' leads to higher money revenues ($\partial R^m/\partial B' > 0$). Since a household's valuation of money increases with B', the government can simply issue more currency to implement a particular price index and thus obtain more revenues from money issuance.

The long-run debt position is determined by the two effects described above.[15] A positive sign for $\partial R^m/\partial B'$ is crucial for non-zero long-run debt.[16] For $\sigma_1 = 1$, money revenues do not respond to borrowing ($\partial R^m/\partial B' = 0$) which eliminates the incentive to borrow in the long run. With $\sigma_1 \in (0,1)$, the government even has an incentive to accumulate assets ($B' < 0$) due to $\partial R^m/\partial B' < 0$.[17] These two cases are not further discussed here because they make default a redundant policy option. When there are productivity shocks, a positive response of money revenues to borrowing also matters for the government's ability to smooth tax distortions across states since, without this effect, only the negative bond price effect would be operative and make it more expensive to issue debt in low productivity states.

When the government can default on its debt, sovereign risk changes the impact of borrowing on public revenues. Panel (c) displays $\left(\partial R^b/\partial B'\right) \times B'$ with sovereign risk. Because a default is more likely for higher amounts of debt, the bond price would respond to B' in a negative way even in the absence of inflation risk. As in Arellano (2008), the debt elasticity

[15] There is also an unstable non-distortionary steady state with $\xi = 0$ (see Martin, 2009 for details).

[16] Looking at Markov-perfect public policy in a real economy setting with endogenous government spending and without default, Debortoli and Nunes (2013) show - for analytical and quantitative examples - that long-run debt only deviates from zero for a small range of parameter values.

[17] For more details see Proposition 5 in Martin (2009) who proves these properties for a deterministic model without default option.

of interest rates is higher in bad (low productivity) than in good (high productivity) states. Panel (d) shows how marginal money revenues respond to B' when there is sovereign risk. The non-monotonic shape results from the optimal mix of default and inflation. Since default and inflation are substitutes, higher borrowing can lower expected inflation by increasing the probability of default. While making money as a store of value more valuable, this interaction also lowers the expected marginal value of money and $\partial R^m/\partial B'$. Due to its adverse effect on marginal bond and money revenues, sovereign risk thus makes debt issuance less attractive.[18] The consequences for the long-run debt position can be illustrated via Figure 1 and Figure 3. By looking at the intersection between the (dotted) 45-degree line and the borrowing policies, one can already see without having simulated the model that average debt is going to be lower in the baseline model with default. The quantitative dimension of the model properties discussed so far is explored in the next section.

3.3 Simulation Results

Table 2 presents the averages of statistics calculated for 1000 simulated economies with 2000 periods each. The time series are filtered using the Hodrick-Prescott filter with a smoothing parameter of 1600. Output is in logs. All simulations are initialized with $S_0 = (0, \mathbb{E}[a])$ and the first 500 observations of each sample are discarded.

Average debt and inflation are lower with default option. More specifically, the average inflation rate in the no-default economy is more than twice as large as in the baseline economy with default. Default risk raises the cost of rolling over even low amounts of debt in recessions. This mechanism restricts the build up of large public debt positions which would make higher inflation more attractive. Less debt also implies that the tax base of the income tax increases relative to that of inflation. Hence, the benefit of raising inflation is lower, leading to a higher average labor tax rate in the baseline economy.[19] While the accumulation of debt crucially depends on the government's ability to collect seigniorage (see the discussion in the previous section), the average seigniorage-to-GDP ratio is moderate and of plausible size.[20] When the government has the option to default, borrowing is more expensive in recessions due to the increased risk of default. The average nominal interest rate however is higher in the no-default economy since it experiences more inflation on average.

The short-run implications of sovereign risk for public policy can be illustrated via Figure 4. For the baseline and the no-default economy, it displays impulse responses of selected model variables to a negative one-time productivity shock. The variables are expressed as absolute deviations from their values in a stationary state to which the economies converge when productivity is kept fixed at its long-run mean.[21] Since productivity is persistent

[18] When borrowing B' is high enough, the probability of default approaches one and $\partial R^m/\partial B'$ as well as $\partial R^b/\partial B'$ go to zero as money and bond revenues hardly change anymore.

[19] For Mexico, Ilzetzki (2011) calculates an average marginal income tax rate of 12.1 percent which is close to the average tax rate in the baseline model (10.47 percent).

[20] Using the same definition of seigniorage as in the model, Aisen and Veiga (2008) calculate that average seigniorage is 2.2 percent of GDP for Mexico.

[21] Of course, the two economies do not exhibit the same stationary state. The variables in such a stationary

	Baseline	No default
Mean		
Default probability (annual)	0.0201	0
Debt-to-GDP	0.1841	0.3299
Tax rate	0.1047	0.0917
Seigniorage-to-GDP	0.0152	0.0293
Inflation rate (annual)	0.2991	0.6467
Nominal interest rate (annual)	0.3933	0.7138
Standard deviation		
Output	0.0236	0.0192
Tax rate	0.0124	0.0012
Inflation rate (annual)	0.1083	0.0545
Nominal interest rate (annual)	0.0777	0.0251
Correlation with output		
Tax rate	-0.8609	-0.9990
Inflation rate (annual)	-0.2971	-0.5587
Nominal interest rate (annual)	-0.4643	-0.9090

Table 2: Selected model moments

($\rho > 0$), the negative shock immediately raises the risk of default as the incentive to default is more likely to be strong in the subsequent period. The high sensitivity of interest rates to changes in debt issuance forces the government to reduce its debt position in order to avoid an even larger decline of the bond price. As a result, the government has to resort to large increases in inflation and taxes to finance debt payments and government spending. Consistent with empirical evidence (see Reinhart and Rogoff, 2009), a sovereign debt crisis thus is highly inflationary. As productivity reverts back to its mean and debt is reduced even further, expected inflation and sovereign risk both decline, leading the government to take advantage of the improved borrowing conditions and accumulate debt again. In the no-default economy, borrowing conditions do not deteriorate very much in response to the negative productivity shock. This property allows the government to effectively smooth tax distortions across states by issuing debt which avoids large increases in taxes and inflation. Because debt cannot be easily rolled over in the baseline model, the impact of productivity shocks on the economy is more pronounced and output volatility is 23 percent higher relative to the model without default.

Since one of the main contributions of this paper is to offer a joint analysis of inflation and sovereign default, it is interesting to compare the cyclical properties of inflation generated by the model with and without default to those observed in the data. Table 3 shows the results. In Mexico, inflation is very volatile and countercyclical. The baseline model with default can replicate these findings. To give the model without default a fair chance, it is recalibrated to match the average inflation rate and the volatility of output in Mexico. While the recalibrated no-default economy yields countercyclical inflation, the baseline model predicts more volatile and less countercyclical inflation than the no-default

state are close to the average values listed in Table 2.

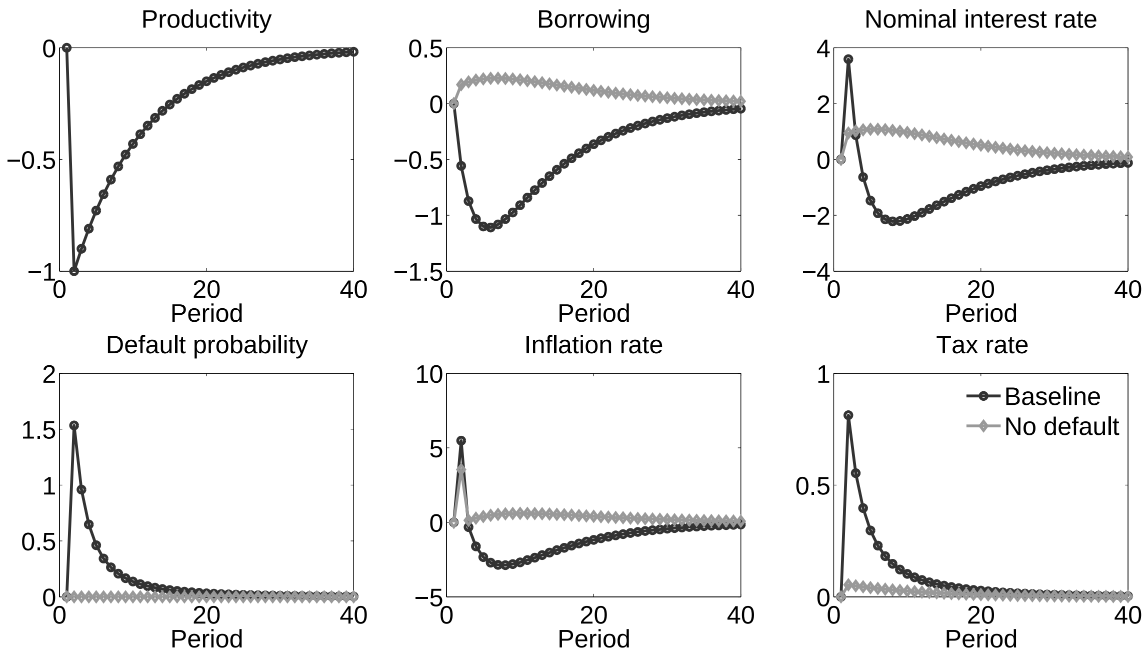

Figure 4: Impulse responses of selected variables to a negative productivity shock. All variables are expressed as absolute deviations (multiplied by 100) from a stationary state to which the respective economy converges when productivity is kept fixed at its unconditional mean.

model which is closer to what is observed empirically.

4 The Welfare Implications of Sovereign Default

This section discusses the welfare implications of sovereign default. With commitment, the option to default will not decrease welfare since the government would otherwise refrain from using it.[22] Without commitment, this is not necessarily the case anymore. Section 3 has shown that the default option has implications for public policy in the short and the long run. On the one hand, by increasing the sensitivity of bond prices with respect to debt and productivity, countercyclical risk of default entails short-run costs because the government loses some of its ability to smooth tax distortions across states. On the other hand, default risk might lead to welfare gains due to its impact on long-run debt. The model features a long-run borrowing motive that stems from the presence of two frictions, lack of commitment and a liquidity constraint (see the discussion in Section 3.2). The government acts in a time-consistent way and does not internalize the effect of its current choice of inflation on the borrowing behavior in previous periods. When household money demand and thus the value of money are increasing in the amount of issued debt, the government persistently chooses positive debt positions which then lead to high average inflation. By limiting public debt

[22]For a real small open economy with incomplete markets and costly sovereign default, Adam and Grill (2012) show that welfare can be increased when the Ramsey planner can commit to a state-contingent default plan.

	Mexico (1980-2007)	Baseline	No default (recalibrated)
Standard deviation	0.2423	0.1083	0.0703
Correlation with output	-0.2734	-0.2971	-0.4587

Table 3: Cyclical properties of inflation

accumulation via more sensitive interest rates, the default option reduces average inflation and the misallocation of consumption compared to the no-default setting.

To evaluate whether the addition of the default option to the set of policy instruments is welfare enhancing, welfare measure Δ is calculated. It measures the percentage increase in credit-good consumption that households in the no-default economy need to be given in each period to achieve the same expected lifetime utility as in the baseline economy with default:

$$\mathbb{E}\left[\sum_{t=0}^{T}\beta^t u(c_{1t}^D, c_{2t}^D, l_t^D)\right] = \mathbb{E}\left[\sum_{t=0}^{T}\beta^t u(c_{1t}^N, c_{2t}^N(1+\Delta), l_t^N)\right].$$

The sequences of consumption and labor supply in the economy with $(j = D)$ and without default option $(j = N)$ are denoted as $\left\{c_{1t}^j, c_{2t}^j, l_t^j\right\}_{t=0}^{T}$. Expected lifetime utility is calculated for both types of economies by averaging realized lifetime utility of 1000 samples with simulated time series of effective length $T = 1500$ each.

The calculated welfare measure is $\Delta = 0.0003$. For the no-default economy, credit-good consumption thus needs to be increased by only 0.03 percent in each period to equalize household welfare in both types of economies. This welfare result is consistent with the finding that, for representative agent models with standard preferences, estimates of the welfare cost of inflation (see e.g. Cooley and Hansen, 1989, Lucas, 2000) tend to exceed estimates of the welfare cost of aggregate fluctuations (see e.g. Otrok, 2001, Lucas, 2003).

While there are welfare gains of having the option to default, these gains are very small, suggesting that, from a welfare perspective, lack of commitment to repayment is not important.

5 Conclusion

To understand the implications of the option to default on debt payments for public policy, this paper has studied optimal monetary and fiscal policy without commitment in a cash-credit economy with nominal debt and endogenous government default. While a default allows the government to reduce inflation and distortionary labor taxation by relaxing its budget constraint, the default option mainly induces lower rates of inflation by constraining debt issuance via endogenous default risk premia. This mechanism reduces the average debt position and the government's incentive to implement high inflation in the long-run. Less debt also implies that the income tax becomes more attractive relative to inflation, resulting in a higher average labor tax rate. Taxes and inflation are more volatile when the default option is available because the government's ability to smooth tax distortions across states is reduced by the presence of default risk. For the case of Mexico, a counterfactual exercise has demonstrated that the option to default still slightly increases household welfare by disciplining the government's incentive to accumulate debt.

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A Appendix

A.1 Derivation of the Generalized Euler Equation

The derivation follows Arellano and Ramanarayanan (2012).[23] For an interior solution to problem (11), the necessary first-order condition for B' is

$$0 = \xi \left(R^b + \frac{\partial R^b}{\partial B'} B' + \frac{\partial R^m}{\partial B'} \right) + \beta \int_{\hat{a}(B')}^{\infty} \frac{\partial W^0(B', a')}{\partial B'} f_a(a'|a) da',$$

with ξ denoting the multiplier on the implementability constraint (9).

When combined with definition

$$R^b = \beta \int_{\hat{a}(B')}^{\infty} \frac{u_2 \left(c_2' \right)}{p'} f_a(a'|a) da',$$

and envelope condition

$$\frac{\partial W^0(B, a)}{\partial B} = -\xi \frac{u_2 \left(c_2 \right)}{p},$$

the first-order condition yields the generalized Euler equation

$$\beta \int_{\hat{a}(B')}^{\infty} \left(\xi' - \xi \right) \frac{u_2 \left(c_2' \right)}{p'} f_a(a'|a) da' = \xi \left(\frac{\partial R^b}{\partial B'} B' + \frac{\partial R^m}{\partial B'} \right).$$

A.2 Numerical Solution

As is known in the literature (see e.g. Krusell and Smith, 2003 or Martin, 2009), there might be multiple Markov-perfect equilibria in models with infinitely-lived agents. In particular, there can be equilibria with discontinuous policy functions which do not arise in the infinite-horizon limit of a finite-horizon model version. To avoid such discontinuous equilibria, I assume differentiability of $W^0 \left(B, a \right)$ and $\Psi_x^0 \left(B, a \right)$, $x \in \{c_2, p\}$, with respect to B and solve for the infinite-horizon limit of a finite-horizon version by backward induction.[24] In practice, this means that I compute the value and policy functions for the final period problem where no borrowing takes place and use these objects as initial values for the value function iteration. The algorithm then iterates on these objects until the maximum absolute difference between value and policy functions obtained in two subsequent iterations is below 10^{-5} for each grid point combination.

The value and policy functions are approximated on equidistant discrete grids. Apart from the default choice, all policies are continuous. To evaluate value and policy functions at states that are off-grid, cubic spline interpolation is used.[25] To approximate expected values in an accurate way, one has to account for the default threshold. This can be seen

[23] See Clausen and Strub (2013) for details on the characterization of optimal debt policies via Euler equations in models with equilibrium default.

[24] The policy functions depicted in Section 3.2 demonstrate that - conditional on the default decision - the assumption of differentiability is indeed justified.

[25] Hatchondo et al. (2010) show that allowing for a continuous state space is crucial for accurate solutions of models with equilibrium default.

by looking at the expected option value of default:

$$\mathbb{E}\left[V(B',a')\right] = \int_0^{\hat{a}(B')} W^1(a') f_a(a'|a) da' + \int_{\hat{a}(B')}^{\infty} W^0(B',a') f_a(a'|a) da'.$$

Gauss-Legendre quadrature nodes and weights are used to approximate the integrals above. The default threshold $\hat{a}(B')$ is computed via bisection method.

For each iteration, the optimal policies are computed as follows. The algorithm makes use of a constrained optimization routine that calculates the optimal static policies c_2, l, and p for given debt and productivity values (B, a) and an arbitrary borrowing value $\tilde{B}'$. More specifically, these static polices are computed using a sequential quadratic programming algorithm (see e.g. Nocedal and Wright, 1999 for details). Using the static policy routine, c_2, l, p and thus period utility $u(\cdot)$ can all be expressed as functions of $(B, a, \tilde{B}')$.[26] For each discrete grid point combination of debt and productivity, the optimal debt policy then is computed via Golden section search, employing the static policy routine to calculate the objective function for each candidate debt value.[27]

As pointed out by Martin (2009), using a Svensson (1985)-type beginning-of-period cash-in-advance constraint in a finite-horizon model requires a terminal money value for a monetary equilibrium to exist. Otherwise, households will not be willing to invest in money in the final period and by backward induction not in any of the previous periods. The impact of the final-period value of money vanishes over time and does not affect the final results.

[26]Recall that cash-good consumption is given by (8).

[27]I also used the Nelder-Mead algorithm instead of Golden section search to find the optimal debt policy which did not change the results but was much slower.

B Online Appendix

B.1 Additional Figures

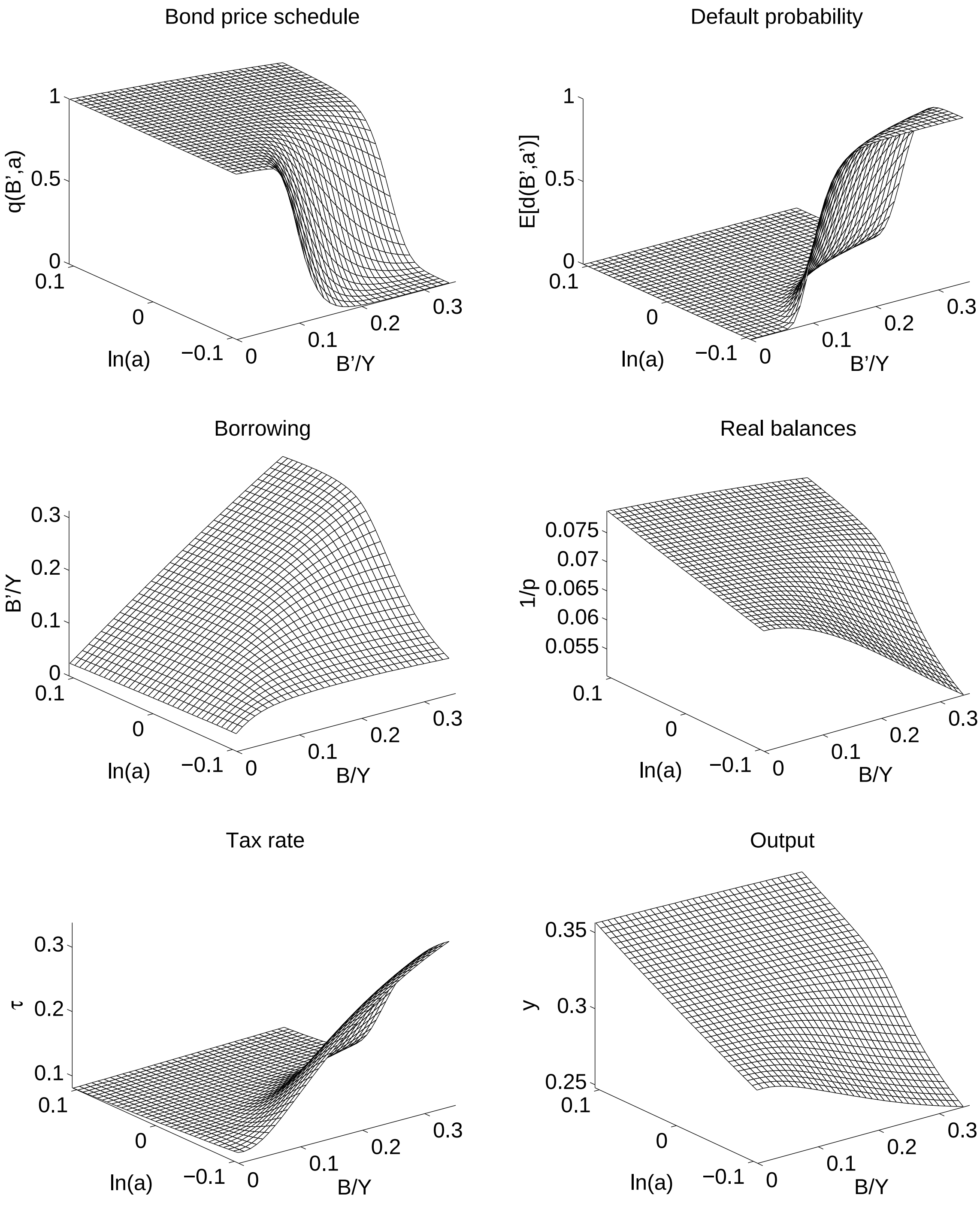

Figure 5: The bond price schedule, the associated default probability and selected policy functions for the baseline economy. All objects are conditional on repayment. Note that by combining the Euler equations (2) and (3), one can see that the bond price, $q = \mathbb{E}\left[(1-d')\left(u_2\left(c_2'\right)/p'\right)\right]/\mathbb{E}\left[u_1\left(c_1'\right)/p'\right]$, only depends on future variables and can thus be expressed for arbitrary B'-values.

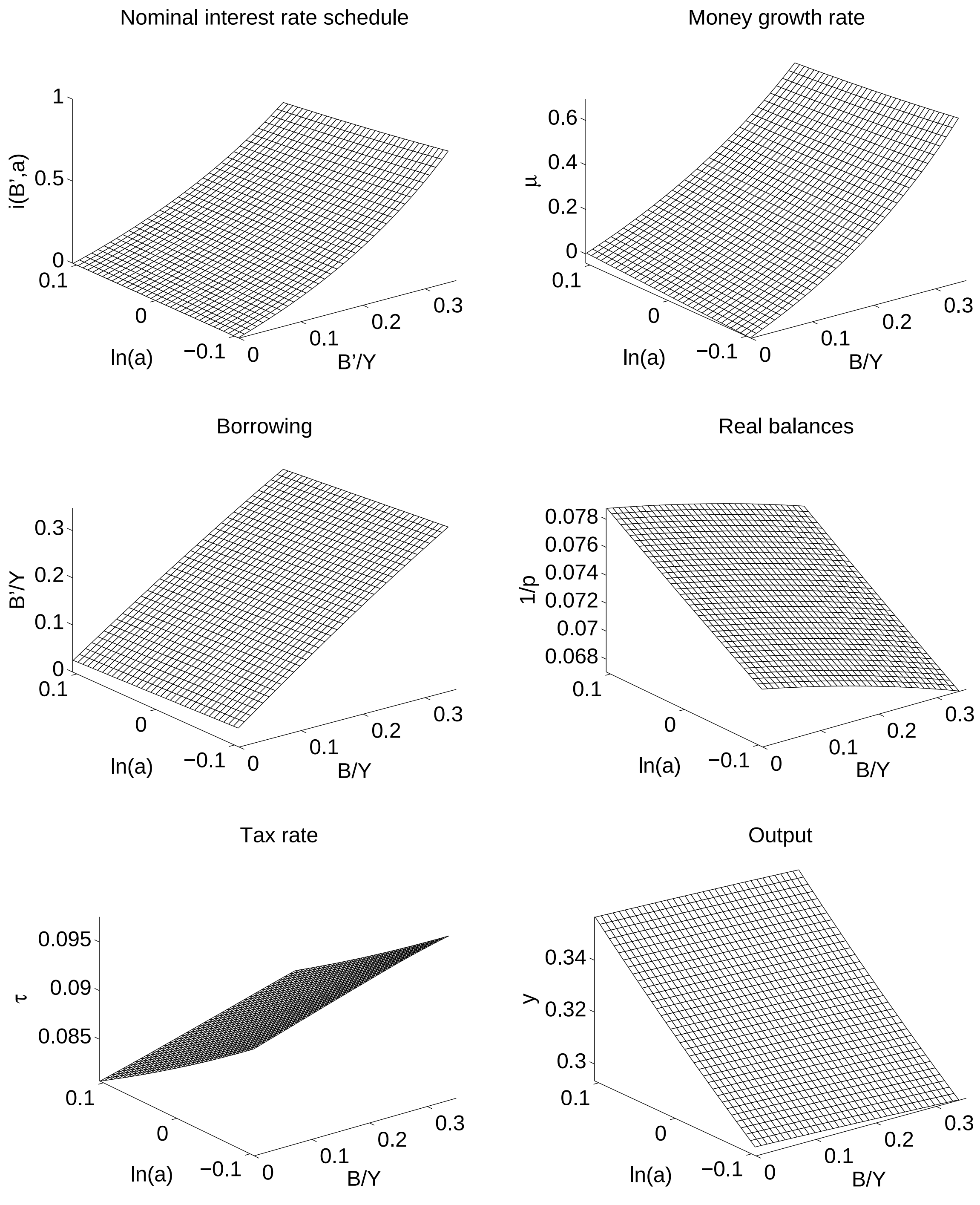

Figure 6: The nominal interest rate schedule and selected policy functions for the no-default economy. The nominal interest rate and the money growth rate are expressed in annual terms.