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DEFAULT RISK PREMIA ON GOVERNMENT BONDS IN A QUANTITATIVE MACROECONOMIC MODEL

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Default Risk Premia on Government Bonds in a Quantitative Macroeconomic Model[i]

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Abstract

We develop a macroeconomic model where the government does not guarantee to repay debt. We ask whether movements in the price of government bonds can be rationalized by lenders' unwillingness to fully roll over debt when the outstanding level of debt exceeds the government's repayment capacity. Investors do not support a Ponzi game and ration credit supply in this case, thus forcing default at an endogenously determined fractional repayment rate. Interest rates on government bonds reflect expectations of this event. Numerical results show that default premia can emerge at moderately high debt-to-GDP ratios where even small changes in fundamentals lead to steeply rising interest rates. The behavior of risk premia broadly accords to recent observations for several European countries that experienced a worsening of fundamental fiscal conditions.

Keywords: Sovereign default; fiscal policy; government debt

JEL: E62, G12, H6

1 Introduction

The recent financial crisis has turned into a fiscal crisis in several European countries. An unusually large adverse shock has reduced tax revenues and has led to higher government spending in an attempt to mitigate the consequences of the shock for aggregate output and employment. The resulting boost in public deficits has led to high levels of government debt, which are already above 100% of annual GDP in some countries and are predicted to rise to even higher levels in the near future. Sizeable yield spreads between government bonds of member countries of the European Monetary Union have emerged and for some countries (in particular for Greece, Ireland, Spain and Portugal) yield spreads in relation to comparatively safe German bonds increased dramatically since 2008.

It is hardly controversial that these spreads reflect the risk that governments might default on their debt obligations. The purpose of the present paper is to analyze the emergence of default risk premia on government bonds based on an ability-to-repay approach. In the literature, a common approach is to determine default as an outcome of an optimizing sovereign borrower who is willing to default if the gains from non-repayment of external debt exceed the costs of autarky and potential resource losses in case of default ("willigness-to-repay approach", see Eaton and Gersovitz, 1981, or Arellano, 2008, among others).[1] In contrast to this, we consider an alternative approach where default is due to credit rationing by lenders when outstanding public debt exceeds the government's debt repayment capacity. We specify fiscal policy in a non-optimizing way and assume that debt is held by investors, who fully take into account the possibility that the present value

[1] Mendoza and Yue (2012) endogenize the effects of default on income via costs of working capital. D'Erasmo and Mendoza (2013) analyze the incentive of governments to use domestic default as a redistribution device.

of government surpluses might not be sufficient to cover outstanding debt. We show that such a set-up is sufficient to obtain interest rates being almost insensitive to low debt-to-gdp ratios and steeply rising for debt-to-gdp ratios comparable to those recently observed in European countries.

We apply a basic closed economy model and consider a government with limited commitment.[2] It levies an exogenously determined proportional tax on labor income and issues one-period bonds to finance real government expenditures, while failing to guarantee repayment of debt. Due to endogenous production, tax revenues are limited and depend on the state of the economy. These bonds are non-state contingent when debt is fully repaid, while default induces a partial state contingency. It is well established that if a government is committed to raising fiscal surpluses in response to rising debt levels (see Bohn, 1998), this can guarantee intertemporal solvency as long as tax rates are below the revenue maximizing level and can always be freely adjusted. Our point of departure, instead, is that the government is not committed to such a policy, such that default triggered by a potential failure of intertemporal solvency becomes a possibility. The present paper studies the qualitative properties and quantitative implications of the resulting equilibrium when investors take this possibility into account.

In particular, if adverse productivity shocks lead to a build-up of public debt and the present value of future surpluses falls short of covering the level of outstanding debt, the government's debt repayment capacity is exceeded.[3] Household-lenders realize that they

[2]We view the assumption of domestically held debt (instead of external debt) as more suited when discussing the experience of Eurozone countries. Reinhart and Rogoff (2008) emphasize the empirical importance of domestic debt.

[3]This approach is similar to Bi (2012) and Bi and Leeper (2012) where default also depends on whether current outstanding debt exceeds a "fiscal limit", which consists of the discouted sum of future maximum

would support a Ponzi game if they further invested in government bonds and thus ration credit supply according to their transversality condition. In this case, default becomes inevitable and the government repays outstanding debt using available revenues from current surpluses and issuance of debt that can be partially rolled-over. Bond holders therefore experience only a partial redemption of their investments at an endogenously determined rate. Each individual lender assesses the probability that this event will occur and, consequently, demands a default risk premium as a compensation for expected losses.

We neither consider governments' incentives to default on external debt nor real costs of default, which might cause governments to raise surpluses that preclude costly defaults. Instead, the essence of the analysis is to explore the impact of fiscal sustainability on equilibrium bond prices. To assess the implications of this approach in the most transparent way, we assume that tax rates and government spending are exogenous (like in the literature on the fiscal theory of the price level, see Sims, 1994, and Woodford, 1994, or on the fiscal theory of sovereign default, see Uribe, 2006).[4] These simplifying assumptions obviously come at the cost that we may underestimate the full impact of fiscal policy on default probabilities to the extent that in reality governments behave in a less mechanical way.

The main results are as follows. We analytically show that there is a unique equilibrium for the price of risky government bonds. We find that the relation between the debt-to-GDP ratio and the default risk premium can be very steep above certain critical levels of

surpluses.

[4] To be more precise, we assume that tax rates are constant, which would actually be an optimal choice under commitment in an economy that is identical to the one considered here, except for public debt being fully state contingent rather than partially state contingent due to default.

debt-to-GDP, consistent with the observation that risk premia may rise suddenly and very strongly when fiscal positions worsen. Of course, the precise value of the critical debt ratio depends on fiscal policy and business cycle parameters. We present calibrated versions of the model intended to capture relevant quantitative features of average European Monetary Union member countries, as well as parameterizations pertaining to those countries that have recently faced sharply rising interest rates on their public debt, such as Greece, Portugal, and Spain. We find that risk premia are generally indistinguishable from zero for low debt-to-GDP ratios and can, if fiscal policy ensures a sufficient amount of surpluses, remain negligible for debt-to-GDP ratios as high as 100%. However, above certain thresholds risk premia tend to rise steeply and in a convex way with higher debt-to-GDP ratios. Additionally, the size of risk spreads on government bonds responds to the state of the business cycle, with a severe slump (due to its adverse consequences for government revenues) being able to trigger substantial increases in spreads even if the risk spreads associated with the same initial debt ratio were negligible outside recessions.

The relation between interest rate premia on government bonds and the debt-to-GDP ratio implied by the model is broadly in line with the experience of those European countries which have been heavily affected by the recent debt crisis. While the model is admittedly too stylized to consider a full-fledged quantitative analysis, its main properties show some interesting similarities with empirical observations. In particular, the model replicates the convex empirical relation between interest rate spreads and debt-to-GDP found in the crisis period following the recession in 2008 to 2009, as well as the observed non-responsiveness of interest rates to changes in debt at lower values of indebtedness in the earlier years of the 2000s decade. We conclude that the quantitative properties of observed interest rate spreads can broadly be understood from the impact of macroeconomic fundamentals.

Our approach to model sovereign default is related to Uribe (2006) "Fiscal Theory of Sovereign Default". He considers nominal debt and exogenous surpluses in an endowment economy to demonstrate that default is inevitable under certain monetary-fiscal policy regimes. For a regime that holds the price level constant, the default rate adjusts in every period to equate the current value of debt to the present value of future surpluses. As shown in Schabert (2010), the intertemporal budget constraint is not sufficient as a criterion to determine the entire sequence of the default rate. To overcome this indeterminacy, Uribe (2006) introduces default rules by which the government defaults when the current surplus falls below some ad-hoc threshold in terms of current debt or output.[5] In the present paper, we instead introduce the assumption that, when outstanding debt exceeds the present value of surpluses, households restrict credit supply to the maximum amount of debt that can be expected to be repaid. This assumption on the lenders' behavior together with the assumption that the government serves lenders with available revenues allows us to determine an entire sequence of default rates. Our approach to determine the default rate differs fundamentally from Bi (2012) and Bi and Leeper (2012), where a default event as well as the default rate are randomly determined, such that default can in principle occur in every state and at every rate (though with different probabilities). Daniel and Shiamptanis (2012) analyze default risk in a monetary union model with an exogenous upper limit on the fiscal surplus. Evans et al. (2013) use an overlapping generations model where the government pays a fixed amount of transfers until the sustainable limit of this policy is reached. Our own approach of relating default risk to the max-

[5] Without such an assumption or Uribe's (2006) fiscal closing rules, default rates can only be determined in the initial period, as shown in Schabert (2010).

imum debt capacity has most recently also been used by Lorenzoni and Werning (2013). These authors analyze the conditions under which multiple self-fulfilling equilibria in the government bond market are possible.[6]

The remainder is organized as follows. Section 2 introduces the model. Section 3.1 describes the determination of equilibrium bond prices in a simplified version where analytical results are available, whereupon section 3.2 presents quantitative results for a calibrated model version. Section 4 concludes.

2 The model

In this section, we present a simple real dynamic general equilibrium model where the government levies income taxes and issues non-state contingent one-period debt. Labor supply is endogenous, which gives rise to a Laffer curve that bounds equilibrium tax revenues. We consider the case where fiscal policy does not guarantee that the government never runs a Ponzi-game.[7] Individual households will ration lending to the government when they realize that a Ponzi scheme is inevitable. Without access to enough credit to completely roll over its debt, the government then defaults while debt is partially repaid. Specifically, the government repays the fraction of maturing debt that can be serviced out of current surpluses and the amount of newly issued debt that household-lenders are willing to roll over. Households consider that partial repayment is possible when adverse productivity shocks lead a to a build-up of public debt. They form expectations of the future fractional rate of repayment of government debt. Accordingly, in an arbitrage-

[6]The working paper version of our paper, which is available upon request, contains a discussion of a scenario leading to multiple equilibria.

[7]This assumption is analogous to the fiscal policy specification in Uribe (2006) and in the fiscal theory of the price level (see Sims, 1994, and Woodford, 1994). In contrast to these studies, in our purely real model the price level is irrelevant.

free equilibrium, default premia exist that compensate household-lenders for expected government default.

2.1 The private sector

There exists a continuum of infinitely lived and identical households of mass one. Their utility increases in consumption c_t and decreases in working time l_t, the latter variable being bounded by a unit time endowment such that $l_t \in (0,1)$. The objective of a representative household is given by

$$\max E_s \sum_{t=0}^{\infty} \beta^t \left[\frac{c_{t+1}^{1-\sigma} - 1}{1-\sigma} + \frac{1 - l_{t+s}}{\gamma} \right], \quad \text{with } \beta \in (0,1), \gamma > 0, \sigma \geq 1, \tag{1}$$

where β denotes the subjective discount factor. Households borrow and lend among each other via one-period state contingent claims. Let $\phi_{t,t+1}$ denote the period t price of one unit of the consumption good in a particular state of period $t+1$ normalized by the probability of occurrence of that state, conditional on the information available in period t. Then, the price of a random payoff d_{t+1} in period $t+1$ is given by $E_t[\phi_{t,t+1} d_{t+1}]$. We restrict our attention to the case where private debt contracts are enforceable and households satisfy the borrowing constraint

$$\lim_{k \to \infty} E_t \phi_{t,t+k} d_{t+k} \geq 0. \tag{2}$$

Utility maximization subject to the borrowing constraint (2) requires the following first order condition for borrowing and lending in terms of state contingent claims to be satisfied, $\phi_{t,t+1} = \beta c_{t+1}^{-\sigma} / c_t^{-\sigma}$. A portfolio that leads to a payoff of one in each state is associated with the risk-free interest rate R_t^{rf}, where $1/R_t^{rf} = E_t \phi_{t,t+1}$, such that

$$1/R_t^{rf} = c_t^{\sigma} \beta E_t \left(c_{t+1}^{-\sigma} \right). \tag{3}$$

Further, the transversality condition holds in the household optimum

$$\lim_{k\to\infty} E_t \phi_{t,t+k} d_{t+k} = 0. \tag{4}$$

Households can further invest in one-period government bonds b_t, subject to $b_{-1} > 0$ and $b_t \geq 0$. Following the literature on sovereign default (see Arellano, 2008), we consider discount bonds, which allow to separately determine the price of bonds and the expected default rate. Specifically, the government offers one-period debt contracts at the price $1/R_t$ in period t that deliver one unit of output in period $t+1$ if the borrower is equipped with a sufficient amount of funds. In contrast to private borrowers, the government does however not guarantee full debt repayment. In case of default, lenders will redeem only an endogenously determined fraction of their investments.

If current and discounted future surpluses are expected to be large enough to repay outstanding debt, the household optimality condition for investment in government bonds is the analogue to the Euler equation (3), namely, $c_t^{-\sigma} = R_t \beta E_t \left(c_{t+1}^{-\sigma}\right)$. The constraint $b_t/R_t \geq 0$ further implies that in the household optimum the transversality condition $\lim_{k\to\infty} E_t \phi_{t,t+k} b_{t+k}/R_{t+k} = 0$ holds. Applying (3) and the law of iterated expectations, the latter can be written as

$$\lim_{t\to\infty} E_s \left(b_{t+s}/R_{t+s}\right) \prod_{i=1}^{t} 1/R^{rf}_{s+i-1} = 0, \tag{5}$$

where we used that the discount factor satisfies $\phi_{t,t+k} = \phi_{t+k-1,t+k} \cdot ... \cdot \phi_{t,t+1}$ and that the exogenous state variable is generated by a first-order Markov process. If beginning-of-period public debt in period t exceeds a level that is too high to be repayable (see section 2.2 for a definition), the government runs into a Ponzi game, which would be inconsistent with the households' transversality condition (5). In this case, households are assumed to ration lending. We assume that they lend to the maximum amount of debt that is expected to be repayable in $t+1$. This necessarily implies that the government defaults in period t, i.e. can honor only a fraction of its debt obligations.

Consider for a moment a deterministic finite horizon framework with a final period T. Optimizing behavior of households implies not to hold assets at the end of period T, such that $b_T = 0$ (which accords to (5)). Hence, the government has to finance its debt obligations in period T by primary surpluses. Rational households will therefore lend to the government in period $T-1$ only up to an amount that promises a repayment of s_T (where s_T denotes the primary surplus in period T). This implies that credit in period $T-1$ will be limited by $b_{T-1} \leq s_T$, such that the government is effectively credit rationed in $T-1$ if it has to issue more debt to fully repay debt b_{T-2} that matures in period $T-1$. Then, default becomes inevitable in $T-1$.

The model presented here generalizes this idea to a stochastic infinite horizon framework in which the transversality condition (5) takes the place of the condition $b_T = 0$ of the finite horizon example. Let Ψ_t be the present value of surpluses (see section 2.2 for a precise definition), a concept which is commonly applied to relate future surpluses to initial indebtedness, e.g. in Auerbach et al. (1994). When outstanding government debt b_{t-1} in period t exceeds the present value of surpluses Ψ_t, such that (5) will be violated, the government will be effectively credit rationed. We assume that households lend the maximum amount that is expected to be repayable by the government in subsequent periods, implying that $b_t = E_t \Psi_{t+1}$ (which accords to $b_{T-1} = s_T$ in the finite horizon example discussed above). This assumption makes the fraction of non-repaid debt as small as possible since those parts of debt that are rationally expected to be repayable can still be rolled over. It should be noted that potential wealth effects stemming from default are neutralized by lump-sum transfers, such that lenders, who ex ante demand risk premia to be compensated for partial repayment, will ex post not be affected by a debt default. Under these assumptions households are indifferent between repayment and default in this framework.

Households realize the possibility of partial default on government bonds and account for the probability of default (of course, since households are atomistic, an individual investor does not take into consideration the influence of his behavior on the probability of default). Let $1-\delta_t$ denote the fraction of government bonds that is redeemed and δ_t the default rate. The household flow budget constraint then reads

$$c_t + (b_t/R_t) + E_t[\phi_{t,t+k} d_{t+1}] \leq (1-\tau_t) w_t l_t + (1-\delta_t)\, b_{t-1} + d_t + pr_t + tr_t,$$

where pr_t are firms' profits and labor income $w_t l_t$ (with the real wage rate w_t) is subject to a proportional tax rate $\tau_t \in (0,1)$. Note that we consider lump-sum transfers $tr_t \geq 0$, which are paid out by the government in periods where debt is smaller than a particular threshold that is never associated with default. Specifically, we assume that the government transfers resources to households over time in an amount which ensures that the intertemporal budget constraint is always satisfied, while the timing of the transfers is chosen so that transfers do not affect the determination of the bond price (see section 2.2 for a discussion). The household optimum is characterized by the first order conditions (3),

$$c_t^{\sigma} = \gamma\,(1-\tau_t)\, w_t, \tag{6}$$

$$c_t^{-\sigma} = R_t \beta E_t \left(c_{t+1}^{-\sigma}\,(1-\delta_{t+1}) \right), \tag{7}$$

and the transversality conditions (4) and (5). Note that the Euler equation for risky government debt, (7), differs from the one for risk-free private debt (3), in that the pricing of government bonds is affected by the fact that repayment is expected to be only partial because of possible future default.

Perfectly competitive firms produce the output good y_t with a simple linear technology

$$y_t = a_t l_t, \tag{8}$$

where labor productivity a_t is generated by a first-order Markov process with mean $\overline{a} = 1$ and a bounded support $[a_l, a_h]$ with $a_h > a_l > 0$. Labor demand satisfies

$$w_t = a_t. \tag{9}$$

2.2 The public sector

The government raises revenues by issuing debt at price $q_t = 1/R_t$ and taxing labor income, and it purchases an exogenously given amount g_t of the final good in each period. Throughout, we assume government spending to be constant, $g_t = g > 0$. The underlying assumption is that political constraints make a certain amount of government spending inevitable. While the government is assumed to be fully committed to its tax and spending policy, it lacks commitment to repay debt. The flow budget constraint is given by

$$b_t R_t^{-1} + s_t = (1 - \delta_t)\, b_{t-1}, \tag{10}$$

where $R_t = 1/q_t$ is the gross real interest rate and surpluses s_t equal tax revenues net of expenditures,

$$s_t = \tau_t w_t l_t - g - tr_t, \tag{11}$$

where the transfers are non-negative $tr_t \geq 0$ (see below). We assume that the government does not guarantee to fully service debt and does not preclude that public debt might evolve on a path that implies a Ponzi scheme. As a simple and most transparent way to implement a fiscal policy of this kind, we assume that the government keeps the tax rate constant, $\tau_t = \tau$. If this implies that debt exceeds the present value of future surpluses, lenders will not roll over debt completely, i.e. they will not supply unlimited credit to the government.

As mentioned above, we are interested in analyzing sovereign default risk premia for governments which do not commit to full debt repayment. Full debt repayment can be guaranteed if the government is able to permanently adjust tax revenues positively in reaction to the volume of outstanding debt, as shown by Bohn (1998). If governments credibly followed a tax rule of this type, and if in addition there were no limits on the size of tax adjustments, default would be known not to occur and hence risk premia on government bonds could not be explained. Hence, we focus on governments which lack this type of commitment. We use the assumptions of constant government spending and tax rates as a simple way to implement the idea that governments may not be willing or flexible enough, probably due to political pressures, to adjust spending quickly enough as to eliminate the risk that deteriorating business cycle conditions engender unstable debt dynamics.

We define the government's maximum debt repayment capacity Ψ_t as the present value of surpluses net of transfers,

$$\Psi_t = E_t \sum_{k=0}^{\infty} \phi_{t,t+k} \left(\tau w_{t+k} l_{t+k} - g \right). \tag{12}$$

The use of the stochastic discount factor $\phi_{t,t+k}$ for discounting ensures that different time profiles of surpluses are made comparable using the household-investor's utility based valuation of payment streams and thus takes household-investor's attitude toward risk into account.

Note our assumption that non-negative transfers (which are introduced to neutralize wealth effects) are neglected in the computation of the maximum debt repayment capacity (12). Thus, using Ψ_t as our measure of the maximum debt repayment capacity takes into account the possibility that the government reduces future transfer promises to increase its net revenues. In this sense, $b_{t-1} = \Psi_t$ is the upper limit for initial debt that can in principle be repaid without default. We assume that households take the debt repayment capacity Ψ_t into account when they lend to the government. The government will fully serve debt obligations if $b_{t-1} \leq \Psi_t$. As long as this is the case, no government default occurs. Default, however, becomes inevitable if the current stock of debt exceeds the repayment capacity:

$$b_{t-1} > \Psi_t. \tag{13}$$

If this is the case, the government is not able to generate enough current and future surpluses to enable full repayment of outstanding debt.

In the case where (13) is satisfied, current outstanding debt cannot be repaid (even when transfers are not paid out). As mentioned above, households then lend $b_t = E_t\Psi_{t+1}$, such that end-of-period debt is expected to be repayable by the government in $t+1$. The government is then unable to fully honor its obligations and redeems as much as possible of its outstanding debt. As a consequence, repayment will only be partial. The non-repayment or default rate δ_t for the case (13) satisfies (see 10)

$$1 - \delta_t = \Psi_t / b_{t-1}, \tag{14}$$

where we used that $\Psi_t = (\tau w_t l_t - g) + E_t\Psi_{t+1}$ (see 12). In the numerical solutions below, we verified that $1-\delta_t$ is positive in all periods for the parameterizations we consider. The price of debt, $1/R_t$, then reflects the probability of default in $t+1$ in form of the expected size of the default rate δ_{t+1} (see 7).

Regardless of default, the household's transversality condition (5) requires the intertemporal government budget constraint to be satisfied in all states and periods, i.e. that current outstanding public debt always equals the present value of future surpluses s_t. Given that fiscal policy behaves in a mechanical way, we therefore assume that the government transfers back residual resources in a lump-sum way, similar to Aiyagari et al. (2002). Specifically, we assume that whenever the present value of surpluses exceeds outstanding debt the households acquire claims on transfer payments, such that the intertemporal budget constraint is always satisfied as an equality. We further assume that these claims are paid out to households not immediately, but only when the initial level of debt b_{t-1} equals or falls below a lower bound $\underline{b}$ that is associated with a negligible default probability (e.g. $\underline{b} < 0$). This assumption allows us to examine the effects of default risk on the bond price without an interference with the transfer scheme. To be more precise, if j is the index for periods where transfers are actually paid, then, in period j the sum of all present valued claims that have been accumulated since the period of the last transfer payment are transferred to the households. Our bond price analysis then focusses on episodes $t \neq j$ where the evolution of debt is not affected by transfer payouts.[8] The assumption that transfers are not paid out immediately allows for isolating bond price effects from adjustments in the intertemporal distribution of resources between the public and the private sector, which are necessary to satisfy the households' transversality condition.

[8] If, by contrast, transfers were paid out immediately in each period where they originate – that is, in each period where the government has more resources than needed to repay its debt – the transfer payout would always restore the level of newly issued debt to the present value of surpluses. In this situation, the next negative shock would inevitably trigger default. Since innovations are mean zero i.i.d, there would be a 50% default probability in every period, and the incidence of default would be i.i.d. itself (see Uribe, 2006).

In the present paper, we focus on the determination of bond prices for a given realization of state variables in situations where the government's refinancing needs preclude transfer payments.

2.3 Equilibrium

In equilibrium, prices adjust to clear markets for goods, labor, and assets and the net stock of risk-free private debt d_t is zero in the aggregate. Households' initial asset endowments are assumed to be positive, i.e. the government is initially indebted. An equilibrium is a set of sequences $\{c_t,\, l_t \in [0,1],\, y_t,\, w_t,\, b_t \geq 0,\, \delta_t \in [0,1], R_t^{rf},\, R_t, s_t\}_{t=0}^{\infty}$ satisfying (3)-(6), (7), (8), (9), (11), (12), and

$$y_t = c_t + g_t, \tag{15}$$

$$b_t = \begin{cases} (b_{t-1} - s_t)\, R_t \text{ if } & \Psi_t \geq b_{t-1} \\ E_t \Psi_{t+1} \quad \text{if } & \Psi_t < b_{t-1} \end{cases}, \tag{16}$$

$$\delta_t = \begin{cases} 0 & \text{if } \Psi_t \geq b_{t-1} \\ 1 - \Psi_t / b_{t-1} & \text{if } \Psi_t < b_{t-1} \end{cases}, \tag{17}$$

a fiscal policy $\tau \in [0,1]$ and $g > 0$, given $\{a_t\}_{t=0}^{\infty}$, and initial debt $b_{-1} > 0$.

The equilibrium allocation is not directly affected by public debt and the (expected) default rate. These properties are due to the assumptions that the current labor income tax rate is assumed not to depend on debt and that default does not lead to resource losses. Of course, the price of government bonds will depend on the expected default rate, which can be seen from the asset pricing equation (7). This reflection of the probability of future default in the interest rate on government bonds is our main object of study.

The equilibrium sequences of consumption, working time, output, the wage rate, the risk-free rate and government surpluses $\{c_t,\, l_t,\, y_t,\, w_t,\, R_t^{rf},\, s_t\}_{t=0}^{\infty}$ are determined for given

τ, g and $\{a_t\}_{t=0}^{\infty}$ by (6), (8), (9), (11) and (15), which can be summarized by

$$c_t = c(a_t, \tau) := [\gamma(1-\tau)a_t]^{1/\sigma}, \tag{18}$$

$$l_t = l(a_t, \tau, g) := (c(a_t, \tau) + g)/a_t, \tag{19}$$

$$s_t = s(a_t, \tau, g) := \tau c(a_t, \tau) - (1-\tau_t)g, \forall t \neq j, \tag{20}$$

$$\text{and } s_j = \tau c(a_j, \tau) - (1-\tau_j)g - tr_j,$$

$$R_t^{rf} = c(a_t, \tau)^{-\sigma}\beta^{-1}/E_t[c(a_{t+1}, \tau)^{-\sigma}], \tag{21}$$

as well as $w_t = a_t$ and $y_t = a_t l(a_t, \tau, g)$, where transfers tr_t are only non-zero if $t = j$.

While the equilibrium sequences $\{c_t,\ l_t,\ y_t,\ w_t,\ s_t\}_{t=0}^{\infty}$ are not affected by sovereign default, these variables are of course correlated with the default rate δ_t due to changes in the productivity level a_t.

In order to determine the bond prices we need to compute expectations about future defaults. We substitute out the stochastic discount factor in (12) using $\phi_{t,t+1} = \beta c_{t+1}^{-\sigma}/c_t^{-\sigma}$, to get $\Psi_t = E_t \sum_{k=0}^{\infty} \beta^k \frac{c_{t+k}^{-\sigma}}{c_t^{-\sigma}} (\tau w_{t+k} l_{t+k} - g)$, and rewrite it by using (18) and (20):

$$\Psi_t = \gamma(1-\tau)a_t E_t \sum_{k=0}^{\infty} \beta^k c(a_{t+k})^{-\sigma}[\tau(c(a_{t+k}, \tau) + g) - g]. \tag{22}$$

The expected default rate, public debt, and the bond price have to be determined simultaneously using the equilibrium conditions (7), (16), and (17). In order to identify these solutions, we have to consider the probabilities of the two distinct cases $\Psi_t \geq b_{t-1}$ and $\Psi_t < b_{t-1}$.

Let a_t^* be the productivity level that leads to a debt repayment capacity that exactly equals beginning-of-period debt b_{t-1},

$$a_t^* : \Psi(a_t^*) = b_{t-1}. \tag{23}$$

Thus, a_t^* is the minimum productivity level that allows full debt repayment and thus precludes default; we will refer to this as the productivity threshold. Further, let $\pi_t(a_{t+1}) = \pi(a_{t+1}|a_t)$ be the probability of a particular value a_{t+1} conditional on a_t. Then, the probabilities of default and of non-default in $t+1$ conditional on the information in t are

$$\begin{aligned} prob\left(\Psi_{t+1} < b_t | a_t, b_t\right) &= \int_{a_l}^{a_{t+1}^*} \pi_t\left(a_{t+1}\right) da_{t+1}, \\ prob\left(\Psi_{t+1} \geq b_t | a_t, b_t\right) &= \int_{a_{t+1}^*}^{a_h} \pi_t\left(a_{t+1}\right) da_{t+1}. \end{aligned}$$

These probabilities can be used to rewrite the asset pricing equation (7), which includes the expectation term $E_t\left[c_{t+1}^{-\sigma}\left(1-\delta_{t+1}\right)\right]$, where we account for the possibility that consumption and the default rate are not independent. According to the assumptions in section 2.2, the default rate δ_{t+1} equals zero if $\Psi_{t+1} \geq b_t$, and $\delta_{t+1} = 1 - \Psi_{t+1}/b_t$ if $\Psi_{t+1} < b_t$. Hence, the term $E_t\left[c_{t+1}^{-\sigma}\left(1-\delta_{t+1}\right)\right]$ satisfies

$$\begin{aligned} & E_t\left[c_{t+1}^{-\sigma}\left(1-\delta_{t+1}\right)\right] \\ = & \int_{a_l}^{a_{t+1}^*} \pi_t\left(a_{t+1}\right)\left[c_{t+1}^{-\sigma} \cdot \left(\Psi_{t+1}/b_t\right)\right] da_{t+1} + \int_{a_{t+1}^*}^{a_h} \pi_t\left(a_{t+1}\right)\left[c_{t+1}^{-\sigma} \cdot (1-0)\right] da_{t+1}. \end{aligned}$$

Using the solutions (18) and (20), the asset pricing equation (7) can thus be written as

$$1/R_t = \frac{\beta}{c_t^{-\sigma}}\left[b_t^{-1}\int_{a_l}^{a_{t+1}^*} \pi_t\left(a_{t+1}\right)\left[c_{t+1}^{-\sigma}\Psi_{t+1}\right] da_{t+1} + \int_{a_{t+1}^*}^{a_h} \pi_t\left(a_{t+1}\right)\left[c_{t+1}^{-\sigma}\right] da_{t+1}\right]. \quad (24)$$

Risk premia can then be computed as follows (further details can be found in appendix A.2): At the beginning of period t, b_{t-1} is known and the stochastic productivity level a_t realizes. We get solutions $\{c_t, s_t\}$ from (18) and (20). Then, we can compute the debt repayment capacity using (22), where the conditional expectation in (22) is calculated using a discrete transition probability matrix for productivity. If $\Psi_t < b_{t-1}$, the government defaults. For $\Psi_t \geq b_{t-1}$, the government does not default in period t. The bond price

$1/R_t$, end-of-period debt b_t, and the productivity threshold a^*_{t+1} then simultaneously solve (24), the updated version of (23) which reads $b_t = \Psi\left(a^*_{t+1}\right)$, and the government's flow budget identity

$$b_t/R_t = (1-\delta_t)b_{t-1} - s_t. \tag{25}$$

We then compute the default risk premium $R_t - R_t^{rf}$ (using 21).

3 Results

In the first part of this section, we apply a simplified version of the model with uniformly distributed productivity values to analytically examine the determination of bond prices. In the second part, we apply a less restrictive version to quantify the relation between debt-to-GDP ratios and interest rate spreads. Throughout the analysis, we restrict our attention to periods $t \neq j$ where transfers are not paid out to households, which allows us to abstract from the impact of the particular transfers scheme on bond prices. We then examine the price of bonds issued in periods $t \neq j$ for the case where the government does not default in the same period t, i.e. for $\delta_t = 0$, and where debt is issued $b_t > 0$ such that default is possible in the subsequent period.

3.1 Determination of bond market equilibrium

To be able to derive analytical results, we apply a simplified version of the model. We assume that a_t is a serially uncorrelated random draw from the uniform distribution with support $[a_l, a_h]$ where a_l and a_h are positive constants. To further simplify the derivation of analytical results, we assume in this section that $\sigma = 1$ and that only the first-order terms of the maximum debt capacity (22) are non-negligible.

With these assumptions, consumption, surpluses, and the debt repayment capacity for all periods $t \neq j$ are linear functions of the current exogenous state a_t:

$$c(a_t) = \gamma(1-\tau)a_t = \theta_2 a_t, \tag{26}$$

$$s(a_t) = (1-\tau)\gamma\tau a_t - (1-\tau)g = \theta_3 a_t - \theta_4, \tag{27}$$

$$\widetilde{\Psi}(a_t) = s(a_t) + (1-\tau)a_t \frac{\beta}{1-\beta}(\gamma\tau - (1/\overline{a})g) = s(a_t) + \theta_1 a_t, \tag{28}$$

where $\widetilde{\Psi}(a_t)$ is the expression for the debt repayment capacity Ψ under the set of simplifying assumptions made in this section. In each line the second equality sign defines the composite parameters $\theta_{1,2,3,4} > 0$. End-of-period debt satisfies $b_t = \widetilde{\Psi}\left(a^*_{t+1}\right)$ (see 23) and the government budget constraint (25) demands $1/R_t = (b_{t-1} - s_t)/b_t = (b_{t-1} - \theta_3 a_t + \theta_4)/\widetilde{\Psi}\left(a^*_{t+1}\right)$. The asset pricing equation (24) can then be written as

$$1/R_t = \beta a_t \left\{ \begin{array}{c} \widetilde{\Psi}\left(a^*_{t+1}\right)^{-1}\left[(\theta_1+\theta_3)\int_{a_l}^{a^*_{t+1}}\pi(a_{t+1})\,da_{t+1} - \theta_4\int_{a_l}^{a^*_{t+1}}\pi(a_{t+1})(1/a_{t+1})\,da_{t+1}\right] \\ +\int_{a^*_{t+1}}^{a_h}\pi(a_{t+1})(1/a_{t+1})\,da_{t+1} \end{array} \right\}.$$

With uniformly distributed productivity levels, this simplifies to

$$1/R_t = \beta\frac{a_t}{a_h - a_l}\left\{\frac{(\theta_1+\theta_3)\left(a^*_{t+1} - a_l\right) - \theta_4\left(\log a^*_{t+1} - \log a_l\right)}{\widetilde{\Psi}\left(a^*_{t+1}\right)} + \left(\log a_h - \log a^*_{t+1}\right)\right\}. \tag{29}$$

Thus, condition (29), which can be interpreted as a credit supply condition, describes the bond price $1/R_t = q_t$ as a function of end-of-period debt $b_t = \widetilde{\Psi}\left(a^*_{t+1}\right)$ for a given exogenous state a_t.

The government's demand for credit is described by the period budget constraint (25), which reads $b_t/R_t = (b_{t-1} - s_t)$. Using (27) it can be written as $1/R_t = (b_{t-1} - \theta_3 a_t + \theta_4)/b_t \Leftrightarrow$

$$1/R_t = (b_{t-1} - \theta_3 a_t + \theta_4)/\Psi\left(a^*_{t+1}\right). \tag{30}$$

Credit supply (29) and credit demand (30) provide two conditions that determine the price

$1/R_t$ and the quantity of debt $b_t = \widetilde{\Psi}\left(a^*_{t+1}\right)$ issued in period t. The bond price can then uniquely be determined, which is summarized in the following proposition.

Proposition 1 *Suppose that productivity is uniformly distributed with $a_t \in [a_l, a_h]$, that $\sigma = 1$ and credit demand exceeds a level below which debt is risk-free, $b_{t-1} - s_t > \widetilde{\Psi}(a_l)/R^{rf}_t$. Then, there exists a unique equilibrium if credit demand satisfies $b_{t-1} - s_t \leq \Omega_{0,t}$, where $\Omega_{0,t} \equiv \widetilde{\Psi}(a_t) - s_t + (1-\tau)\, g(a_t\beta - 1/R^{rf}_t)$.*

Proof. See appendix A.1. ■

To illustrate the results summarized in proposition 1, we plot credit supply (29) and credit demand (30) as well as the threshold $\Omega_{0,t}$, beyond which credit demand is too large to be supported by an equilibrium bond price. For this, we apply some example parameter values. With the uniform distribution for productivity assumed in this section, the graphical exercise is meant to be merely illustrative and serves to highlight the determination of equilibrium bond prices in a tractable environment. Specifically, we apply the following parameter values: As in section 3.2, the discount factor is set to $\beta = 0.97$. The uniform distribution for the productivity level is characterized by $a_l = 0.01$ and $a_h = 1.99$. We set $\gamma = 0.354$ to match a working time in steady state, $l(\overline{a}) = 1/3$. The average government share is set to $g/y(\overline{a}) = 0.20$, and the average tax rate is $\tau = 0.35$. In the next section, we consider a parameterization intended to match certain characteristics of European data in the context of a more realistic specification of the process governing productivity.

For these parameter values, figure 1 shows credit supply (29) (solid line) and demand (30) (dashed line), where both sides of (29) and (30) are multiplied with next period debt, $b_t = \widetilde{\Psi}\left(a^*_{t+1}\right)$, such that their LHS give the market value of issued debt $q_t b_t$ $(= b_t/R_t)$. We assume that the current productivity level a_t equals its unconditional mean of 1. The dotted line further shows the threshold $\Omega_{0,t}$. For the beginning-of-period debt-to-GDP ratio determining credit demand by the government (see 30), we assume $b_{t-1}/y_t = 0.8$, which is an arbitrary example value in this context due to the extreme assumption of uniformly distributed productivity. Figure 1 shows the amount of revenues raised by issuing debt $q_t b_t$ for all possible realizations of a^*_{t+1} (over the entire support of the productivity distribution). The intersection of credit demand and supply determines the (future) productivity threshold a^*_{t+1} (and thus $q_t b_t = q_t \widetilde{\Psi}\left(a^*_{t+1}\right)$).

<< Figure 1 about here >>

Consider first credit demand, $1/R_t = (b_{t-1} - s_t)/b_t$. It implies end-of-period debt to be proportional to the interest rate for a given stock of initial debt b_{t-1} and the exogenous state a_t. This reflects the fact that the government has to issue more bonds b_t if the price q_t is lower, or the interest rate $R_t = 1/q_t$ is higher. In figure 1, where credit demand (dashed line) is multiplied by next period debt, $b_t = \widetilde{\Psi}\left(a^*_{t+1}\right)$, the market value of debt $q_t b_t = b_{t-1} - s_t$ is a horizontal line as the current productivity level and initial debt are given. A higher beginning-of-period debt level b_{t-1}, or a lower current productivity level a_t (leading to lower current surpluses) both imply larger financing needs of the government, and thus an increased credit demand by the government. In figure 1 this would lead to a parallel upward shift of the dashed horizontal line corresponding to (30).

Credit supply (29) also implies a positive relation between b_t and R_t, since future surpluses that suffice to repay debt become less likely for higher thresholds a^*_{t+1} $(= \widetilde{\Psi}^{-1}(b_t))$. This tends to reduce the expected return from bonds (since it increases the probability of default) and induces investors to demand a higher risk premium as a compensation. Yet, with higher end-of-period debt levels the risk premium and, thus, the interest rate increase more than proportionally. The solid line in figure 1 shows credit supply in terms of the market value $q_t b_t$. Debt is repaid up to the amount that equals the present value of surpluses. In turn, the repayment rate is large enough (haircuts are small enough) such that the relation between (compensating) interest rates and debt leads to a unique equilibrium (see proposition 1).[9]

3.2 Quantitative results

In this section, we relax the simplifications made in the previous section and solve the model numerically for a more realistic parametrization. In particular, we replace the assumption of uniformly distributed productivity shocks (which was only used for analytical tractability above) with a more conventional assumption of normally distributed innovations to a first-order Markov process governing productivity. The debt capacity Ψ_t is computed numerically, without relying on local approximation. Details on the computation are described in appendix A.2. In a first step, we calculate the pricing curves implied by the model. These curves show the relation between the model's state variables, initial debt and productivity, and the implied equilibrium interest rate premium on government bonds. Then, we compare the quantitative model predictions to observed data on sovereign bond spreads for various countries.

For the quantitative analysis, we consider different calibrations of the model. As a point of departure, we calibrate the model for the average member country of the European Monetary Union. In addition, we calibrate the model for Greece, Portugal, and Spain, which are countries that currently face substantial fiscal stress. Finally, we use a parameterization intended to capture the fiscal stance of Italy and Belgium, which are interesting cases

[9] Note that if $b_{t-1} > \widetilde{\Psi}(a_t) + (1-\tau)\, g(a_t\beta - 1/R_t^{rf})$, there exists no bond market equilibrium. The possibility of multiple bond price equilibria has recently rekindled the interest of macroeconomists, see Lorenzoni and Werning (2013). In the working paper version, we showed that multiple bond market equilibria can arise under a scenario where it is assumed that a default event triggers investors to stop lending to the government. These results are available upon request.

Table 1: Summary of parameter values.

	Eurozone average	Greece	Portugal	Spain	Italy	Belgium
$g/y(\overline{a})$ (government share)	0.205	0.172	0.196	0.183	0.192	0.224
τ (tax rate)	0.250	0.208	0.225	0.219	0.283	0.298
Implied output volatility	0.028	0.033	0.031	0.025	0.017	0.015

to compare because spreads for these countries are relatively low although their debt-to-GDP ratio is quite high. While model periods are typically interpreted as quarters in the business cycle literature, we deviate from this assumption and interpret one period as a year, which facilitates computing theoretical predictions for government bonds with one-year maturity (without considering multi-period bonds). Throughout the quantitative analysis, we set the discount rate at $\beta = 0.97$ to match a standard average value for a risk-free annual real interest rate.[10] The mean working time share is $l(\overline{a}) = 1/3$.[11] The level of relative risk aversion is set to $\sigma = 2$.[12]

Table 1 shows the country-specific values for the tax rates and government expenditure

[10] The annual yield on German one year government bonds (from the Bundesbank, see http://www.bundesbank.de) has been 2.91%, when deflated with the growth rate of the GDP deflator (from AMECO, see http://ec.europa.eu/economy_finance/db_indicators/ameco/index_en.htm) and averaged over the longest pre-crisis period available (1974-2008).

[11] This is achieved by setting the utility parameter γ as $\gamma = [(1-(g/y(\overline{a}))) \cdot l(\overline{a})] / (1-\tau)$, where the government share and the tax rate take the country-specific values given in table 1.

[12] In our model, the quantitative effect of risk aversion is relatively small. The reason is that larger degrees of risk aversion, on the one hand, tend to lead to a larger premium due to the uncertainty of payoffs for a given consumption sequence (in accordance with the logic of standard asset pricing theory), and, on the other hand, to less volatile consumption sequences in equilibrium, which tends to reduce the risk premium. We find that both effects are of similar size, such that the total effect of changes in σ on bond prices is relatively small. This holds in particular compared to the size of the price effect of the default probability which is the primary focus of our analysis.

shares. These are sample averages of annual data from Eurostat's national accounts, where the sample period is 1995 to 2010 due to the limited availability of tax data. The government share is measured as the ratio of government consumption over GDP, and the tax rate is measured as total tax revenues over GDP (in both cases, referring to total government at all federal levels).[13] For all calibrations, we assume that productivity follows a first-order Markov process with an autocorrelation of 0.9 and normally distributed innovations. We choose the innovation variance such that the realized standard deviation of HP-filtered log output from stochastically simulated model runs conforms with the standard deviation of HP-filtered log real annual GDP for the countries that we consider (using an HP-filter smoothing parameter 100; annual real GDP is available from 1960 to 2011 from the European Commission's AMECO data base[14]).

Figure 2 shows the model's implied equilibrium relation between the interest rate spread of risky government bonds over the riskless interest rate, $R_t - R_t^{rf}$, and the beginning-of-period ratio of debt to output for a given productivity level.[15] Note that these spreads refer to states of the economy where the government does not default on its debt. In the figure, the solid lines display risk premia when the current state is at the mean productivity level ($a_t = \overline{a} = 1$), while the dashed and dotted lines correspond to lower productivity levels that indicate moderate or deep recessions, respectively (the dashed line is for $a_t = 0.975$ and the dotted line is for $a_t = 0.95$). When productivity declines, government surpluses and thus the debt capacity decline. Both effects shift the equilibrium bond pricing rule to the left. To facilitate a direct comparison of the various

[13] See http://epp.eurostat.ec.europa.eu/portal/page/portal/statistics/search_database.

[14] See http://ec.europa.eu/economy_finance/db_indicators/ameco/index_en.htm.

[15] Appendix A.2 presents details on the computation of equilibrium bond prices.

calibrations, we use the same scaling for the axes for the various specifications, the exceptions being the specifications for Italy and Belgium (discussed below). Note that no model parameter is specifically chosen to let the model match observations related to the level or the price of public debt.

<< Figure 2 about here >>

First, figure 2 shows that risk premia are generally indistinguishable from zero for low debt-to-GDP ratios. This can can even hold for debt-to-GDP ratios exceeding 100% if surpluses are sufficiently high, as is the case for the EU average or for Belgium and Italy. Second, above certain thresholds that depend on the country-specific fiscal parameters, risk premia tend to rise sharply with higher debt-to-GDP ratios, and in a convex way. Both model characteristics are compatible with the recent history of interest rate spreads on government debt in the Eurozone. Empirically, spreads were close to non-existent prior to the 2008 recession, but then increased steeply when the shortfall in government revenues let public deficits and debt levels soar (a more detailed discussion on the empirical evidence on risk spreads follows below). It is noteworthy that this qualitative pattern can be generated by the model: The flat part of the pricing curve is explained as the near-absence of default risk in situations when debt levels are far away from the government's repayment capacity. The convexity of the increase in the risk premium is explained as the interaction of debt levels approaching the repayment capacity with the expected amount of repayment in the case of default. Third, and unsurprisingly, there exist levels of initial indebtedness which lead to practically zero risk premia in normal times (productivity at its mean, solid lines), while triggering strong increases in spreads in a recession (dashed and dotted lines) due to the decrease in government revenues.

For a country with the average Eurozone member's parameter values (concerning taxes, spending, and output volatility) the level of initial debt above which noticeable risk spreads begin to emerge is rather high (about 130% debt-to-GDP). By contrast, in line with empirical evidence, our model predicts that risk premia begin to rise much earlier (i.e. a lower levels of debt-to-gdp) in those countries that recently have been in serious fiscal trouble, like Greece, Spain, and Portugal. Thus, the model, while admittedly stylized, is in principle able to explain (based on the parameters concerning tax revenues and government spending as well as the level of output volatility) why some countries experience sharply rising interest rates on government debt. In this sense, the main implications of the model are broadly in line with recent experiences of some Eurozone countries. On the other hand, according to our model, countries like Belgium or Italy should not be in noteworthy fiscal difficulties at observed debt-to-GDP ratios, because in these countries the level of taxation is particularly high (see table 1).

The comparison between the model predictions and recent European fiscal history can be pushed a bit further by looking at figure 3, which shows empirical evidence concerning the relation between government debt-to-GDP ratios on the horizontal and interest rate spreads on government bonds on the vertical axis. The data are from the OECD Economic Outlook database and are quarterly from 2005q1 to 2012q1. The spread is calculated as the difference between the yield to maturity on long-term (typically ten year) government bonds of the country shown in each panel and the corresponding German rate (which amounts to the common practice of considering the German government bond rate as pertaining to a relatively safe asset), expressed in basis points. Note that the empirical patterns displayed in figure 3 are measured over different states of the business cycle, which renders a direct comparison with the theoretical pricing curves in figure 2 impossible (which are drawn for constant values of productivity each). However, with this caveat in mind, some interesting conclusions can be drawn. Most notably, the data do seem to imply a convex relation between debt-to-GDP ratios and interest rate spreads. Thus, the evidence is compatible with this key prediction of the model, although the model tends to imply a steeper rise in spreads than found in the data.

Further, some observations are roughly in line with the model's predictions. In particular, the rise in spreads on Greek debt for debt ratios larger than 100% is well accounted for by our model. Similarly, the model is able to explain that spreads on Portuguese spreads begun to emerge at a smaller debt-to-GDP ratio of around 80%. The case of Spain is more difficult. Empirically, Spain experiences large risk premia already at relatively low debt ratios around 70% of GDP, while the model suggests that this should not be the case for values much below 100% of GDP (see the fourth panel in figure 2). One possible explanation is that investors expect the Spanish government to eventually guarantee parts of the debt of the country's banking sector, which would imply that observed debt figures understate the dimension of the fiscal problem. In the case of Belgium, the model predictions are in line with observations in so far as empirical spreads are much lower in Belgium than in Portugal or Spain despite similar debt-to-GDP ratios, the reason being Belgium's high level of taxation. Yet, the model implies that Belgian spreads are practically zero for debt-to-GDP ratios below 200%, whereas in reality they are relatively small but clearly different from zero. The same is true in the case of Italy, for which, according to our model, default risk should not be an issue for debt-to-gdp ratios smaller than 2.5 times GDP, given Italy's exceptionally high historical level of taxation. However, Italy has experienced non-negligible spreads in the recent past. This discrepancy between model prediction and empirical evidence might be due to the fact that markets doubt Italy's commitment to Eurozone membership and thus reflect convertibility risk, or could point out that investors are not convinced that the government will repay its debt as long as it is solvent.

<< Figure 3 about here >>

4 Conclusion

This paper shows that risk premia on government bonds can be rationalized in a simple macroeconomic framework with lenders rationing lending when outstanding public debt exceeds a government's debt repayment capacity, i.e. the present value of primary surpluses. A government that does not commit to repay debt is then unable to fully roll over debt. As a consequence, default occurs when the state of the economy worsens sufficiently such that the build-up of public debt exceeds the present value of future surpluses. The default risk premium on government bonds reflects the probability of this event and the expected rate of partial repayment in case of default. We assume that households provide the maximum volume of credit to the government compatible with expected full repayment tomorrow. The model predicts that above a critical debt-to-GDP ratio, risk premia begin to rise steeply, since market participants expect that further adverse macroeconomic shocks may lead to a situation where the government's debt repayment capacity is exceeded. The critical debt-to-GDP ratio depends on fiscal parameters, for which we apply values that accord to average European Monetary Union member countries as well as to countries that have recently faced severe refinancing problems, such as Greece, Portugal, and Spain. The quantitative predictions of the model are broadly in line with the experience of those countries most affected by the recent debt crisis. In particular, the model can replicate the critical range of indebtedness, the convex relation between interest rate spreads and debt-to-GDP (though it overstates its slope), as well as the observed non-responsiveness of interest rates to changes in debt-to-GDP ratios that relate to pre-crisis years.

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A Appendix

A.1 Proof of proposition 1

To lighten the notation in this section, we drop the time index and define $b = b_t$, $b_{-1} = b_{t-1}$, $a = a_t$, $a' = a_{t+1}$, and $a^* = a^*_{t+1}$. For $a^* \to a_l$, the RHS of (29) is given by $a\beta \frac{\log a_h - \log a_l}{a_h - a_l}$, which is the inverse of the risk-free rate $R^{rf} = 1/\left(a\beta \frac{\log(a_h/a_l)}{a_h - a_l}\right)$. Now combine credit supply (29) with credit demand $1/R = (b_{-1} - \theta_3 a + \theta_4)/\widetilde{\Psi}(a^*)$ to get the following condition for a^*:

$$b_{-1} - (\theta_3 a - \theta_4) = \kappa \left\{ [\theta_1 + \theta_3](a^* - a_l) - \theta_4 (\log a^* - \log a_l) + \widetilde{\Psi}(a^*)(\log a_h - \log a^*) \right\} \tag{31}$$

where $\kappa = \beta \frac{a}{a_h - a_l}$. To examine the solution(s) for a^*, we define $F(a^*)$ as

$$\begin{aligned} F(a^*) \equiv \kappa & \left\{ [\theta_1 + \theta_3](a^* - a_l) - \theta_4 (\log a^* - \log a_l) + \widetilde{\Psi}(a^*)(\log a_h - \log a^*) \right\} \\ & - (b_{-1} - (\theta_3 a - \theta_4)), \end{aligned}$$

and solve for $F(a^*) = 0$. At the lower bound of the support, $F(a^*)$ is given by $F(a_l) = \frac{1}{R^{rf}} \widetilde{\Psi}(a_l) - (b_{-1} - (\theta_3 a - \theta_4))$ implying that $F(a_l) < 0$ if credit demand is sufficiently large

$$b_{-1} - (\theta_3 a - \theta_4) > \frac{1}{R^{rf}} \widetilde{\Psi}(a_l). \tag{32}$$

At the upper bound of the support, $F(a_h)$ satisfies $F(a_h) = \beta a \left([\iota \cdot \theta_1 + \theta_3] - \frac{\log a_h - \log a_l}{a_h - a_l} \theta_4 \right) + \theta_3 a - \theta_4 - b_{-1}$, implying that $F(a_h) \geq 0$ if credit demand does not exceed a particular threshold

$$(b_{-1} - (\theta_3 a - \theta_4)) \leq a\beta (1 - \tau) \left[\frac{\beta(\gamma\tau - g)}{1 - \beta} + \gamma\tau \right] - \frac{1}{R^{rf}} (1 - \tau) g, \tag{33}$$

where we used $\theta_1 = (1-\tau)\frac{\beta}{1-\beta}(\gamma\tau - g)$, $\theta_3 = (1-\tau)\gamma\tau$, $\theta_4 = (1-\tau)g$, and $1/R^{rf} = a\beta\frac{\log(a_h/a_l)}{a_h - a_l}$. Next, we examine the slope of $F(a^*)$, which is given by

$$F'(a^*) = \kappa\widetilde{\Psi}'(a^*)\left[-\Xi + (\log a_h - \log a^*)\right], \tag{34}$$

where $\widetilde{\Psi}'(a^*) > 0$ (see 28). At the upper bound, the derivative of $F(a^*)$ equals zero $F'(a_h) = 0$. Otherwise, the derivative is strictly positive, $F'(a^*) > 0\ \forall a^* < a_h$. Hence, there exists a unique bond market equilibrium if (32) and (33) are satisfied, where the latter can be written as $b_{-1} - (\theta_3 a - \theta_4) \leq \beta a(\theta_1 + \theta_3 - \frac{\log a_h - \log a_l}{a_h - a_l}\theta_4)$, or using $\widetilde{\Psi}(a) = \theta_1 a + \theta_3 a - \theta_4$ as

$$b_{-1} \leq \widetilde{\Psi}(a) + (1-\tau)g(a\beta - 1/R^{rf}).$$

If the latter is violated, $F(a_h) < 0$ and there exists no equilibrium. ■

A.2 Computation of equilibrium bond prices

We consider a discrete valued version of the problem.[16] We use Tauchen's (1986) algorithm to approximate the first order Markov process for productivity by a discrete-valued Markov chain. We provide the size of the interval $I_a = [a_1, a_n]$ and the number of grid points, n (we use $n = 801$). Given the autocorrelation ρ, the interval I_a is chosen to include ± 4 standard deviations of the productivity process. Tauchen's algorithm then delivers the exogenous state space of the model $S = \{a_1, a_2, ..., a_n\}$, $a_i < a_{i+1}, i = 1, 2, ..., n-1$, and the associated transition probability matrix $P = (p_{ij})$, whose row i and column j element is the probability of moving from state a_i state to state a_j.

We calculate the debt repayment capacity $\Psi_t = E_t \sum_{k=0}^{\infty} \beta^k \frac{c_{t+k}^{-\sigma}}{c_t^{-\sigma}} s_{t+k}$ using simulation techniques. Specifically, we draw time series of productivity realizations for a sample

[16] As an alternative to the discretization, one can solve the integrals in the asset pricing equation (24) using numerical integration.

length of 1,000 (conditional on the starting value a_t) and then determine the associated sequences of consumption and surpluses. These series are used to approximate the infinite sum of future discounted surpluses. We further approximate the conditional expectation by repeating this simulation 10,000 times and taking the mean over all repetitions.

For a given combination of initial debt b_{t-1} and current productivity level a_t, the equilibrium interest rate spread on government bonds is determined as follows:

- For a given current productivity level, a_t, current consumption $c_t = c(a_t)$, surpluses $s_t = s(a_t)$, and the maximum debt repayment capacity of the current period, $\Psi_t = \Psi(a_t)$ are computed with (18), (20), and (22). The risk-free rate is determined using $R_t^{rf} = c_t^{-\sigma} / \left(\beta E_t c_{t+1}^{-\sigma}\right)$, where the conditional expectation $E_t c_{t+1}^{-\sigma}$ is computed by $E_t c(a_{t+1})^{-\sigma} = \sum_{j=1}^{n} p_{ij} \cdot c(a_j)^{-\sigma}$ and i denotes the index number for today's stochastic state, a_t.

- Then, we check whether the government defaults in period t or not. If $\Psi_t < b_{t-1}$, the government defaults, end-of-period debt equals zero, $b_t = 0$, and there is no borrowing. If $\Psi_t > b_{t-1}$, the government does not default in period t and the bond market equilibrium price is determined as follows:

  - The bond market equilibrium consists of a price $1/R_t$ and end-of-period debt b_t. Replacing the integrals in (24) by sums over the finite number of states, the asset pricing equation reads

$$\frac{b_{t-1} - s_t}{b_t} = \frac{\beta}{c_t^{-\sigma}} \begin{bmatrix} b_t^{-1} \sum_{a_{t+1}=a_1}^{a_{t+1}^*} \pi_t(a_{t+1}) \left[c(a_{t+1})^{-\sigma} \Psi(a_{t+1}) \right] \\ + \sum_{a_{t+1}=a_{t+1}^*}^{a_n} \pi_t(a_{t+1}) \left[c(a_{t+1})^{-\sigma} \right] \end{bmatrix} \tag{35}$$

Use the updated version of (23), $b_t = \Psi\left(a^*_{t+1}\right)$, to replace b_t in (35):

$$b_{t-1} - s_t = \frac{\Psi\left(a^*_{t+1}\right)\beta}{c_t^{-1}} \left[\begin{array}{c} \Psi\left(a^*_{t+1}\right)^{-1} \sum_{a_{t+1}=a_1}^{a^*_{t+1}} \pi_t\left(a_{t+1}\right)\left[c\left(a_{t+1}\right)^{-\sigma}\Psi\left(a_{t+1}\right)\right] \\ +\sum_{a_{t+1}=a^*_{t+1}}^{a_n} \pi_t\left(a_{t+1}\right)\left[c\left(a_{t+1}\right)^{-\sigma}\right] \end{array} \right]. \tag{36}$$

Equation (36) is then solved for the unknown productivity threshold in the next period, a^*_{t+1}.

– Given the solution for a^*_{t+1}, next-period's debt level b_t and the asset price $1/R_t$ are determined by $b_t = \Psi\left(a^*_{t+1}\right)$ and

$$\frac{1}{R_t} = \frac{\beta}{c_t^{-\sigma}} \left[\begin{array}{c} \Psi\left(a^*_{t+1}\right)^{-1} \sum_{a_{t+1}=a_1}^{a^*_{t+1}} \pi_t\left(a_{t+1}\right)\left[c\left(a_{t+1}\right)^{-\sigma}\Psi\left(a_{t+1}\right)\right] \\ +\sum_{a_{t+1}=a^*_{t+1}}^{a_n} \pi_t\left(a_{t+1}\right)\left[c\left(a_{t+1}\right)^{-\sigma}\right] \end{array} \right]$$

– The risk premium on government bonds for given states b_{t-1} and a_t is calculated as $R_t - R_t^{rf}$.

Figures

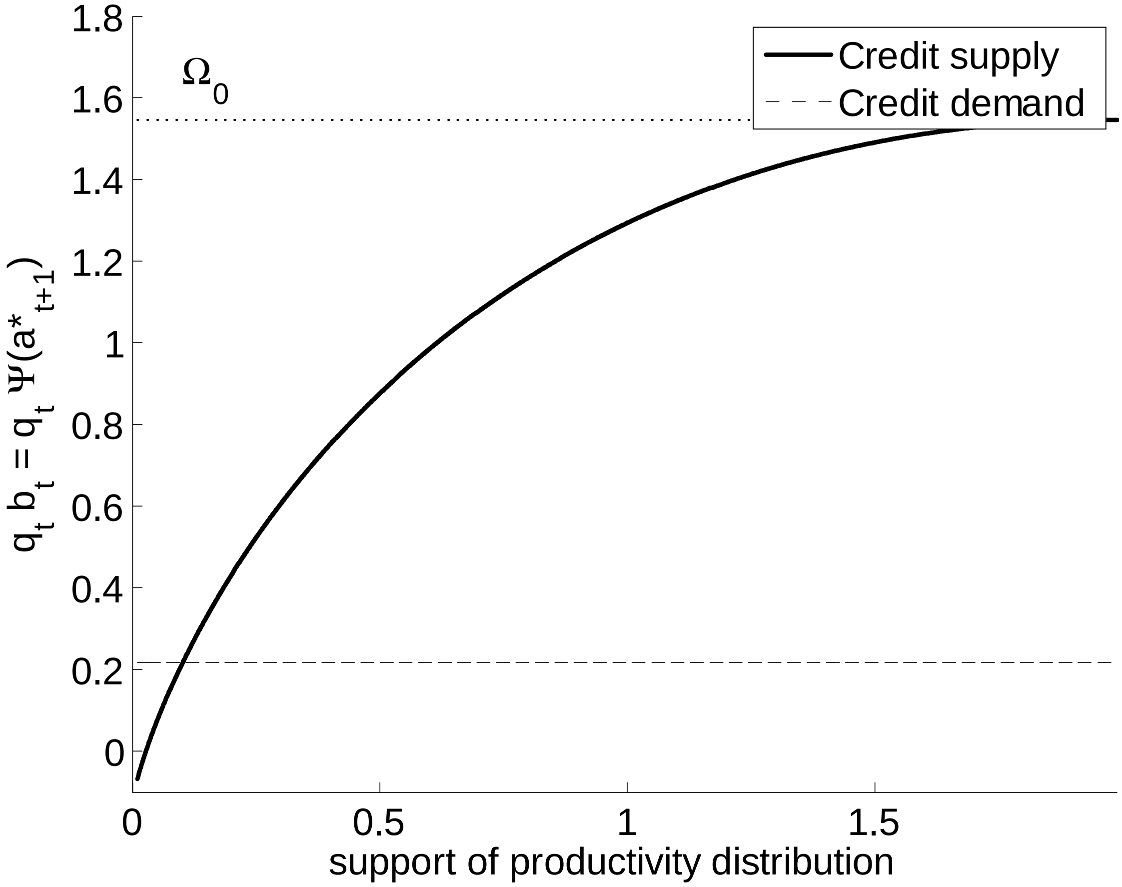

Figure 1: Bond market equilibrium (uniformly distributed productivity shocks)

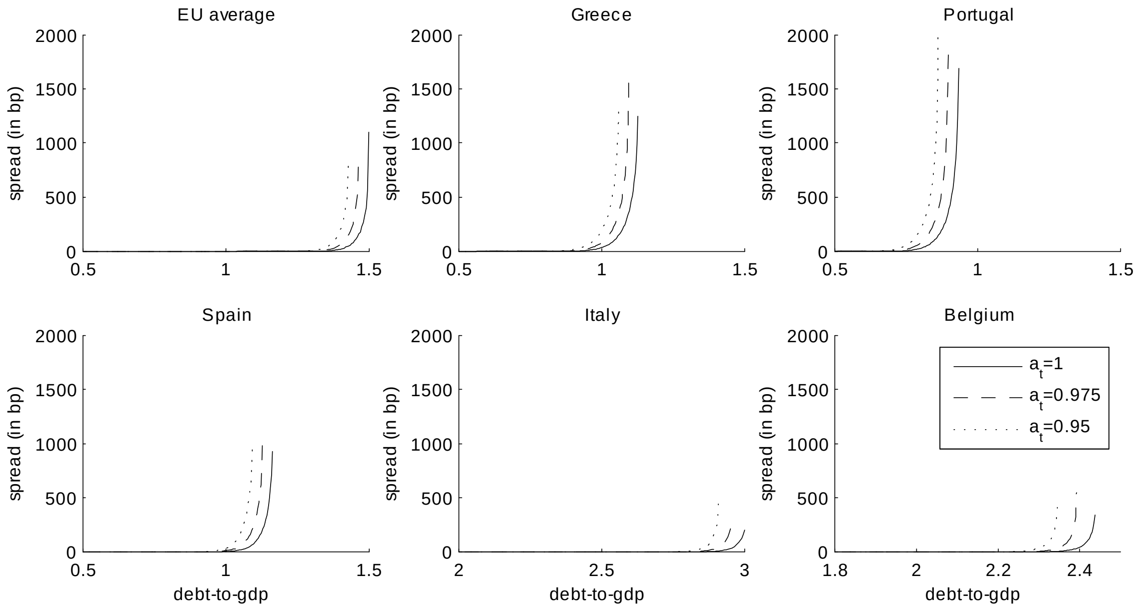

Figure 2: Model-predicted interest rate spreads for various countries

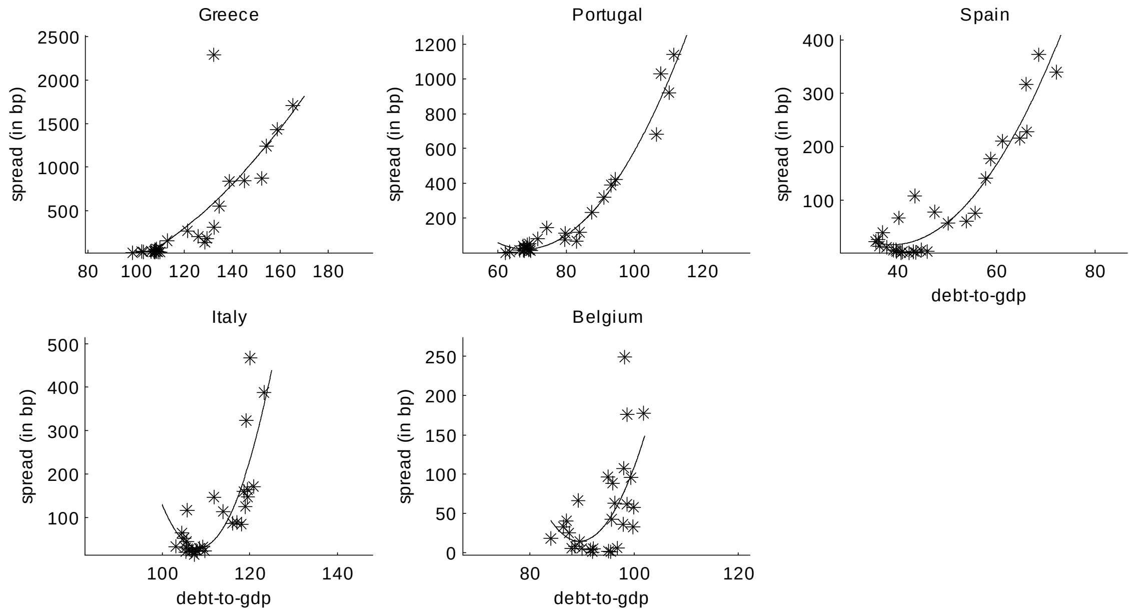

Figure 3: Empirical interest rate spreads on government bonds