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A MONETARY ANALYSIS OF BALANCE SHEET POLICIES

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A Monetary Analysis of Balance Sheet Policies[1]

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Abstract

We augment a standard macroeconomic model to analyze the effects and limitations of balance sheet policies. We show that the central bank can stimulate real activity by changing the size or the composition of its balance sheet, when interest rate policy is ineffective. Specifically, the central bank can stabilize the economy by increasing money supply against eligible assets even when the policy rate is at the zero lower bound. By changing the composition of its balance sheet, it can affect interest rates and, for example, neutralize increases in firms' borrowing costs, which is not possible under a single instrument regime. We further analyze the limitations of balance sheet policies and show that they are particularly useful under liquidity demand shocks.

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Keywords: Unconventional monetary policy, collateralized lending, quantitative easing, liquidity premium, zero lower bound.

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1 Introduction

Central banks in industrialized countries have responded to the recent financial crisis with unconventional monetary policies. The Bank of England (BoE) and the US Federal Reserve (Fed), for example, have set the policy rate at its zero lower bound (ZLB) and introduced various lending facilities as well as direct asset purchases.[3] These policies, which have been summarized by the term "balance sheet policy" (see Borio and Disyatat, 2009), were aimed at reducing spreads attributable to illiquidity (see Kocherlakota, 2011), stabilizing stressed credit markets (see Yellen, 2009), and stimulating spending and real activity (see Bean, 2009). However, they have been implemented with only little theoretical or empirical guidance available. In particular, conventional macroeconomic models are unable to explain how liquidity providing operations can be effective at the ZLB, where money demand is typically not well defined.

In this paper, we augment a standard macroeconomic model to be applicable for the analysis of balance sheet policies in addition to pure interest rate policy, on which the New Keynesian paradigm has focussed. Given that we aim at providing a basic framework that facilitates a generic analysis of the effects and the limitations of balance sheet policies, we specify the model in a sufficiently simple way to derive analytical results. We thereby focus on monetary policy implementation and money supply by the central bank, while we disregard the possibility of central banks to mitigate disruptions of private financial intermediation.[4] We show that changing the size and the composition of the central bank balance sheet can be non-neutral, as long as money is positively valued and assets eligible for central bank liquidity providing operations are scarce; the latter property being reflected by the existence of a liquidity premium. We show that balance sheet policies are particularly useful when the implementation of a stabilizing policy via policy rate adjustments reaches its limits. This is demonstrated for exogenously driven shifts in firms' borrowing costs that cannot be neutralized by policy rate adjustments alone and for the case where the policy rate hits the ZLB. We further examine the scope of balance sheet policies and quantify their maximum effects. The analysis, in particular, rationalizes the types of liquidity providing facilities that were introduced by the BoE or the Fed in 2008-2009.

[3] Under the Asset Purchase Facility the Bank of England purchased commercial papers, corporate bonds, and government bonds. The US Federal Reserve, for example, introduced the Term Auction Facility, which provided short-term credit to depository institutions, the Commercial Paper Funding Facility, where three-month commercial paper were purchased, and the Treasury Securities Lending Facility, which provided Treasury securities in exchange for mortgage-backed securities and commercial paper.

[4] This has been analyzed in related studies, where central banks provide financial intermediation (e.g. direct central bank lending) in situations where private financial intermediation is more costly due to severe financial frictions (see Curdia and Woodford, 2011, Gertler and Karadi, 2011, and Gertler and Kiyotaki, 2011).

The term *quantitative easing* refers to an increase in the supply of reserves via purchases of securities, such as government bonds (see Bernanke et al., 2004). Conducting such a policy when the policy rate is at the ZLB should be ineffective according to conventional macroeconomic models since money demand is not well defined or assumed to equal a satiation level at the ZLB (see Krugman, 1998, Walsh, 2010). Specifically, quantitative easing in terms of treasury securities should be irrelevant as long as they do not change expectations about the future conduct of monetary and fiscal policy (see Eggertsson and Woodford, 2003, or Curdia and Woodford, 2011). Moreover, a policy that exclusively changes the composition of the central bank's balance sheet, which will be labelled *collateral policy* in this paper,[5] is obviously neutral in single interest rate models, where assets are perfect substitutes. Hence, standard macroeconomic models are hardly able to account for broad empirical evidence, which suggests that the above mentioned lending facilities of the BoE and the Fed have been effective, in particular, by easing money supply and by reducing liquidity premia (see e.g. Joyce, 2010, and Fleming, 2012, for an overview).

We apply a macroeconomic model that mainly differs from a canonical New Keynesian model by accounting for the scarcity of assets eligible in open market operations. We assume that government bonds as well as corporate debt can serve as collateral for central bank operations, whereas other assets (like debt issued by households) are not eligible. The central bank sets the policy rate, i.e. the price of money in terms of eligible assets, and decides on the size and the composition of its balance sheet. Private agents rely on money for goods market purchases, while money is supplied only in exchange for eligible assets, which leads to a spread between the interest rate on non-eligible and eligible assets, i.e. a liquidity premium. Thus, interest rates on non-eligible assets can be positive, even if the policy rate is at the ZLB, which is consistent with the empirical observation that interest rates on non-money market securities typically do not hit the ZLB. This implies positive opportunity costs of money holdings, such that money demand is well defined and expansionary balance sheet policies can be non-neutral.

Firms are assumed to demand loans for working capital and to issue debt subject to default risk. An increase in default risk, which is induced by shocks to the distribution of idiosyncratic productivity (like in Christiano et al., 2013), raises firms' costs of borrowing and thereby exerts a downward pressure on production. We further consider demand shocks, e.g. shocks to the rate of time preference and liquidity demand shocks, which both can induce an endogenously adjusted

[5] A credit easing policy has been defined in a broader way by Bernanke (2009) as: "the Federal Reserve's credit easing approach focuses on the mix of loans and securities that it holds and on how this composition of assets affects credit conditions for households and businesses".

policy rate to hit the ZLB. In this framework, we examine quantitative easing (i.e. an increase in the amount of eligible assets), which raises money supply like a conventional money injection, and collateral policy (i.e. accepting loans as collateral while keeping the size of the balance sheet constant), which can lower the firms' cost of borrowing by reducing the (il-)liquidity premium on loans. We show that both types of balance sheet policies affect the equilibrium allocation and prices when eligible assets are scarce (or, phrased in technical terms, when the collateral constraint in open market operations is binding), which is reflected by a liquidity premium on these assets.[6] If, however, an expansionary monetary policy is conducted in an excessive way, balance sheet policies can become ineffective when the valuation of liquidity falls to zero, indicating that collateral is abundant.

Our main results can be summarized as follows. Quantitative easing and collateral policy are in general not equivalent to policy rate adjustments and can enhance the ability of the central bank to stabilize inflation and output compared to a pure interest rate policy. We show that a collateral policy, i.e. exchanging corporate debt against government bonds held by the central bank, directly affects firms' borrowing costs and therefore the marginal costs of production. In contrast to a pure interest rate policy, a collateral policy can thus neutralize an increase in borrowing costs of firms induced by adverse (default risk) shocks.[7] Quantitative easing can enable a central bank to implement a stabilizing policy even when the policy rate is at the ZLB and the central bank cannot commit to future policies. To explore the limits of balance sheet policies, which are reached when a stimulating policy drives down the liquidity premium to zero, we present numerical results for an augmented version of the model with capital accumulation. We find that the maximum effect of an isolated quantitative easing policy on output is equivalent to the output effect of a 7 basis point reduction in the policy rate. We further consider a shock to the liquidity demand for investment, which has been suggested by Del Negro et al. (2013) as major factor in the crisis of 2008. This shock drives downs the policy rate to the ZLB and leads to a pronounced output contraction as well as a to strong increase in the liquidity premium. We find that even a maximum quantitative easing policy cannot neutralize this shock, though, it can mitigate the output contraction by 50%.

[6] A liquidity premium exists when eligible assets can be exchanged against money at a price (i.e. the policy rate) that is lower than the consumption Euler equation rate, which measures private agents' marginal valuation of money. Based on US data, Canzoneri et al. (2007) provide evidence in favor of a positive average spread between a standard consumption Euler equation rate and the policy rate, which they identify with the Federal Funds rate.

[7] In a companion paper, Schabert (2012) applies a closely related model and shows that the additional monetary policy instruments can help to overcome the well-known monetary policy trade-off between stabilizing prices and closing output-gaps.

Thus, balance sheet policies are particularly powerful when the economy is hit by liquidity demand shocks, which increase liquidity premia, as in the recent financial crisis.

The paper is related to a large literature on monetary policy options at the ZLB, which typically advocates providing monetary stimulus by shaping expectations on future policies is (see e.g. Krugman, 1998, Eggertsson and Woodford, 2003, and Adam and Billi, 2007). Motivated by central bank responses to the recent financial crisis, a literature on non-standard policies under financial market imperfections has recently developed (see Gertler and Karadi, 2011, Gertler and Kiyotaki, 2011, or Curdia and Woodford, 2011), where financial intermediation by the central bank is shown to be beneficial under severe financial market disruptions. Applying a an overlapping generations model where investment in assets are subject to margin requirements, Ashcraft et al. (2011) show that the required return on an eligible asset falls when the central bank reduces the haircut applied to this asset. Chen et al. (2012) examine output and inflation effects of large scale asset purchases in an estimated model with segmented asset markets. Del Negro et al. (2013) consider a negative shock to the resaleability of assets to match the U.S. economy in late 2008, and find that the Fed's policy interventions prevented a second Great Depression.

The paper is organized as follows. Section 2 presents the model. In Section 3, we describe the conditions under which balance sheet policies are effective, and demonstrate that monetary policy instruments are in general not equivalent. In Section 4, we show how balance sheet policies can be applied in response to default risk shocks and in the case where the ZLB on the policy rate is binding. In Section 5, we examine the limits to balance sheet policies. Section 6 concludes.

2 The model

In this Section, we present a sticky price model where money demand is induced by households facing a cash-in-advance constraint and firms requiring working capital. To account for common central bank practice, we assume that money is supplied by the central bank only in exchange for eligible assets, which is modelled by a collateral constraint for open market operations.[8] The central bank sets the policy rate and decides on the size (quantitative easing) and the composition (collateral policy) of its balance sheet. In particular, it controls the fractions of assets that are eligible in open market operations (which can alternatively be interpreted as haircuts on assets under discount window lending). Households' investment decisions take these policies into account, which gives rise to interest rate spreads resulting from liquidity premia. Quantitative easing and

[8] Although, the term collateral only applies to repurchase agreements and not to outright purchases, we follow central banks' practice and we use the term collateral constraint, for convenience.

collateral policy can lower these liquidity premia and can stimulate aggregate demand. To present the problems of households and firms in a transparent way, we introduce indices for individual households and firms.

For analytical convenience, we consider three types of firms.[9] Perfectly competitive intermediate goods producing firms face idiosyncratic productivity shocks, require working capital, and issue intraperiod loans that are subject to default risk. Monopolistically competitive retailers buy intermediate goods and sell a differentiated good at prices set in a staggered way. Competitive bundlers buy the differentiated goods from the retailers and assemble the final good.

2.1 Timing of events

Households enter period t with money, government bonds, and state contingent claims, $M_{i,t-1}^H + B_{i,t-1} + D_{i,t-1}$. They further dispose of a time-invariant time endowment. They supply labor to intermediate goods producing firms, which do not hold any financial wealth. At the beginning of the period, aggregate shocks (including default risk shocks) are realized. Then, the central bank sets its instruments, i.e. it announces the fractions of government bonds and corporate loans that are accepted as collateral in open market operations, $\kappa_t^B \in (0,1]$ and $\kappa_t \in [0,1]$, and sets the policy rate $R_t^m \geq 1$. The remainder of the period can be divided into four subperiods.

1. The labor market opens, where a perfectly competitive intermediate goods producing firm j hires workers $n_{j,t}$. We assume that it has to pay workers their wages in cash before goods are sold. Since the firm does not hold any financial wealth, it has to borrow cash, while it does not commit to repay. Firm j thus faces the constraint

$$L_{j,t}/R_{j,t}^L \geq P_t w_t n_{j,t}, \tag{1}$$

where w_t denotes the real wage rate, P_t denotes the final goods price and $L_{j,t}/R_{j,t}^L$ the amount received by the borrowing firm. Lenders sign standard debt contracts with ex-ante identical firms at the same price $1/R_t^L$, taking into account that a fraction κ_t of all loans can be used as collateral for repurchase agreements (repos) and that a fraction δ_t^e of firms default.

2. Open market operations are conducted, where the central bank sells or purchases assets outright or supplies money via repos against collateral at the rate R_t^m. In contrast to debt issued by households, corporate loans and government bonds can be eligible. In period t,

[9] This allows separating the intratemporal borrowing decision of intermediate goods producing firms from the intertemporal pricing decision of retailers.

household i receives new money (injections) from the central bank $I_{i,t}$ in exchange for eligible assets, where loans are only held under repos.[10] Specifically, the central bank supplies money against fractions of randomly selected bonds κ_t^B and loan contracts κ_t, such that money supply is rationed according to the following collateral constraint:

$$I_{i,t} \leq \kappa_t^B \left(B_{i,t-1}/R_t^m\right) + \kappa_t \left(L_{i,t}/R_t^m\right). \tag{2}$$

After receiving money $I_{i,t}$ from the central bank, household i delivers the amount $L_{i,t}/R_t^L$ to firms according to the debt contract. Its holdings of money, bonds, and loans then are $M_{i,t-1}^H + I_{i,t} - (L_{i,t}/R_t^L)$, $B_{i,t-1} - \Delta B_{i,t}^c$, and $L_{i,t} - L_{i,t}^R$, where $\Delta B_{i,t}^c$ are bonds received by the central bank and $L_{i,t}^R$ are loans under repos, such that $I_{i,t} = (\Delta B_{i,t}^c/R_t^m) + (L_{i,t}^R/R_t^m)$.

3. Wages are paid, idiosyncratic productivities are drawn, and intermediate as well as final goods are produced. Then, the final goods market opens, where purchases of consumption goods require cash holdings. Hence, household i faces the following cash-in-advance constraint in the goods market:

$$P_t c_{i,t} \leq I_{i,t} + M_{i,t-1}^H - \left(L_{i,t}/R_t^L\right) + P_t w_t n_{i,t}. \tag{3}$$

Household $i's$ stock of money then equals $\widetilde{M}_{i,t} = M_{i,t-1}^H + I_{i,t} - (L_{i,t}/R_t^L) + P_t w_{i,t} n_{i,t} - P_t c_{i,t} \geq 0$, while its stock of bonds amounts to $\widetilde{B}_{i,t} = B_{i,t-1} - \Delta B_{i,t}^c \geq 0$.

4. Before the asset market opens, household i receives government transfers $P_t \tau_{i,t}$ and dividends of firms and retailers, which sum up to $P_t v_{i,t}$. Repos are settled, i.e. household i buys back loans $L_{i,t}^R = R_t^m M_{i,t}^L$ and bonds $B_{i,t}^R = R_t^m M_{i,t}^R$ from the central bank. In the asset market, households receive payoffs from maturing assets and the government issues new bonds at the price $1/R_t$. Household i issues (or invests in) state contingent debt and can buy bonds from the government, while transactions in the asset market are constrained by

$$\begin{aligned} &(B_{i,t}/R_t) + E_t[\varphi_{t,t+1} D_{i,t}] + M_{i,t}^H \\ &\leq \widetilde{B}_{i,t} + B_{i,t}^R + \widetilde{M}_{i,t} - R_t^m \left(M_{i,t}^R + M_{i,t}^L\right) + \left(1 - \delta_t^e\right) L_{i,t} + D_{i,t-1} + P_t v_{i,t} + P_t \tau_{i,t}, \end{aligned} \tag{4}$$

where $\varphi_{t,t+1}$ denotes a stochastic discount factor (which will be defined in Section 2.3). The central bank reinvests its payoffs from maturing bonds into new government bonds and leaves money supply unchanged at this stage, $\int_0^1 M_{i,t}^H di = \int_0^1 (M_{i,t-1}^H + I_{i,t} - M_{i,t}^R - M_{i,t}^L) di$.

[10] Note that the central bank does not hold loans until maturity, which allows abstracting from central bank losses.

2.2 Firms

There is a continuum of intermediate goods producing firms indexed with $j \in [0,1]$. They are perfectly competitive, produce (identical) intermediate goods $z_{j,t}$ with labor, and are owned by the households. Production depends on random idiosyncratic productivity levels $\omega_{j,t} \geq 0$, which materialize after the labor market closes. Firm j produces according to the production function $z_{j,t} = \omega_{j,t} n_{j,t}^{\alpha}$, where $\alpha \in (0,1)$, and sells the intermediate good to retailers at the price $P_{z,j,t}$. We assume that wages have to be paid in advance, i.e. before intermediate goods are sold. For this, firm j borrows cash $L_{j,t}$ from households at the price $1/R_{j,t}^L$ and repays the loan at the end of the period.

To account for credit default risk in a simple way, we assume that the realizations of the idiosyncratic productivity levels can freely be observed by borrowers, while lenders can only observe the realized idiosyncratic productivity level at proportional monitoring costs $\varrho \geq 0$. We then consider the following standard debt contract: Firm j offers a loan at the price $1/R_{j,t}^L$ that leads to a pay-off of 1 when its productivity level is sufficiently high $\omega_{j,t} \geq \overline{\omega}_{j,t}$, where $\overline{\omega}_{j,t}$ is the minimum productivity level that enables full repayment. Otherwise, if $\omega_{j,t} < \overline{\omega}_{j,t}$, firm j goes bankrupt and the lender can seize total revenues. For simplicity, we consider the following maximization problem of firm j

$$\max E_t[P_{z,j,t}\omega_{j,t} n_{j,t}^{\alpha} - P_t w_t n_{j,t} - L_{j,t}(R_{j,t}^L - 1)/R_{j,t}^L], \quad \text{s.t. (1)}, \tag{5}$$

where it disregards that loan repayments are contingent on idiosyncratic states.[11] The expectations operator E_t is based upon the information at the beginning of the period after aggregate state variables, but not productivity levels $\omega_{j,t}$, are realized. After wages are paid, these idiosyncratic productivity levels are drawn from the same potentially time-varying distribution with density function $f_t(\omega_{j,t})$ and a mean of one, $E_t(\omega_{j,t}) = 1$. Since firms are ex-ante identical, loan contracts for different firms are signed at the same rate $R_{j,t}^L = R_t^L$ and the same size $L_{j,t} = L_t$. The first order conditions to the problem (5) are therefore given by $R_t^L - 1 = \mu_{j,t}$, $(P_{z,j,t}/P_t)\,\alpha n_{j,t}^{1-\alpha} = w_t + \mu_{j,t} w_t$, (1), and $\mu_{j,t}[(L_{j,t}/R_t^L) - P_t w_t n_{j,t}] = 0$, where $\mu_{j,t} \geq 0$ is the multiplier on (1). Hence, intermediate goods producing firms do not borrow more than required to pay wages $w_t n_{j,t}$ if $R_t^L > 1 \Rightarrow \mu_{j,t} > 0$, which will be satisfied throughout the analysis. Given that $\mu_{j,t} = \mu_t$, $n_{j,t} = n_t$, and $P_{z,j,t} = P_{z,t}$,

[11] If the firm internalized limited liability for its maximization problem, its credit demand would be larger. This would slightly modify its first order conditions, but leaves the main results and the conclusions unchanged.

all firms behave in an identical way and the following conditions describe labor demand and loans:

$$(P_{z,t}/P_t)\,\alpha n_t^{\alpha-1} = w_t R_t^L, \tag{6}$$

$$l_t/R_t^L \geq w_t n_t, \tag{7}$$

where $l_t = L_t/P_t$. After idiosyncratic productivity shocks are realized, firm j fully repays loans $l_t = \alpha\,(P_{z,t}/P_t)\,n_t^\alpha$ if $\omega_{j,t} \geq \alpha$ or lenders receive $(1-\varrho)\omega_{j,t}\,(P_{z,t}/P_t)\,n_t^\alpha$ if $\omega_{j,t} < \alpha$, where $\varrho\omega_{j,t}\,(P_{z,t}/P_t)\,n_t^\alpha$ denotes monitoring costs. Hence, the expected pay-off for a lender is given by $\int_\alpha^\infty \alpha\,(P_{z,t}/P_t)\,n_t^\alpha f_t\,(\omega_{j,t})\,d\omega_{j,t} + (1-\varrho)\int_0^\alpha \omega_{j,t}\,(P_{z,t}/P_t)\,n_t^\alpha f_t\,(\omega_{j,t})\,d\omega_{j,t}$, and the expected rate of repayment $1-\delta_t^e \in [0,1)$ on loans equals

$$1-\delta_t^e = 1 - F_t\,(\alpha) + (1-\varrho)\alpha^{-1}E_t\left[\omega_{j,t}|\omega_{j,t} \leq \alpha\right], \tag{8}$$

and is therefore exogenous. Firms drawing a productivity level that exceeds α transfer their profits to households. Following Christiano et al. (2013), we assume that the distribution of the idiosyncratic productivity shocks can vary stochastically over time in a mean preserving way. Hence, these shocks to the distribution, which will be called default risk shocks, shift the mass of defaulting firms over time (i.e. change the standard deviation $\sigma_{\omega,t}$ of idiosyncratic productivity) without affecting the expected productivity. Realizations of default risk shocks, which will be considered in Section 4.1, are revealed at the beginning of the period t, and therefore shift the current period expected rate of repayment $1-\delta_t^e$.

Monopolistically competitive retailers buy intermediate goods $z_t = \int_0^1 z_{j,t}dj$ at the common price $P_{z,t}$. A retailer $k \in [0,1]$ relabels the intermediate good to $y_{k,t}$ and sells it at the price $P_{k,t}$ to perfectly competitive bundlers, who bundle the goods $y_{k,t}$ to the final consumption good y_t with the technology, $y_t^{\frac{\varepsilon-1}{\varepsilon}} = \int_0^1 y_{k,t}^{\frac{\varepsilon-1}{\varepsilon}}dk$, where $\varepsilon > 1$. The cost minimizing demand for $y_{k,t}$ is therefore given by $y_{k,t} = (P_{k,t}/P_t)^{-\varepsilon}\,y_t$. We assume that each period a measure $1-\phi$ of randomly selected retailers may reset their prices independently of the time elapsed since the last price setting, while a fraction $\phi \in [0,1)$ of retailers do not adjust their prices. A fraction $1-\phi$ sets their price to maximize the present value of profits. For $\phi > 0$, the first order condition for their price $\widetilde{P}_t$ is

$$\widetilde{P}_t = \frac{\varepsilon}{\varepsilon-1}\frac{E_t\sum_{s=0}^\infty(\phi\beta)^s\,q_{t,t+s}y_{t+s}P_{t+s}^\varepsilon mc_{t+s}}{E_t\sum_{s=0}^\infty(\phi\beta)^s\,q_{t,t+s}y_{t+s}P_{t+s}^{\varepsilon-1}}, \tag{9}$$

where $mc_t = P_{z,t}/P_t$ denotes retailers' real marginal costs. With perfectly competitive bundlers, the price index P_t for the final good satisfies $P_t^{1-\varepsilon} = \int_0^1 P_{k,t}^{1-\varepsilon}dk$. Using that $\int_0^1 P_{k,t}^{1-\varepsilon}dk = (1-\phi)\sum_{s=0}^\infty \phi^s \widetilde{P}_{t-s}^{1-\varepsilon}$ holds, and taking differences, leads to $P_t^{1-\varepsilon} = (1-\phi)\widetilde{P}_t^{1-\varepsilon} + \phi P_{t-1}^{1-\varepsilon}$.

2.3 Households

There is a continuum of infinitely lived households indexed with $i \in [0,1]$. Households have identical preferences and asset endowments. Household i maximizes the expected sum of a discounted stream of instantaneous utilities

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t u(c_{i,t}, n_{i,t}), \text{ with } u(c_{i,t}, n_{i,t}) = \left[(c_{i,t}^{1-\sigma} - 1) / (1-\sigma) \right] - \left[\theta n_{i,t}^{1+\sigma_n} / (1+\sigma_n) \right], \tag{10}$$

where $\theta > 0, \sigma \geq 1, \sigma_n \geq 0$ and E_0 is the expectation operator conditional on the time 0 information set, and $\beta \in (0,1)$ is the subjective discount factor. The term ξ_t is a stochastic preference parameter with an autocorrelation coefficient $\rho_\xi \in (0,1)$, which is typically used in the literature to drive the policy rate down to the ZLB (see e.g. Eggertsson, 2012). We examine this shock in Section 4.2.

A household i is initially endowed with money $M_{i,-1}^H$, government bonds $B_{i,-1}$, and state contingent claims $D_{i,-1}$. In each period, it supplies labor, lends funds to all intermediate goods producing firms (such that the loan portfolio is perfectly diversified) and trades assets with the central bank in open market operations. Before household i enters the goods market, where it needs money as the only accepted means of payment, it can get additional money in open market operations. Loans to firms can be refinanced in case the central bank accepts these loans as collateral in open market operations. Given that idiosyncratic productivity shocks are not realized at this moment and that random draws of eligible loan contracts are made after loan contracts are signed, the price of loans is $1/R_t^L$ for all firms j.

We restrict our attention to the case where the central bank supplies a sufficiently large share of money via repos, which implies that money will never be withdrawn from the private sector $I_{i,t} \geq 0$. Hence, households rely on positive holdings of bonds and loans to satisfy the collateral constraint (2). In the goods market, household i can use money holdings net of lending for its consumption expenditures (see 3). Before the asset market opens, household i buys back assets under repos. In the asset market, it receives payoffs from maturing assets (including loans), buys bonds from the government, borrows (and lends) using a full set of nominally state contingent claims, and trades all assets with other households. Dividing the period t price of one unit of nominal wealth in a particular state of period $t+1$ by the period t probability of that state gives the stochastic discount factor $\varphi_{t,t+1}$. The period t price of a payoff $D_{i,t}$ in period $t+1$ is then given by $E_t[\varphi_{t,t+1} D_{i,t}]$. Substituting out the stock of bonds and money held before the asset market opens, $\widetilde{B}_{i,t}$ and $\widetilde{M}_{i,t}$, in (4), the asset market constraint of household i can be rewritten as

$$0 \leq M^H_{i,t-1} - M^H_{i,t} + B_{i,t-1} - (B_{i,t}/R_t) + (1-\delta^e_t)\, L_{i,t} - \left(L_{i,t}/R^L_t\right) \quad (11)$$
$$+ D_{i,t-1} - E_t[\varphi_{t,t+1} D_{i,t}] - (R^m_t - 1)\, I_{i,t} + P_t w_t n_{i,t} - P_t c_{i,t} + P_t v_{i,t} + P_t \tau_{i,t},$$

where household $i's$ borrowing is restricted by $M^H_{i,t} \geq 0$, $B_{i,t} \geq 0$, and the no-Ponzi game condition $\lim_{s\to\infty} E_t \varphi_{t,t+s} D_{i,t+s} \geq 0$. The term $(R^m_t - 1)\, I_{i,t}$ in (11) measures the cost of money acquired in open market operations, i.e. household i receives new cash $I_{i,t}$ in exchange for $R^m_t I_{i,t}$ assets. Maximizing the objective (10) subject to the collateral constraint (2), the goods market constraint (3), the asset market constraint (11) and the borrowing constraints, for given initial values $M_{i,-1}$, $B_{i},_{-1}$, and $D_{i,-1}$ leads to the following first order conditions for consumption, working time, additional money, and loans

$$\xi_t c^{-\sigma}_{i,t} = \lambda_{i,t} + \psi_{i,t}, \quad (12)$$
$$\theta \xi_t n^{\sigma_n}_{i,t} = w_t \left(\lambda_{i,t} + \psi_{i,t}\right), \quad (13)$$
$$\psi_{i,t} = (R^m_t - 1)\, \lambda_{i,t} + R^m_t \eta_{i,t}, \quad (14)$$
$$\left(\lambda_{i,t} + \psi_{i,t}\right)/R^L_t = (1-\delta^e_t)\, \lambda_{i,t} + \eta_{i,t}\kappa_t, \quad (15)$$

as well as for investment in government bonds, money, and contingent claims

$$\lambda_{i,t} = \beta R_t E_t \frac{\lambda_{i,t+1} + \kappa^B_{t+1}\eta_{i,t+1}}{\pi_{t+1}}, \quad (16)$$
$$\lambda_{i,t} = \beta E_t \frac{\lambda_{i,t+1} + \psi_{i,t+1}}{\pi_{t+1}}, \quad (17)$$
$$\varphi_{t,t+1} = \frac{\beta}{\pi_{t+1}} \frac{\lambda_{i,t+1}}{\lambda_{i,t}}, \quad (18)$$

where $\lambda_{i,t} \geq 0$ denotes the multiplier on (11), $\eta_{i,t} \geq 0$ the multiplier on $\kappa^B_t B_{i,t-1} + \kappa_t L_{i,t} \geq R^m_t I_{i,t}$, and $\psi_{i,t} \geq 0$ the multiplier on (3). Further, (2), (3),

$$\psi_{i,t}[I_{i,t} + M^H_{i,t-1} - \left(L_{i,t}/R^L_t\right) + P_t w_t n_{i,t} - P_t c_{i,t}] \geq 0, \quad (19)$$
$$\eta_{i,t}[\kappa^B_t B_{i,t-1} + \kappa_t L_{i,t} - R^m_t I_{i,t}] \geq 0, \quad (20)$$

and (11) with equality hold as well as the transversality conditions. The risk free R^{rf}_t, i.e. the rate of return on a portfolio of contingent claims that guarantees a payoff of one unit for all states, is defined as $R^{rf}_t = 1/E_t \varphi_{t,t+1}$. Comparing the first order conditions with regard to investment in bonds (16) and contingent claims (18) shows that the risk free rate can differ from the bond rate R_t by a liquidity premium, which relies on a binding collateral constraint, $\eta_{i,t+1} > 0$, and increases with the future fraction of eligible bonds κ_{t+1}^B. Combining (14) and (15) to $R_t^m \left(\lambda_{i,t} + \eta_{i,t}\right) = R_t^L \left((1 - \delta_t^e) \lambda_{i,t} + \eta_{i,t}\kappa_t\right)$, shows that the loan rate compensates for default risk and tends to decrease with the expected repayment rate $1 - \delta_t^e$ as well as with the fraction of eligible loans κ_t, if $\eta_{i,t} > 0$. Notably, when loans are not fully eligible $\kappa_t < 0$, there will be a spread between the policy rate and the loan rate due to a liquidity premium, even if there is no default risk, $\delta_t^e = 0$. Combining (14), (16), and (17), leads to

$$R_t E_t \left[\left(\lambda_{i,t+1} + \kappa_{t+1}^B \eta_{i,t+1}\right) / \pi_{t+1}\right] = E_t \left[R_{t+1}^m \left(\lambda_{i,t+1} + \eta_{i,t+1}\right) / \pi_{t+1}\right], \tag{21}$$

The no-arbitrage condition (21) shows that households are indifferent between investing in money or investing in government bonds and converting these (partially) into cash in the next period at the rate R_{t+1}^m. For $\kappa_{t+1}^B = 1$, the interest rate on government bonds is closely linked to next period's expected policy rate, i.e. R_t equals $E_t R_{t+1}^m$ up to first order. When not all bonds are eligible, $\kappa_t^B < 1$, bonds are less liquid and become more akin to debt issued by households.

2.4 Public sector

The central bank transfers seigniorage revenues $P_t \tau_t^m$ to the government, which issues one-period bonds. Government bonds grow at a constant rate, $B_t^T = \Gamma B_{t-1}^T$, where $\Gamma \geq 1$ and B_t^T summarizes the total supply of government bonds, which are typically considered to be eligible for open market operations in normal times. To abstract from fiscal policy effects via tax distortions, we assume that the government has access to lump-sum transfers $P_t \tau_t$. Its budget constraint reads $(B_t^T / R_t) + P_t \tau_t^m = B_{t-1}^T + P_t \tau_t$, where bonds B_t^T are either held by households, B_t, or the central bank, $B_t^C : B_t^T = B_t + B_t^C$.

The central bank supplies money outright $M_t^H = \int_0^1 M_{i,t}^H di$, and under repos against bonds, $M_t^R = \int_0^1 M_{i,t}^R di$, and loans, $M_t^L = \int_0^1 M_{i,t}^L di$. Given that corporate loans are not held by the central bank until maturity, default on loans do not lead to central bank losses. Alternatively, if it holds risky corporate loans until maturity, the central bank could impose haircuts equal to the default probability in order to avoid losses. The central bank transfers its interest earnings to the government, $P_t \tau_t^m = B_t^C - (B_t^C / R_t) + (R_t^m - 1) \left(M_t^H - M_{t-1}^H + M_t^R + M_t^L\right)$, and reinvests its wealth exclusively in new government bonds, which accords to common central bank practice. Its budget constraint thus reads $\left(B_t^C / R_t\right) - B_{t-1}^C + P_t \tau_t^m = R_t^m \left(M_t^H - M_{t-1}^H\right) + (R_t^m - 1) \left(M_t^R + M_t^L\right)$. Substituting out central bank transfers, its bond holdings evolve according to $B_t^C - B_{t-1}^C = M_t^H - M_{t-1}^H$. The central bank controls three main instruments. Like in standard models, it controls the policy rate $R_t^m \geq 1$. It can further adjust the fractions of randomly selected eligible loans $\kappa_t \in [0,1]$ and eligible bonds $\kappa_t^B \in (0,1]$, which both affect the size and the composition of the central bank balance sheet. We consider two particular balance sheet policies for the subsequent analysis, i.e. quantitative easing and collateral policy, which are defined as follows.[12]

- *Quantitative easing* increases money supply against eligible assets in open market operations. Quantitative easing can be conducted in terms of government bonds or corporate loans and is implemented by an increase in κ_t or κ_t^B, respectively.
- *Collateral policy* changes the composition of the central bank's balance sheet without affecting its size. It is implemented by a change in κ_t, accompanied by a neutralizing change in κ_t^B.

The central bank further sets the inflation target π and controls how money is supplied in exchange for bonds in repos or outright (while loans are only traded under repos). Specifically, it sets a constant share of bond repos $\Omega \geq 0$, defined as $M_t^R = \Omega M_t^H$. In the Sections 3.2 and 4.2, we consider the limiting case $\Omega \to \infty$ in order to facilitate the derivation of analytical results.

3 Equilibrium properties

In this Section, we present some main properties of the rational expectations (RE) equilibrium (see Definition 3 in Appendix A.1). In the first part of this Section, we explain when balance sheet policies are effective. In the second part, we demonstrate that they are in general not equivalent to changes in the policy rate.

3.1 When are balance sheet policies effective?

The goods market constraint, which reads $P_t c_t \leq M_t^H + M_t^R + M_t^L$ in equilibrium, is relevant for the non-neutrality of monetary policy. Changes in money supply can affect prices and the allocation only if this constraint is binding. Further, the collateral constraint, which in equilibrium reads

$$M_t^H - M_{t-1}^H + M_t^R + M_t^L \leq \kappa_t^B \left(B_{t-1}/R_t^m\right) + \kappa_t \left(L_t/R_t^m\right), \tag{22}$$

is decisive for the effectiveness of quantitative easing and collateral policy. The instruments κ_t^B and κ_t can affect the equilibrium allocation only by relaxing the collateral constraint (22) (see

[12] Among the liquidity facilities created by the BoE or the Fed during 2008-09, many had elements of both quantitative easing and collateral policy, as defined above. Under the Asset Purchase Facility, the BoE purchased commercial papers, corporate bonds, and government bonds, which corresponds to our definition of quantitative easing. The Fed's purchases of treasury securities and the extension of credit to depository institutions through the Term Auction Facility come closest to quantitative easing by increasing κ_t^B, whereas programs such as the Term Securities Lending Facility and the Commercial Paper Funding Facility relate to our definition of collateral policy.

Definition 3 in Appendix A.1). Hence, if $\eta_t = 0$, such that (22) is slack, balance sheet policies will not affect the equilibrium allocation and the associated price system. To see when this is the case, we first use the conditions (12) and (17), which in equilibrium imply $\xi_t c_t^{-\sigma} = \beta E_t \frac{\xi_{t+1} c_{t+1}^{-\sigma}}{\pi_{t+1}} + \psi_t$ and that the multiplier on the goods market constraint ψ_t satisfies

$$\psi_t(c_t^{\sigma}/\xi_t) = 1 - (1/R_t^{Euler}) \geq 0, \tag{23}$$

where R_t^{Euler} denotes the Euler equation rate, which is defined as $1/R_t^{Euler} = \beta E_t \frac{\xi_{t+1} c_{t+1}^{-\sigma}}{\xi_t c_t^{-\sigma} \pi_{t+1}}$ (see Canzoneri et al., 2007).[13] If $R_t^{Euler} > 1$, households are willing to pay a positive price to transform one unit of an illiquid asset into one unit of money. Then, $\psi_t > 0$ (see 23) and the goods market constraint is binding (see 19), indicating that money is positively valued by households and they will not hold more money than required for consumption expenditures. If, however, $R_t^{Euler} = 1$, then the marginal valuation of money equals zero and the goods market constraint is slack, $\psi_t = 0$, such that changes in money supply are neutral.

The conditions (12), (14), and (17) further imply $\xi_t c_t^{-\sigma} = R_t^m(\lambda_t + \eta_t)$ and $\lambda_t = \beta E_t \frac{\xi_{t+1} c_{t+1}^{-\sigma}}{\pi_{t+1}}$. Eliminating λ_t, shows that the multiplier for the collateral constraint η_t satisfies

$$\eta_t(c_t^{\sigma}/\xi_t) = (1/R_t^m) - (1/R_t^{Euler}) \geq 0, \tag{24}$$

in equilibrium. Condition (24) shows that when the policy rate is strictly smaller than the Euler equation rate, $R_t^m < R_t^{Euler}$, the multiplier η_t is positive and the collateral constraint is binding (see 20). Then, the policy rate R_t^m does not determine whether the goods market constraint is binding or not, since it is not identical to R_t^{Euler}. When $R_t^m < R_t^{Euler}$, the goods market constraint is binding as well, $\psi_t > 0$ (see 23), given that $R_t^m \geq 1$. Households can then get money in exchange for an eligible asset at a price, $R_t^m - 1$, which is below their marginal valuation of money, $R_t^{Euler} - 1$. Hence, they use eligible assets as much as possible to get money in open market operations, such that (22) is binding. If, however, the policy rate equals the Euler equation rate, $R_t^m = R_t^{Euler}$, households are indifferent between transforming eligible assets into money or holding them until maturity and the collateral constraint is slack, $\eta_t = 0$ (see 24).[14] These results are summarized in the following proposition.

[13] The Euler equation rate differs from the risk free rate, which refers to an investment leading to a non-cash payoff.

[14] In this case, the model reduces to a standard model where the (real) policy rate governs the intertemporal rate of substitution (see Definition 5 in Appendix A.1). Then, the policy instruments κ_t and κ_t^B do neither affect the allocation nor the price system, such that quantitative easing and collateral policy are ineffective, which accords to the conventional view on quantitative easing (see e.g. Eggertsson and Woodford, 2003).

Proposition 1 *For a given consumption sequence $\{c_t\}_{t=0}^{\infty}$, the demand for real balances is uniquely determined iff the Euler equation rate satisfies $R_t^{Euler} > 1$. Quantitative easing and collateral policy then affect the equilibrium allocation and the associated price system iff the policy rate is smaller than the Euler equation rate, $R_t^{Euler} > R_t^m$.*

Proof. In equilibrium, the cash constraint (3) implies $c_t \leq m_t^H + m_t^R + m_t^L$, which is binding iff $\psi_t > 0$ (see 19). According to (23), this is the case iff $R_t^{Euler} > 1$. Then, the demand for real balances $m_t^H + m_t^R + m_t^L$ is determined for a given sequence $\{c_t\}_{t=0}^{\infty}$. The collateral constraint (22) is further binding, iff $\eta_t > 0$ (see 20), which is the case iff $R_t^{Euler} > R_t^m$ (see 24). Then, $[c_t - \kappa_t (l_t/R_t^m)]\pi_t = \kappa_t^B (b_{t-1}/R_t^m) + m_{t-1}^H$ holds and changes in κ_t and κ_t^B affect consumption, loans, and inflation for a given policy rate R_t^m and asset endowments, $b_{t-1} > 0$ and $m_{t-1}^H > 0$. ■

Money demand can be uniquely determined, even if the policy rate is at the ZLB $R_t^m = 1$, as long as the Euler equation rate is larger than one (see Proposition 1), which implies a positive valuation of money. Both rates are in general not identical, since money supply is restricted by the collateral constraint. However, an increase in money supply, which stimulates consumption, tends to drive down the Euler equation rate, ultimately until it equals the policy rate and liquidity premia disappear. Hence, non-neutrality of balance sheet policies depends on the state of the economy and on monetary policy itself, which will be examined in Section 5. For $R_t^m = 1$, both multipliers η_t and ψ_t are identical (see 23 and 24), since eligible assets can costlessly be transformed into money, while balance sheet policies can nevertheless affect aggregate demand and prices as long as the Euler equation rate exceeds one, $R_t^{Euler} > 1$. This is not possible in standard models, where money supply is not rationed, such that the price of money has to be equal to its marginal valuation by households, i.e. $R_t^m = R_t^{Euler}$ (see Definition 5 in Appendix A.1).

3.2 Are balance sheet policies equivalent to interest rate policy?

We now demonstrate that balance sheet policies are in general not equivalent to policy rate adjustments. For this preliminary analysis, we apply a simplified version of the model, which allows analyzing the model without relying on approximation methods: We disregard idiosyncratic productivity shocks, $\omega_{j,t} = 1$, set preference parameters equal to $\sigma = 1$ and $\sigma_n = 0$, and assume that prices are perfectly flexible, $\phi = 0$, production is linear $\alpha = 1$, and that money is only supplied under repos, $\Omega \to \infty$.[15] A RE equilibrium with a binding collateral constraint ($\eta_t > 0$), which requires the policy rate to be set below the Euler equation rate (see Proposition 1), can then be

[15] The central bank then holds eligible assets only under repos, such that the total stock of government bonds will be held by households, $B_t = B_t^T$.

summarized in the following way (see Appendix A.1).

Definition 1 *For $\sigma = 1$, $\sigma_n = 0$, $\alpha = 1$, $\omega_{j,t} = 1$, $\phi = 0$, $\Omega \to \infty$, a RE equilibrium with a binding collateral constraint is a set of sequences $\{y_t, \pi_t, R_t^L, b_t\}_{t=0}^{\infty}$ and $P_0 > 0$ satisfying*

$$y_t = \left[(\mu/\theta)\left(1/R_t^L\right)\right]^{\alpha/(1+\sigma_n)}, \tag{25}$$

$$1/R_t^L = \kappa_t(1/R_t^m) + (1-\kappa_t)\beta E_t\left[\xi_{t+1}y_t/\left(\xi_t y_{t+1}\pi_{t+1}\right)\right], \tag{26}$$

$$y_t = \left[\kappa_t^B/(R_t^m - \kappa_t\mu)\right] b_{t-1}/\pi_t, \tag{27}$$

$$b_t = \Gamma b_{t-1}\pi_t^{-1} \ \forall t \geq 1 \quad \text{and} \quad \Gamma P_0 b_0 = B_{-1}, \tag{28}$$

where $\mu = \frac{\varepsilon-1}{\varepsilon}\alpha < 1$, for a monetary policy setting $1 \leq R_t^m < 1/\left[\xi_t^{-1}y_t\beta E_t\left(\xi_{t+1}y_{t+1}^{-1}\pi_{t+1}^{-1}\right)\right]$, κ_t, and κ_t^B for a given sequence $\{\xi_t\}_{t=0}^{\infty}$ and an initial stock of bonds $B_{-1} > 0$.

Condition (25), which is based on labor market equilibrium (i.e. $\theta n_t^{\sigma_n} = c_t^{-\sigma}$ and 6), aggregate production, and goods market clearing, shows that the loan rate R_t^L reduces aggregate output. Condition (26), which is based on (12), (14), (15), and (17), shows that the loan price $1/R_t^L$ is a linear combination of the inverses of the policy rate, $1/R_t^m$, and of the Euler equation rate, $1/R_t^{Euler} = \beta E_t[\xi_{t+1}y_t/\left(\xi_t y_{t+1}\pi_{t+1}\right)]$, where the former is weighted with the fraction of eligible loans κ_t and the latter with $1-\kappa_t$. Thus, if R_t^m is set below R_t^{Euler}, the central bank can lower the loan rate by increasing the fraction of eligible loans κ_t. This exerts a positive effect on output (see 25) by reducing the firms' marginal costs. If loans are fully eligible, $\kappa_t = 1$, the loan rate R_t^L equals R_t^m, whereas it equals R_t^{Euler} if they are not eligible, $\kappa_t = 0$. Combining the cash constraints (1) and (3) with the collateral constraint (22) leads to (27), which shows that aggregate demand tends to increase when money supply is increased by raising the fractions of eligible bonds and loans or by lowering the policy rate. The evolution of privately held bonds is further governed by the total supply of bonds (see 28).[16]

The policy instruments R_t^m, κ_t, and κ_t^B enter the equilibrium conditions (25)-(28) in different ways. The effects of changes in these instruments are therefore in general not equivalent. To make this property more transparent, we substitute out R_t^L in (25) with (26), and then y_t with (27). We further define a term Υ_t as $\Upsilon\left(R_t^m, \kappa_t^B, \kappa_t\right) = \kappa_t^B/(R_t^m - \kappa_t\mu)$, which depends only on monetary policy instruments and measures the generosity of money supply. We then obtain the following

[16] It should be noted that long-run inflation π is affected by the availability of eligible assets, when the collateral constraint is binding. As bonds grow with the rate Γ, the price level tends to grow with the same rate when bonds are eligible. To control long-run money supply and thereby long-run inflation, the central bank can adjust the fraction of accepted bonds κ_t^B in an appropriate way. Specifically, the central bank can implement an inflation target independent of fiscal policy and can, for example, ensure long-run price stability by setting $\kappa_t^B/\kappa_{t-1}^B = \Gamma^{-1}$ (see Appendix B, which is made available online).

representations, for inflation and output (see 27):

$$\pi_t = b_{t-1} (\mu/\theta)^{-1} \Phi_t^{-1}, \text{ and } y_t = \Upsilon \left(R_t^m, \kappa_t^B, \kappa_t\right) (\mu/\theta) \Phi_t, \tag{29}$$

$$\text{where } \Phi_t = [\kappa_t/R_t^m] \Upsilon \left(R_t^m, \kappa_t^B, \kappa_t\right)^{-1} + (1-\kappa_t) (\beta/\Gamma) E_t[\left(\xi_{t+1}/\xi_t\right) \Upsilon \left(R_{t+1}^m, \kappa_{t+1}^B, \kappa_{t+1}\right)^{-1}].$$

The term Φ_t in (29) can be separately affected by Υ_t and the instruments R_t^m and κ_t. When loans are not eligible, $\kappa_t = 0$, the terms $\Upsilon_t = \kappa_t^B/R_t^m$ and $\Phi_t = (\beta/\Gamma) E_t[\left(\xi_{t+1}/\xi_t\right) \Upsilon_{t+1}^{-1}]$ imply that changes in the policy rate and inverse changes in the fraction of eligible bonds κ_t^B affect output and inflation in an identical way. Hence, both instrument can be used equivalently unless one of them cannot be adjusted due to feasibility constraints, like the ZLB (see Section 4.2). If, however, loans are eligible $\kappa_t > 0$, the conditions in (29) reveal that policy instruments are not equivalent. In particular, changes in the policy rate R_t^m as well as in the fraction of eligible loans κ_t can alter the term Φ_t, and therefore output and inflation, differently from Υ_t via their direct effects on the loan rate (see Section 4.1).

4 Limits to conventional monetary policy

In the previous Section, we have demonstrated that monetary policy instruments are in general not equivalent. In this Section, we consider two particular scenarios, where this property is exploited to use quantitative easing and collateral policy in order to implement equilibria that are preferable to equilibria which are implementable when only conventional interest rate policy is available. For the first scenario, we consider default risk shocks, i.e. shocks to the variance of idiosyncratic productivity, and show that the central bank can fully neutralize these shocks with collateral policy, which is not possible under a pure interest rate policy. For the second scenario, we consider a contractionary preference shock and show that quantitative easing can serve as a substitute for reductions in the policy rate, when the latter is at the ZLB. Throughout the analysis, we consider the more realistic case of imperfectly flexible prices, $\phi > 0$.

4.1 Collateral policy and default risk shocks

For the first scenario, we examine default risk shocks, i.e., mean preserving changes in the distribution of idiosyncratic productivity of borrowers (as in Christiano et al., 2013), while we disregard preference shocks, $\xi_t = 1$, for convenience. Specifically, we consider an increase in the standard deviation of idiosyncratic productivity $\sigma_{\omega,t}$ that increases the probability of default, $F_t(\alpha)$, and reduces the expected repayment rate of loans (see 8), which induces lenders to demand a higher loan rate. Since changes in the loan rate affect marginal costs of firms (see 6), the default risk shock has a cost push effect on the production sector, which tends to increase the price level, giving rise to welfare losses due to imperfect price adjustments. Thus, default risk shocks exert purely distortionary effects. Put differently, shocks to the distribution of idiosyncratic productivity would not affect aggregate variables in a frictionless economy when firms have access to a complete asset market, while they cause inflation and output responses in this model, which are entirely welfare reducing. Hence, a central bank that aims at maximizing welfare should neutralize these shocks.

When the standard deviation of idiosyncratic productivity shocks is positive and a non-zero fraction of intermediate goods producing firms default, $F_t(\alpha) > 0$, lenders take the default probability into account (see 15). Combining (12), (14), (15), and (17), the loan rate then satisfies

$$\frac{1}{(1-\delta_t^e)\,R_t^L} = \frac{\kappa_t}{1-\delta_t^e}\frac{1}{R_t^m} + \left(1 - \frac{\kappa_t}{1-\delta_t^e}\right)\beta E_t \frac{c_{t+1}^{-\sigma}}{\pi_{t+1} c_t^{-\sigma}}, \tag{30}$$

instead of (26). Under a time varying distribution of idiosyncratic productivity, the expected repayment rate $1-\delta_t^e$ also varies over time (see 8). According to the assumption that changes in the distribution of idiosyncratic productivity shocks are revealed at the beginning of the period, shocks to the expected default rate δ_t^e affect the loan rate in the same period. In particular, the loan rate then tends to increase with the expected default rate (see 30).[17]

The right hand side of (30) shows that the central bank can in principle offset default risk shocks, i.e. changes in δ_t^e, by adjusting the instruments κ_t or R_t^m. Suppose that the central bank only adjusts the policy rate R_t^m and keeps the fraction of eligible loans at a positive constant, $\kappa > 0$. It can then offset a decrease in the repayment rate by lowering the policy rate. Alternatively, the central bank can lower the loan rate by accepting more loans as collateral in open market operations (see 30), i.e. by raising κ_t when $R_t^m < R_t^{Euler}$. However, changes in the policy instruments simultaneously affect aggregate demand under a binding collateral constraint (either by reducing the price of money or by supplying more money against loans in open market operations), such that default risk shocks are not completely neutralized.

If, however, the central bank simultaneously reduces the fraction of eligible bonds κ_t^B, it can compensate for the change in κ_t or in R_t^m in a way such that money supply and thus aggregate demand are held constant. Given that the policy instruments R_t^m, κ_t, and κ_t^B affect the private sector behavior only via the collateral constraint (22) and the pricing conditions for bonds (21) and loans (30), the central bank can completely neutralize the effects of the default risk shocks

[17] Even when all loans are eligible, $\kappa_t = 1$, the default rate tends to increase the loan rate, $1/R_t^L = (1/R_t^m) - \delta_t^e(1/R_t^{Euler})$ (see 30), given that the central bank is not exposed to the risk of default.

on the equilibrium allocation, since the latter is not affected by changes in the bond price (see Definition 4 in Appendix A.1). This result for a collateral policy is summarized in the following proposition.[18]

Proposition 2 *Under a binding collateral constraint, the central bank can fully neutralize default risk shocks by collateral policy, but not by policy rate adjustments alone.*

Proof. See Appendix A.2 ■

It should be noted that the success of collateral policy in this scenario is limited to small default risk shocks. Specifically, the maximum size of default risk changes that can completely be neutralized by collateral policy is determined by the size of the liquidity premium (see proof of Proposition 2).[19] The central bank can nevertheless mitigate the effects of larger default risk shocks through collateral policy, i.e. by reducing the illiquidity premium on loans.[20]

4.2 Quantitative easing at the zero lower bound

The second scenario refers to policy options at the ZLB. For this, we consider a shock to the preference parameter ξ_t following other studies on public policy at the ZLB, like Eggertsson (2012), while we disregard idiosyncratic productivity shocks, $\omega_{j,t} = 1$, for convenience. We further disregard central bank lending against corporate debt, $\kappa_t = 0$, such that only government bonds are eligible. Changes in the policy rate and in the fraction of eligible bonds then exert equivalent effects on the equilibrium allocation (see 29). However, quantitative easing will be particularly useful for the central bank when the policy rate cannot be adjusted in the desired way, i.e. when reductions in the policy rate are not feasible due to the ZLB. Notably, a conventional macroeconomic model would predict quantitative easing to be entirely ineffective in this case, given that changes in money supply are neutral when all interest rates are at the ZLB.

To facilitate the derivation of analytical results, we apply a local analysis at a steady state with a binding collateral constraint. In the steady state, which is described in Appendix A.2, all real variables are constant and are denoted by small letters without a time index. The steady state Euler equation rate satisfies $R^{Euler} = \pi/\beta$ as usual, while the loan rate equals the Euler equation rate, $R^L = R^{Euler}$, since $\kappa_t = 0$ (see 26). The central bank sets the policy rate below the Euler equation rate in the steady state. Hence, there is a liquidity premium, as revealed by the steady

[18] In a similar way, the central bank can neutralize default risk shocks by simultaneous adjustments in the policy rate and the fraction of eligible bonds.

[19] According to Longstaff et al. (2005), this roughly equals 50 b.p. (see Section 5.1 for further details).

[20] Arguably, borrowing by firms might be affected by this type of policy, e.g. through their willingness to take risk, which is however beyond the scope of our analysis.

state version of (24), $\eta = c^{-\sigma}[(1/R^m) - (\beta/\pi)] \geq 0$. We assume that government bonds are initially not fully eligible, leaving room for maneuver for quantitative easing,[21] and that the inflation target is consistent with long-run price stability, $\pi = 1$.[22] Given that $\pi = 1$ implies $R^{Euler} > 1$ and that $R^m < R^{Euler}$, the goods market constraint and the collateral constraint are binding in the steady state (see Proposition 1). In a neighborhood of this steady state, the equilibrium sequences are approximated by the solutions to the linearized equilibrium conditions (see Appendix A.2). A RE equilibrium is then defined as follows, where $\widehat{a}_t$ denotes relative deviations of a generic variable a_t from its steady state value $a : \widehat{a}_t = \log(a_t/a)$.

Definition 2 *For $\Omega \to \infty$, $\Gamma = \pi = \omega_{j,t} = 1$, $R^m \in [1, 1/\beta)$, $\kappa^B < 1$, and $\kappa_t = 0$, a RE equilibrium is a set of convergent sequences $\{\widehat{y}_t,\ \pi_t,\ \widehat{b}_t,\ \widehat{R}^L_t\}_{t=0}^{\infty}$ satisfying*

$$\widehat{y}_t = \widehat{b}_{t-1} - \widehat{\pi}_t - \widehat{R_t}^m + \widehat{\kappa}^B_t, \tag{31}$$

$$\sigma\widehat{y}_t = \sigma E_t\widehat{y}_{t+1} - \widehat{R}^L_t + E_t\widehat{\pi}_{t+1} + \left(1 - \rho_\xi\right)\widehat{\xi}_t, \tag{32}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \chi\left(\varpi - 1\right)\widehat{y}_t + \chi\widehat{R}^L_t, \tag{33}$$

$$\widehat{b}_t = \widehat{b}_{t-1} - \widehat{\pi}_t, \tag{34}$$

where $\chi = (1-\phi)(1-\beta\phi)/\phi$ and $\varpi = \frac{1+\sigma_n}{\alpha} + \sigma > 1$, for monetary policy setting $\{\widehat{\kappa}^B_t, \widehat{R_t}^m\}_{t=0}^{\infty}$ and preference shocks satisfying $\widehat{\xi}_t = \rho_\xi\widehat{\xi}_{t-1} + \varepsilon_t$, $E_{t-1}\varepsilon_t = 0$ and $\rho_\xi \in [0, 1)$, given $b_{-1} > 0$.

The linear model summarized in Definition 2 is similar to a New Keynesian model with the "cost channel" (see Ravenna and Walsh, 2007). In particular, the conditions (32) and (33) resemble standard conditions for aggregate demand and for aggregate supply, where the latter is affected by the cost of loans due to the working capital assumption. The crucial difference to the canonical New Keynesian model is, however, that this is not a single interest rate framework. Specifically, the policy rate, which is not identical to the loan rate (since $\kappa_t \neq 1$, see 26), neither enters (32) nor (33). Nevertheless, the policy rate affects the equilibrium allocation via the (consolidated version of the) money supply constraint (31). Thus, an increase in the policy rate tends – for a given amount of eligible bonds – to reduce the amount of money and thereby aggregate demand.

For the central bank's objective, we apply Ravenna and Walsh's (2007) approximated household welfare function of an isomorphic model.[23] The efficient output level, which would be realized under flexible prices (see 25 for the $\sigma = 1$ case) and a policy rate pegged at one, is constant

[21] Alternatively, we can set κ^B_t equal to one and allow for long-term treasury securities to be accepted as collateral.

[22] Precisely, the central bank implements long-run price stability by long-run adjustments of κ^B_t contingent on the supply of government bonds. We can therefore disregard the case of a growing supply of bonds, which can be neutralized by a shrinking fraction of eligible bonds, and we assume – without loss of generality – that $\Gamma = 1$.

[23] This presumes long-run price stability and fixed fiscal transfers that compensate for average price mark-ups and the average loan rate, which are not modelled in this paper.

$y^{*}=(\alpha/\theta)^{\alpha/(\sigma_n+\sigma\alpha-1+\alpha)}$ and leads to output gaps y_t^g satisfying $y_t^g=y_t/y^*$ and, thus, $\widehat{y}_t^g=\widehat{y}_t$. We assume that the central bank cannot fully commit to future policies. Taking expectations and fiscal policy as given, it minimizes a loss function L_t in a discretionary way, $L_t=E\sum_{t=0}^{\infty}\beta^t\frac{1}{2}(\widehat{\pi}_t^2+\Lambda\widehat{y}_t^2)$, where $E\sum_{t=0}^{\infty}\beta^t u_t\approx[u/(1-\beta)]-\Omega L$, $\Omega>0$, and $\Lambda=(\chi/\varepsilon)(\varpi-1)$ (see Ravenna and Walsh, 2007), subject to the private sector equilibrium conditions (32) and (33). By eliminating the lending rate, (32) and (33) can be combined to a single constraint to the policy problem, $\widehat{\pi}_t=(\beta+\chi)E_t\widehat{\pi}_{t+1}+\chi\eta\widehat{y}_t+\chi\sigma E_t\widehat{y}_{t+1}+\chi\left(1-\rho_{\xi}\right)\widehat{\xi}_t$, where $\gamma=(1+\sigma_n)/\alpha>1$. The central bank then faces a trade-off between stabilizing the price level and output even if only preference shocks are present.[24] Minimizing L_t with respect to $\widehat{\pi}_t$ and $\widehat{y}_t$ in a discretionary way, leads to

$$\widehat{\pi}_t=-\left[\Lambda/\left(\chi\gamma\right)\right]\widehat{y}_t. \tag{35}$$

The optimal discretionary plan of the central bank then is a set of sequences $\{\widehat{R}_t^L,\widehat{\pi}_t,\widehat{y}_t\}_{t=0}^{\infty}$ satisfying (32), (33), and (35). When preference shocks $\widehat{\xi}_t$ are small, such that $R_t^m>1$, the central bank can implement this optimal plan solely by adjusting the policy rate according to (31) and (34) for a given fraction of eligible assets κ^B. In particular, a decline in ξ_t, which leads to a fall in inflation under the optimal plan, calls for a reduction in the policy rate R_t^m (see 31). Hence, if the economy is hit by a sufficiently large contractionary ξ_t-shock, the ZLB can render the implementation of the optimal plan by policy rate adjustments impossible. In this case, the central bank can still implement the plan via quantitative easing, i.e. by increasing κ_t^B. This is shown for the parameter restrictions $\sigma<(1+\sigma_n)/\alpha$ and $\varepsilon>2$, which ensure equilibrium uniqueness and unambiguous responses under the optimal plan.[25]

Proposition 3 *Consider a RE equilibrium as given in Definition 2 for $\sigma<(1+\sigma_n)/\alpha$ and $\varepsilon>2$. A central bank acting without commitment can implement its policy plan, even if the policy rate is at the ZLB, by conducting quantitative easing.*

Proof. See Appendix A.2. ■

Proposition 3 implies that quantitative easing can increase the set of states, for which the central bank can implement its optimal plan. Quantitative easing, however, also reaches its limits either when κ_t^B approaches one or when the collateral constraint becomes slack. Easing money supply tends to increase current aggregate demand, which implies a decreasing Euler equation rate. When the latter falls to a level that equals the policy rate, collateral is abundant and quantitative easing

[24] This effect of the "cost channel" for the central banks' trade-off is stressed by Ravenna and Walsh (2007).

[25] An analysis of equilibrium determinacy under the optimal plan can be found in Appendix B, which is made available online.

becomes neutral. A condition for a positive multiplier on the collateral constraint at the ZLB is given in the proof of Proposition 3.[26] Notably, quantitative easing in terms of treasuries, i.e. an exogenous and autocorrelated increase of κ_t^B, further leads to a decline in the bond rate as well as in the loan rate, since it reduces the marginal valuation of liquid assets.[27] This pattern is consistent with the empirical evidence provided by Krishnamurthy and Vissing-Jorgensen (2011) for the Fed's asset purchases between 2008 and 2011.

5 Limits to balance sheet policies

It has been shown in the previous Section that quantitative easing and collateral policy can be useful additional instruments for the central bank. Here, we explore how the effectiveness of balance sheet policies is limited. We apply a numerical analysis, where we disregard preference and productivity shocks ($\xi_t = \omega_{j,t} = 1$), which have been examined before. To allow for a reasonable quantification of the effects and to facilitate the calibration of the model, we introduce investment in physical capital, which is described in the first part of this Section. In the second part, we analyze effects of isolated balance sheet policies, for which we consider a policy experiment where quantitative easing is exogenously conducted. In the third part, we examine responses of balance sheet policies to a shock to the liquidity demand for investment. This shock has been described as a major factor in the crisis of 2008 (as argued by Del Negro et al., 2013) and leads, in particular, to a large liquidity premium between eligible and non-eligible assets, as observed during the crisis.

5.1 Extension and calibration

For a quantitative analysis, we extend the model presented in Section 2 by introducing capital accumulation, while disregarding idiosyncratic productivity shocks and preference shocks, for convenience. Households own the stock of capital, $k_t = \int k_{i,t} di$, and rent it to firms at the rate r_t^k. The capital stock of household i evolves according to $k_{i,t} = (1 - \delta_k) k_{i,t-1} + x_{i,t} S(x_{i,t}/x_{i,t-1})$, where $\delta_k \in (0,1)$ denotes the depreciation rate and x_t investment expenditures. Following large parts of the literature on quantitative macroeconomic models (see e.g. Christiano et al., 2005), we introduce investment adjustment costs to avoid extreme investment responses to aggregate shocks, $S(x_t/x_{t-1}) = 1 - \frac{\vartheta}{2}(x_t/x_{t-1} - 1)^2$, where $\vartheta > 0$. We assume that households rely on cash for

[26] Specifically, this condition (see 68) is satisfied if the steady state spread between the Euler equation rate and the policy rate, $(\pi/\beta) - R^m$, is sufficiently large. For the parameter values applied in Definition 1 and $\pi = 1$, it simplifies to $R^m < \Omega$ with $\Omega = \frac{2\chi\varepsilon + 2(1-\beta\rho)}{2\chi\varepsilon + 2(1-\beta\rho) - \chi(\varepsilon - 2)(1-\beta)}$, where the threshold for the policy rate is strictly larger than one, $\Omega > 1$, if the substitution elasticity ε lies between 2 and 1572.

[27] This is shown in Appendix B, which is made available online.

purchases of investment goods up to a time-varying fraction $\upsilon_{x,t} \geq 0$. We further introduce a parameter $\upsilon_c > 0$, which determines the fraction of purchases of consumption goods that require cash. Thus, the cash constraint (3) is replaced by

$$\upsilon_c P_t c_{i,t} + \upsilon_{x,t} P_t x_{i,t} \leq I_{i,t} + M^h_{i,t-1} - \left(L_{i,t}/R^L_t\right) + P_t w_t n_{i,t}, \tag{36}$$

such that money demand of households is increasing in υ_c and $\upsilon_{x,t}$. These parameters allow relating expenditures to the monetary base in accordance with empirical counterparts. Intermediate goods producing firms rent capital from households. Firm j produces with technology $z_{j,t} = n^{\alpha}_{j,t} k^{1-\alpha}_{j,t-1}$ and pays the rents after their goods are sold, facing (1). Its first order conditions for $R^L_t > 1$ are given by $mc_{j,t}\alpha\left(n_{j,t}/k_{j,t-1}\right)^{\alpha-1} = w_t R^L_t$, $mc_{j,t}\left(1-\alpha\right)\left(n_{j,t}/k_{j,t-1}\right)^{\alpha} = r^k_t$, and (7). We further introduce (exogenous) government spending, such that market clearing requires $y_t = c_t + x_t + g_t$.[28]

For the numerical analysis, we use standard parameter values as much as possible. The parameters of the utility function equal $\sigma = 2$ and $\sigma_n = 1$, the labor share equals $\alpha = 0.66$, the steady state markup $1/mc = 11\%$ ($\varepsilon = 10$), steady state working time $n = 1/3$, the fraction of non-optimally price adjusting firms $\phi = 0.75$, the share of government spending $g/y = 0.19$, and the adjustment cost parameter $\vartheta = 2.5$ (see e.g. Christiano et al., 2005). The steady state values of $\upsilon_{x,t}$ and υ_c are calibrated to the observed ratios $Px/M0 = 1.15$ and $Pc/M0 = 2.71$, and the depreciation rate is set to $\delta_k = 0.023$ to match the observed ratio of consumption to investment, $c/x = 2.36$.[29] The long-run policy rate is set at $R^m = 1.0133$ (or 5.41% in terms of annualized rates), which equals the average of the federal funds rate, and the inflation target is set at its average value $\pi = 1.00647$ (or 2.61% at an annual rate).[30]

The policy rate is either held at its long-run mean R^m (in Section 5.2) or set according to a simple Taylor rule $\widehat{R}^m_t = \rho_\pi \widehat{\pi}_t + \rho_y \widehat{y}_t$ with $\rho_\pi = 1.5$ and $\rho_y = 0.5^{1/4}$ (in Section 5.3). We consider the case where loans are not eligible in the steady state, i.e. $\kappa = 0$, which accords to the Fed's pre-crisis "Treasuries only" regime. In contrast, bonds are fully eligible, $\kappa^B = 1$, where we assume – without explicitly specifying – that the supply of bonds is consistent with the steady state inflation rate. We further set the repo share to $\Omega = 1.5$ to match the observed ratio between total reserves

[28] The full set of equilibrium conditions can be found in Appendix B that is made available online.

[29] Data on consumption, investment, government spending and gdp are taken from NIPA Table 1.15, where durable consumption goods are included into investment, and data on the monetary base are from the Federal Reserve Board's H3 Statistical Release. All data are seasonally adjusted and refer to averages for 25 years over the period QI/1981–QIV/2006.

[30] The data are taken from the U.S. Bureau of Economic Analysis for the sampe peroid QIV/1982–QIII/2008 (to exclude the pre-Volcker period).

and reserves supplied under repurchase agreements, which was almost constant in the 2000s.[31]

The spread between the policy rate and the loan rate, which equals the Euler equation rate $R^L = R^{Euler} = \pi/\beta$ in a steady state with $\kappa = 0$, is crucial for the size of monetary policy effects. According to the literature on the "corporate bond yield spread" (see Christensen, 2008), the yield spread between treasury securities and corporate bonds can be attributed to a default risk component and a liquidity component (see e.g. Longstaff et al., 2005). Since we disregard default risk shocks in this Section, we focus on the liquidity component. Specifically, we refer to Longstaff et al.'s (2005) estimate of the liquidity premium for the spread between corporate bonds and treasury securities, who report that, for AAA rated corporate bonds, 51% of the credit spread can be explained by default risk. Given that the average short-term spread for AAA corporate bonds equals 104 basis points at annualized rates (see Longstaff et al., 2005), we apply a liquidity premium of $(1 + 49\% \cdot 0.0104)^{1/4} - 1 = 13$ basis points (in terms of quarterly rates), which implies a discount factor of $\beta = \frac{\pi}{R^m + 13 \cdot 10^{-4}} = 0.992$ and a (quarterly) loan rate of 1.0146. Since the central bank then sets its targets according to $\pi > \beta$ and $1 \leq R^m \in [1, \pi/\beta)$, the collateral constraint and the cash constraint are binding in the steady state (see Proposition 1).

5.2 Maximum effects of quantitative easing

The calibrated model is solved by applying a first-order approximation at the deterministic steady state. To examine the effects of an isolated balance sheet policy, we consider a policy experiment with exogenous quantitative easing. Specifically, we compute impulse responses to an unexpected shift in the fraction of eligible loans κ_t, which is assumed to follow an AR(1) process with mean κ and an autocorrelation $\rho_\kappa > 0$.[32] Notably, a corresponding experiment with quantitative easing in terms of treasuries (i.e. shocks to κ_t^B) leads to virtually identical effects. Variables are either given in terms of percentage deviations from steady state, e.g. $\widetilde{y}_t = 100 \cdot \widehat{y}_t$, or in absolute terms.

The policy rate is held at its mean, $R_t^m = R^m$. Balance sheet policies are then effective as long as the collateral constraint is binding, i.e. the Euler equation rate exceeds the mean policy rate. Easing money supply will however increase consumption until $R_t^{Euler} \to R^m$, such that households' and firms' cash demand will be satiated and collateral becomes abundant. To see this, recall that the multiplier on the collateral constraint η_t (see 24) has to satisfy

$$\eta_t = (c_t^{-\sigma}/R_t^m) - \beta E_t c_{t+1}^{-\sigma} \pi_{t+1}^{-1} > 0, \tag{37}$$

[31] See Federal Reserve Bank of New York, Domestic Open Market Operations, various issues, and FRED database.

[32] This experiment corresponds to the analysis of responses to innovations to policy rate rules, which is typically conducted to isolate the effects of monetary policy actions.

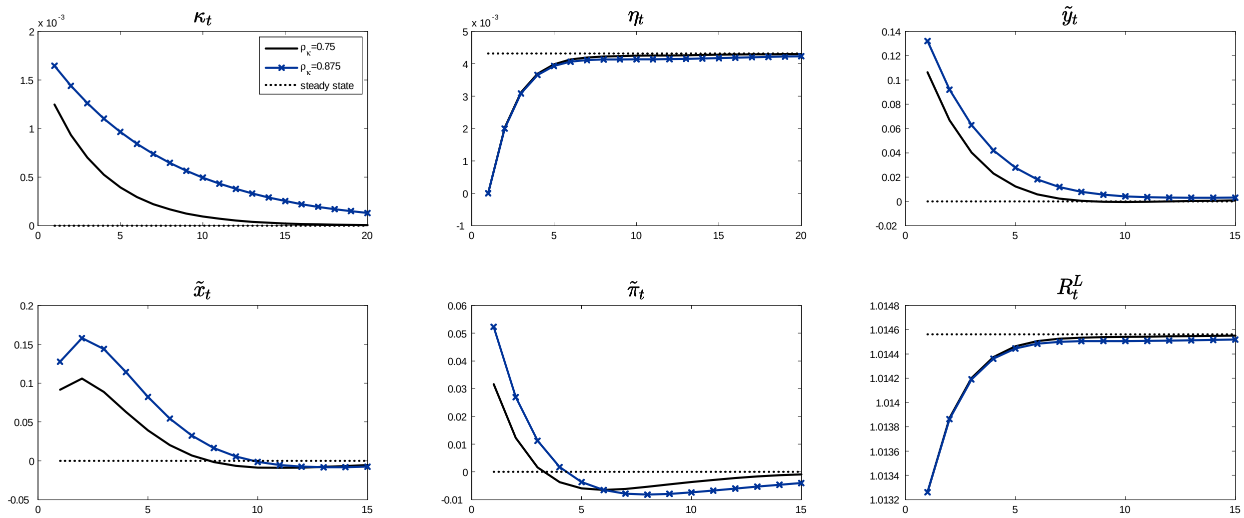

Figure 1: Maximum effects of quantitative easing

for balance sheet policies to be effective (see Proposition 1). Thus, (37) shows that the range over which the collateral constraint is binding is particularly large at low policy rates, and that the multiplier approaches zero if an increase in current consumption is sufficiently large and not too persistent. Beyond this point, $\eta_t = 0$ and balance sheet policies are neutral.

We consider effects of an unexpected increase in κ_t with an autocorrelation of $\rho_\kappa = 0.75$ (0.875), which corresponds to an expected duration of the policy of one year (two years). Figure 1 shows impulse responses to the maximum quantitative easing policy, which is defined as an increase in κ_t that just lets the collateral constraint be binding on impact, $\eta_1 \to 0$, which implies an increase of $\triangle\kappa_t = 0.12\%$ for $\rho_\kappa = 0.75$ (see solid line). This policy induces the loan rate to fall close to the policy rate and leads to a rise in output by 0.106% and in inflation by 3.2 basis points. When the policy is more persistent, $\rho_\kappa = 0.875$ (see starred line) a larger intervention is possible according to (37). Quantitative easing can then be conducted at a larger scale ($\triangle\kappa_t = 0.16\%$), such that output rises by 0.13% and inflation by 5.2 basis points. Compared to conventional monetary policy, the maximum output effect of quantitative easing (for $\rho_\kappa = 0.875$) is rather small and corresponds to the output effect of a reduction in the policy rate of about 7 basis points.

5.3 Quantitative easing under liquidity demand shocks

We now consider a second scenario where the economy is hit by an increase in the fraction $\upsilon_{x,t}$ of investment goods that have to be purchased with cash (see 36). This shock, which implies that less investment can be financed on credit, can be interpreted as an increase in financial distress

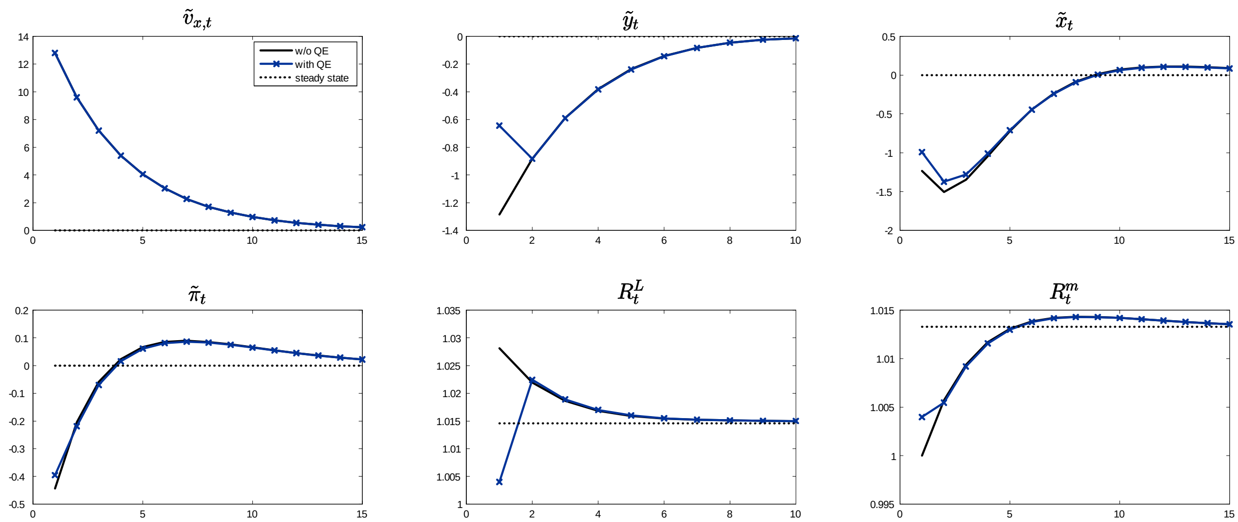

Figure 2: Liquidity demand shock with and without quantitative easing

that lowers the extent to which investment can be pledged as collateral, which – as argued by Del Negro et al. (2013) – has been as a major factor in the crisis of 2008. We apply an AR(1) process for $\upsilon_{x,t}$ with mean $\upsilon_x > 0$ and an autocorrelation of 0.75. The policy rate is endogenously adjusted according to the Taylor rule. We consider a shock that drives the policy rate to the ZLB in the impact period, which requires $\widetilde{\upsilon}_{x,t} = 12.78\%$. The solid line in Figure 2 shows the impulse responses to this shock without quantitative easing. Investment and consumption fall, so that output declines by 1.29% despite the endogenous reduction of the policy rate. Inflation falls, while the spread between the policy rate and the loan rate increases in a substantial way, which increases the scope for quantitative easing.

The starred line shows the responses for the case where the central bank applies a maximum quantitative easing policy in terms of corporate debt at the ZLB, which is again identified by a multiplier on the collateral constraint approaching zero on impact, $\eta_1 \to 0$. Though, the central bank can accommodate the increase in money demand by raising κ_t, it cannot completely neutralize the liquidity demand shocks, even when the collateral constraint is binding. The reason is that an increase in $\upsilon_{x,t}$ does not only affect money demand via (36), but also tends to raise the required return on investment in physical capital, given that investment – as "cash goods" – become more costly.[33] Quantitative easing is only conducted in the first period, as the policy rate increases

[33]This can be seen from the first order condition for investments x_t, which for a simplified case with $S_t = 1$ and $S'_t = 0$ can be written as $\lambda_t + \upsilon_{x,t}\psi_t = \beta E_t \left[\lambda_{t+1} r^k_{t+1} + (1-\delta_k)\left(\lambda_{t+1} + \upsilon_{x,t+1}\psi_{t+1}\right)\right]$, where the RHS measures the marginal costs of investment in physical capital.

afterwards. This policy substantially mitigates the contractionary output effect of the shock, as output falls on impact by only 0.65% (while it is virtually unaffected afterwards). Inflation falls by slightly less (5 basis points) than in the case without intervention. Since the impact output contraction is reduced by 50%, the analysis shows that quantitative easing can exert output effects in response to liquidity demand shocks that are much larger than in the case where it is exogenously conducted (see Section 5.2). The reason is that the liquidity premium, which determines the scope of balance sheet policies, is here much larger than in the latter case.

6 Conclusion

Balance sheet policies have recently been introduced by several central banks, while policy rates were set at the ZLB. At the same time, conventional macroeconomic analysis of monetary policy predicts that balance sheet policies at the ZLB are irrelevant as long as they do not affect expectations about future polices. In this paper, we augment a standard monetary model to be applicable for the analysis of the effects as well as the limitations of balance sheet policies. We show that they can be non-neutral, even when the central bank cannot commit to future policies and financial intermediation is not disrupted. As a main principle, we find that this relies on the scarcity of eligible assets, which is reflected by a liquidity premium. We further show that quantitative easing (i.e. increasing the balance sheet's size) and collateral policy (i.e. changing the balance sheet's composition) are not equivalent to policy rate adjustments and are particularly useful when pure interest rate policy reaches its limits: Collateral policy can neutralize shifts in firms' borrowing costs, while quantitative easing allows to implement a stabilizing policy even when the policy rate is at the ZLB. A numerical analysis shows that quantitative easing is particularly helpful in response to a surge in liquidity demand as in the crisis of 2008.

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A Appendix

A.1 Rational expectations equilibrium

In equilibrium, markets clear, $\int_0^1 n_{j,t}dj = \int_0^1 n_{i,t}di$, $\int_0^1 y_{j,t}dj = \int_0^1 c_{i,t}di$, $\int_0^1 L_{i,t}di = \int_0^1 L_{j,t}dj$, $\int_0^1 D_{i,t}di = 0$, $\int_0^1 M_{i,t}^H di = M_t^H$, $\int_0^1 M_{i,t}^R di = M_t^R$, $\int_0^1 M_{i,t}^L di = M_t^L$, $\int_0^1 B_{i,t}di = B_t$, $\int_0^1 I_{i,t}di = I_t = M_t^H - M_{t-1}^H + M_t^R + M_t^L$, and $B_t^T = B_t + B_t^C$, and (9) can be written as $Z_t = \frac{\varepsilon}{\varepsilon-1} Z_{1,t}/Z_{2,t}$, where $Z_t = \widetilde{P}_t/P_t$, $Z_{1,t} = c_t^{-\sigma} y_t mc_t + \phi\beta E_t \pi_{t+1}^{\varepsilon} Z_{1,t+1}$ and $Z_{2,t} = c_t^{-\sigma} y_t + \phi\beta E_t \pi_{t+1}^{\varepsilon-1} Z_{2,t+1}$. The price index $P_t = (\int_0^1 P_{k,t}^{1-\varepsilon} dk)^{1/(1-\varepsilon)}$ implies $1 = (1-\phi)\, Z_t^{1-\varepsilon} + \phi\pi_t^{\varepsilon-1}$. Aggregate output satisfies $x_t = n_t^{\alpha}$, where $n_t = \int_0^1 n_{j,t}dj$, since $E_t(\omega_{j,t}) = 1$, and $x_t = \int_0^1 y_{k,t}dk$, implies $n_t^{\alpha} = \int_0^1 (P_{k,t}/P_t)^{-\varepsilon} y_t dk \Leftrightarrow y_t = n_t^{\alpha}/s_t$, where $s_t = \int_0^1 (P_{k,t}/P_t)^{-\varepsilon} dk$ and $s_t = (1-\phi) Z_t^{-\varepsilon} + \phi s_{t-1}\pi_t^{\varepsilon}$ given s_{-1}.

Definition 3 *A RE equilibrium is given by a set of sequences* $\{c_t,\, y_t,\, n_t,\, \lambda_t,\, m_t^R,\, m_t^H,\, m_t^L,\, b_t,\, b_t^T,\, l_t,\, w_t,\, mc_t,\, \widetilde{Z}_t,\, s_t,\, \pi_t,\, R_t,\, R_t^{Euler},\, R_t^L\}_{t=0}^{\infty}$ *satisfying*

$$\theta n_t^{\sigma_n} = w_t c_t^{-\sigma}, \tag{38}$$

$$\left[(1-\delta_t^e)\, R_t^L\right]^{-1} = \kappa_t \left[(1-\delta_t^e)\, R_t^m\right]^{-1} + \left[1 - \kappa_t/(1-\delta_t^e)\right] \beta c_t^{\sigma} E_t \left[\xi_{t+1} c_{t+1}^{-\sigma} / \left(\xi_t \pi_{t+1}\right)\right], \tag{39}$$

$$\lambda_t = \beta E_t \left[\xi_{t+1} c_{t+1}^{-\sigma} / \pi_{t+1}\right], \tag{40}$$

$$\lambda_t = \beta R_t E_t [\left(\lambda_{t+1}\left(1-\kappa_{t+1}^B\right) + \kappa_{t+1}^B \xi_{t+1} c_{t+1}^{-\sigma}/R_{t+1}^m\right) \pi_{t+1}^{-1}], \tag{41}$$

$$1/R_t^{Euler} = \beta E_t \left[\left(\xi_{t+1} c_{t+1}^{-\sigma}\right) / \left(\xi_t c_t^{-\sigma} \pi_{t+1}\right)\right], \tag{42}$$

$$c_t = m_t^H + m_t^R + m_t^L, \text{ if } R_t^{Euler} > 1, \tag{43}$$

$$\text{or } c_t \leq m_t^H + m_t^R + m_t^L, \text{ if } R_t^{Euler} = 1,$$

$$\kappa_t^B b_{t-1}/\left(R_t^m \pi_t\right) = m_t^H - m_{t-1}^H \pi_t^{-1} + m_t^R, \text{ if } R_t^{Euler} > R_t^m, \tag{44}$$

$$\text{or } \kappa_t^B b_{t-1}/\left(R_t^m \pi_t\right) \geq m_t^H - m_{t-1}^H \pi_t^{-1} + m_t^R, \text{ if } R_t^{Euler} = R_t^m,$$

$$\kappa_t l_t / R_t^m = m_t^L \text{ if } R_t^{Euler} > R_t^m \text{ or } \kappa_t l_t / R_t^m \geq m_t^L \text{ if } R_t^{Euler} = R_t^m, \tag{45}$$

$$b_t - b_{t-1}\pi_t^{-1} = (\Gamma - 1) b_{t-1}^T \pi_t^{-1} - \left(m_t^H - m_{t-1}^H \pi_t^{-1}\right), \tag{46}$$

$$mc_t \alpha n_t^{\alpha-1} = w_t R_t^L, \tag{47}$$

$$l_t / R_t^L = w_t n_t, \tag{48}$$

$$\widetilde{Z}_t \left(\varepsilon - 1\right)/\varepsilon = Z_{1,t}/Z_{2,t}, \tag{49}$$

$$\text{where } Z_{1,t} = c_t^{-\sigma} y_t mc_t + \phi\beta E_t \pi_{t+1}^{\varepsilon} Z_{1,t+1} \text{ and } Z_{2,t} = c_t^{-\sigma} y_t + \phi\beta E_t \pi_{t+1}^{\varepsilon-1} Z_{2,t+1},$$

$$1 = (1-\phi)(\widetilde{Z}_t)^{1-\varepsilon} + \phi\pi_t^{\varepsilon-1}, \tag{50}$$

$$m_t^R = \Omega_t m_t^H, \tag{51}$$

$$b_t^T = \Gamma b_{t-1}^T / \pi_t, \tag{52}$$

$$y_t = n_t^{\alpha} / s_t, \tag{53}$$

$$y_t = c_t, \tag{54}$$

$$s_t = (1-\phi)\widetilde{Z}_t^{-\varepsilon} + \phi s_{t-1} \pi_t^{\varepsilon}, \tag{55}$$

the transversality conditions, a monetary policy setting $\{R_t^m \geq 1,\, \kappa_t^B \in (0,1],\, \kappa_t \in [0,1]\}_{t=0}^{\infty}$, $\Omega > 0$ *and* $\pi \geq \beta$, *and a fiscal policy setting* $\Gamma \geq 1$, *for given sequences* $\{\xi_t, \delta_t^e\}_{t=0}^{\infty}$ *and initial values* $m_{-1}^H > 0$, $b_{-1} > 0$, $b_{-1}^T > 0$, *and* $s_{-1} \geq 1$.

The price of government bonds enters the set of equilibrium conditions in Definition 3 only via (41) and is irrelevant for the equilibrium allocation. We can therefore redefine the RE equilibrium for the case where the cash and collateral constraints are binding, which requires that the Euler equation rate R_t^{Euler} exceeds the policy rate $R_t^m \geq 1$ (see 43, 44, and 45).

Definition 4 *A RE equilibrium with a binding cash constraint and a binding collateral constraint is given by a set of sequences* $\{c_t,\ y_t,\ n_t,\ m_t^R,\ m_t^H,\ m_t^L,\ b_t,\ b_t^T,\ l_t,\ w_t,\ mc_t,\ Z_t,\ s_t,\ \pi_t,\ R_t^L\}_{t=0}^{\infty}$ *satisfying (38), (39), (46)-(55),*

$$c_t = \kappa_t^B b_{t-1} \left(R_t^m \pi_t\right)^{-1} + m_{t-1}^H \pi_t^{-1} + \kappa_t l_t / R_t^m, \tag{56}$$

the transversality conditions, a fiscal policy setting $\Gamma \geq 1$*, and a monetary policy setting* $\{R_t^m,\ \kappa_t^B,\ \kappa_t\}_{t=0}^{\infty}$*, where* $R_t^m \in \left[1, 1/\left\{\beta E_t\left[\xi_{t+1} c_{t+1}^{-\sigma} / \left(\xi_t c_t^{-\sigma} \pi_{t+1}\right)\right]\right\}\right)$*,* $\Omega_t > 0$*, and* $\pi \geq \beta$*, for given sequences* $\{\xi_t, \delta_t^e\}_{t=0}^{\infty}$*, initial values* $M_{-1}^H > 0$*,* $B_{-1} > 0$*,* $B_{-1}^T > 0$*, and* $s_{-1} \geq 1$*.*

When money supply is not effectively rationed, i.e. when the collateral constraints (44) and (45) are slack, and there are no idiosyncratic productivity shocks, the model reduces to a standard sticky price model (with only one interest rate $R_t^L = R_t^m = R_t^{Euler}$), for which a RE equilibrium can be defined as follows.

Definition 5 *A RE equilibrium for* $\eta_t = 0$ *and* $\omega_{j,t} = 1$ *is a set of sequences* $\{c_t,\ y_t,\ n_t,\ l_t,\ w_t,\ mc_t,\ \widetilde{Z}_t,\ s_t,\ \pi_t,\ R_t^L\}_{t=0}^{\infty}$ *satisfying* $\mu_t \theta n_t^{\sigma_n} = w_t c_t^{-\sigma}$*,* $\xi_t c_t^{-\sigma} = \beta R_t^m E_t[\xi_{t+1} c_{t+1}^{-\sigma} \pi_{t+1}^{-1}]$*,* $R_t^L = R_t^m$*, (47)-(50), (53)-(55), the transversality conditions, a monetary policy setting* $\{R_t^m \geq 1\}_{t=0}^{\infty}$ *and the inflation target* $\pi \geq \beta$*, for a given sequence* $\{\xi_t\}_{t=0}^{\infty}$ *and* $s_{-1} \geq 1$*.*

The policy instruments κ_t and κ_t^B enter the set of equilibrium conditions listed in Definition 4 only via (39) and (56), while they are apparently irrelevant for the equilibrium given in Definition 5.

Next, we consider the model given in Definition 4 for the simplifying case where $\omega_{j,t} = 1$, $\phi = 0$, $\sigma = 1$, $\sigma_n = 0$, $\alpha = 1$, and $\Omega \to \infty$, such that $M^H \to 0$. Then, a RE equilibrium given in Definition 4 is a set of sequences $\{c_t,\ y_t,\ n_t,\ b_t,\ l_t,\ w_t,\ \pi_t,\ R_t^L\}_{t=0}^{\infty}$ satisfying (38), (48), (54), $b_t = \Gamma b_{t-1} \pi_t^{-1}$,

$$\frac{1}{R_t^L} = \frac{\kappa_t}{R_t^m} + (1 - \kappa_t)\beta E_t \frac{\xi_{t+1} c_t}{\xi_t c_{t+1} \pi_{t+1}},\ c_t = \frac{\kappa_t^B b_{t-1}}{R_t^m \pi_t} + \frac{\kappa_t l_t}{R_t^m},\ w_t R_t^L = \frac{\varepsilon - 1}{\varepsilon} \alpha n_t^{\alpha - 1},\ c_t = n_t^{\alpha}, \tag{57}$$

and $R_t^m < 1/\left[\xi_t^{-1} c_t \beta E_t\left(\xi_{t+1} c_{t+1}^{-1} \pi_{t+1}^{-1}\right)\right]$. Eliminating n_t, w_t, c_t and l_t in (57) gives the set of equilibrium conditions listed in Definition 1.

A.2 Appendix to Section 4

Proof of proposition 2. Consider a RE equilibrium as given in Definition 4 for $\xi_t = 1$, where the monetary policy instruments R_t^m, κ_t and κ_t^B affect the equilibrium allocation and price system via (39) and (56). According to (39), changes in δ_t^e tend to alter the loan rate and therefore labor demand (see 47). Define $\overline{a}_t$ as the equilibrium value of a generic variable a_t that prevails for $\omega_{j,t} = 0$ (such that $\delta_t^e = 0$). Then, (39) implies that $\overline{R}_t^L = \{(1 - \overline{\kappa}_t)\, \beta E_t[(\overline{c}_{t+1}^{-\sigma}/\overline{c}_t^{-\sigma})\, \overline{\pi}_{t+1}^{-1}] + \overline{\kappa}_t/\overline{R}_t^m\}^{-1}$ holds for some $\overline{\kappa}_t \in [0,1]$. Let $\widetilde{\kappa}_t$ be a particular value for the fraction of eligible loans given by

$$\widetilde{\kappa}_t = \left[1 - (1 - \delta_t^e)\,(\overline{R}_t^L/\overline{R}_t^{Euler})\right] / \left[(\overline{R}_t^L/\overline{R}_t^m) - (\overline{R}_t^L/\overline{R}_t^{Euler})\right] \in [0,1], \tag{58}$$

where $\overline{R}_t^{Euler} = \left(\beta E_t[(\overline{c}_{t+1}^{-\sigma}/\overline{c}_t^{-\sigma})\, \overline{\pi}_{t+1}^{-1}]\right)^{-1} \geq \overline{R}_t^L$. By setting $\kappa_t = \widetilde{\kappa}_t$, the central bank thus offsets the immediate impact of δ_t^e on the loan rate. Further, κ_t affects money supply and thereby consumption via (56). Setting $\kappa_t = \widetilde{\kappa}_t$ is however consistent with the equilibrium allocation and prices under $\delta_t^e = 0$ if κ_t^B is adjusted to neutralize the effect of $\widetilde{\kappa}_t$ on money supply and consumption, i.e. if $\kappa_t^B = \widetilde{\kappa}_t^B$ where

$$\widetilde{\kappa}_t^B = \frac{\overline{R}_t^m \overline{\pi}_t}{\overline{b}_{t-1}} \left[\overline{c}_t - \overline{m}_{t-1}^H \overline{\pi}_t^{-1} - \frac{\overline{l}_t}{\overline{R}_t^m} \frac{1 - (1 - \delta_t^e)\,(\overline{R}_t^L/\overline{R}_t^{Euler})}{(\overline{R}_t^L/\overline{R}_t^m) - (\overline{R}_t^L/\overline{R}_t^{Euler})}\right],$$

for which we eliminated κ_t with $\widetilde{\kappa}_t$ (see 58) in (56). Hence, if κ_t and κ_t^B are set equal to $\widetilde{\kappa}_t \in [0,1]$ and $\widetilde{\kappa}_t^B \in [0,1]$, default risk shocks are completely neutralized. If however only the policy rate R_t^m is adjusted to offset the impact of δ_t^e on the loan rate, the RHS of (56) is altered such that consumption cannot be held at $\overline{c}_t$. ■

We now characterize the steady state. The central bank sets $R^m \geq 1$, $\kappa^B \in (0,1]$, $\kappa \in [0,1]$ and the long-run (target) value for the inflation rate $\pi \geq \beta$. In a steady state, all endogenous variables grow with a constant rate and time-invariant policy targets have to be consistent with long-run equilibrium values. In what follows, we examine the properties of all other endogenous variables in a steady state with $\delta^e = 0$. Given steady state inflation π, (50) implies that $\widetilde{Z} = \left((1 - \phi\pi^{\varepsilon-1}) / (1 - \phi)\right)^{1/(1-\varepsilon)}$, and (49) that Z_1/Z_2 is constant. The price dispersion term s_t, satisfying (55), thus converges in the long-run to $s = \frac{1-\phi}{1-\phi\pi^{\varepsilon}} \widetilde{Z}^{-\varepsilon}$, given that $\phi\pi^{\varepsilon} < 1 \Leftrightarrow \pi < (1/\phi)^{1/\varepsilon}$. Since s is bounded from below and neither productivity nor labor supply exhibit trend growth, real resources cannot permanently grow with a non-zero rate, $y = c = n^{\alpha}/s$. Then, $Z_{2,t}$ converges to $Z_2 = yc^{-\sigma}/\left(1 - \phi\beta\pi^{\varepsilon-1}\right)$ if $\phi\beta\pi^{\varepsilon-1} < 1 \Leftrightarrow \pi < [1/(\phi\beta)]^{1/(\varepsilon-1)}$. Given that Z_1/Z_2 as well as $\widetilde{Z}$ are constant and $Z_{1,t} = Z_1 = \frac{yc^{-\sigma}mc}{1-\phi\beta\pi^{\varepsilon}}$ holds, real marginal costs are also constant and given by $mc = \widetilde{Z}\,(\varepsilon - 1)\,\varepsilon^{-1}\,(1 - \phi\beta\pi^{\varepsilon}) / \left(1 - \phi\beta\pi^{\varepsilon-1}\right)$. Since steady state consumption is constant, (42) determines the steady state Euler equation rate in the usual way, $R^{Euler} = \pi/\beta$ (see 18). Condition (39) further implies for the steady state loan rate

$$\left(1/R^L\right) = \kappa\,(1/R^m) + (1 - \kappa)\beta/\pi. \tag{59}$$

Given that the loan rate, marginal costs, and working time are constant in a steady state, (47) implies a constant steady state wage rate, $w = mc\alpha n^{\alpha-1}/R^L$. Moreover, the steady state is characterized by $\theta n^{\sigma_n} = wc^{-\sigma}$, $c = n^\alpha$, and $l = R^L wn = R^L \theta c^{\sigma+(1+\sigma_n)/\alpha} = \frac{\varepsilon-1}{\varepsilon}\alpha c$ (see 38, 53, 54, and 48). We now consider the simplified case, where $\Omega \to \infty$ and $\Gamma = \pi = 1$. Log-linearizing (38), (47), (49), (50), (53), (54), and $b_t = \Gamma b_{t-1}\pi_t^{-1}$ at the steady state gives

$$\sigma\widehat{c}_t + \sigma_n\widehat{n}_t = \widehat{w}_t,\ \widehat{mc}_t = \widehat{w}_t + \widehat{R}_t^L + (1-\alpha)\,\widehat{n}_t,\ \widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \chi\widehat{mc}_t,\ \alpha^{-1}\widehat{y}_t = \widehat{n}_t, \tag{60}$$

$$\widehat{c}_t = \widehat{y}_t,\ \widehat{b}_t = \widehat{b}_{t-1} - \widehat{\pi}_t, \tag{61}$$

where $\chi = (1-\phi)(1-\beta\phi)/\phi$. Further, log-linearizing (39) for $\delta_t^e = 0$, using (59), and defining $\varkappa = \frac{\kappa}{1-\kappa}\frac{\pi/\beta}{R^m}$, we get

$$\sigma E_t\widehat{c}_{t+1} - \sigma\widehat{c}_t + \left(1-\rho_\xi\right)\widehat{\xi}_t + E_t\widehat{\pi}_{t+1} - \left(\varkappa - \kappa/(1-\kappa)\right)\widehat{\kappa}_t + \varkappa\widehat{R}_t^m = (1+\varkappa)\,\widehat{R}_t^L, \tag{62}$$

where we used that the preference shock is autocorrelated: $E_t\widehat{\xi}_{t+1} = \rho_\xi\widehat{\xi}_t$. Combining (44) and (45) with (48), to get $\kappa_t R_t^L w_t n_t + \kappa_t^B b_{t-1}/\pi_t = R_t^m(m_t^R + m_t^L)$, gives in linearized form

$$(\varsigma\varpi - 1)\,\widehat{c}_t + \varsigma\widehat{\kappa}_t + \varsigma\widehat{R}_t^L + (1-\varsigma)\,\widehat{\kappa}_t^B + (1-\varsigma)\,\widehat{b}_{t-1} - (1-\varsigma)\,\widehat{\pi}_t = \widehat{R_t}^m, \tag{63}$$

where we eliminated wages (with 60), and $\varsigma = \kappa\frac{\varepsilon-1}{\varepsilon}\alpha/R^m > 1$ and $\varpi = \frac{1+\sigma_n}{\alpha} + \sigma > 1+\sigma$ hold. Eliminating $\widehat{w}_t$, $\widehat{n}_t$, and $\widehat{mc}_t$ in (60)-(63), we can summarize the RE equilibrium as a set of sequences $\{\widehat{R}_t^L, \widehat{c}_t, \widehat{y}_t, \pi_t, \widehat{b}_t\}_{t=0}^\infty$ that converge to the steady state and satisfy (61), (62), (63), and

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \chi\,(\varpi-1)\,\widehat{c}_t + \chi\widehat{R}_t^L, \tag{64}$$

for $\{\widehat{\kappa}_t, \widehat{\kappa}_t^B, \widehat{R}_t^m\}_{t=0}^\infty$ set by the central bank, given $\{\widehat{\xi}_t\}_{t=0}^\infty$ and $b_{-1} > 0$. The conditions (61)-(64), reduce to (31)-(34) for $\kappa = 0 \Rightarrow \varsigma = \varkappa = 0$.

Proof of proposition 3. Consider a RE equilibrium as given in Definition 2. To establish the claims made in the proposition, we first combine the constraint to the policy problem, $\widehat{\pi}_t = (\beta+\chi)\,E_t\widehat{\pi}_{t+1} + \chi\gamma\widehat{y}_t + \chi\sigma E_t\widehat{y}_{t+1} + \chi\left(1-\rho_\xi\right)\widehat{\xi}_t$, with the targeting rule (35) to get

$$\left((\beta+\chi)\,\varepsilon^{-1}\,(1+\sigma/\gamma) - \chi\sigma\right)\widehat{y}_{t+1} = \left(\chi\gamma + \varepsilon^{-1}\,(1+\sigma/\gamma)\right)\widehat{y}_t + \chi\widehat{u}_t, \tag{65}$$

where $\widehat{u}_t = \left(1-\rho_\xi\right)\widehat{\xi}_t$. There is a unique solution if $(\beta+\chi)\,\frac{1}{\varepsilon}\frac{\sigma+\gamma}{\gamma} = \chi\sigma$, i.e. $\widehat{y}_t = \{\chi/[\chi\gamma + \varepsilon^{-1}\,(1+\sigma/\gamma)]\}\widehat{u}_t$ and $\widehat{\pi}_t = -\{\varepsilon^{-1}\,(1+\sigma/\gamma)\,\chi/[\chi\gamma + \varepsilon^{-1}\,(1+\sigma/\gamma)]\}\widehat{u}_t$ or, otherwise, if the eigenvalue of (65) lies outside the unit circle, $\left|\frac{\chi\gamma+\varepsilon^{-1}(1+\sigma/\gamma)}{(\beta+\chi)\varepsilon^{-1}(1+\sigma/\gamma)-\chi\sigma}\right| > 1$. If $(\beta+\chi)\,\varepsilon^{-1}\left(\frac{\sigma+\gamma}{\gamma}\right) <$

$\chi\sigma$, this requires $\chi(\gamma-\sigma)+\varepsilon^{-1}(1+\sigma/\gamma)(1+\beta+\chi)>0$, which is satisfied for $\gamma>\sigma$ where $\gamma=(1+\sigma_n)/\alpha$. If $(\beta+\chi)\varepsilon^{-1}(1+\sigma/\gamma)>\chi\sigma$, uniqueness requires $\varepsilon^{-1}(1+\sigma/\gamma)[\chi(\varepsilon\gamma-1)+(1-\beta)]>0$, which is always satisfied given that $\varepsilon>1$ and $\gamma>1$. The unique solutions for output and inflation take the form $\widehat{y}_t=\delta_y\widehat{u}_t$ and $\widehat{\pi}_t=\delta_\pi\widehat{u}_t$, where the coefficients δ_y and δ_π can be identified via the method of undetermined coefficients: For $(\beta+\chi)\varepsilon^{-1}(1+\sigma/\gamma)\neq\chi\sigma$, the coefficients are

$$\delta_y=-\varepsilon\chi/\Theta<0 \text{ and } \delta_\pi=(1+\sigma/\gamma)\chi/\Theta>0, \tag{66}$$

where $\Theta=(1-\beta\rho)(1+\sigma/\gamma)+\chi(\rho(\varepsilon\gamma-1)(1+\sigma/\gamma)+\varepsilon\gamma(1-\rho))>0$. To implement this solution, the central bank has to set its instruments R_t^m and κ_t^B according to

$$\widehat{R_t}^m-\widehat{\kappa}_t^B=\widehat{b}_{t-1}-[\delta_\pi+\delta_y]\left(1-\rho_\xi\right)\widehat{\xi}_t, \tag{67}$$

where we used (31). For $\varepsilon>(\sigma/\gamma)+1$, which is satisfied if $\varepsilon>2$ for $\gamma>\sigma$, the term in the square brackets in (67) is negative, such that a decline in $\widehat{\xi}_t$ requires the central bank either to lower R_t^m or to raise κ_t^B to implement its plan. The former is possible as long as $R_t^m>1$ and thus for shocks satisfying $\widehat{\xi}_t\geq\overline{\xi}$, where $\overline{\xi}=\frac{1-1/R^m}{\chi[(\sigma/\gamma)+1-\varepsilon]\left(1-\rho_\xi\right)}\Theta<0$ (see 66 and 67). For $\widehat{\xi}_t<\overline{\xi}$, the policy plan cannot be implemented by a policy rate reduction. The central bank can then implement its plan by rasing $\widehat{\kappa}_t^B$, which requires that $\kappa^B<1$ and that the collateral constraint is binding, i.e. that $\eta_t>0$ implying $R_t^{Euler}=R_t^L>R_t^m=1$ (see 24). Using (32) and the solutions (see 66), it can easily be shown that $\eta_t>0$ holds if $\widehat{\xi}_t\geq\underline{\xi}$, where $\underline{\xi}=-\frac{(1-\beta/\pi)\Theta}{[(\sigma+\gamma)\varepsilon\chi+(1-\beta\rho)(1+\sigma/\gamma)]\left(1-\rho_\xi\right)}<0$. Thus, for $\widehat{\xi}_t\in(\underline{\xi},\overline{\xi})$, where $(\underline{\xi},\overline{\xi})\neq\varnothing$ if

$$\frac{1-\beta/\pi}{1-1/R^m}>\frac{(\sigma+\gamma)\varepsilon\chi+(1-\beta\rho)(1+\sigma/\gamma)}{\chi[\varepsilon-(1+\sigma/\gamma)]}, \tag{68}$$

the central bank can implement the policy plan at the ZLB with quantitative easing, but not with policy rate adjustments. ■

B Additional Appendix (*not intended for publication*)

In this Appendix, we present two propositions on long-run inflation and interest rate effects of quantitative easing, and a definition of the RE equilibrium of the extended version with capital accumulation (where we disregard preference shocks and idiosyncratic productivity shocks).

Proposition 4 (Long-run inflation) *Consider a steady state of the RE equilibrium given in Definition 1. The central bank can control the steady state inflation rate via the growth rate $g_{\kappa,t} = \kappa_t^B/\kappa_{t-1}^B$, and it can implement long-run price stability by setting $g_{\kappa,t}$ equal to the inverse of the growth rate of government bonds Γ^{-1}.*

Proof. Consider the set of equilibrium conditions given in Definition 1 for $\xi_t = 1$. Use (27) or $\kappa_t^B b_{t-1}/\pi_t = (R_t^m - \kappa_t\mu)\, y_t$ as well as its time $t+1$ version and $b_t = \Gamma b_{t-1}\pi_t^{-1}$, to eliminate y_t and y_{t+1} in (26):

$$\frac{1}{R_t^L} = \frac{\kappa_t}{R_t^m} + (1-\kappa_t)\frac{\beta}{\Gamma}E_t\left[\frac{R_{t+1}^m - \kappa_{t+1}\mu}{R_t^m - \kappa_t\mu}\frac{\kappa_t^B}{\kappa_{t+1}^B}\right], \tag{69}$$

where $\mu = \frac{\varepsilon-1}{\varepsilon}\alpha \in (0,1)$. Then, divide both sides of $\kappa_t^B b_{t-1}/\pi_t = (R_t^m - \mu\kappa_t)\, y_t$ by its period $t-1$ version $\kappa_{t-1}^B b_{t-2}/\pi_{t-1} = \left(R_{t-1}^m - \mu\kappa_{t-1}\right) y_{t-1}$, and use $b_{t-1} = \Gamma b_{t-2}\pi_{t-1}^{-1}$ to express inflation as $\pi_t = \Gamma\frac{\kappa_t^B}{\kappa_{t-1}^B}\frac{R_{t-1}^m - \mu\kappa_{t-1}}{R_t^m - \mu\kappa_t}\frac{y_{t-1}}{y_t}$. Replacing output with (27) in the latter and in $R_t^m < 1/\left[y_t\beta E_t\left(y_{t+1}^{-1}\pi_{t+1}^{-1}\right)\right]$, and real bonds with (28), leads to the following four equilibrium conditions for R_t^L, y_t, π_t and η_t: (69), $y_t = (\mu/\theta)^{\alpha/(1+\sigma_n)}\left(1/R_t^L\right)^{\alpha/(1+\sigma_n)}$,

$$\pi_t = \Gamma\frac{\kappa_t^B}{\kappa_{t-1}^B}\frac{R_{t-1}^m - \mu\kappa_{t-1}}{R_t^m - \mu\kappa_t}\left(\frac{R_t^L}{R_{t-1}^L}\right)^{\alpha/(1+\sigma_n)}, \tag{70}$$

$\frac{\eta_t}{y_t^{-1}} = \frac{1}{R_t^m} - \frac{\beta}{\Gamma}E_t\frac{\kappa_t^B}{\kappa_{t+1}^B}\frac{R_{t+1}^m - \mu\kappa_{t+1}}{R_t^m - \mu\kappa_t} > 0$, which can be solved sequentially. Now, consider a steady state, where all variables grow with a constant rate (so that time indices are considered), and suppose that the central bank holds R_t^m and κ_t constant in the long-run. Then, (69) and (70) imply that steady state inflation depends only on the growth rate of κ_t^B and on $\Gamma : \pi = \Gamma g_{\kappa,t}$, where $\kappa_t^B = g_{\kappa,t}\kappa_{t-1}^B$. The central bank can therefore control the steady state inflation rate by setting $g_{\kappa,t}$ contingent on Γ, while the steady state price level is stable, $\pi = 1$, iff $g_\kappa = \Gamma^{-1}$. ■

Proposition 5 (QE-effects on interest rates) *Consider a RE equilibrium as given in Definition 2 for $\sigma < (1+\sigma_n)/\alpha$. When the policy rate is at the ZLB, the equilibrium is uniquely determined and an exogenous increase in κ_t^B leads to a decline in the bond rate as well as in the loan rate.*

Proof of proposition 5. To establish the claims made in the proposition, we consider the effects of an exogenous and autocorrelated increase in κ_t^B (where ρ is the coefficient of autocorrelation) when the policy rate is held at the ZLB, $R_t^m = 1$. We start by examining the bond rate. Using (14) and (18), we can rewrite (21) in equilibrium as $\frac{1}{R_t} = E_t\left[\left(1-\kappa_{t+1}^B\right)\varphi_{t,t+1}\right] + \beta E_t[\kappa_{t+1}^B \frac{1}{R_{t+1}^m}\frac{\psi_{t+1}+\lambda_{t+1}}{\lambda_t \pi_{t+1}}]$. Using (17) and $R_t^{rf} = 1/E_t\varphi_{t,t+1}$, we get $(1/R_t) = \left(1 - E_t\kappa_{t+1}^B\right)(1/R_t^{rf}) + E_t\kappa_{t+1}^B \cdot E_t\left(1/R_{t+1}^m\right) + \mathcal{O}^2$, where $\mathcal{O}^2$ summarizes terms of higher than first order. Since $R_t^{rf} > R_t^m$ is initially satisfied in a steady state (where $R_t^{rf} = R^{Euler} = \pi/\beta$) with a binding collateral constraint, the bond rate R_t is, up to first order, decreasing in the expected fraction of eligible bonds $E_t\kappa_{t+1}^B$ for $R_t^m = 1$.

To examine the loan rate, we now consider the model given in definition 2 for $\widehat{\xi}_t = 0$, which can be further reduced to (34), $\widehat{\pi}_t = (\beta+\chi)E_t\widehat{\pi}_{t+1} + \chi(\varpi - 1 - \sigma)\widehat{y}_t + \chi\sigma E_t\widehat{y}_{t+1}$ and $\widehat{y}_t = \widehat{b}_{t-1} - \widehat{\pi}_t + \widehat{\kappa}_t^B$. Substituting output, leads to $\zeta_1 E_t\widehat{\pi}_{t+1} + \zeta_2\widehat{b}_t = \widehat{\pi}_t - \zeta_3\widehat{\kappa}_t^B$, which together with (34) can be written as

$$\begin{pmatrix} E_t\widehat{\pi}_{t+1} \\ \widehat{b}_t \end{pmatrix} = A\begin{pmatrix} \widehat{\pi}_t \\ \widehat{b}_{t-1} \end{pmatrix} + \begin{pmatrix} \zeta_1 & \zeta_2 \\ 0 & 1 \end{pmatrix}^{-1}\begin{pmatrix} -\zeta_3 \\ 0 \end{pmatrix}\widehat{\kappa}_t^B, \text{ where } A = \begin{pmatrix} \zeta_1 & \zeta_2 \\ 0 & 1 \end{pmatrix}^{-1}\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}.$$

where $\zeta_1 = \beta + \chi(1-\sigma)$, $\zeta_2 = \chi(\varpi - 1) > 0$, and $\zeta_3 = \chi(\varpi - 1 - \sigma(1-\rho)) > 0$. Given that there exists exactly one predetermined variable, $\widehat{b}_{t-1}$, local determinacy requires one stable and one unstable eigenvalue. The characteristic polynomial of A is given by $F(X) = X^2 - \left(1 + \zeta_1^{-1} + \zeta_2\zeta_1^{-1}\right)X + \zeta_1^{-1}$, where $F(0) = 1/\zeta_1$ and $F(1) = -\zeta_2/\zeta_1$ and $signF(0) = -signF(1)$. Hence, there exists at least one real stable eigenvalue between zero and one. Further, $F(X)$ at $X = -1$ is given by $F(-1) = \{-\chi(\sigma - [1 - \alpha + \sigma_n]/\alpha) + 2(1+\beta+\chi)\}/\zeta_1$, where the term in the curly brackets is strictly positive, such that $signF(0) = signF(-1)$, if and only if $\frac{1}{2}\left(1 + \sigma - \frac{1+\sigma_n}{\alpha}\right) < 1 + \frac{1+\beta}{\chi}$, which is ensured by $\sigma < (1+\sigma_n)/\alpha$. Then, there exists exactly one stable eigenvalue, between zero and one, and one unstable eigenvalue, indicating local determinacy.

Given that the stable eigenvalue is strictly positive, we know that the unique solution to the system (31)-(34), is given by the generic form $\widehat{\pi}_t = \delta_1\widehat{b}_{t-1} + \delta_2\widehat{\kappa}_t^B$, $\widehat{y}_t = \delta_3\widehat{b}_{t-1} + \delta_4\widehat{\kappa}_t^B$, $\widehat{R}_t^L = \delta_5\widehat{b}_{t-1} + \delta_6\widehat{\kappa}_t^B$, and $\widehat{b}_t = (1-\delta_1)\widehat{b}_{t-1} - \delta_2\widehat{\kappa}_t^B$, where the stable eigenvalue is $1 - \delta_1 \in (0,1)$. Applying the method of undetermined coefficients, it can easily be shown that the coefficients satisfy $\delta_3 = 1 - \delta_1 \in (0,1)$, $\delta_2 = (\chi\gamma + \sigma\chi\rho)/\Xi > 0$, and $\delta_4 = [(1-\rho\beta) + \delta_1\beta + (\sigma - 1)\chi\delta_3 + \chi(1-\rho)]/\Xi > 0$, where $\Xi = (1-\rho\beta) + \delta_1(\beta+\chi) + \chi\gamma + (\sigma-1)\chi\rho + \sigma\chi\delta_3 > 0$. Inserting these solutions in (32) and combining terms leads to the following coefficients for the loan rate solution: $\delta_5 = -\delta_1(1-\delta_1)(\sigma-1) < 0$ and $\delta_6 = -[(\sigma-1)\delta_3\delta_2 + (\delta_2 + \sigma\delta_4)(1-\rho)] < 0$. Given that $\partial\widehat{R}_t^L/\partial\widehat{\kappa}_t^B = \delta_6 < 0$, an increase in $\widehat{\kappa}_t^B$ reduces the loan rate on impact. ■

Definition 6 *A RE equilibrium with capital accumulation is given by a set of sequences* $\{c_t,\ y_t,\ k_t,\ x_t,\ n_t,\ \lambda_t,\ \psi_t,\ \eta_t,\ q_t,\ m_t^R,\ m_t^H,\ m_t^L,\ b_t,\ b_t^T,\ l_t,\ w_t,\ r_t^k,\ mc_t,\ \widetilde{Z}_t,\ s_t,\ \pi_t,\ R_t^L\}_{t=0}^{\infty}$ *satisfying (44)-(46), (48)-(52), (55), and*

$$c_t^{-\sigma} = \lambda_t + \upsilon_c \psi_t, \tag{71}$$

$$\theta n_t^{\sigma_n} = (\lambda_t + \psi_t)\, w_t, \tag{72}$$

$$\lambda_t + \psi_t = R_t^m (\lambda_t + \eta_t), \tag{73}$$

$$\lambda_t + \psi_t = R_t^L (\lambda_t + \eta_t \kappa_t), \tag{74}$$

$$\begin{aligned} \lambda_t + \upsilon_{x,t}\psi_t = {} & c_t^{-\sigma} q_t \left[S(x_t/x_{t-1}) + (x_t/x_{t-1})\, S'(x_t/x_{t-1}) \right] \\ & - \beta E_t c_{t+1}^{-\sigma} q_{t+1} \left[(x_{t+1}/x_t)^2\, S'(x_{t+1}/x_t) \right], \end{aligned} \tag{75}$$

$$c_t^{-\sigma} q_t = \beta E_t \left[\lambda_{t+1} r_{t+1}^k + (1-\delta_k)\, q_{t+1} c_{t+1}^{-\sigma} \right], \tag{76}$$

$$\lambda_t = \beta E_t \left[(\lambda_{t+1} + \psi_{t+1}) / \pi_{t+1} \right], \tag{77}$$

$$w_t R_t^L = mc_t \alpha (n_t/k_{t-1})^{\alpha-1}, \tag{78}$$

$$r_t^k = mc_t (1-\alpha)(n_t/k_{t-1})^{\alpha}, \tag{79}$$

$$k_t = (1-\delta_k)\, k_{t-1} + x_t S(x_t/x_{t-1}), \tag{80}$$

$$y_t = n_t^{\alpha} k_{t-1}^{1-\alpha} / s_t, \tag{81}$$

$$y_t (1 - g/y) = c_t + x_t, \tag{82}$$

$$\upsilon_c c_t + \upsilon_{x,t} x_t = m_t^H + m_t^R + m_t^L, \text{ if } \psi_t > 0, \tag{83}$$

$$\text{or } \upsilon_c c_t + \upsilon_{x,t} x_t \le m_t^H + m_t^R + m_t^L, \text{ if } \psi_t = 0, \tag{84}$$

(where q_t *denotes the value of installed capital relative to consumption goods and the adjustment cost function is given by* $S(x_t/x_{t-1}) = 1 - \frac{\vartheta}{2}(x_t/x_{t-1} - 1)^2$*) as well as the transversality conditions, a monetary policy setting* $\{R_t^m \ge 1,\ \kappa_t^B,\ \kappa_t \in [0,1]\}_{t=0}^{\infty}$*,* $\Omega > 0$*, and* $\pi \ge \beta$*, and a fiscal policy setting* $\Gamma \ge 1$ *and* $g/y > 0$*, for a given sequence* $\{\upsilon_{x,t}\}_{t=0}^{\infty}$ *and initial values* $m_{-1}^H > 0$*,* $b_{-1} > 0$*,* $b_{-1}^T > 0$*,* $k_{-1} > 0$*,* $x_{-1} > 0$*, and* $s_{-1} \ge 1$*.*

Table 1 Parameter and steady state values

Subjective discount factor	$\beta = 0.992$	Working time	$n = 0.33$
Relative risk aversion	$\sigma = 2$	Policy rate	$R^m = 1.0133$
Inverse of Frisch elasticity	$\sigma_n = 1$	Government share	$g/y = 0.19$
Substitution elasticity	$\varepsilon = 10$	Share of repos	$\Omega = 1.5$
Labor income share	$\alpha = 0.66$	Inflation rate	$\pi = 1.00647$
Investment adjustment cost parameter	$\vartheta = 2.48$	Fraction of eligible loans	$\kappa = 0$
Rate of depreciation of capital stock	$\delta_k = 0.03$	Fraction of eligible bonds	$\kappa^B = 1$
Growth rate of bonds	$\Gamma = 1.00647$	Cash-share of consumption	$\upsilon_c = 0.7399$
Fraction of non-price-adjusting firms	$\phi = 0.75$	Cash-share of investment	$\upsilon_x = 0.4292$