No. 59

UNIVERSITY OF COLOGNE

WORKING PAPER SERIES IN ECONOMICS

AGING AND PENSION REFORM: EXTENDING THE RETIREMENT AGE AND HUMAN CAPITAL FORMATION

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Aging and Pension Reform: Extending the Retirement Age and Human Capital Formation*

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This version: February 5, 2013

Abstract

Projected demographic changes in industrialized and developing countries vary in extent and timing but will reduce the share of the population in working age everywhere. Conventional wisdom suggests that this will increase capital intensity with falling rates of return to capital and increasing wages. This decreases welfare for middle aged agents with assets accumulated for retirement. This paper addresses three important adjustments channels to dampen these detrimental effects of ageing: investing abroad, endogenous human capital formation and increasing the retirement age. Although non of these suggestions is new in itself, we examine their effects jointly in one coherent model. Our quantitative finding is that openness has a relatively mild effect. In contrast, endogenous human capital formation in combination with an increase in the retirement age has strong effects. Under these adjustments maximum welfare losses of demographic change for households alive in 2010 are reduced by about 3 percentage points.

JEL classification: C68, E17, E25, J11, J24
Keywords: population aging; human capital; welfare; pension reform; retirement age; open economy

*This research was supported by the U.S. Social Security Administration through grant # 5RRC08098400-03-00 to the National Bureau of Economic Research (NBER) as part of the SSA Retirement Research Consortium. The findings and conclusions expressed are solely those of the authors and do not represent the views of the SSA, any agency of the Federal Government, the NBER, or of the European Central Bank (ECB). Further financial support by the State of Baden-Württemberg and the German Insurers Association (GDV) is gratefully acknowledged.

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1 Introduction

The world will experience major changes in its demographic structure in the next decades. In all countries, this process is driven by increasing life expectancy and falling birth rates. The fraction of the population in working-age will decrease and the fraction of people in old-age will increase. This process is already well under way in industrialized countries with many developing countries following suit in a few decades. Standard economic analyses predict that these demographic processes will increase the capital-labor ratio. Hence, rates of return to capital will decrease and wages increase, which has adverse welfare consequences for current cohorts who will be retired when the rate of return on assets is low.

The purpose of this paper is to ask how strongly three channels of adjustment to these ongoing developments and their interactions dampen such adverse welfare effects. First, compared to industrialized countries, developing countries are relatively young. In autarky, rates of return to capital in these economies are therefore higher. From the perspective of industrialized countries, globalization and investing capital abroad may therefore stabilize the return to capital. Second, as raw labor will become a relatively scarce factor and as life expectancy increases, strong incentives to invest in human capital emanate. This improves productivity. Such endogenous human capital adjustments may thereby substantially mitigate the effects of demographic change on macroeconomic aggregates and individual welfare. Third, while human capital adjustment increases the *quality* of the factor labor, a parametric pension reform through increasing the retirement age will increase the *quantity* of labor. This will increase per capita productivity further. In addition to this direct effect, increasing the retirement age will also extend the worklife planning horizon of households. This amplifies the incentives to accumulate human capital.

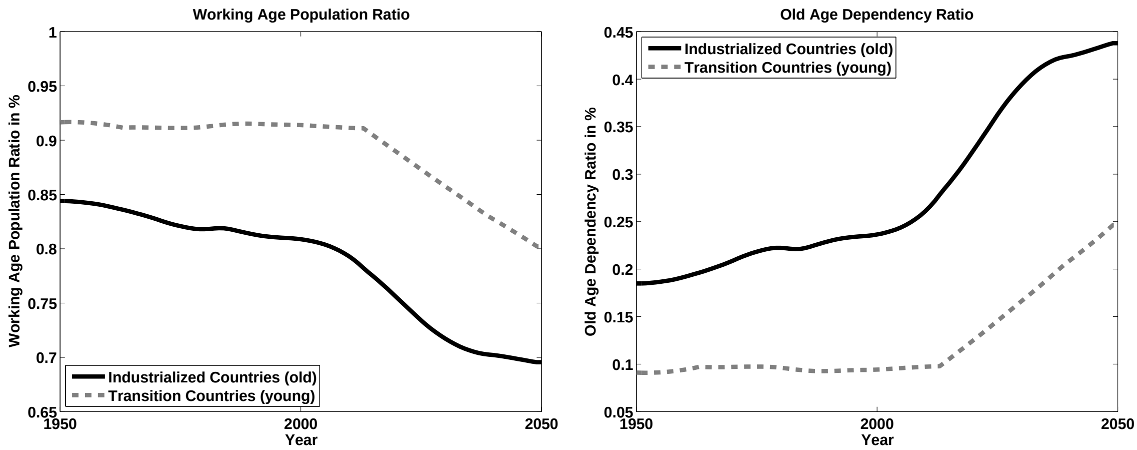
Point of departure of our analysis is the demographic evolution in two world regions, the major industrial countries and the rest of the world. As the distinctive feature of the two regions is their population structure, we will call the industrialized countries “old” and the developing countries “young”. The left panel of figure 1 illustrates the impact of demographic change on the working-age population ratio — the ratio of the working-age population (of age 16 – 64) to the total adult population (of age 16 – 90) — and the right panel the old-age dependency ratio — the ratio of the old population of age 65 – 90 to the working-age population — in these regions. As the figure shows, the demographic structure is subject to significant changes over time in both regions. Currently, there are large level differences but overall demographic trends are very similar.

We feed these demographic data into an Auerbach and Kotlikoff (1987) style overlapping generations (OLG) model with two integrated world regions, endogenous labor supply decisions and endogenous human capital formation. Our model builds on Ludwig, Schelkle, and Vogel (2012) who focus at the US as a closed economy and ignore any adjustments of the retirement age. Our extensions of this earlier work allow us to compare different adjustments — which have been identified as important in previous literature — within one coherent framework and to highlight interactions. Despite these conceptual differences to earlier work, we also take a broader view in that we focus on the group of industrialized “old” countries in an integrated world and not the US in isolation.

As the central part of our analysis we work out the quantitative differences between a benchmark model — with open economies and endogenous human capital formation — and counterfactual models where countries operate as closed economies and where human capital may be

Figure 1: Old-Age Dependency Ratio and Working-Age Population Ratio

(a) Working-Age Population Ratio (b) Old-Age Dependency Ratio



Notes: Data taken from United Nations (2007) and own projections.

exogenous. Along this line we emphasize the role of pension policy. We combine our pension reform of increasing the retirement age with two pension scenarios of a stylized pay-as-you-go (PAYGO) pension system. In these scenarios either the contribution or the benefit level is held constant and — given a balanced budget and the demographic trends as displayed in figure 1 — benefits or contributions adjust.

Our main findings about the general equilibrium feedback effects of aging — concentrating on our constant contribution rate pension scenario — can be summarized as follows. First, newborn agents gain from increasing wages and decreasing returns. Expressed as consumption equivalent variations, these welfare gains are between 0.8% and 1.1% of lifetime utility. Middle-aged and asset rich households experience losses between 3.6% and 6.5%. The size of this range strongly depends on the adjustment of human capital and the retirement age. These losses must be compared with strong welfare gains for all future generations. Second, while openness to international capital markets affects our predictions for per capita GDP and other macroeconomic aggregates, the impact of openness on rates of return to physical capital and wages is relatively small. This is due to the fact that the group of old countries is relatively open to the rest of the world already today and demographic trends across the world regions regions which cause price changes over time are so similar, cf. figure 1. Consequently, welfare of generations that live through the demographic transition is relatively little affected by the degree of capital market openness. Third, endogenous human capital formation has strong welfare effects. In the open economy, maximum welfare losses of middle-aged households shrink from 6.5% to 4.4% when human capital can endogenously adjust. Fourth, increasing the retirement age has similarly strong welfare effects: maximum welfare losses of middle-aged households further shrink from 4.4% to 3.6% when the retirement age is increased. We also document that changes in retirement legislation lead to very small feedback effects at the intensive margin. Hence, reform "backlashes" (Börsch-Supan and Ludwig 2009), are relatively small. We therefore conclude that increasing the retirement age is a very effective reform. Our qualitative finding — that endogenous human capital is an important adjustment mechanism — holds also in a world where we keep benefits constant and increase contributions. However, as this pushes up contribution rates the potentially welfare improving effect of human capital is significantly dampened.

In closed economies İmrohoroğlu et al. (1995), Fuster, İmrohoroğlu, and İmrohoroğlu (2007) Huang et al. (1997), and De Nardi et al. (1999) quantify the effects of social security adjustments on factor prices and welfare. In open economies, Domeij and Flodén (2006), Bösch-Supan et al. (2006), Fehr et al. (2005), Attanasio et al. (2007) and Krüger and Ludwig (2007), among others, investigate the role of international capital flows during the demographic transition. Storesletten (2000) examines the effect of migration to industrialized countries as a means to take pressure from social security systems. The effects of increased human capital accumulation is examined by Fougère and Mérette (1999), Sadahiro and Shimasawa (2002), Buyse et al. (2012), Ludwig, Schelkle, and Vogel (2012) and Heijdra and Reijnders (2012). This work uses some version of the seminal paper by Ben-Porath (1967)[1] and concludes that human capital adjustments may significantly mitigate the adverse consequences of demographic change.

While evidence of the effect of changes in the mandatory retirement age in the quantitative literature is scarce, there is a growing number of empirical papers estimating the effect of pension system reforms on old-age labor supply and actual retirement age. For instance, Mastrobuoni (2009), Hurd and Rohwedder (2011) and French and Jones (2012) document that the response of older workers to changes in retirement age legislation is large (extensive margin) whereby younger workers do not react much (intensive margin), just as we find.[2] While in this paper we ignore the link between human capital accumulation endogenous growth in the long-term, there is a considerable number of contributions shedding light on this topic.[3]

The remainder of our analysis is organized as follows. In section 2 we present the formal structure of our quantitative model. Section 3 describes the calibration strategy and our computational solution method. Our results are presented in section 4. Finally, section 5 concludes the paper. Detailed descriptions of computational methods and additional results are relegated to separate appendices.

2 The Model

We use a large scale multi-country OLG model in the spirit of Auerbach and Kotlikoff (1987) with endogenous labor supply, human capital formation and a standard consumption-saving decision. Our model extends Ludwig, Schelkle, and Vogel (2012) to an open economy setup and a flexible treatment of the retirement age. The population structure is exogenously determined by time and region specific demographic processes for fertility, mortality, and migration, the exogenous driving force of the model.[4] The world population is divided into 2 regions. We group countries into one of the two regions according to their demographic and economic

[1] The model developed by Ben-Porath (1967) is the workhorse model to understand questions linked to any sort of human capital accumulation and wage growth over the life cycle (see Browning, Hansen, and Heckman (1999) for a review). Further, Heckman, Lochner, and Taber (1998), Guvenen and Kuruscu (2009), and Huggett, Ventura, and Yaron (2012) used the model to explain changes in income inequality.

[2] Similarly, Imrohoroglu and Kitao (2009) find in a calibrated life cycle model that privatizing the social security system has large effects on the reallocation over the life cycle but small effects on aggregate labor supply.

[3] See, e.g., de la Croix and Licandro (1999), Echevarría and Iza (2006), Ludwig, Schelkle, and Vogel (2007), Heijdra and Romp (2009a) and Lee and Mason (2010).

[4] Although changes in prices may have — via numerous mechanisms — feedback effects on life expectancy, fertility, and migration we abstract from examining these channels. See Liao (2011) for a decomposition of economic growth into effects caused by demographics (endogenous fertility) and technological progress.

stage of development. Our first region — which we label "old" — consists of industrialized nations: USA, Canada, Japan, Australia, New Zealand, Switzerland, Norway and the EU-15. The second ("young") region consists of all other countries. Demographic processes *within* the set of countries are rather synchronized. Therefore intra-regional demographic differences do not matter much for international capital flows. The quantitatively important capital flows will occur *between* the two regions. Further — although timing and extent vary — most countries in Europe currently implement pension reforms aiming at an increase in the retirement age. Hence, assuming a perfectly aligned pension reform for the whole region captures the general dynamics of these adjustments.

We follow Buiter and Kletzer (1995) and assume that physical capital is perfectly mobile whereas human capital (labor) is immobile. Firms produce with a standard constant returns to scale production function in a perfectly competitive environment. Agents contribute a share of their wage to the pension system and retirees receive a share of current net wages as pensions. Growth of labor productivity in the steady-state is exogenous but it fluctuates during the transition as agents adjust their pattern of human capital investment.

2.1 Timing, Demographics and Notation

The model is cast in discrete time with time t being measured in calender years. Each year, a new cohort enters the economy. Since agents are inactive before they enter the labor market, entering the economy refers to the first time agents make own decisions and is set to real life age of 16 (model age $j = 0$). In the benchmark scenario agents retire at an exogenously given age of 65 (model age $jr = 49$) and live at most until age 90 (model age $j = J = 74$). Both numbers are identical across regions. At a given point in time t, individuals of age j in country i survive to age $j+1$ with probability $\varphi_{t,j,i}$, where $\varphi_{t,J,i} = 0$. The number of agents of age j at time t in country i is denoted by $N_{t,j,i}$ and $N_{t,i} = \sum_{j=0}^{J} N_{t,j,i}$ is total population in t,i. In the demographic projections migration happens at the age of 16. Thus, we implicitly assume that new migrants are born with the initial human capital endowment and human capital production function of natives. This assumption is consistent with Hanushek and Kimko (2000) who show that individual productivity (and thus human capital) of workers appears mainly to be related to a country's level of schooling and not to cultural factors.

2.2 Households

Households are populated by one representative agent deciding about consumption, saving, labor supply, and time investment into human capital formation. The remaining time is consumed as leisure. A household in region i maximizes lifetime utility at the beginning of economic life ($j = 0$) in period t,

$$\max \sum_{j=0}^{J} \beta^j \pi_{t,j,i} \frac{1}{1-\sigma} \{c_{t+j,j,i}^{\phi}(1-\ell_{t+j,j,i}-e_{t+j,j,i})^{1-\phi}\}^{1-\sigma}, \quad \sigma > 0, \tag{1}$$

where the per period utility function takes consumption c, working hours ℓ and time spent on increasing the stock of human capital e, as inputs. Standardizing the time endowment to unity leaves $1-\ell-e$ as leisure time. ϕ is the consumption elasticity in utility, β is the raw time discount factor, and σ is the inverse of the inter-temporal elasticity of substitution with respect to the consumption-leisure aggregate. $\pi_{t,j,i}$ denotes the unconditional probability to survive until age j, $\pi_{t,j,i} = \prod_{k=0}^{j-1} \varphi_{t+k,k,i}$, for $j > 0$ and $\pi_{t,0,i} = 1$.

Agents earn labor income (pensions if retired), interest payments on their physical assets, and receive accidental bequests. Social security contributions are a share $\tau_{t,i}$ of their gross wages. Net wage income in period t of an agent of age j living in region i is given by $w^n_{t,j,i} = \ell_{t,j,i} h_{t,j,i} w_{t,i}(1-\tau_{t,i})$, where $w_{t,i}$ is the (gross) wage per unit of supplied human capital at time t in region i. Annuity markets are missing and accidental bequests are distributed by the government as lump-sum payments to households. The household's dynamic budget constraint is given by

$$a_{t+1,j+1,i} = \begin{cases} (a_{t,j,i} + tr_{t,i})(1+r_t) + w^n_{t,j,i} - c_{t,j,i} & \text{if } j < jr \\ (a_{t,j,i} + tr_{t,i})(1+r_t) + p_{t,j,i} - c_{t,j,i} & \text{if } j \geq jr, \end{cases} \tag{2}$$

where $a_{t,j,i}$ denotes assets, $p_{t,j,i}$ is pension income, $tr_{t,i}$ are transfers from accidental bequests, and r_t is the real interest rate, the rate of return to physical capital. Household start without assets ($a_{t,0,i} = 0$) and do not intend to leave bequests to the next generation ($a_{t,J+1,i} = 0$).

2.3 Formation of Human Capital

The initial level of human capital $h_{t,0,i} = h_0$ is exogenously given, identical across households of a birth cohort and cohort invariant. Then, at any point in time agents can spend a fraction of their time to build human capital. We employ a frequently used twist of the Ben-Porath (1967) human capital technology given by

$$h_{t+1,j+1,i} = h_{t,j,i}(1-\delta^h_i) + \xi_i (h_{t,j,i} e_{t,j,i})^{\psi_i} \quad \psi_i \in (0,1), \;\; \xi_i > 0, \;\; \delta^h_i \geq 0, \tag{3}$$

where ξ_i is a scaling factor, ψ_i determines the curvature of the human capital technology and δ^h_i is the depreciation rate of human capital. Parameters of the production function vary across countries to allow for region-specific human capital profiles during our calibration period.[5] Since we do not model any other labor market frictions[6] or costs of human capital acquisition this is the only way to replicate observed differences in age-wage profiles. However, we adjust parameters such that they are eventually identical in both regions and thus agents will have — everything else equal — the same life cycle human capital profile in the final steady state (see section 3.3).

Investment into human capital requires only the input of time. Opportunity costs of human capital accumulation are not only forgone wages but also the utility loss due to less leisure. As we do not model formal education and on-the-job-experience (learning-by-doing) separately, the accumulation of human capital is a mixture of formal and informal training programs. Human capital can be accumulated at all stages of the life-cycle but optimal behavior implies that agents will spend more time on building human capital early in life and stop investing some years before retirement.

[5] See Browning, Hansen, and Heckman (1999) for a summary of the literature and an overview over empirical estimates of the parameters.

[6] de la Croix, Pierrard, and Sneessens (2013) emphasize the role of labor market frictions in the context of demographic change.

2.4 Firms

There is a large number of firms in a perfectly competitive environment producing a homogenous good (which can be consumed or invested) using the Cobb-Douglas technology

$$Y_{t,i} = K_{t,i}^{\alpha}(A_{t,i}L_{t,i})^{1-\alpha}. \tag{4}$$

Here, α denotes the share of capital used in production. $K_{t,i}, L_{t,i}$ and $A_{t,i}$ are region-specific stocks of physical capital, effective labor and the level of technology, respectively. Labor inputs and human capital of different agents (of different age) are perfect substitutes. Aggregate effective labor input $L_{t,i}$ is then given by $L_{t,i} = \sum_{j=0}^{jr-1} \ell_{t,j,i} h_{t,j,i} N_{t,j,i}$. Factors of production are paid their marginal products, i.e.,

$$w_{t,i} = (1-\alpha)A_{t,i}k_t^{\alpha} \qquad r_t = \alpha k_t^{\alpha-1} - \delta, \tag{5a}$$

$$k_t = k_{t,i} = \frac{K_{t,i}}{A_{t,i}L_{t,i}} \tag{5b}$$

where $w_{t,i}$ is the gross wage per unit of efficient labor, r_t is the interest rate and δ_t denotes the depreciation rate of physical capital. Since we have frictionless international capital markets, capital stocks $k_{t,i}$ adjust such that the rate of return is equalized across regions and are therefore determined by the global capital stock relative to global output (see section on aggregation and equilibrium for more details). Since agents and their human capital are immobile by assumption, wages differ across regions and are a function of the country specific productivity $A_{t,i}$. Total factor productivity, $A_{t,i}$, is growing at the region-specific exogenous rate $g_{t,i}^A$: $A_{t+1,i} = A_{t,i}(1 + g_{t,i}^A)$.

2.5 Capital Markets

We assume that both regions are initially closed economies. Opening up of capital markets is modeled as a non-expected event. We first solve for the equilibrium transition path of both economies with agents using only prices and transfers from the closed economy scenario. Then, we "surprise" agents by opening up capital markets in 1970. Hence, from 1970 onwards there is only one frictionless capital market and thus the marginal product of capital is equalized across regions. Our choice is motivated by the fact that 1970 is commonly viewed as the beginning of the opening up process of capital markets (Broner, Didier, Erce, and Schmukler (2011), Lane and Milesi-Ferretti (2007) or Reinhart and Rogoff (2008)). The historic event initiating the liberalization process is, in fact, the break-up of the Bretton Woods System in 1971. Since we model opening up as a non-expected (zero probability) event, agents can re-optimize only for their remaining lifetime.

2.6 The Pension System

The pension system is a pay-as-you-go system which is balanced every period by adjusting inflows (i.e. the contribution rate) or outflows (i.e. the replacement rate). Contributions are a fraction $\tau_{t,i}$ of gross wages and retirees receive a fraction $\rho_{t,i}$ of current average net wages of workers as pensions.[7] Hence, pensions in each period are given by

$$p_{t,j,i} = \rho_{t,i}(1-\tau_{t,i})w_{t,i}\bar{h}_{t,i}$$

[7] Pension systems across countries differ along many dimensions all of which we ignore for simplicity. See for instance (Diamond and Gruber 1999) or Whitehouse (2003) for an overview.

where $\bar{h}_{t,i} = \frac{\sum_{j=0}^{jr-1} \ell_{t,j,i} h_{t,j,i} N_{t,j,i}}{\sum_{j=0}^{jr-1} \ell_{t,j,i} N_{t,j,i}}$ denotes average human capital of workers. Observe that we ignore an earnings related linkage in our pension benefit formula as done, e.g., in Ludwig, Schelkle, and Vogel (2012). Hence, we bias upwards the labor supply and human capital accumulation distortions of our model. Consequently, we bias downwards the endogenous labor supply and human capital accumulation responses to increases in the retirement age. The budget constraint of the system is given by

$$\tau_{t,i} w_{t,i} \sum_{j=0}^{jr-1} \ell_{t,j,i} h_{t,j,i} N_{t,j,i} = \sum_{j=jr}^{J} p_{t,j,i} N_{t,j,i}$$

for all t. Substituting $p_{t,j,i}$, the equation from above simplifies to

$$\tau_{t,i} \sum_{j=0}^{jr-1} \ell_{t,j,i} h_{t,j,i} N_{t,j,i} = \rho_{t,i}(1-\tau_{t,i}) \sum_{j=jr}^{J} N_{t,j,i} \quad \forall t. \tag{6}$$

We consider two policy scenarios in order to ensure the long-term sustainability of the public pension system. In our first reform scenario we keep the retirement age at the baseline level (65 years). Then, we either hold the contribution rate constant $\tau_{t,i} = \bar{\tau}_i$ (labeled "const. τ"), and endogenously adjust the replacement rate to balance the budget of the pension system or we hold the replacement rate constant, $\rho_{t,i} = \bar{\rho}_i$ (labeled "const. ρ"), and endogenously adjust the contribution rate. As the second dimension of pension reforms we increase the normal retirement age. This reform scenario captures two effects on incentives to acquire human capital: a lengthening of the working life combined with — everything else equal — lowering the tax burden on currently working individuals. In fact, most governments currently implement a mix of the two strategies. In order to highlight the most extreme economic impact of different reforms, we perform the two types of policy experiments in isolation.

2.7 Equilibrium

Denoting current period/age variables by x and next period/age variables by x', a household of age j solves in region i, at the beginning of period t, the maximization problem

$$V(a,h,t,j,i) = \max_{c,e,a',h'} \{u(c, 1-\ell-e) + \varphi_i \beta V(a',h',t+1,j+1,i)\} \tag{7}$$

subject to $w^n_{t,j,i} = \ell_{t,j,i} h_{t,j,i} w_{t,i}(1-\tau_{t,i})$, (2), (3) and the constraint $e \in [0, 1-\ell)$.

Definition 1. *Given the exogenous population distribution and survival rates in all periods* $\{\{\{N_{t,j,i}, \varphi_{t,j,i}\}_{j=0}^{J}\}$ *an initial physical capital stock and an initial level of average human capital* $\{K_{0,i}, \bar{h}_0\}_{i=1}^{I}$, *and an initial distribution of assets and human capital* $\{\{a_{t,0,i}, h_{t,0,i}\}_{j=0}^{J}\}_{i=1}^{I}$, *a competitive equilibrium is sequences of individual variables* $\{\{\{c_{t,j,i}, e_{t,j,i}, a_{t+1,j+1,i}, h_{t+1,j+1,i}\}_{j=0}^{J}\}_{t=0}^{T}\}_{i=1}^{I}$, *sequences of aggregate variables* $\{\{L_{t,i}, K_{t+1,i}, Y_{t,i}\}_{t=0}^{T}\}_{i=1}^{I}$, *government policies* $\{\{\rho_{t,i}, \tau_{t,i}\}_{t=0}^{T}\}_{i=1}^{I}$, *prices* $\{\{w_{t,i}, r_t\}_{t=0}^{T}\}_{i=1}^{I}$, *and transfers* $\{\{tr_{t,i}\}_{t=0}^{T}\}_{i=1}^{I}$ *such that*

1. *given prices, bequests and initial conditions, households solve their maximization problem as described above,*

2. *interest rates and wages are paid their marginal products, i.e.* $w_{t,i} = (1-\alpha)\frac{Y_{t,i}}{L_{t,i}}$ *and* $r_t = \alpha\frac{Y_t}{K_t} - \delta$,
3. *per capita transfers are determined by*

$$tr_{t,i} = \frac{\sum_{j=0}^{J} a_{t,j,i}(1-\varphi_{t-1,j-1,i})N_{t-1,j-1,i}}{\sum_{j=0}^{J} N_{t,j,i}}, \tag{8}$$

4. *government policies are such that the budget of the social security system is balanced every period and region, i.e. equation (6) holds* $\forall t, i$, *and household pension income is given by* $p_{t,j,i} = \rho_{t,i}(1-\tau_{t,i})w_{t,i}\bar{h}_{t,i}$,
5. *all regional labor markets clear at respective wage rate* $w_{t,i}$, *the world capital market clears at world interest rate* r_t *and allocations are feasible in all periods:*

$$L_{t,i} = \sum_{j=0}^{jr-1} \ell_{t,j,i} h_{t,j,i} N_{t,j,i} \tag{9a}$$

$$Y_{t,i} = \sum_{j=0}^{J} c_{t,j,i}N_{t,j,i} + K_{t+1,i} - (1-\delta)K_{t,i} + F_{t+1,i} - (1+r_t)F_{t,i}, \tag{9b}$$

$$Y_t = \sum_{i=1}^{I} Y_{t,i} \tag{9c}$$

$$K_{t+1} = \sum_{i=1}^{I}\sum_{j=0}^{J} a_{t+1,j+1,i}N_{t,j,i} \tag{9d}$$

6. *and the sum of foreign assets* $F_{t,i}$ *in all regions is zero*

$$\sum_{i=1}^{I} F_{t,i} = 0. \tag{10}$$

Definition 2. *A stationary equilibrium is a competitive equilibrium in which per capita variables grow at constant rate* $1+\bar{g}^A$ *and aggregate variables grow at constant rate* $(1+\bar{g}^A)(1+n)$.

2.8 Thought Experiments

The exogenous driving force of our model is the time-varying and region specific demographic structure. The solution of our model is done in two steps. We firstly assume that both regions are closed and solve for the region specific artificial initial steady state. We then compute the closed economy equilibrium transition paths to the new steady state. While computing the transition paths, we include sufficiently many "phase-in" and "phase-out" periods[8] to ensure convergence. Then, we use the distribution of physical capital, human capital and population from the two closed regions in year 1970 and recompute the equilibrium transition path until the new — open economy — steady state is reached. Finally, we report simulation results for the main projection period of interest, from 2010 to 2050. We use data from 1960 – 2005 in order to calibrate the vector of structural model parameters (cf. section 3).

[8] In fact, changes in variables which are constant in steady state are numerically irrelevant already around 100 periods before the we impose the steady state restriction.

When reporting our results, we will compare time paths of variables of interest across various model variants for different social security reform scenarios for the old countries.[9] Our baseline model variant (which is also used in calibration) is one with agents adjusting their human capital, open economy and benchmark retirement age. Hence, our strategy is to first solve and calibrate for the transitional dynamics using the model as described above. Then, we use the results from this model to compute average time investment and the associated human capital profile which is used as input in the alternative model with a fixed productivity profile.[10] We obtain the life-cycle profile of time investment into education $\bar{e}_{j,i}$ for each age $j = 1,2,\ldots,J$ by averaging over all life-cycle profiles of agents living during the calibration period.[11] The human capital profile is then computed by substituting the time series $\bar{e}_{j,i}$ into (3). Then, we use this profile in all our experiments with exogenous human capital.

3 Calibration and Computation

The calibration of the model is standard. We choose parameters such that simulated moments match their counterparts in the data. For the wage profile, we choose parameters such that the endogenous wage profiles match the empirically observed wage profile during the calibration period $1960-2005$ (cf. section 3.3).[12] We provide a condensed overview over all parameters in table 1.

3.1 Demographics

Population data from $1950-2005$ are taken from the United Nations (2007). For the period until 2050 we use the same data source and choose the UN's "medium" variant for the fertility projections. However, we have to forecast population dynamics beyond 2050 to solve our model. The key assumptions of our projection are as follows: First, for both regions total fertility rate is constant at 2050 levels until 2100. Then we adjust fertility such that the number of newborns is constant for the rest of the simulation period. Second, we use the life expectancy forecasted by the United Nations (2007) and extrapolate it until 2100 at the same (region and gender-specific) linear rate.[13] Then we assume that life expectancy in the old nations stays constant. Life expectancy in the rest of the world keeps rising until it reaches the level of the old countries by the year 2300. This choice ensures that in the final steady state, the population structure is identical across world regions. By delaying this adjustment process to 2300 we make sure that we exclude any anticipation effects of currently living generations and have

[9] Results for the young countries are available upon request.

[10] We restrict time investment into human capital production to be identical for all cohorts (instead of using each cohort's own endogenous profile and keeping it fixed). We do this in order not to change the time endowment available to households from cohort to cohort.

[11] Formally, we compute $\bar{e}_{j,i} = \frac{1}{t_1-t_0+1}\sum_{t=t_0}^{t_1} e_{t,j,i}$.

[12] We do the moment matching exercise in the model variant with endogenous human capital and constant contribution rate scenario with the benchmark retirement age. We do not re-calibrate model parameters across social security scenarios or for the alternative human capital model, mainly because any parametric change would make comparisons (especially welfare analysis) across models impossible.

[13] Life expectancy estimated by the UN for cohort born in 2050 is in the industrialized nations 81.5 year for men and 86.8 year for women. In the rest of the world, life expectancy is 71.7 for men and 75.7 for women. The estimates of the trend are as follows: in the industrialized countries life expectancy at birth increasers for each cohort at a linear rate of 0.12 years for men and 0.117 years for women. For the rest of the world the slope coefficient for is 0.204 for men and 0.217 for women. See also Oeppen and Vaupel (2002) for the evolution of life expectancy.

enough periods left to test convergence properties of the model. These assumptions imply that a stationary population structure is reached in about 2200 in the old nations and in 2300 in the rest of the world. While the assumptions from above seem to be arbitrary, they are close to what is done in the rest of the literature.

3.2 Households

We set σ to 2. This corresponds to a standard estimate of the IES of 0.5 (Hall 1988). The pure time discount factor β is chosen to match the empirical capital-output ratio of 2.8 in the old countries which requires $\beta = 0.98$. To calibrate the weight of consumption in the utility function, we set $\phi = 0.37$ by targeting an average labor supply of 1/3 of the total available time. We constrain the parameters of the utility function to be identical across regions.

3.3 Individual Productivity and Labor Supply

We follow Ludwig, Schelkle, and Vogel (2012) and choose the parameters of the human capital production function such that average wage profiles resulting from endogenous human capital model replicate empirically observed wage profiles. The estimates of wage profiles are based on PSID data, adopting the procedure of Huggett et al. (2012). After normalizing the initial value of human capital to $h_0 = 1$ we determine the value of the structural parameters $\{\xi_i, \psi_i, \delta_i^h\}_{i=1}^I$ using indirect inference methods (Smith 1993; Gourieroux et al. 1993). To do this, we run regressions on the wage profiles obtained from the simulation and the observed data on a 3rd-order polynomial in age defined as

$$\log w_{j,i} = \lambda_{0,i} + \lambda_{1,i} j + \lambda_{2,i} j^2 + \lambda_{3,i} j^3 + \varepsilon_{j,i}. \tag{11}$$

where $w_{j,i}$ denotes age specific productivity. We write the coefficient vector from the regression on the observed wage data as $\lambda_i^d = [\lambda_{1,i}, \lambda_{2,i}, \lambda_{3,i}]'$ and the one from the simulated human capital profile of cohorts born in $1960 - 2005$ by $\hat{\lambda}^s = [\hat{\lambda}_{1,i}, \hat{\lambda}_{2,i}, \hat{\lambda}_{3,i}]'$. The vector $\hat{\lambda}^s$ is then a function of the deep structural parameters $\{\xi_i, \psi_i, \delta_i^h\}_{i=1}^I$. We choose the values for the structural parameters by minimizing the distance between the values of the polynomial obtained from the regression on the actual data and the simulated data, i.e. minimizing $\|\lambda_i^d - \hat{\lambda}_i^s\| \ \forall i$, see subsection 3.6 for computational details.

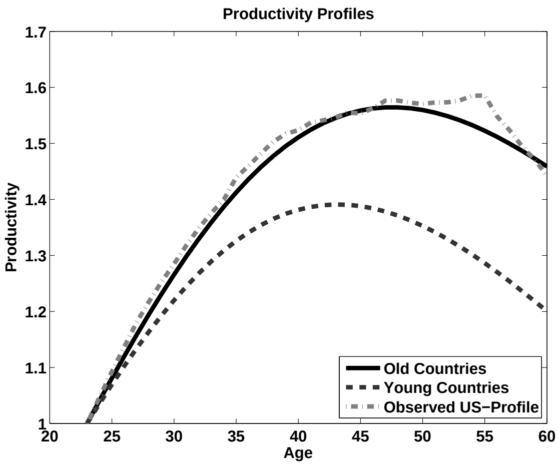
Figure 2 presents the empirically observed productivity profile and the estimated polynomials for the different regions. The coefficients[14] and the shape of the wage profile are in line with the literature, e.g. number reported by Altig et al. (2001) and Hansen (1993). The value of $\psi \approx 0.60$ is also in the middle of the range reported in Browning, Hansen, and Heckman (1999). The depreciation rate of human capital is $\delta^h = 1.4\%$ for young and $\delta^h = 0.9\%$ for old countries. Although there is a considerable disagreement about δ^h in the literature, our numbers are in reasonable range (see e.g. Arrazola and de Hevia (2004), Browning, Hansen, and Heckman (1999)).

Due to lack of reliable individual wage data or good estimates for age-wage profiles we cannot apply the same technique to young countries. Instead, we take the polynomial estimated on the U.S.-profile and scale coefficient λ_1 by a factor of 0.95. The resulting age-wage profile

[14] The coefficient estimates from the regression on the US profiles are λ_0: -1.6262, λ_1: 0.1054, λ_2: -0.0017 and λ_3: 7.83e-06. The coefficients for the young countries are identical except for λ_1 which is scaled by 0.95.

corresponds to a profile estimated on Mexican data by Attanasio, Kitao, and Violante (2007). The main difference between the two profiles is that wages in the U.S. drop by 10% and Mexican wages by 20% from their peak to retirement age and that the maximal wage in the U.S. is about 100% higher than the wage at entry into the labor market. The same number in Mexico is about 90%. Attanasio et al. (2007) attribute these differences — US profiles are steeper and drop less towards the end of working life — to differences in the physical requirements in the two economies. Working in the probably less human capital intensive Mexican labor market requires relatively more physical strength. This is likely to imply that the peak is reached earlier and that productivity decreases faster afterwards. Further supportive evidence on flatter profiles is provided by Lagakos, Moll, Porzio, and Qian (2012). Using a panel with 48 developing and developed countries, they find that age-experience profiles are much steeper in developed countries.

Figure 2: Wage Profiles



Notes: Data standardized by the wage at the age 23. Source: PSID, own calculations.

To minimize biases, we adjust the parameters of the human capital production function such that they are eventually identical in both regions. To this end we parameterize the adjustment path and calibrate it such that parameters start to change for the cohort born in year 2100 and are identical for the cohort born in year 2300. We denote the vector of parameters $\{\xi_i, \psi_i, \delta_i^h\} = \vec{\chi}_i$ and assume that

$$\vec{\chi}_{i,k} = \vec{\chi}_{i \neq j,k} + \Delta(\chi_{j,k}) \cdot t \quad k = 1,2,3, \tag{12}$$

for the adjustment process where $\Delta(\chi_{j,k})$ denotes the per period linear adjustment of the parameter, t is the length of the adjustment period, and k is an element from χ_i.

3.4 Production

The share of capital in production is set to $\alpha = 0.33$ such that we match the share of capital income in national accounts. The average growth rate of total factor productivity, $\bar{g}_i^A$, is calibrated such that we match the region-specific growth rate of GDP per capita, taken from Maddison (2003). Growth of output per capita in the old countries during our calibration period is 2.8%. Accordingly, we set the growth rate of TFP to 1.85% to meet our calibration target. To match the observed growth of GDP per capita of 2.2% in the young countries, we let TFP grow at a rate of 1.5%. From 2100 onwards we let the growth rate of TFP in the young countries adjust smoothly to the growth rate in the old countries. This adjustment process is assumed to be completed in 2300. Further, we compute relative GDP per capita from Maddison (2003) for both regions in 1950 and use this ratio to calibrate the relative productivity levels at the beginning of the calibration period. Initially, per capita GDP in the young countries is only 20% of income per capita in the old nations. Finally, we calibrate δ such that our simulated data match an average investment output ratio of 20% in the old countries which requires $\delta = 0.035$.

3.5 The Pension System

In our first social security scenario ("const. τ") we fix contribution rates and adjust replacement rates of the pension system. Since there is no yearly data on contribution rates for sufficiently many countries, we use data from Palacios and Pallarés-Miralles (2000) for the mid 1990s and assume that the contribution rate was constant through the entire calibration period. On the individual country level, we use the pension tax as a share of total labor costs weighted by the share of contributing workers to compute a national average. Then we weight these numbers by total GDP to compute a representative number for the two world regions. The contribution rate in the young (old) region is then 4.1% (10.9%). Given the initial demographic structures, the replacement rate is 13.8% (20.4%) in the young (old) region. In our baseline social security scenario we freeze the contribution rate at the level used for the calibration period for all following years. When simulating the alternative social security scenario with constant replacement rates ("const. ρ") we feed the equilibrium replacement rate obtained in the "const. τ" scenario into the model and hold it constant at the level of year 2000 for all remaining years. Then, the contribution rate endogenously adjusts each period to balance the budget of the social security system. In both scenarios we assume that the retirement age is fixed at 65 years and agents do not expect any change. We label this scenario as "Benchmark" ("BM") in the following analysis.

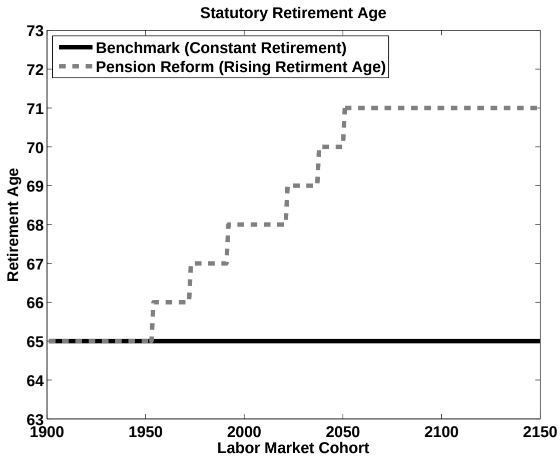
For the second type of policy reform we increase the retirement age by linking it to remaining life expectancy at age 65 (the current retirement age). We assume that for an increase in conditional life expectancy by 1.5 years, retirement increases by one year. We model this change — labeled "Pension Reform" ("PR") — by assuming that this reform affects already workers in the labor market in 1955 (birth cohort 1939) by raising their retirement age immediately by one year and thereby effectively increasing the number of workers already in 2001. We then apply this rule for all following cohorts. This pattern mimics recent pension reforms in many old countries.[15] This reform has direct effects via lengthening expected lifetime labor supply of workers and changing prices for retirees. Given our projections of life expectancy, the retirement age will eventually settle down at 71 years, a value also discussed in the public debate about pension reforms. We show the stepwise increase in the retirement age in figure 3 as a function of the respective labor market cohort.

3.6 Computational Method

For a given set of structural model parameters, we solve the model by iterating on household related variables (inner loop) and aggregate variables (outer loop). In the outer loop, we solve

[15] See for instance, http://www.ssa.gov/oact/ProgData/nra.html for the U.S. system.

Figure 3: Retirement Age



Notes: The jumps in the broken line indicate the cohort which is affected by the change in the retirement age and not the actual time when the number of workers is increasing.

for the equilibrium by making an initial guess about the time path of the following variables: the capital intensity, the ratio of bequests to wages, the replacement rate (or contribution rate) of the pension system and the average human capital stock for all periods from $t = 0, 1, \ldots, T$. For the open economy we impose the restriction of identical capital intensity for both regions but require all other variables from above to converge for each country separately. On the households level (inner loop), we start by guessing $\{c_J, h_J\}$, i.e. the terminal values for consumption and human capital. Then we iterate on them until convergence of the inner loop as defined by some metric. In each outer loop, household variables are aggregated in each iteration for all periods. Values for the aggregate time series are then updated using the Gauss-Seidel-Quasi-Newton algorithm suggested in Ludwig (2007) until convergence.

To calibrate the model (we do this in the"const. τ" scenario, benchmark retirement), we run additional "outer outer" loops on the vector of structural model parameters in order to minimize the distance between moments computed from the simulated data and their corresponding calibration targets for the calibration period $1960-2005$. In a nutshell, the common parameter values determined in this procedure are β, ϕ, δ, and the country and specific parameters of the human capital production function $\{\xi_i, \psi_i, \delta_i^h\}$.

4 Results

We divide the presentation of results into three parts. In the first part, in subsection 4.1, we look at the evolution of economic aggregates such as the rate of return of detrended GDP per capita along the transition. When we evaluate future trends of economic aggregates in this first part, we do so in two steps.

In our first step, we look at a comparison of open and counterfactual closed economy versions of our model for two pension scenarios (a "fixed contribution rate" and a "fixed replacement rate" system) and two human capital scenarios ("exogenous" versus "endogenous" human capital). What we learn from this exercise is that openness matters for the evolution of aggregates such as GDP per capita but, probably surprisingly, not so much for rates of return. What

Table 1: Model Parameters

			Young	Old
Preferences	σ	Inverse of Inter-Temporal Elasticity of Substitution	2.00	
	β	Pure Time Discount Factor	0.985	
	ϕ	Weight of Consumption	0.370	
Human Capital	ξ	Scaling Factor	0.176	0.166
	ψ	Curvature Parameter	0.576	0.586
	δ^h	Depreciation Rate of Human Capital	1.4%	0.9%
	h_0	Initial Human Capital Endowment	1.00	1.00
Production	α	Share of Physical Capital in Production	0.33	
	δ	Depreciation Rate of Physical Capital	3.5%	
	g^A	Exogenous Growth Rate		
		Calibration Period	1.5%	1.9%
		Final Steady State	1.9%	1.9%

Notes: "Young" and "Old" refer to the region. Only one value in a column indicates that the parameter is identical for both regions.

matters more for the time path of the latter is whether human capital is endogenous and how the pension system is designed.

In our second step, we evaluate how our findings are affected by increasing the retirement age. To distinguish increases in the retirement age semantically from the aforementioned pension "scenarios", we label the experiment of increasing the retirement age as a "pension reform" ("PR"). Given our findings from the first step, we focus our analysis only at the more realistic and policy relevant open economy model version of our model. We find that increasing the retirement age, by increasing labor supply and human capital, will significantly alter the time paths of future rates of return to capital and wages.

In the second part, subsection 4.2, we shed more light on how the increase in retirement age affects effective labor supply. We ask how much of the exogenous increase in the retirement age (extensive margin) is potentially offset by adverse endogenous labor supply reactions at the intensive margin. Overall, we find little response at the intensive labor supply margin. Consequently, increasing the retirement age is a very effective reform. However, we ignore the endogenous adjustment of retirement to policy, as done by, e.g., Heijdra and Romp (2009b) and Buyse et al. (2012). We leave extensions of our work along these dimensions for future research.

In the third part, subsection 4.3, we evaluate welfare of households who live through the demographic transition and show that endogenous formation of human capital and the design of the pension system are — via future time paths of wages and returns — key for the welfare effects of aging. Consistently with the literature[16], we find that, when the contribution rate is held constant, increasing wages dominate for newborn households who experience welfare gains whereby the converse applies to old and asset rich households. Gains of the young (and losses of the old) are significantly higher (lower) when human capital can endogenously adjust and when the retirement age increases. On the contrary, welfare differences between our closed

[16] E.g., Krüger and Ludwig (2007), Ludwig, Schelkle, and Vogel (2012).

and open economy scenarios are small.

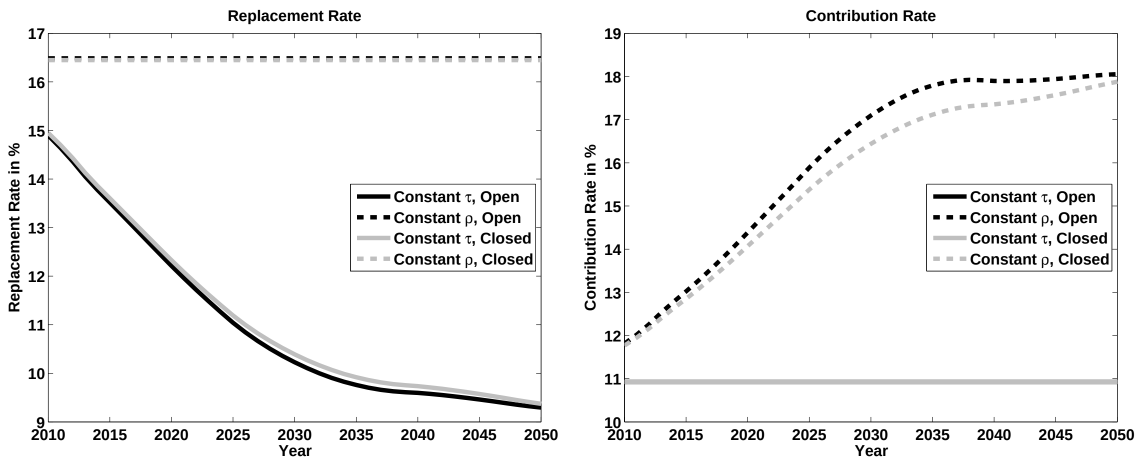
4.1 Macroeconomic Aggregates

Aggregate Variables for the Benchmark Model

Figures 4(a) and 4(b) depict the evolution of contribution and replacement rates for the benchmark pension system. Holding the replacement rate constant at 16.4% requires an increase of the contribution rate to 18% in 2050. Conversely, keeping the contribution rate unchanged during the entire period at 10.9% requires a drop in the replacement rate to 9.4% until 2050. Small differences emanate in the graphs for the closed and open economy scenarios of our model. These are induced by differential paths of wages and labor supply as well as human capital formation in the respective model variants.

Figure 4: Adjustment of Pension System, Benchmark Pension System

(a) Replacement Rate (b) Contribution Rate



Notes: "Open" and "Closed" refer to the results obtained from the closed and open economy versions. All results obtained from the endogenous human capital scenario, benchmark pension system with constant retirement age ("BM").

The evolution of important macroeconomic variables for the scenario with constant contribution rate ("const. τ") is presented in figure 5. As in this subsection we concentrate on the model comparison across capital market scenarios we report results for the alternative adjustment of the pension system ("const. ρ") in figure 10 in appendix B.

Figure 5(a) shows the evolution of the rate of return to physical capital for different model variants. Looking at the development of the rate of return in a closed economy we observe the well known result of falling returns due to population aging. However, we also observe that the drop in the interest rate is much lower in the model where human capital is endogenous as opposed to the "standard" model with an exogenously given life cycle productivity profile. This effect is the result of higher investment into human capital due to falling returns to physical capital. In the open economy variant, we observe a qualitatively similar result with two differences. As the young economy is importer of capital, the interest rate in the open economy is initially higher than in the closed economy. This pattern is reversed after year 2030 when baby boomers retire in the old economy and withdraw their funds (see below). Quantitatively, initial differences are relatively small when human capital formation is endogenous.

We next report the level of de-trended GDP per capita in figure 5(d) where we have standardized the value for the year 2010 to 100. In both model versions with endogenous human capital the income level in 2050 is by an economically significant amount above the level of the "standard" model. This effect is due to more investment into human capital. Turning to the comparison between closed and open economies we observe that income in 2050 in the open economy tends to be higher compared to the closed economy (for endogenous and exogenous human capital).

GDP per capita in the closed economy is initially higher than in the open economy. The reason is the inflow and outflow of physical capital during the transition period. As can be seen from figure 5(b), the old region has a positive net foreign asset position until 2035 (2040) in the version with endogenous (exogenous) human capital. This initial outflow of capital decreases production at home and thereby GDP per capita. However, as the capital flows are reversed and capital is repatriated, GDP per capita is surpassing the level of the closed economy.

These differences between the open and closed economy scenarios are most pronounced when human capital adjusts. This can be seen in figure 5(c). Average human capital is higher in the open economy compared to the closed economy after 2035. The initially lower time investment into human capital accumulation in the open economy scenario can be attributed to the lower wage growth and higher rate of return compared to the closed economy scenario. In the closed economy, the demographic structure of the old economy leads to relatively more pronounced accumulation of physical capital which can be only used domestically. Therefore, opening up the economy and thereby exporting physical capital to regions with higher returns is initially detrimental for investment into human capital at home.

The most striking result displayed in figure 5(d) is the increase of de-trended per capita GDP, despite the seemingly detrimental effects of aging on raw labor supply. Endogenous human capital formation in combination with a constant contribution rate pushes growth rates along the transition above the long-run trend level of 1.8 percent. On the contrary, de-trended GDP per capita decreases until about 2030 in the open economy when replacement rates are held constant. In this case the associated increase in contribution rates — cf. figure 4(b) — reduces incentives to invest in human capital, cf. figure 10 in appendix B.

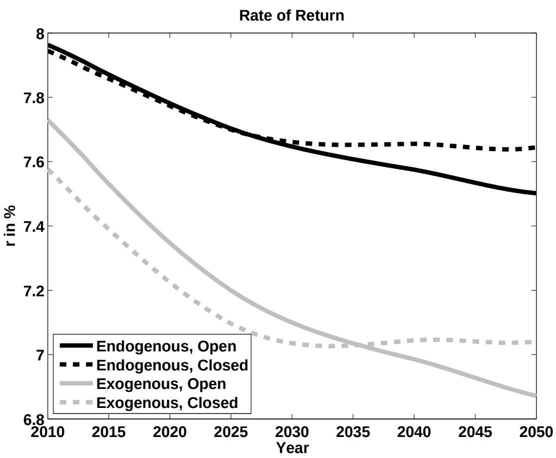
Aggregate Variables when the Retirement Age is Increased

We now investigate how our results are affected once we let the retirement age increase according to the pattern shown in figure 3. To focus our analysis, we do so in our benchmark open economy model with endogenous human capital formation. We continue with our comparison across the two pension designs — constant contribution and constant replacement rates.

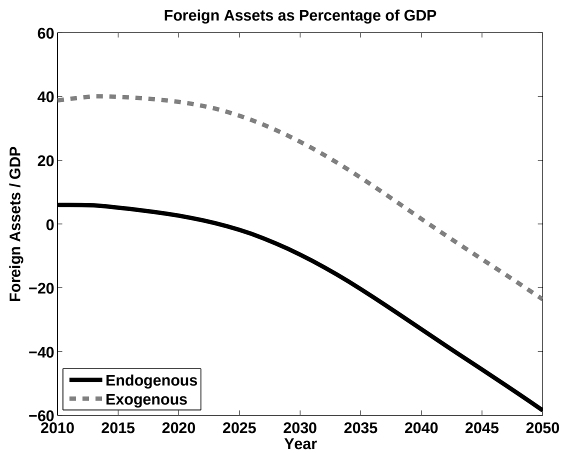
Figure 6 displays the associated time paths of replacement and contribution rates. Observe that, in case of a fixed contribution (replacement) rate, the base replacement (contribution) rate is higher when the retirement age adjusts. This is so for mainly two reasons. First, there is a higher exogenous amount of raw labor in the economy. Second, as shown below, higher retirement ages increase the incentive to invest in human capital. Hence, also the quality of labor is higher when retirement ages are raised. The figure also displays jumps in the relevant variables. This is due to the fact that our smallest unit of time is one calendar year and it

Figure 5: Aggregate Variables for Constant Contribution Rate Scenario, Benchmark Pension System

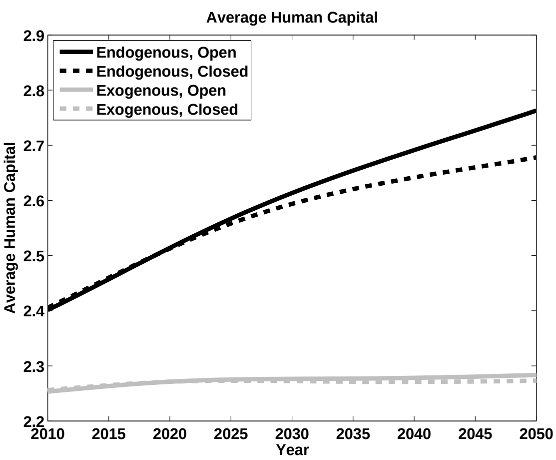
(a) Rate of Return to Physical Capital



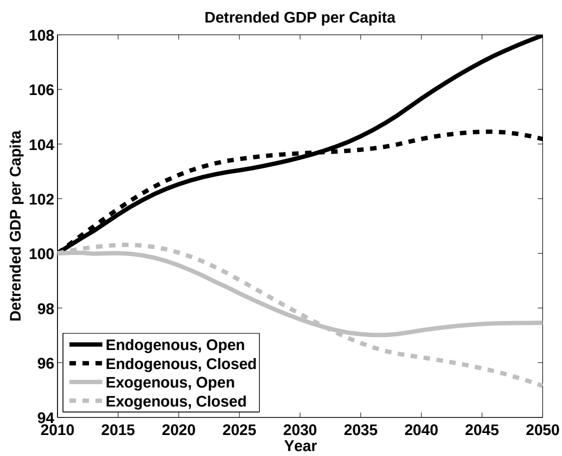
(b) Net Foreign Assets



(c) Average Human Capital



(d) Detrended GDP per Capita



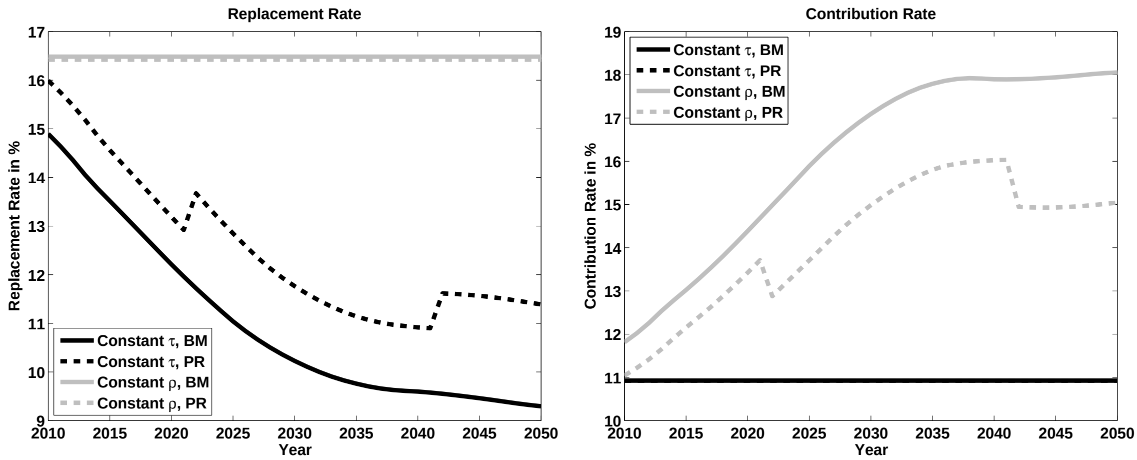
Notes: "Endogenous" and "Exogenous" refer to the human capital production profile. "Open" and "Closed" refer to the results obtained from the closed and open economy versions. All results obtained from the constant contribution rate scenario, benchmark pension system with constant retirement age ("BM").

is therefore not possible to implement more gradual changes in the retirement age leading to smoother transitions.

Figure 6: Adjustment of Pension System, Pension Reform

(a) Constant Replacement Rate

(b) Constant Contribution Rate



Notes: Results from the model with endogenous human capital and open capital markets.

Our results of this experiment on macroeconomic variables are summarized in figure 7. Increasing the retirement age has strong effects. The level of rates of returns are significantly higher than in our benchmark scenario. This is so because of the aforementioned effects of increasing the retirement age on total effective labor supply. As a consequence, even when replacement rates are held constant — and contribution rates adjust correspondingly — GDP per capita is now found to increase.

Notice that in the endogenous human capital model the effect of increasing the retirement age is stronger in the constant replacement rate scenario (e.g., compare panels (c) and (d) of figure 7). To understand this, recall from figure 6 that the increase of contribution rates is substantially dampened in the reform "PR". The overall tax burden is therefore lowered which induces additional incentives to invest in human capital. On the contrary, in the constant contribution rate scenario, the replacement rate decreases by less. This just has the opposite incentive effects on human capital formation.

In figure 11 in appendix B we plot changes in the same variables for the model with exogenous human capital. As expected, when human capital cannot adjust the effect of a higher retirement age are generally found to be smaller.

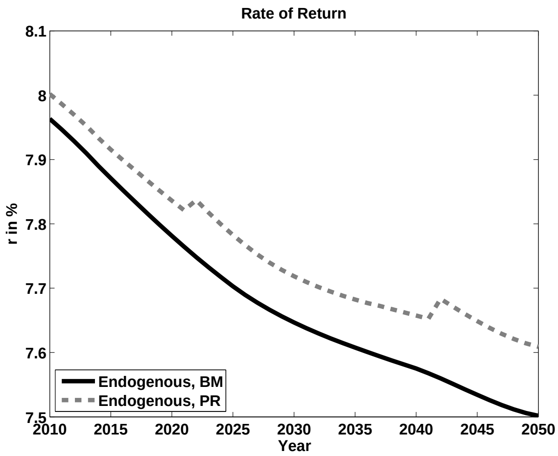
4.2 Labor Supply over the Life Cycle

To understand the effects of the pension reform on aggregate hours and human capital, it is key to understand the response over the life cycle. Theoretical insights into these life cycle effects are developed in appendix B.2 by use of a simple two-period model. To shed light on these adjustments quantitatively, we isolate the effects induced by a change in the retirement age from changes induced by general equilibrium feedback.

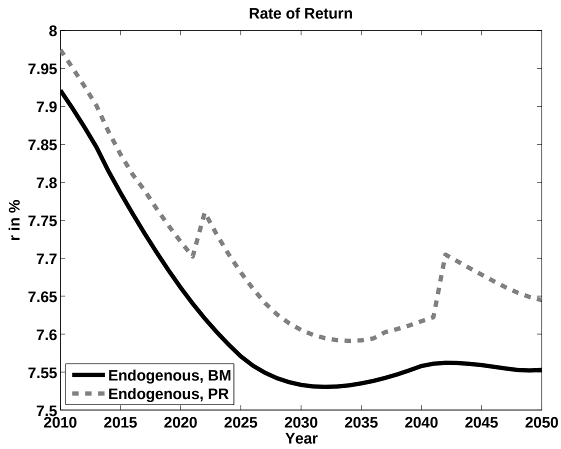
First, we define our benchmark to be the cohort entering the labor market in 1955. This is the first cohort affected by an increase of the normal retirement age. Then, we take general

Figure 7: Aggregate Variables in Open Economy Scenario, Benchmark vs. Pension Reform

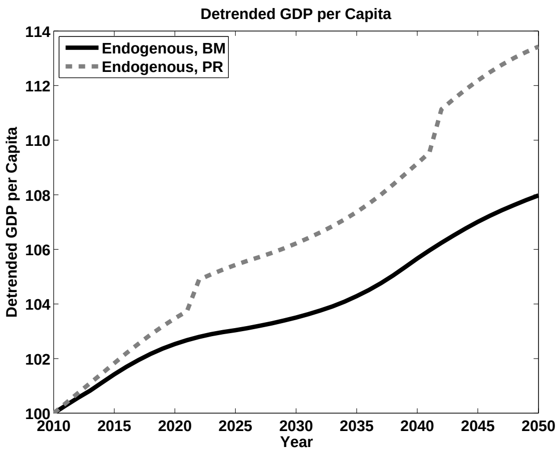
(a) Rate of Return to Physical Capital, Constant τ



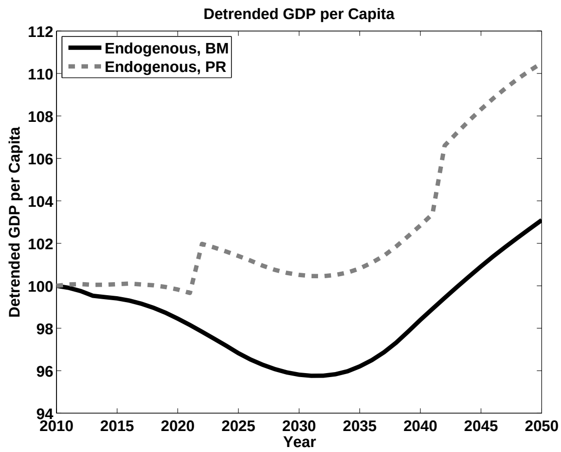
(b) Rate of Return to Physical Capital, Constant ρ



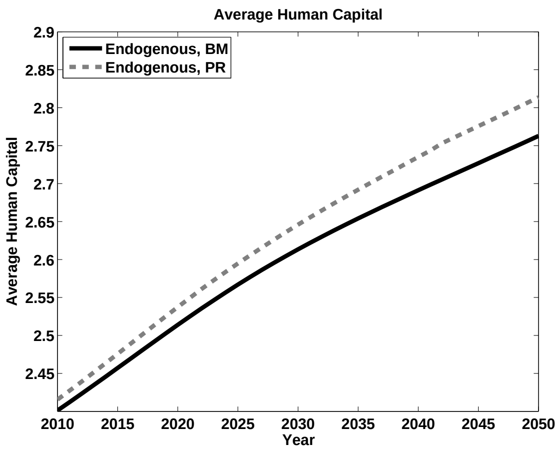
(c) Detrended GDP per Capita, Constant τ



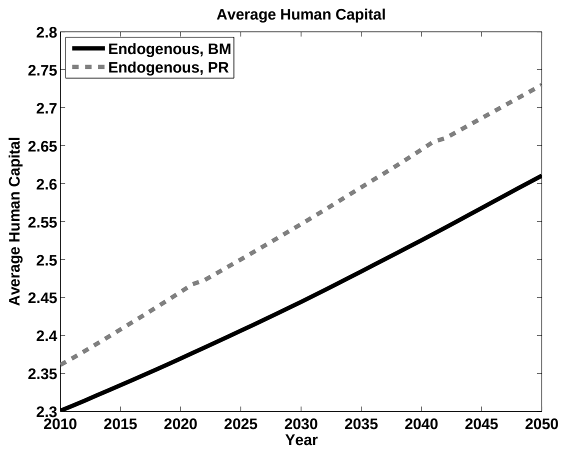
(d) Detrended GDP per Capita, Constant ρ



(e) Average Human Capital, Constant τ



(f) Average Human Capital, Constant ρ



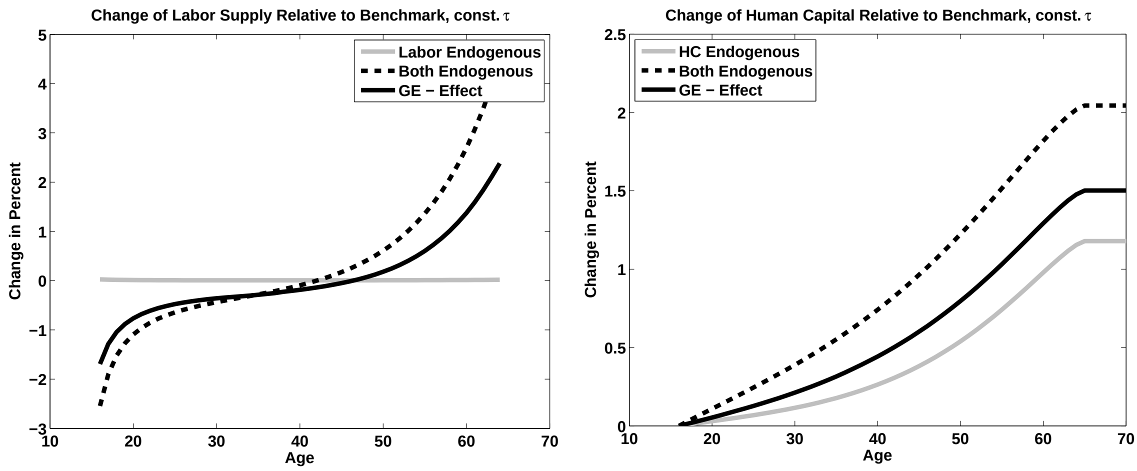
Notes: Results from the model with endogenous human capital and open capital markets.

equilibrium prices and policy instruments from the benchmark economy (with retirement age at 65) and increase the retirement age. This enables us to compare household decisions in partial equilibrium. In order to isolate the different adjustment channels — labor supply and human capital — in this partial equilibrium setting we perform three experiments. We start by holding constant the human capital profile and endogenously compute labor supply. We next hold the labor supply profile fixed and allow human capital to adjust. Finally, we allow labor supply and human capital to be endogenous and thereby capture the total effect (in partial equilibrium). As a last step, we compare our results to the life cycle profiles of households observed in general equilibrium after implementation of the pension reform.

Results of this decomposition analysis are summarized in figure 8 as percent deviations from the benchmark life cycle profile. Ignoring general equilibrium adjustments and human capital accumulation, an increase in the retirement age has a negligible impact on labor supply at the intensive margin. With both margins of adjustment at work ("both endogenous") the young invest more time in education and hence labor supply decreases relative to the benchmark. Older agents work more to reap the benefits of higher levels of human capital. These predictions are consistent with the theoretical model in the appendix (cf.equation (29)). Finally, in general equilibrium, the endogenous labor supply and human capital response leads to higher pension payments, lower increases in wages and relatively higher interest rates. This mitigates the incentives to invest into human capital and increases (decreases) labor supply when young (old) relative to the case with constant prices. As a consequence, once all general equilibrium adjustments are considered, incentives to work and invest into human capital are weakened and lead only to a small change of labor supply at the intensive margin.

Figure 8: Reallocation over the Life Cycle with PR, Partial vs. General Equilibrium

(a) Change in Labor Supply, Constant τ (b) Change in Human Capital, Constant τ



Notes: "Endogenous" and "Exogenous" refer to the human capital production profile. Results with open capital markets.

Table 2 documents the total effect of these behavioral adjustments as the sum of changes of life cycle allocations shown in the previous figure. Moving from left to right in the table, we observe a reaction of labor supply at the intensive margin in partial equilibrium only when education and hours worked are simultaneously endogenous. This is offset in general equilibrium. We can therefore conclude that the exogenous increase of the retirement age has a significant impact on total aggregate effective labor supply only by inducing agents to work more in the marginal year and investing more into human capital. On the contrary, reactions at the intensive margin only shift allocations over the life cycle without affecting the aggregate much.[17]

These quantitative responses are in line with empirical estimates. As Mastrobuoni (2009) shows, more than half of the increase in retirement age in the US is taken up by agents. In table 2 (rightmost column) we show that the increase in hours in the *additional* year corresponds to an implicit pass-through of roughly 64% which we believe to be a reasonable approximation of the empirical results.[18] Therefore, although our model misses endogenous retirement choice as a decision at the extensive margin, we are confident that our quantitative predictions resulting from endogenous labor supply adjustments are about right.

A key aspect in this context is the elasticity of labor supply. As discussed in Ludwig, Schelkle, and Vogel (2012) — especially section 4.1 — alternative measures of labor supply elasticities range between 1.1 and 1.9 in this type of model. In light of adjustments at the extensive margins and the empirical work by Imai and Keane (2004) — who argue that standard estimates are downward-biased by not considering endogenous human-capital accumulation explicitly — this is a reasonable magnitude.

Table 2: Change in Life Cycle Allocations (Constant τ)

	Endogenous Variable			
	ℓ	e	ℓ, e	ℓ, e (GE)
Labor (Intensive)	0.0%	0.0%	0.3%	0.0%
Labor (Total)	1.2%	1.2%	1.6%	1.3%

Notes: "Intensive" refers to the total amount of effective labor supply up to the "old" retirement age, "total" adds the additional working year. The variables reported (title of column) is always the endogenous variable, the other is assumed to be exogenous (see description of methodology above).

4.3 Welfare

In our model, households are affected by three distinct consequences of demographic change and policy reform. First, for given prices utility increases because survival probabilities increase. Second, households are affected by changes in prices and transfers due to general equilibrium effects of aging. For cohorts currently alive, these profound changes can have — depending on the position in the life cycle — positive or negative welfare effects. Third, when the retirement age is increased, a constraint is relaxed which will lead to welfare gains. Furthermore, as shown above, increasing the retirement age leads to higher levels of rates of return

[17] With the exception of the first few years after the reform during which increased human capital investments through eduction crowd out labor supply. The effect is, however, small and therefore not shown.

[18] If workers would work full time, an increase in the retirement age by one year constitutes an increase of total time by about $\frac{1}{65-16} \approx 0.02$. However, total hours worked over the working life increase by only about 1.3% which is a pass-through of about $1.3/2 \approx 0.64$. An alternative estimate of the pass-through would be to argue that hours worked are roughly $1/3$ on average and households of age 65 work about $1/4$ of their time in our model, see figure 16 in the appendix so that the pass-through would be $4/3 \cdot 64\% \approx 85\%$. We could also view each worker at age 66 as a potential full-time worker who then decides to spend roughly 25% of time working, then the pass-through would be 25%.

(lower wages) and lower decreases of the rate of return as societies are aging — cf. figures 7(a) and 7(b) — with associated feedback into welfare consequences

We want to isolate and quantify the effect of changing prices, taxes and transfers as well as increasing the retirement age on households' lifetime utility. To this end, we first compute the (remaining) lifetime utility of an agent of age j born in year t using the full set of (time varying) general equilibrium prices, taxes and transfers. Then, we hold all prices and transfers constant at their respective year 2010 value and recompute agent's remaining lifetime utility. For both scenarios we keep constant survival probabilities at their year 2010 values. We compute the consumption equivalent variation, $g_{t,j,i}$, i.e., the percentage of consumption that needs to be given to the agent at each date for her remaining lifetime at prices from 2010 in order to make her indifferent between the two scenarios. Positive values of $g_{t,j,i}$ thus indicate welfare gains from the general equilibrium effects of aging.[19] In order to isolate the effects of changing prices, taxes and transfers, we do not account for the gain in households' lifetime utility during the additional life years generated by the increase in life expectancy.

In the remainder of this section we stick to the structure from the previous section. We first report numbers from our welfare analysis for agents living in the benchmark pension scenario holding the retirement age constant. Then we advance to the comparison of the effects of increasing the retirement age. To work out distributional consequences across generations, we first look at welfare consequences for agents alive in 2010, followed by an analysis of welfare of future generations.

Welfare of Generations Alive in 2010 in the Benchmark Model

The analysis in this section performs an inter-generational welfare comparison of the consequences of demographic change holding the retirement age at the current benchmark level. The results for the "const ρ" ("const τ") scenario are shown in figures 9(a) and 9(b). They can be summarized as follows: for constant contribution rates, newborns in the endogenous (exogenous) model benefit up to 0.7% (0.4%) of lifetime consumption from the general equilibrium effects of aging. This is due to increasing wages and decreasing interest rates. Higher wage growth makes investment into human capital more attractive and falling interest rates decrease the costs of borrowing. Middle aged agents with high levels of physical assets incur welfare losses due to falling interest rates. Further, their ability to change human capital investment is restricted as they are in a relatively advanced stage of their life cycle. Retired agents additionally incur losses from falling pensions (due to constant contribution rates).

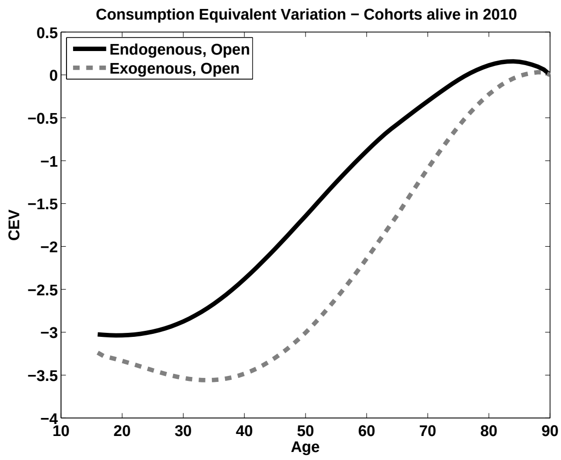
Of particular interest is that agents in the endogenous human capital investment model incur much lower losses compared to households with a fixed human capital profile. With endogenous human capital, maximum losses are about 32% lower compared to the model with a fixed human capital profile. Thus, agents with the possibility to react to changing prices will do so and this decreases their potential losses by a considerable amount.

On the contrary, openness per se does not have strong effects on welfare. This is directly related to our insights from the previous part where we have shown that time paths of rates of

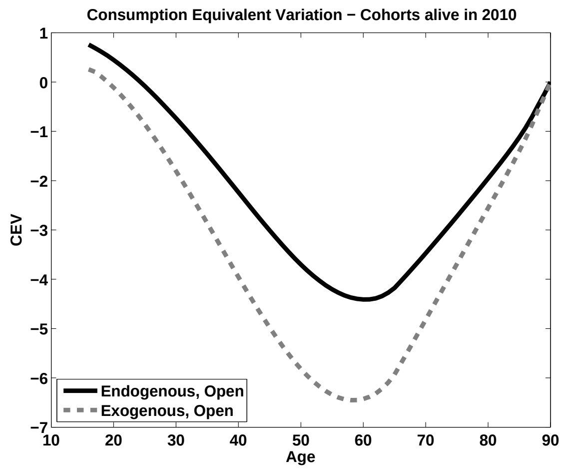
[19] Using the functional form from equation (1) the consumption equivalent variation is given by $g_{t,j,i} = \left(\frac{V_{t,j,i}}{V_j^{2010}}\right)^{\frac{1}{\phi(1-\sigma)}} - 1$ where $V_{t,j,i}$ denotes lifetime utility using general equilibrium prices and V_j^{2010} is lifetime utility using constant prices from 2010.

Figure 9: Consumption Equivalent Variation of Agents alive in 2010, Benchmark Pension System

(a) Constant Replacement Rate



(b) Constant Contribution rate



Notes: "Endogenous" and "Exogenous" refer to the human capital production profile. Results from the model with endogenous human capital and open capital markets.

return to physical capital (and thereby wages) do not differ much between our open and our counterfactual closed economy scenario. Welfare results for the closed economy are shown in figure 12 in appendix B.

In the constant replacement rate scenario the majority of the population loses from the effects of demographic change. Young agents are worse off as constant replacement rates require rising contributions which depresses net wages and incentives to invest in human capital. Very old agents experience small gains in lifetime utility due to rising pensions. Also notice that welfare consequences are more evenly distributed under constant replacement rates where maximum losses are considerably smaller than with constant contribution rates.[20]

We also summarize welfare consequences for newborns in our benchmark economy with open capital markets in table 3 and the maximum welfare losses in table 4. Results for the closed economy and can be found in appendix B in tables 5 and 6.

Table 3: Welfare Gains / Losses - Newborns 2010

	Open Economy			
Pension System	Constant τ		Constant ρ	
	Endog.	Exog.	Endog.	Exog.
BM	0.8%	0.3%	-3.0%	-3.2%
PR	1.2%	0.6%	-1.7%	-2.3%

Notes: "Endog." and "Exog." refer to the endogenous and exogenous human capital production profile. Results from the open economy scenario. "BM" denotes the benchmark pension system (constant retirement age), and "PR" denotes the pension reform (increase in the retirement age).

[20]See Glomm and Kaganovich (2008) for the relationship between changes in social security, public education and welfare in a heterogenous agents model.

Table 4: Maximum Welfare Losses - Agents alive 2010

	Open Economy			
Pension System	Constant τ		Constant ρ	
	Endog.	Exog.	Endog.	Exog.
BM	-4.4%	-6.5%	-3.0%	-3.6%
PR	-3.6%	-6.0%	-1.9%	-3.0%

Notes: "Endog." and "Exog." refer to the endogenous and exogenous human capital production profile. Results from the open economy scenario."BM" denotes the benchmark pension system (constant retirement age), and "PR" denotes the pension reform (increase in the retirement age).

Welfare of Generations Alive in 2010 when the Retirement Age is Increased

We next look at differences in welfare changes for agents living in 2010 across all pension reform scenarios. For sake of brevity, we focus on the summary of results in tables 3 and 4 and relegate the corresponding figure 13 to appendix B. Gains to newborn households increase by 0.3 to 0.4 percentage points under constant replacement rates and losses decrease by more than one percentage point under constant replacement rates.[21]

Welfare of Future Generations in the Benchmark Model

We now report welfare changes for newborns between 2010 and 2050, see figure 14 in appendix B. Holding the replacement rate constant has a strong negative impact on lifetime utility for future generations. Even with endogenous investment into human capital, maximum welfare losses can be up to 7% (5.9%) in the open (closed) economy. With an exogenous efficiency profile, losses are even higher. With constant contribution rates, newborns gain up to 2.5% (1.5%) in the open (closed) economy and endogenous human capital investment.

Welfare of Future Generations when the Retirement Age is Increased

In case of constant replacement rates, increasing the retirement age cannot totally eliminate welfare losses from demographic change, see figure 15 in appendix B. However, the effect is strong. In case of endogenous human capital, welfare losses of the cohort born in 2050 are now at -2.6%, compared to -5.9% without the reform.

5 Conclusion

This paper revisits the literature on the consequences of demographic change — aging – for welfare of generations who live through the demographic transition in industrialized countries

[21] GDP per capita and welfare increase due to higher capital accumulation and exogenous technical progress. Our methodology to measure general equilibrium feedback effects, however, neglects these effects as we compare changing with constant prices ("purged" from aging induced effects) for any *level* of GDP per capita. In other words, for any cohort the level of GDP is identical for both scenarios at the beginning of the life-cycle but prices diverge later on.

by asking a straightforward question: How can the potentially detrimental consequences of aging of the group of industrialized countries be mitigated by two margins of adjustment, namely investing abroad and human capital formation? We address this question in combination with pension policy. That is, we ask how the design of pension policy may contribute to dampening via these endogenous channels.

We conclude with a negative and a positive finding. Our negative finding is that openness to international capital markets has only modest welfare implications vis-a-vis a counterfactual closed economy world. The reason for this is that overall demographic trends across our two highly aggregate world regions are very similar (despite strong differences in levels and the heterogeneity within each region). To the extent that the group of industrialized countries is relatively open to the rest of the world already today, this leads to relatively small price changes over time when we contrast an open economy scenario with a counterfactual closed economy scenario. Small price differences in turn lead to small welfare changes.

Our positive finding is that endogenous human capital formation in combination with constant contribution rates to the pension system and increases of the statutory retirement age has strong welfare effects. We find that, in general equilibrium, the exogenous increase of the retirement age translates almost one for one into labor market participation of the elderly while there is almost no reaction at the intensive margin of those already working. Hence, "reform backlashes" (Börsch-Supan and Ludwig 2009) are small and effective labor supply is consequently increased quite strongly. This leads to strong welfare gains for all future generations (under constant contribution rates). Welfare losses for middle aged and asset rich households who suffer from decreasing returns on savings and decreasing pension payments are 6.5% if human capital cannot adjust and if the statutory retirement age is held constant. These losses decrease to 3.6% if human capital endogenously adjusts and increases of the retirement age roughly track life expectancy.

Our main conclusion is therefore that labor market policies focusing at the extensive margin (by increasing the retirement age) and adjustments at the education margin are key to mitigate the adverse effects of demographic change. However, we only model labor supply decisions at the intensive margin, treating retirement as exogenous. While we document that our estimates of endogenous labor supply adjustments to changes in the exogenous reform of the retirement age are reasonable, one may still argue that it is important to explicitly model the endogenous choice of retirement at the extensive margin. We plan to extend our model along these dimensions in our future research.

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A Computational Appendix

A.1 Household Problem

To simplify the description of the solution of the household model for given prices (wage and interest rate), transfers and social security payments, we focus on steady states and therefore drop the time index t and the country index i. Furthermore, we focus on a de-trended version of the household problem in which all variables x are transformed to $\tilde{x} = \frac{x}{A}$ where A is the technology level growing at the exogenous rate g. To simplify notation, we do not denote variables by the symbol $\tilde{\cdot}$ but assume that the transformation is understood. The de-trended version of the household problem is then given by

$$
\begin{aligned}
V(a,h,j) = \max_{c,\ell,e,a',h'} & \left\{ u(c, 1-\ell-e) + \beta s (1+g)^{\phi(1-\sigma)} V(a',h',j+1) \right\} \\
\text{s.t.} & \\
a' &= \frac{1}{1+g} \left((a+tr)(1+r) + y - c \right) \\
y &= \begin{cases} \ell h w (1-\tau) & \text{if } j < jr \\ p & \text{if } j \geq jr \end{cases} \\
h' &= g(h,e) \\
\ell &\in [0,1], \quad e \in [0,1].
\end{aligned} \tag{13}
$$

Here, $g(h,e)$ is the human capital technology.

Let $\tilde{\beta} = \beta s (1+g)^{\phi(1-\sigma)}$ be the transformation of the discount factor. Using the budget constraints, now rewrite the above as

$$
\begin{aligned}
V(a,h,j) = \max_{c,\ell,e,a',h'} & \left\{ u(c, 1-\ell-e) + \tilde{\beta} V\left(\frac{1}{1+g} \left((a+tr)(1+r) + y - c \right), g(h,e), j+1 \right) \right\} \\
\text{s.t.} & \\
\ell &\geq 0.
\end{aligned}
$$

where we have also replaced the bounded support of time investment and leisure with a one-side constraint on ℓ because the upper constraints, $\ell = 1$, respectively $e = 1$, and the lower constraint, $e = 0$, are never binding due to Inada conditions on the utility function and the functional form of the human capital technology (see below). Denoting by μ_ℓ the Lagrange multiplier on the inequality constraint for ℓ, we can write the first-order conditions as

$$
c: \quad u_c - \tilde{\beta} \frac{1}{1+g} V_{a'}(a',h';j+1) = 0 \tag{14a}
$$

$$
\ell: \quad -u_{1-\ell-e} + \tilde{\beta} h w (1-\tau) \frac{1}{1+g} V_{a'}(a',h',j+1) + \mu_\ell = 0 \tag{14b}
$$

$$
e: \quad -u_{1-\ell-e} + \tilde{\beta} g_e V_{h'}(a',h',j+1) = 0 \tag{14c}
$$

and the envelope conditions read as

$$
a: \quad V_a(a,h,j) = \tilde{\beta} \frac{1+r}{1+g} V_{a'}(a',h',j+1) \tag{15a}
$$

$$
h: \quad V_h(a,h,j) = \tilde{\beta} \left(\ell w (1-\tau) \frac{1}{1+g} V_{a'}(a',h',j+1) + g_h V_{h'}(a',h',j+1) \right). \tag{15b}
$$

Note that for the retirement period, i.e. for $j \geq jr$, equations (14b) and (14c) are irrelevant and equation (15b) has to be replaced by

$$V_h(a,h,j) = \tilde{\beta} g_h V_{h'}(a',h',j+1).$$

From (14a) and (15a) we get

$$V_a = (1+r)u_c \tag{16}$$

and, using the above in (14a), the familiar inter-temporal Euler equation for consumption follows as

$$u_c = \tilde{\beta}\frac{1+r}{1+g}u_{c'}. \tag{17}$$

From (14a) and (14b) we get the familiar intra-temporal Euler equation for leisure,

$$u_{1-\ell-e} = hw(1-\tau)u_c + \mu_\ell. \tag{18}$$

From the human capital technology (3) we further have

$$g_e = \xi\psi(eh)^{\psi-1}h \tag{19a}$$

$$g_h = (1-\delta^h) + \xi\psi(eh)^{\psi-1}e. \tag{19b}$$

We loop backwards in j from $j = J-1,\ldots,0$ by taking an initial guess of $[c_J, h_J]$ as given and by initializing $V_{a'}(\cdot,J) = V_{h'}(\cdot,J) = 0$. During retirement, that is for all ages $j \geq jr$, our solution procedure is by standard backward shooting using the first-order conditions. However, during the period of human capital formation, that is for all ages $j < jr$, the first order conditions would not be sufficient if the problem is not a convex-programming problem. And thus, our backward shooting algorithm will not necessarily find the true solution. In fact this may be the case in human capital models such as ours because the effective wage rate is endogenous (it depends on the human capital investment decision). For a given initial guess $[c_J, h_J]$ we therefore first compute a solution via first-order conditions and then, for each age $j < jr$, we check whether this is the unique solution. As an additional check, we consider variations of initial guesses of $[c_J, h_J]$ on a large grid. In all of our scenarios we never found any multiplicities.

The details of our steps are as follows:

1. In each j, $h_{j+1}, V_{a'}(\cdot, j+1), V_{h'}(\cdot, j+1)$ are known.
2. Compute u_c from (14a).
3. For $j \geq jr$, compute h_j from (3) by setting $e_j = \ell_j = 0$ and by taking h_{j+1} as given and compute c_j directly from equation (23) below.
4. For $j < jr$:
   (a) Assume $\ell \in [0,1)$ so that $\mu_\ell = 0$.
   (b) Combine (3), (14b), (14c) and (19a) to compute h_j as

$$h_j = \frac{1}{1-\delta^h}\left(h_{j+1} - \xi\left(\frac{\xi\psi\frac{1}{1+g}V_{h'}(\cdot,j+1)}{\omega(1-\tau)V_{a'}(\cdot,j+1)}\right)^{\frac{\psi}{1-\psi}}\right) \tag{20}$$

(c) Compute e from (3) as

$$e_j \quad = \frac{1}{h_j}\left(\frac{h_{j+1}-h_j(1-\delta^h)}{\xi}\right)^{\frac{1}{\psi}}. \tag{21}$$

(d) Calculate $lcr_j = \frac{1-e_j-\ell_j}{c_j}$, the leisure to consumption ratio from (18) as follows: From our functional form assumption on utility marginal utilities are given by

$$u_c = \left(c^{\phi}(1-\ell-e)^{1-\phi}\right)^{-\sigma}\phi c^{\phi-1}(1-\ell-e)^{1-\phi}$$
$$u_{1-\ell-e} = \left(c^{\phi}(1-\ell-e)^{1-\phi}\right)^{-\sigma}(1-\phi)c^{\phi}(1-\ell-e)^{-\phi}$$

hence we get from (18) the familiar equation:

$$\frac{u_{1-\ell-e}}{u_c} = hw(1-\tau) = \frac{1-\phi}{\phi}\frac{c}{1-\ell-e},$$

and therefore:

$$lcr_j = \frac{1-e_j-\ell_j}{c_j} = \frac{1-\phi}{\phi}\frac{1}{hw(1-\tau)}. \tag{22}$$

(e) Next compute c_j as follows. Notice first that one may also write marginal utility from consumption as

$$u_c = \phi c^{\phi(1-\sigma)-1}(1-\ell-e)^{(1-\sigma)(1-\phi)}. \tag{23}$$

Using (22) in (23) we then get

$$\begin{aligned} u_c &= \phi c^{\phi(1-\sigma)-1}(lcr\cdot c)^{(1-\sigma)(1-\phi)} \\ &= \phi c^{-\sigma}\cdot lcr^{(1-\sigma)(1-\phi)}. \end{aligned} \tag{24}$$

Since u_c is given from (14a), we can now compute c as

$$c_j = \left(\frac{u_{c_j}}{\phi\cdot lcr_j^{(1-\sigma)(1-\phi)}}\right)^{-\frac{1}{\sigma}}. \tag{25}$$

(f) Given c_j, e_j compute labor, ℓ_j, as

$$\ell_j = 1 - lcr_j\cdot c_j - e_j.$$

(g) If $\ell_j < 0$, set $\ell_j = 0$ and iterate on h_j as follows:

i. Guess h_j

ii. Compute e as in step 4c.

iii. Noticing that $\ell_j = 0$, update c_j from (23) as

$$c = \left(\frac{u_c}{\phi(1-e)^{(1-\sigma)(1-\phi)}}\right)^{\frac{1}{\phi(1-\sigma)-1}}.$$

iv. Compute μ_ℓ from (14b) as

$$\mu_\ell = u_{1-\ell-e} - \tilde{\beta}hw(1-\tau)V_{a'}(\cdot, j+1)$$

v. Finally, combining equations (14b), (14c) and (19a) gives the following distance function f

$$f = e - \left(\frac{\tilde{\beta} \xi \psi h^{\psi} \frac{1}{1+g} V_{h'}(\cdot)}{\tilde{\beta} \omega h (1-\tau) V_{a'}(\cdot) + \mu_{\ell}} \right)^{\frac{1}{1-\psi}}, \tag{26}$$

where e is given from step 4(g)ii. We solve for the root of f to get h_j by a non-linear solver iterating on steps 4(g)ii through 4(g)v until convergence.

5. Update as follows:
   (a) Update V_a using either (15a) or (16).
   (b) Update V_h using (15b).

Next, loop forward on the human capital technology (3) for given h_0 and $\{e_j\}_{j=0}^{J}$ to compute an update of h_J denoted by h_J^n. Compute the present discounted value of consumption, PVC, and, using the already computed values $\{h_j^n\}_{j=0}^{J}$, compute the present discounted value of income, PVI. Use the relationship

$$c_0^n = c_0 \cdot \frac{PVI}{PVC} \tag{27}$$

to form an update of initial consumption, c_0^n, and next use the Euler equations for consumption to form an update of c_J, denoted as c_J^n. Define the distance functions

$$g_1(c_J, h_J) = c_J - c_J^n \tag{28a}$$

$$g_2(c_J, h_J) = h_J - h_J^n. \tag{28b}$$

In our search for general equilibrium prices, constraints of the household model are occasionally binding. Therefore, solution of the system of equations in (28) using Newton based methods, e.g., Broyden's method, is instable. We solve this problem by a nested Brent algorithm, that is, we solve two nested univariate problems, an outer one for c_J and an inner one for h_J.

Check for uniqueness: Observe that our nested Brent algorithm assumes that the functions in (28) exhibit a unique root. As we adjust starting values $[c_J, h_J]$ with each outer loop iteration we thereby consider different points in a variable box of $[c_J, h_J]$ as starting values. For all of these combinations our procedure always converged. To systematically check whether we also always converge to the unique optimum, we fix, after convergence of the household problem, a large box around the previously computed $[c_J, h_J]$. Precisely, we choose as boundaries for this box $\pm 50\%$ of the solutions in the respective dimensions. For these alternative starting values we then check whether there is an additional solution to the system of equations (28). We never detected any such multiplicities.

A.2 The Aggregate Model

To solve the open economy general equilibrium transition path we proceed as follows: for a given $r \times 1$ vector $\vec{\Psi}$ of structural model parameters, we first solve for an "artificial" initial steady state in period $t = 0$ which gives initial distributions of assets and human capital. We thereby presume that households assume prices to remain constant for all periods $t \in \{0, \ldots, T\}$ and are then surprised by the actual price changes induced by the transitional dynamics. Next, we solve for the final steady state of our model which is reached in period T and supported by our demographic projections. In the sequel, the superscripts c and o refer to the closed or open economy and M denotes the number of regions.

For the closed economy steady state, for each region j we solve for the equilibrium of the aggregate model by iterating on the $m^c \times 1$ steady state vector $\vec{P}^c_{ss,j} = \left[p_{1,j}, \ldots, p_{m^c,j}\right]'$. $p_{1,j}$ is the capital intensity, $p_{2,j}$ are transfers (as a fraction of wages), $p_{3,j}$ are social security contribution (or replacement) rates and $p_{4,j}$ is the average human capital stock for region j. We perform this procedure separately for both world regions.

To compute the open economy steady state we solve for the equilibrium of the aggregate model by iterating on the $m^o \times 1$ steady state vector $\vec{P}^o_{ss} = \left[p_1, \ldots, p_{m^o,j}\right]'$ where the number of variables is given by $m^o = M(m^c - 1) + 1$. p_1 is the *common (world)* capital intensity, $p_{2,j}$ are transfers (as a fraction of wages), $p_{3,j}$ are social security contribution (or replacement) rates and $p_{4,j}$ is the average human capital stock for region j. Notice that all elements of $\vec{P}^c_{ss}$ and $\vec{P}^o_{ss}$ are constant in the steady state.

Solution for the steady states for each *closed* region j (where we drop the region index for brevity) of the model involves the following steps:

1. In iteration q for a guess of $\vec{P}^{c,q}_{ss}$ solve the household problem.
2. Update variables in $\vec{P}^c_{ss}$ as follows:
   (a) Aggregate across households to get aggregate assets and aggregate labor supply to form an update of the capital intensity, p^n_1.
   (b) Calculate an update of bequests to get p^n_2.
   (c) Using the update of labor supply, update social security contribution (or replacement) rates to get p^n_3.
   (d) Use labor supply and human capital decisions to form an update of the average human capital stock, p^n_4.
3. Collect the updated variables in $\vec{P}^{c,n}_{ss}$ and notice that $\vec{P}^{c,n}_{ss} = H(\vec{P}^c_{ss})$ where H is a vector-valued non-linear function.
4. Define the root-finding problem $G(\vec{P}^c_{ss}) = \vec{P}^c_{ss} - H(\vec{P}^c_{ss})$ and iterate on $\vec{P}^c_{ss}$ until convergence. We use Broyden's method to solve the problem and denote the final approximate Jacobi matrix by B_{ss}.

Solution for the steady states of the *open* economy of the model involves the following steps:

1. In iteration q for a guess of $\vec{P}^{o,q}_{ss}$ solve the household problem.
2. Update variables in $\vec{P}^o_{ss}$ as follows:
   (a) Use the guess for the global capital intensity to compute the capital stock for region j compatible with the open economy, perfect competition setup. Use this aggregate capital stock with the aggregate labor supply to form an update of the global capital intensity, p^n_1.
   (b) Calculate an update of bequests to get $p^n_{2,j}$ $\forall j$.
   (c) Using the update of labor supply, update social security contribution (or replacement) rates to get $p^n_{3,j}$ $\forall j$.
   (d) Use labor supply and human capital decisions to form an update of the average human capital stock, p^n_4 $\forall j$.

3. Collect the updated variables in $\vec{P}_{ss}^{o,n}$ and notice that $\vec{P}_{ss}^{o,n} = H(\vec{P}_{ss}^{o})$ where H is a vector-valued non-linear function.
4. Define the root-finding problem $G(\vec{P}_{ss}^{o}) = \vec{P}_{ss}^{o} - H(\vec{P}_{ss}^{o})$ and iterate on $\vec{P}_{ss}^{o}$ until convergence. We use Broyden's method to solve the problem and denote the final approximate Jacobi matrix by B_{ss}.

Next, we solve for the transitional dynamics for each of the closed economies (where we again drop the region index j) by the following steps:

1. Use the steady state solutions to form a linear interpolation to get the starting values for the $m^c(T-2) \times 1$ vector of equilibrium prices, $\vec{P}^c = [\vec{p}_1', \ldots, \vec{p}_{m^c}']'$, where $p_i, i = 1, \ldots, m^c$ are vectors of length $(T-2) \times 1$.
2. In iteration q for guess $\vec{P}^{c,q}$ solve the household problem. We do so by iterating backwards in time for $t = T-1, \ldots, 2$ to get the decision rules and forward for $t = 2, \ldots, T-1$ for aggregation.
3. Update variables as in the steady state solutions and denote by $\tilde{\vec{P}}^c = H(\vec{P}^c)$ the $m^c(T-2) \times 1$ vector of updated variables.
4. Define the root-finding problem as $G(\vec{P}^c) = \vec{P}^c - H(\vec{P}^c)$. Since T is large, this problem is substantially larger than the steady state root-finding problem and we use the Gauss-Seidel-Quasi-Newton algorithm suggested in Ludwig (2007) to form and update guesses of an approximate Jacobi matrix of the system of $m^c(T-2)$ non-linear equations. We initialize these loops by using a scaled up version of B_{ss}.

We then solve for the transitional dynamics for the open economy setup by the following steps:

1. Use the equilibrium transition solutions from the closed economies to get the starting values for the $m^o(T-\tilde{t}-2) \times 1$ vector of equilibrium prices, $\vec{P}^o = [\vec{p}_1', \ldots, \vec{p}_{m^o}']'$, where $p_i, i = 1, \ldots, m^o$ are vectors of length $(T-\tilde{t}-2) \times 1$ where $\tilde{t}$ is the year of opening up.
2. In iteration q for guess $\vec{P}^{o,q}$ solve the household problem. We do so by iterating backwards in time for $t = T-\tilde{t}-1, \ldots, 2$ to get the decision rules and forward for $t = 2, \ldots, T-\tilde{t}-1$ for aggregation. For agents already living in year $\tilde{t}$ we use their holdings of physical assets and human capital from year $\tilde{t}$ as state variables and solve their household problem only for their remaining lifetime.
3. We then proceed as in the case for the closed economies (updating) but define the root-finding problem now for the open economy as $G(\vec{P}^o) = \vec{P}^o - H(\vec{P}^o)$ which we solve by the same method as above.

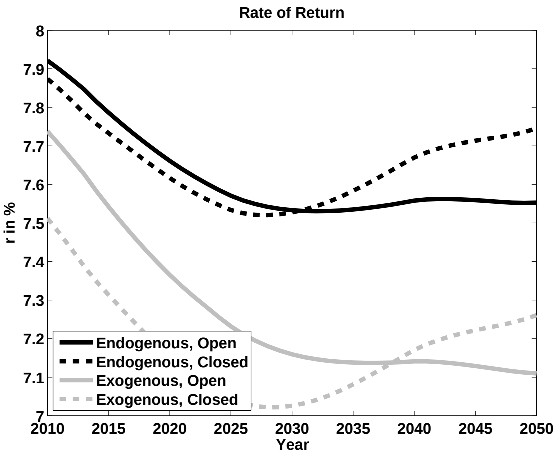
B Supplementary Appendix

B.1 Additional Results

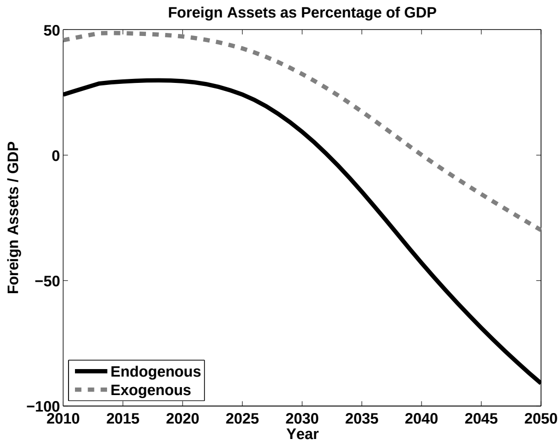
Aggregate Variables

Figure 10: Aggregate Variables for Constant Replacement Rate Scenario, Benchmark Pension System

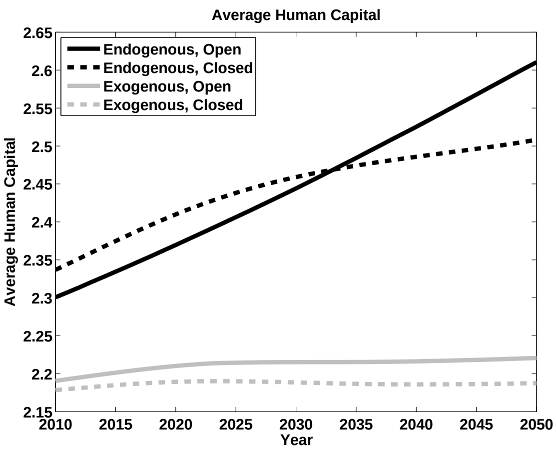
(a) Rate of Return to Physical Capital



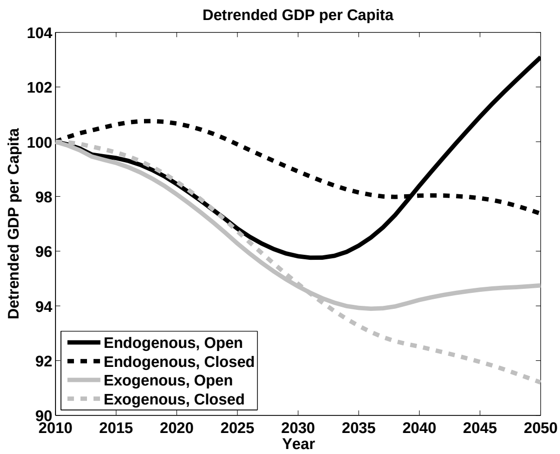
(b) Net Foreign Assets



(c) Average Human Capital



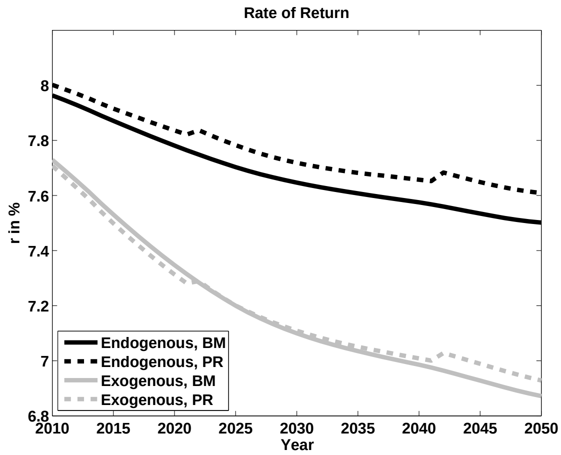
(d) Detrended GDP per Capita



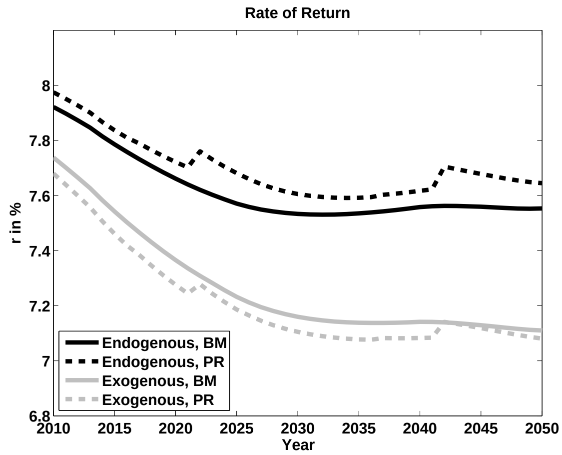
Notes: “Endogenous” and “Exogenous” refer to the human capital production profile. “Open” and “Closed” refer to the results obtained from the closed and open economy versions. All results obtained from the constant replacement rate scenario, benchmark pension system with constant retirement age (“BM”).

Figure 11: Aggregate Variables in Open Economy Scenario, Benchmark vs. Pension Reform

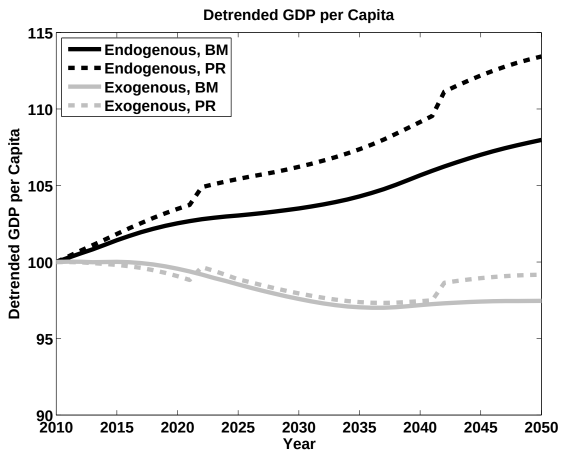
(a) Rate of Return to Physical Capital, Constant τ



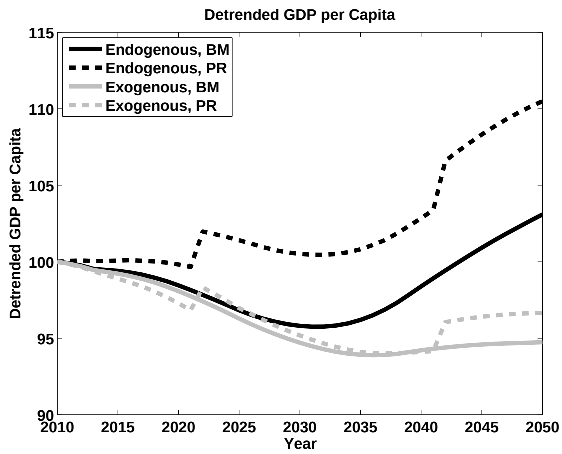
(b) Rate of Return to Physical Capital, Constant ρ



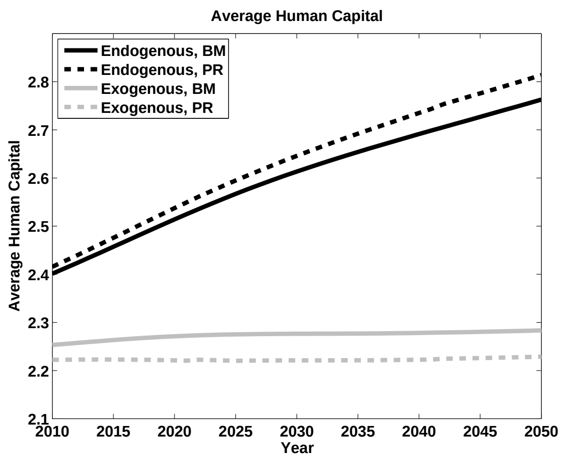
(c) Detrended GDP per Capita, Constant τ



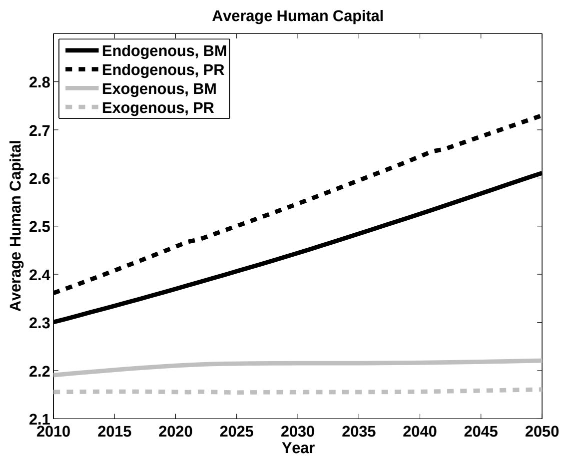
(d) Detrended GDP per Capita, Constant ρ



(e) Average Human Capital, Constant τ



(f) Average Human Capital, Constant ρ

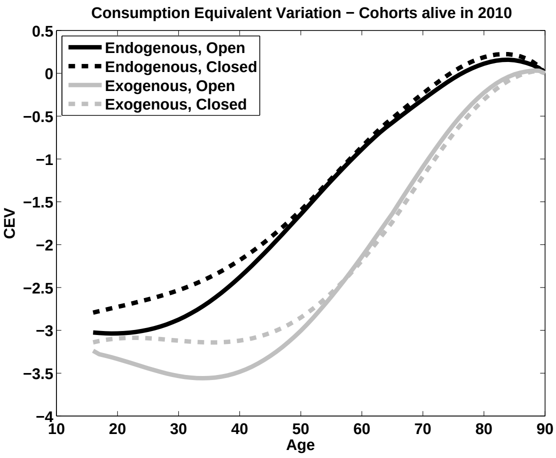


Notes: "Endogenous" and "Exogenous" refer to the human capital production profile. Results with open capital markets.

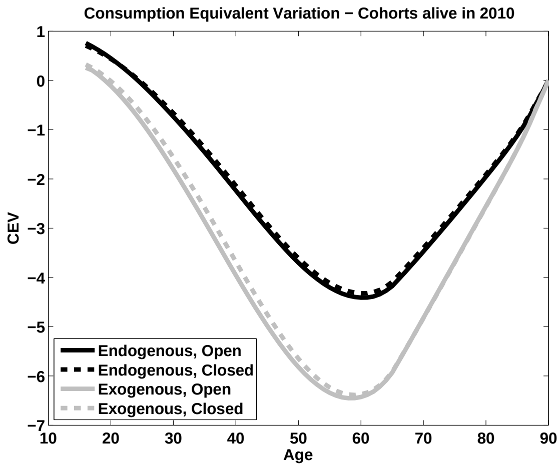
Welfare of Generations Alive in 2010 (Benchmark Model & Pension Reform)

Figure 12: Consumption Equivalent Variation of Agents alive in 2010, Benchmark Pension System

(a) Constant Replacement Rate



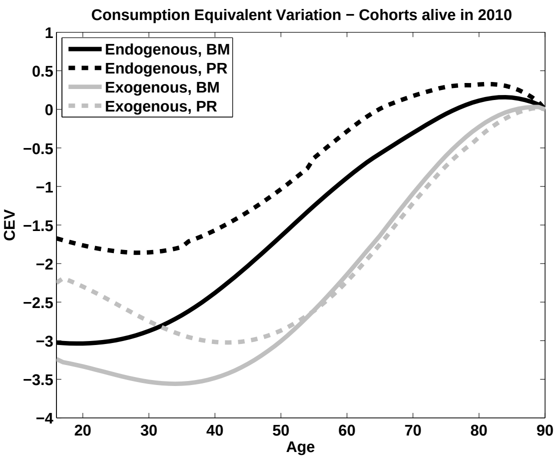
(b) Constant Contribution rate



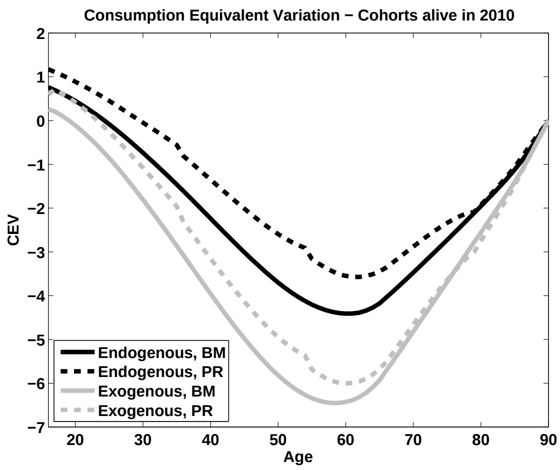
Notes: "Endogenous" and "Exogenous" refer to the human capital production profile. "Open" and "Closed" refer to the results obtained from the closed and open economy versions.

Figure 13: Consumption Equivalent Variation of Agents alive in 2010, Pension Reform

(a) Constant Replacement Rate



(b) Constant Contribution Rate

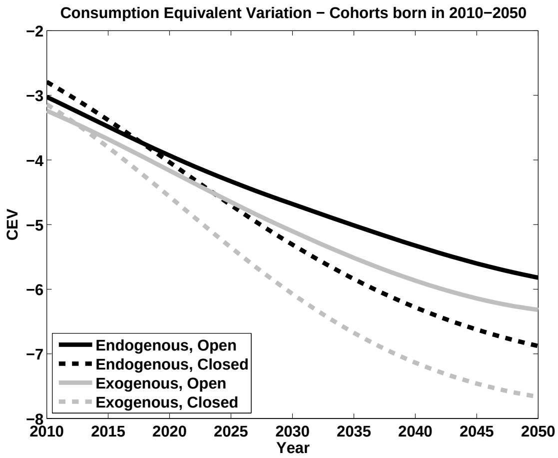


Notes: "Endogenous" and "Exogenous" refer to the human capital production profile. Results with open capital markets.

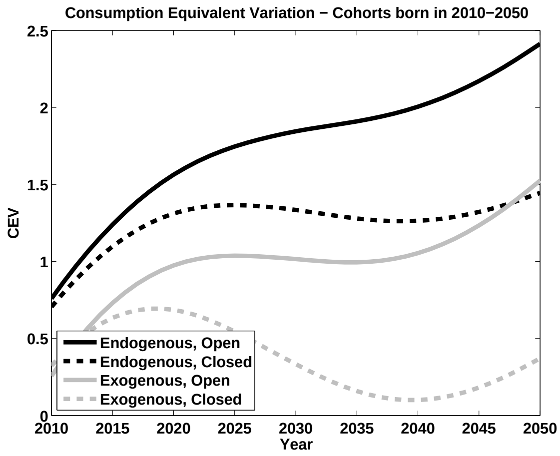
Welfare of Future Generations (Benchmark Model & Pension Reform)

Figure 14: Consumption Equivalent Variation of Agents born 2010-2050, Benchmark Pension System

(a) Constant Replacement Rate



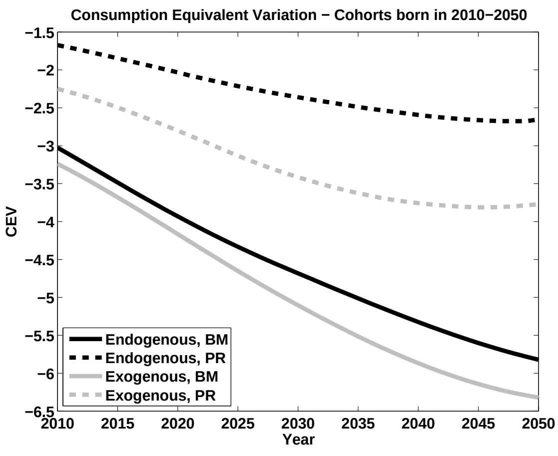
(b) Constant Contribution rate



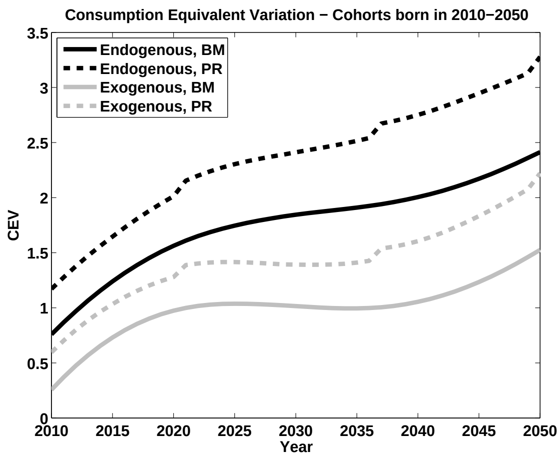
Notes: "Endogenous" and "Exogenous" refer to the human capital production profile. "Open" and "Closed" refer to the results obtained from the closed and open economy versions. "BM" denotes the benchmark pension system (constant retirement age), and "PR" denotes the pension reform (increase in the retirement age).

Figure 15: Consumption Equivalent Variation of Agents born 2010-2050, Pension Reform

(a) Constant Replacement Rate



(b) Constant Contribution Rate



Notes: "Endogenous" and "Exogenous" refer to the human capital production profile. Results with open capital markets.

Welfare in Closed and Open Economy

Table 5: Welfare Gains / Losses - Newborns 2010

	Open Economy			
Pension System	Constant τ		Constant ρ	
	Endog.	Exog.	Endog.	Exog.
BM	0.8%	0.3%	-3.0%	-3.2%
PR	1.2%	0.6%	-1.7%	-2.3%

	Closed Economy			
Pension System	Constant τ		Constant ρ	
	Endog.	Exog.	Endog.	Exog.
BM	0.7%	0.3%	-2.8%	-3.1%
PR	0.9%	0.7%	-1.8%	-2.1%

Notes: "Endog." and "Exog." refer to the endogenous and exogenous human capital production profile. "Open" and "Closed" refer to the results obtained from the closed and open economy versions. "BM" denotes the benchmark pension system (constant retirement age), and "PR" denotes the pension reform (increase in the retirement age).

Table 6: Maximum Welfare Losses - Agents alive 2010

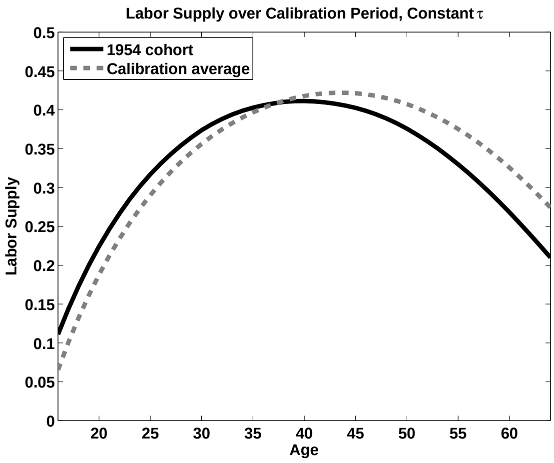
	Open Economy			
Pension System	Constant τ		Constant ρ	
	Endog.	Exog.	Endog.	Exog.
BM	-4.4%	-6.5%	-3.0%	-3.6%
PR	-3.6%	-6.0%	-1.9%	-3.0%

	Closed Economy			
Pension System	Constant τ		Constant ρ	
	Endog.	Exog.	Endog.	Exog.
BM	-4.3%	-6.4%	-2.8%	-3.1%
PR	-3.6%	-5.5%	-1.8%	-2.5%

Notes: "Endog." and "Exog." refer to the endogenous and exogenous human capital production profile. "Open" and "Closed" refer to the results obtained from the closed and open economy versions. "BM" denotes the benchmark pension system (constant retirement age), and "PR" denotes the pension reform (increase in the retirement age).

Life Cycle Labor Supply for Calibration Period

Figure 16: Life Cycle Labor Supply for Calibration Period, Constant τ



Notes: "Calibration average" refers to the unweighted average of the labor supply profiles during the calibration period and "1954 cohort" refers to the life-cycle labor supply of the cohort born in 1954.

B.2 Simple Model: Reallocation of Time over the Life Cycle

We want to understand theoretically the effects on labor supply and human capital at the intensive and extensive margin. We further decompose household's reaction when we keep prices and policy instruments fixed — i.e., wages, interest rates, the contribution rate and pension payments — and when we allow for general equilibrium feedback.

To understand the mechanisms theoretically, consider a simplified two-period version of the model used in the quantitative part. Households maximize utility

$$\begin{aligned} U &= \phi \ln(c_1) + (1-\phi)\ln(1-\ell_1) + \beta\left(\phi \ln(c_2) + (1-\phi)\mathbb{1}\ln(1-\ell_2)\right) \\ &\text{s.t.} \\ c_1 + \frac{c_2}{1+r} &= w_1\ell_1(1-e) + \frac{1}{1+r}(w_2\ell_2 h(e)\mathbb{1} + (1-\mathbb{1})p) \end{aligned}$$

with standard notation. $h(e) \geq 1$ is a strictly concave human capital production function where e is time investment which has to be made in the first period. $\mathbb{1}$ is an indicator function taking on the value of 1 if the agent is working in the last (second) period and 0 if he is retired and receives a pension p. Hence, changing the value of the indicator function from 0 to 1 mimics the pension reform of the quantitative model in a consistent way. Without loss of generality we normalize $w_1 = 1$. Denote first-period labor supply in the benchmark model — where $\mathbb{1} = 0$ — by ℓ_1^{BM} and labor supply with the higher retirement age — where $\mathbb{1} = 1$ — by ℓ_1^{PR}. Then, the difference in labor supply after increasing the retirement age is

$$\ell_1^{PR} - \ell_1^{BM} = \frac{\beta(1-\phi)^2}{(1+\beta)(1+\beta\phi)} - \frac{1-\phi}{R(1+\beta)}\left(w_2\frac{h(e^*)}{1-e^*} - p\frac{1+\beta}{1+\beta\phi}\right) \tag{29}$$

with e^* being the equilibrium investment into human capital. This means that — keeping human capital constant — increasing the retirement age can in general either increase or decrease labor supply in the first period. Labor supply in the first period increases if

$$\frac{\beta(1-\phi)}{(1+\beta\phi)} > \frac{1}{R}\left(w_2\frac{h(e^*)}{1-e^*} - p\frac{1+\beta}{1+\beta\phi}\right)$$

whereby the right-hand-side of this condition can be interpreted as reflecting the (adjusted) difference between human capital wealth — i.e., the discounted value of future income — between the model without retirement — in term $w_2\frac{h(e^*)}{1-e^*}$ — and with retirement — in term $p\frac{1+\beta}{1+\beta\phi}$. If future wages are relatively small, i.e., if $w_2\frac{h(e^*)}{1-e^*} < p\frac{1+\beta}{1+\beta\phi}$, then the discounted value of future income in case of the reform is small such that labor supply in the first period increases. Effects are however ambiguous if future labor income is sufficiently high. An unambiguous finding is that allowing for endogenous human capital increases $\frac{h(e^*)}{1-e^*}$ and therefore decreases labor supply when the retirement age increases.