No. 54

UNIVERSITY OF COLOGNE

WORKING PAPER SERIES IN ECONOMICS

TOO MUCH INFORMATION SHARING? WELFARE EFFECTS OF SHARING ACQUIRED COST INFORMATION IN OLIGOPOLY

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Too Much Information Sharing? Welfare Effects of Sharing Acquired Cost Information in Oligopoly*

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March 2012

Abstract

By using general information structures and precision criteria based on the dispersion of conditional expectations, we study how oligopolists' information acquisition decisions may change the effects of information sharing on the consumer surplus. Sharing information about individual cost parameters gives the following trade-off in Cournot oligopoly. On the one hand, it decreases the expected consumer surplus for a given information precision, as the literature shows. On the other hand, information sharing increases the firms' incentives to acquire information, and the consumer surplus increases in the precision of the firms' information. Interestingly, the latter effect may dominate the former effect.

Keywords: Information acquisition, Information sharing, Information structures, Oligopoly, Consumer surplus.
JEL classification numbers: D82, D83, L13, L40.

*We are grateful for comments by Larry Ausubel, Giacomo Calzolari, Luis Corchon, Christoph Engel, Maria Angeles de Frutos, Matthias Lang, Johannes Münster, Sandro Shelegia, seminar participants at UPF (Barcelona), University of Bologna and University of Cologne, and conference attendees at the JEI (Vigo), the Madrid Summer Workshop on Economic Theory (2009), the ACLE meeting on "Information, Communication and Competition" (Amsterdam), the 11th SFB/TR 15 Conference (Caputh), the EEA Congress (Glasgow), the EARIE conference (Stockholm), and the MaCCI Inaugural Conference (Mannheim). Jansen gratefully acknowledges support of the MPI (Bonn), where part of the research for this paper was done. Naturally, all errors are ours.

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1 Introduction

The role of trade associations in facilitating firms sharing information, has always been an important and controversial topic for economic theorists, practitioners and antitrust authorities. On the practical side, the controversy starts with some contradictory decisions taken by US Courts (e.g., see Vives (1990) for details). Currently, although antitrust authorities do not forbid explicitly the exchange of information (as long as it is not used to facilitate collusion or deter entry), suspicions remain.[1] On the theoretical side, there is a classical literature (see, e.g., Kühn and Vives (1995), Raith (1996), and Vives (1999) for surveys) that provides a taxonomy of regulatory recommendations depending on the type of strategic interaction, and the type of information.

This paper tries to look at this old issue with new methodological techniques and taking the new perspective of information economics, which allows agents to some extent to determine their information structures. In particular, we focus on a simple question that has a clear answer in the previous literature: Should Cournot oligopolists be allowed to share information about their private costs of production? The answer of the classical literature on information sharing in oligopoly seems to be unambiguously negative. In the first place, information sharing among competing firms decreases the consumer surplus when the firms compete in quantities (Shapiro, 1986, Sakai and Yamato, 1989). Moreover, information sharing may facilitate collusion between firms, which also hurts consumers.[2] Hence, a policy maker, who maximizes expected consumer surplus, should prohibit agreements among Cournot oligopolists to share information about their costs. However, we do not observe many regulatory restrictions on information sharing in reality.[3]

However, this conclusion was drawn in settings where firms receive information exogenously. In this paper we show that the policy conclusion may become ambiguous when information is endogenous, i.e., firms invest in acquiring information.[4] This may provide a rationale for the

[1]See Vives (1990), Kühn and Vives (1995), and Kühn (2001) for a discussion on the decisions of antitrust authorities regarding information-sharing policies.

[2]Information sharing may help firms to detect deviations from collusive agreements (Green and Porter (1984)).

[3]We claim that this result should lead to regulatory constraints not only because there are economic sectors as the automobile industry that are typically associated with Cournot competition. But also, since with Bertrand competition it can be shown that competing firms do not have an interest in sharing information.

[4]There are several situations where a firm may not have complete information about its cost of production. For example, a firm's production process may be complex (e.g., in the car industry, and high-tech industries), and firms take their output decisions before all of the production contracts are signed and all input costs are known. The firm's costs may depend on exchange rate fluctuations (e.g., if firms import some of their inputs from different countries), geological or meteorological conditions (e.g., if they extract natural resources or farm), or the fluctuating

observed lenient antitrust policy regarding information sharing. This counter-intuitive result is based on the fact that allowing firms to share information has two effects on consumer surplus. On the one hand, as previous literature pointed out, there is a negative direct effect. For an exogenously given level of information precision, allowing information sharing between firms has a negative effect (positive effect) on the consumer surplus (profits of the firms). However, on the other hand, there is a positive indirect effect of sharing information. This is due to the fact that the incentives of acquiring information are larger when firms are allowed to share information. The higher investments by firms that share information have a positive effect on the consumer surplus. On top of that, we provide an example in which allowing firms to share information increases the consumer surplus, i.e., the positive indirect effect dominates the negative direct effect.

One of the main difficulties in translating the theoretical results of information sharing to regulatory policies is that it is difficult to obtain unambiguous results. We have to acknowledge that, although making information structures endogenous is a necessary step in understanding the welfare consequences of information-sharing policies, it may make this task more complex. However, the second main contribution of this paper is to clarify the driving forces of our and existing results. Basically, we show that most of the results are due to the fact that the objective functions of the firms (i.e., profits) and antitrust authority (i.e., consumer surplus) are convex functions of the firms' outputs. This implies that the dispersion of the firms' outputs has a clear impact on the outcomes. In turn, information sharing and information acquisition strategies determine the quality of information held by firms (i.e., the distribution of posterior beliefs), and indirectly, the dispersion of outputs.

Recently, Ganuza and Penalva (2010) has provided a family of precision criteria for ranking information structures according to the effect that information has on the dispersion of conditional expectations. The basic principle of these precision criteria is that a more accurate information structure leads to a more disperse distribution of the conditional expectation. Applying these informativeness measures allows us to obtain very general results in terms of the information structures under consideration.

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prices of raw materials. Further, a firm may have developed a process innovation, and it may not be clear how far this innovation reduces the firm's cost. In these cases, the production decisions are based on the expected cost. In addition, our model can be reinterpreted as a model in which firms have independently distributed demand functions, and each firm does not know the exact size of of its market.

Besides this conceptual contribution, we attempt to contribute to the literature on information acquisition in oligopoly. Li *et al.* (1987), Hwang (1995), Hauk and Hurkens (2001) study the information acquisition incentives of Cournot oligopolists. These papers assume that firms do not share their acquired information, and make complementary comparisons.[5] By contrast, we focus on the interaction between the incentives to acquire information and to share information. Fried (1984), Kirby (2004), and Jansen (2008) study the effects of this interaction on the expected profits of firms. In contrast to these papers, we focus on welfare effects, and we consider more general information structures.

Persico (2000) studies the interaction between information acquisition and information aggregation in an auction model with affiliated values. For a given information structure the second price auction yields a higher expected revenue to an auctioneer than the first price auction. But the first price auction gives a greater incentive to acquire information, which may reverse the expected revenue ranking. As in our paper, Persico (2000) also considers general information structures but ordered according to an alternative informativeness criterion (i.e., Lehmann (1986)).

In the next section we describe the model. Section 3 defines the concept of Integral Precision for signals. Section 4 briefly describes the equilibrium strategies. Section 5 compares expected consumer surplus levels in equilibrium. Section 6 extends the analysis in some relevant directions. Finally, section 7 concludes the paper. All proofs are relegated to the Appendix. The Supplementary Appendix presents some results related to the model's extensions.

2 The Model

2.1 Preferences and Technology

Consider an industry where two risk-neutral firms (i.e., firms 1 and 2) compete in quantities of differentiated goods.[6] The representative consumer's gross surplus from consuming (x_1, x_2) is:

$$u(x_1, x_2) \equiv \alpha(x_1 + x_2) - \frac{1}{2}(x_1 + x_2)^2 + (1 - \beta)x_1 x_2, \tag{1}$$

[5]For example, Hwang (1995) observes that information acquisition incentives are important for the welfare comparison between perfect competition, oligopoly, and monopoly. Although perfect competition yields the highest expected welfare for any exogenously given precision of information, it may fail to do so when the precision is determined endogenously, since firms in perfectly competitive markets may have a lower incentive to acquire information. Whereas Hwang changes the mode of competition while keeping information sharing constant, we do the opposite.

[6]Section 6 considers an oligopoly with N firms ($N \geq 2$). All duopoly results also hold with more than two firms.

with $0 < \beta \leq 1$. Hence, the inverse demand function for good i is linear in the outputs, i.e., $P_i(x_i, x_j) = \alpha - x_i - \beta x_j$. The demand intercept α is sufficiently high. The parameter β represents the degree of substitutability between goods 1 and 2. For $\beta = 1$ the goods are perfect substitutes, while for $\beta = 0$ the markets for the goods are independent. The consumption of the bundle (x_1, x_2) gives the representative consumer a net surplus of:

$$v(x_1, x_2) \equiv u(x_1, x_2) - \sum_{i=1}^{2} P_i(x_i, x_j)x_i = \frac{1}{2}(x_1 + x_2)^2 - (1 - \beta)x_1 x_2. \tag{2}$$

Firms have constant marginal costs of production. Firm i's profit of producing quantity x_i at marginal cost θ_i is simply: $\pi_i(x_i, x_j; \theta_i) = [P_i(x_i, x_j) - \theta_i]\, x_i$ for $i, j = 1, 2$ and $j \neq i$.[7]

2.2 Firms' Information Structures

Firms' marginal costs are initially unknown. The cost of firm i is distributed according to c.d.f. $F_i : [0, \theta^h] \to [0, 1]$ with mean $\overline{\theta}_i$. Firm i can acquire a costly signal S_i^δ about θ_i, where $S_i^\delta \in \mathcal{S}$ for some set $\mathcal{S}$. Signal S_i^δ is characterized by the family of distributions $\{H_\delta(s|\theta_i)\}_{\theta_i}$. That is, given the marginal cost θ_i, which is a realization of the random variable Θ_i, S_i^δ is represented by the conditional distribution $H_\delta(s|\theta_i) = \Pr(S_i^\delta \leq s|\Theta_i = \theta_i)$. The prior distribution $F_i(\theta)$ and the signal distribution $\{H_\delta(s|\theta_i)\}_{\theta_i}$ define the information structure, i.e., the joint distribution of (Θ_i, S_i^δ). Parameter δ orders the signals in the sense of Integral Precision (see section 3). We denote the cost of acquiring a signal S_i^δ of precision δ by $c(\delta)$, where c is increasing in δ.

We assume that $H_\delta(s|\theta_i)$ admits a density $h_\delta(s|\theta_i)$. The marginal distribution of S_i^δ is denoted by $H_i^\delta(s)$ and satisfies:

$$H_i^\delta(s) = \int_{\{y \in \mathcal{S} | y \leq s\}} \int_0^{\theta^h} h_\delta(y|\theta) dF(\theta) dy.$$

Let $F_i^\delta(\theta_i|s_i^\delta)$ and $E_i[\theta|s_i^\delta]$ denote the posterior distributions and the conditional expectation of Θ_i conditional on $S_i^\delta = s_i^\delta$.

[7] This model is equivalent to a model in which firms have known costs, and consumers have an unknown utility (and demand) for the goods, where $U(x_1, x_2; \theta_1, \theta_2) \equiv (\alpha - \theta_1)x_1 + (\alpha - \theta_2)x_2 - \frac{1}{2}(x_1 + x_2)^2 + (1 - \beta)x_1 x_2$. Hence, all results can be reinterpreted in terms of demand information too.

2.3 *Firms' Information-Sharing Policies*

If the antitrust authority allows information sharing between firms, the firms simultaneously choose their information-sharing policy *vis-à-vis* their competitor before they acquire the signal.[8] As is common in the literature (e.g., Raith (1996)), a firm either shares its information truthfully or it keeps the information secret. We focus on a parametric family of information-sharing policies. Firm i chooses $\rho_i \in [0,1]$, which implies that firm j receives the informative message, $m_i = s_i^\delta$ (the private realization of the signal S_i^δ), with probability ρ_i, and the non-informative message, $m_i = \varnothing$, with the complementary probability, $1 - \rho_i$, for $i = 1, 2$.[9]

2.4 *Timing*

1. Initially, an antitrust authority chooses whether to allow or prohibit information sharing between the firms in the industry. The authority maximizes the expected consumer surplus.
2. In the second stage, firms simultaneously choose their information-sharing policy *vis-à-vis* their competitor, $\rho_i \in [0,1]$, taking into account the decision of the antitrust authority.
3. The marginal costs of firms 1 and 2 are determined by two independent draws from their corresponding distributions F_1 and F_2, respectively.
4. Firms simultaneously choose information acquisition investments: δ_i at a cost of $c(\delta_i)$ for $i = 1, 2$. Firm i's investment δ_i determines the precision of the firm's cost signal S_i^δ.
5. Firms send messages about their signal in accordance with their information-sharing policies

[8] In other words, firms unilaterally choose whether to precommit to information sharing. Alternative assumptions could be to allow the firms to precommit cooperatively to share information (through a *quid pro quo* agreement), or to assume that firms make strategic information sharing choices (i.e., each firm chooses whether to share information after it learns its signal). As it turns out, in equilibrium the information sharing choices are not affected by considering cooperative information-sharing choices instead of non-cooperative choices (see section 4.3). Moreover, if the firms' signals can be ordered, then strategic information-sharing choices tend to coincide with the choices of precommitting firms (e.g., see Okuno-Fujiwara *et al.* (1990)). By contrast, if signals cannot always be ordered, then the equilibrium disclosure choices tend to differ from the choices of precommitting firms (e.g., in Jansen (2008) firms choose a selective disclosure strategy, since an uninformative signal prevents the complete ordering of signals).

[9] We focus on this information-sharing technology to keep the analysis tractable. This assumption is less restrictive since we expect a corner solution in a more general setting, as in the advertising literature, where Johnson and Myatt (2006) show that profits are quasi-convex in the information disclosed for general information structures. In our setting, we will obtain a corner solution in which firms share all their information. As we discuss later, this result is not driven by the linearity of the information structure but by the fact that firms' profits (with some qualification) tend to be monotonically increasing in the information-sharing choice.

in stage 2.

6. In the final stage firms simultaneously choose their output levels, $x_i \geq 0$ for firm i, to maximize the expected value of $\pi_i(x_i, x_j; \theta_i)$, i.e., firms are Cournot competitors.

We solve the game backwards, and restrict the analysis to perfect Bayesian equilibria. Before solving the model, we want to discuss how the choice of information acquisition investment δ_i determines the information structure.

3 Information Criteria: Integral Precision

In this paper we assume that the variable δ_i ranks signals according to Integral Precision. Precision criteria (introduced by Ganuza and Penalva, 2010) are based on the principle that an information structure, i.e., the joint distribution of the state of the world and the signal, is more informative (*more precise*) than another if it generates more dispersed conditional expectations. This dispersion effect arises because the sensitivity of conditional expectations to the realized value of the signal depends on the informational content of the signal. If the informational content of the signal is low, conditional expectations are concentrated around the expected value of the prior. By contrast, if the informational content is high, conditional expectations depend to a large extent on the realization of the signal which increases their variability.

In our context, given the prior distribution $F_i(\theta)$, we assume that if $\delta_i > \delta_i'$ then $E_i[\theta|S_i^{\delta}]$ is "more spread out" than $E_i[\theta|S_i^{\delta'}]$. In the present paper, we use the *Integral Precision* criterion, which combines this approach with the convex order (Ganuza and Penalva, 2010):

Definition 1 (Convex Order) *Let Y and Z be two real-valued random variables with distribution F and G respectively. Then Y is greater than Z in the* convex order *($Y \geq_{cx} Z$) if for all convex real-valued functions ϕ, $E[\phi(Y)] \geq E[\phi(Z)]$ provided the expectation exists.*

Using the convex order, Ganuza and Penalva define Integral Precision to order signals in terms of their informativeness:

Definition 2 (Integral Precision) *Given a prior $F_i(\theta)$ and two signals S_1 and S_2, signal S_1 is more* integral precise *than S_2 if $E_i[\theta|S_1]$ is greater than $E_i[\theta|S_2]$ in the convex order.*

Ganuza and Penalva (2010) show that *Integral Precision* is weaker than (is implied by) all common informativeness orders based on the value of information for a decision maker (Blackwell, 1951, Lehmann, 1988, and Athey and Levin, 2001). In other words, if S_1 is more valuable for a decision maker than S_2, then S_1 is more integral precise than S_2. The following information models are consistent with *Integral Precision*.

Normal Experiments: Let $F_i(\theta) \sim \mathcal{N}(\mu, \sigma_v^2)$ and $S_i^\delta = \theta_i + \epsilon_\delta$, where $\epsilon_\delta \sim \mathcal{N}(0, \sigma_\delta^2)$ and is independent of θ_i. The variance of the noise, σ_δ^2, orders signals in the usual way: we assume that $\delta > \delta' \iff \sigma_\delta^2 < \sigma_{\delta'}^2$ and the signal with a noise term that has lower variance is more informative in terms of Integral Precision.

Linear Experiments: Let the signal be perfectly informative, $S_i^\delta = \theta_i$, with probability δ, and the signal is pure noise, $S_i^\delta = \epsilon$ where $\epsilon \sim F_i(\theta)$ and is independent of θ_i, with probability $1-\delta$. Let S_i^δ and $S_i^{\delta'}$ be two such signals. If $\delta > \delta'$, i.e. S_i^δ reveals the truth with a higher probability than $S_i^{\delta'}$, then S_i^δ is more informative than $S_i^{\delta'}$ in terms of Integral Precision.

Binary Experiments: Let θ_i be equal to θ^h with probability q and θ^l with probability $1-q$. The signal, S_i^δ, can take two values h or l, where $\Pr[S_i^\delta = k|\theta_i = \theta^k] = \frac{1}{2}(1+\delta_i)$ for $i \in \{1,2\}$ and $k \in \{l,h\}$, where $0 \leq \delta_i \leq 1$. The parameter δ_i orders signals in the usual way: higher δ implies greater Integral Precision.

Uniform Experiments: Let $F_i(\theta)$ be the uniform distribution on $[0,1]$ and let $H_\delta(s|\theta_i)$ be uniform on $\left[\theta_i - \frac{1}{2\delta}, \theta_i + \frac{1}{2\delta}\right]$. For any δ, δ' with $\delta > \delta'$, S_i^δ is more informative than $S_i^{\delta'}$ in terms of Integral Precision.

Partitions: Let $F_i(\theta)$ have support equal to $[0,1]$. Consider two signals generated by two partitions of $[0,1]$, $\mathcal{A}$ and $\mathcal{B}$, where $\mathcal{B}$ is finer than $\mathcal{A}$.[10] Using these partitions, one can define signals S_i^δ and $S_i^{\delta'}$ in the usual way: signal S_i^δ [$S_i^{\delta'}$] tells you which set in the partition $\mathcal{A}$ [$\mathcal{B}$] contains θ_i.[11] If a larger δ means a finer partition, δ orders signals according to Integral Precision.

[10] A partition, $\mathcal{A}$, divides $[0,1]$ into disjoint subsets, $\mathcal{A} = \{A_1, .., A_k\}$, i.e., $\cup_{j=1}^k A_j = [0,1]$ and $A_i \cap A_j = \emptyset$ for all $i, j = 1, .., k$ with $i \neq j$. Partition $\mathcal{B}$ is finer than $\mathcal{A}$, when for all $B \in \mathcal{B}$, there exists $A \in \mathcal{A}$ such that $B \subseteq A$.

[11] However, observing A_j [B_j] does not allow you to distinguish between different states of the world within that set.

4 Solving the Model: Equilibrium Strategies

We solve the perfect Bayesian equilibrium by backwards induction. First, we characterize the equilibrium output levels $(x_1^*(s_1; m_1, m_2), x_2^*(s_2; m_2, m_1))$ for any realization of signals and messages, (s_1, s_2, m_1, m_2). Second, we analyze the information acquisition strategies of firms, $(\delta_1^*(\rho_1, \rho_2), \delta_2^*(\rho_2, \rho_1))$ for any firms' information-sharing choices, (ρ_1, ρ_2). Finally, we determine the equilibrium information-sharing strategies, (ρ_1^*, ρ_2^*).

4.1 Output Levels

Each firm chooses its output level on the basis of its own information, s_i, and the information received from its competitor, $m_j \in \{s_j, \varnothing\}$. In order to save notation we do not make explicit the dependence of s_i on δ_i. The expected cost given the uninformative message $m_j = \varnothing$ is $E\{\theta_j|\varnothing\} = \overline{\theta}_j$.

For any combination of messages m_i and m_j, firm i with signal s_i maximizes its expected profit, which yields the following first-order condition:

$$x_i(s_i; m_i, m_j) = \frac{1}{2}\Big(\alpha - E\{\theta_i|s_i\} - \beta E\{x_j(s_j; m_j, m_i)|m_i, m_j\}\Big) \tag{3}$$

for $i, j = 1, 2$ with $i \neq j$. Solving the system of equations (3) for $i = 1, 2$ gives the following equilibrium output levels.[12]

Proposition 1 *For any feasible combination of signals (s_1, s_2) and messages (m_1, m_2), the product market stage has a unique equilibrium $(x_1^*(s_1; m_1, m_2),\ x_2^*(s_2; m_2, m_1))$, where:*

$$x_i^*(s_i; m_i, m_j) = \frac{1}{4-\beta^2}\left((2-\beta)\alpha - 2E\{\theta_i|s_i\} + \beta E\{\theta_j|m_j\} + \frac{\beta^2}{2}\left[E\{\theta_i|s_i\} - E\{\theta_i|m_i\}\right]\right) \tag{4}$$

for $i, j = 1, 2$ with $i \neq j$, with $E\{\theta_i|m_i\} = E_{s_i}\{E(\theta_i|s_i)|m_i\}$.

First, notice that the last term is the distortion due to the asymmetric information between firms. If $m_i = s_i$, there is no distortion. If $m_i = \varnothing$ and s_i gives bad news (high θ_i), the term is positive since the firm j is reacting to the average cost, producing less than it would have produced with perfect information. Conversely, concealed good news gives a negative distortion.

[12] The proof of this proposition is standard (e.g., see Vives (1999)), and therefore it is ommitted.

Second, notice that the expected equilibrium output level is independent of the information-acquisition and information-sharing variables:[13]

$$\overline{x_i}^* \equiv E_{s_i,m_i}\left\{E_{s_j,m_j}[x_i^*(s_i;m_i,m_j)]\right\} = \frac{1}{4-\beta^2}\left((2-\beta)\alpha - 2\overline{\theta_i} + \beta\overline{\theta_j}\right) \tag{5}$$

where

$$E_{s_i,m_i}\left\{E_{s_j,m_j}[x_i^*(s_i;m_i,m_j)]\right\} = \rho_i E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i;s_i,m_j)]\right\} + (1-\rho_i)E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i;\varnothing,m_j)]\right\}$$

and $E_{s_j,m_j}[.]$ is defined likewise. Hence, information acquisition and sharing have no effect on the average output level. They only have an effect on the output dispersion.

The expected equilibrium product market profit of firm i with signal s_i, and messages m_i and m_j equals: $\pi_i^*(s_i;m_i,m_j) = x_i^*(s_i;m_i,m_j)^2$. Hence, the firm's expected profit equals:

$$\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j) \equiv E_{s_i,m_i}\left\{E_{s_j,m_j}[x_i^*(s_i;m_i,m_j)^2]\right\} - c(\delta_i) \tag{6}$$

Notice that the profit function of firm i is convex in its own output. This feature of the objective function is important for our future results. It implies that firms prefer more dispersed individual outputs. As we show below, the information-sharing policies as well as the information-acquisition strategies affect the dispersion of the outputs.

4.2 Information Acquisition

In this subsection, we study the effects of information-acquisition investments on firms. First, we analyze the effects of information-acquisition investments on the expected profit.

PROPOSITION 2 *For $i,j = 1,2$ with $i \neq j$, firm i's expected equilibrium profit from the product market, $E_{s_i,m_i}\left\{E_{s_j,m_j}[x_i^*(s_i;m_i,m_j)^2]\right\}$, is: (i) increasing in the own information acquisition investment δ_i, and (ii) weakly increasing in the competitor's investment δ_j.*

Proposition 2(i) confirms that a firm generates a positive revenue by acquiring information. The firm trades off this marginal revenue from investment (i.e., $\partial E_{s_i,m_i}\left\{E_{s_j,m_j}[x_i^*(s_i;m_i,m_j)^2]\right\}/\partial\delta_i$) against the marginal cost of investment $c'(\delta_i)$. In Proposition 2(ii) we show that also the information-acquisition investment of the competitor increases a firm's expected profit.

[13] This result is due to the linearity of the demand and cost functions.

Second, we characterize the equilibrium in the information-acquisition stage by analyzing the relationship between the information-acquisition investments and the information-sharing policy.

PROPOSITION 3 *For any* (ρ_1, ρ_2), *there exists an equilibrium in the information-acquisition stage* $(\delta_1^*(\rho_1, \rho_2), \delta_2^*(\rho_2, \rho_1))$. *For* $i, j = 1, 2$ *with* $i \neq j$, *firm* i*'s equilibrium information-acquisition investment,* $\delta_i^*(\rho_i, \rho_j)$, *is: (i) independent of the competitor's information-sharing choice,* ρ_j, *and (ii) increasing in the own choice,* ρ_i.

Proposition 3(i) follows from the fact that the marginal profit from information acquisition (i.e., $\partial\Pi_i/\partial\delta_i$) is independent of ρ_j. This is due to two reasons. First, ρ_j has no impact on the conditional distribution of θ_i, given the independence of the firms' costs and signals. Moreover, in our setting, what firm i learns from j only affects the intercept of firm i's residual demand and the adjustment of equilibrium output, $x_i^*(s_i; m_i, m_j) - x_i^*(s_i; m_i, \varnothing) = \frac{\beta}{4-\beta^2}\left(E\{\theta_j|m_j\} - E\{\theta_j|\varnothing\}\right)$, is therefore independent of δ_i.

Proposition 3(ii) follows from the fact that the expected profit $\Pi_i(\delta_i, \delta_j; \rho_i, \rho_j)$ is supermodular in (δ_i, ρ_i).[14] This implies that information-sharing firms have a greater incentive to acquire information than concealing firms. Regarding (ii), firstly notice that we have not made any explicit assumption to warrant uniqueness and therefore $\delta_i^*(\rho_i, \rho_j)$ may not be a singleton. In this case, our monotonicity result applies to the extremal equilibrium investments. For further results of the paper, if $\delta_i^*(\rho_i, \rho_j)$ is not unique, we require to focus on a specific selection of $\delta_i^*(\rho_i, \rho_j)$. Given the positive effect of $\delta_i^*(\rho_i, \rho_j)$ on welfare, we assume that firm i chooses the maximum of the set of equilibrium investments.[15] In such a case, the proposition states that the selected equilibrium investment (the maximum of the set of equilibrium investments) is monotonically increasing.[16]

The next sections analyze incentives of firms to share information, and give intuition for Propositions 2 and 3(ii).

[14]Formally, for $\rho_i > \rho_i'$, the marginal profit $\Pi_i(\delta_i, \delta_j; \rho_i, \rho_j) - \Pi_i(\delta_i, \delta_j; \rho_i', \rho_j)$ is weakly increasing in δ_i for all δ_j and ρ_j.

[15]In other words, we select the Pareto-dominant equilibrium, since firm i is indifferent between its optimal investments, firm i's competitor is best off with the highest investment δ_i^* (Proposition 2(ii)), and also the consumers are best off with the highest investment of firm i as we show later (Proposition 5).

[16]In the proof we use a result of Milgrom and Shanon (1994) that states that supermodularity leads to the monotonicity of the set of maximizers according to the the Veinott's strong set order. Let $\rho_i > \rho_i'$, then $\delta_i^*(\rho_i, \rho_j) \geq \delta_i^*(\rho_i', \rho_j)$ in the following sense, for all $\delta \in \delta_i^*(\rho_i, \rho_j)$ and all $\delta' \in \delta_i^*(\rho_i', \rho_j)$, $\max\{\delta, \delta'\} \in \delta_i^*(\rho_i, \rho_j)$ and $\min\{\delta, \delta'\} \in \delta_i^*(\rho_i', \rho_j)$. This implies that the maximum of the set of optimal solutions is monotonically increasing.

For a given precision, information sharing is a dominant strategy for a firm (Gal-Or (1986), Shapiro (1986)). We confirm that sharing information is also a dominant strategy in our model.[17]

PROPOSITION 4 *For $i, j = 1, 2$ with $i \neq j$, (i) the expected profit $\Pi_i(\delta_i, \delta_j; \rho_i, \rho_j)$ is increasing in ρ_i and ρ_j for all (δ_i, δ_j), (ii) Firm i's unique equilibrium information-sharing strategy is to share all its information, $\rho_i^* = 1$.*

Proposition 4(i) holds for an exogenously given precision. The proof of Proposition 4(ii) requires the evaluation of the overall effect of information sharing.[18] For the overall effect, we recognize that information sharing also affects the choice of a firm's information precision. The effect of information sharing on a firm's own profit can be decomposed as follows:

$$\frac{d\Pi_i(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{d\rho_i} = \frac{\partial\Pi_i(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{\partial\delta_i} \cdot \frac{\partial\delta_i^*}{\partial\rho_i} + \frac{\partial\Pi_i(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{\partial\delta_j} \cdot \frac{\partial\delta_j^*}{\partial\rho_i} + \frac{\partial\Pi_i(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{\partial\rho_i} > 0. \quad (7)$$

The first term of (7) is zero, since the firm chooses its information-acquisition investment optimally (i.e., $\partial\Pi_i/\partial\delta_i = 0$). Also the second term of (7) is zero. This follows from Proposition 3(i), which shows that the competitor's equilibrium information-acquisition investment is independent of the firm's information-sharing choice (i.e., $\partial\delta_j^*/\partial\rho_i = 0$). Finally, Proposition 4(i) shows that the last term of (7) is positive. Hence, information sharing is also a dominant strategy in our model.[19]

The propositions have another implication. We can decompose the effects of information sharing on the profit of a firm's competitor as follows:

$$\frac{d\Pi_i(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{d\rho_j} = \frac{\partial\Pi_i(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{\partial\delta_i} \cdot \frac{\partial\delta_i^*}{\partial\rho_j} + \frac{\partial\Pi_i(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{\partial\delta_j} \cdot \frac{\partial\delta_j^*}{\partial\rho_j} + \frac{\partial\Pi_i(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{\partial\rho_j} > 0. \quad (8)$$

As before, Proposition 3(i) implies that the first term of (8) is zero (i.e., $\partial\delta_i^*/\partial\rho_j = 0$). The second term of (8) captures an indirect effect of information sharing. This effect is non-negative

[17] Gal-Or (1986) and Shapiro (1986) show this result, as we do in the Proposition 4(i), for some particular information-sharing technology. In the conclusion we will discuss the robustness of this result by considering arbitrary information structures.

[18] Notice that in the information-acquision stage there could exist multiple equilibria. Proposition 4(ii) holds for all equilibria since sharing information is a dominant strategy for the firms.

[19] Proposition 4(ii) implicitly assumes that firms' information sharing choices are not restricted, $\rho_i \in [0, 1]$. If the antitrust authority restricts information sharing, e.g., constraining the choices of the firms to the interval $[0, \overline{\rho}]$, then each firm chooses the maximal probability, $\rho_i^* = \overline{\rho}$.

for the following reasons. First, a firm's profit is weakly increasing in the competitor's information-acquisition investment (i.e., $\partial \Pi_i / \partial \delta_j \geq 0$), as Proposition 2(ii) shows. Second, Proposition 3(ii) shows that the competitor's investment is increasing in a firm's information-sharing probability (i.e., $\partial \delta_j^* / \partial \rho_j > 0$). Finally, the third term of (8) is positive due to Proposition 4(i). Hence, the overall effect of information sharing on a competitor's expected profit is positive. This observation implies that firms would also have an incentive to share information cooperatively (e.g., by entering a *quid pro quo* agreement), since information sharing increases the industry profits.

In short, firms will share information if they are allowed to do so. In the remainder of this section we illustrate the intuition of Propositions 2-4 by means of a simple example.

4.4 Binary Example

Consider a simple version of our model in which two risk-neutral firms compete in quantities of a homogenous good ($\beta = 1$). There is uncertainty only regarding firm 1's cost. Nature draws θ_1 from the set $\{\theta^l, \theta^h\}$ with equal probability and sends a private signal to firm 1:

$$S^\delta = \begin{cases} \theta_1 & \text{with probability } \delta_1 \\ \varnothing & \text{with probability } 1 - \delta_1. \end{cases}$$

Firm 2's cost θ_2 is common knowledge.[20] Information-sharing policies and information-acquisition strategies are binary, i.e. $\delta_1, \rho_1 \in \{0, 1\}$.

Illustration of the Profit Results. In our binary example, we have to consider three regimes. First, there is the information-sharing regime (s), in which firm 1 learns perfectly its cost and shares this information with its competitor (i.e., $\delta_1 = 1$, $\rho_1 = 1$). This allows both firms to adjust their outputs to the true productivity of firm 1. Fig. 1(a) illustrates this. In particular, if the firms learn that $\theta_1 = \theta^h$, then firm 1's best response is $r_1(x_2; \theta^h)$. Firm 2's best response is the bold curve $r_2(x_1; \theta_2)$. The equilibrium corresponds to point A. Similarly, if the firms learn that $\theta_1 = \theta^l$, then firm 1 expands its output by adopting best response $r_1(x_2; \theta^l)$ while firm 2 reduces its output, and they reach equilibrium point B. The output adjustments of the firms create output dispersion $\Delta x_1^s \equiv x_1^s(\theta^l) - x_1^s(\theta^h)$ for firm 1, and dispersion $\Delta x_2^s \equiv x_2^s(\theta^h) - x_2^s(\theta^l)$ for firm 2.

[20] In Appendix B.5, we extend the binary example to the case in which there is uncertainty about the cost of both firms, and both firms can acquire and share information.

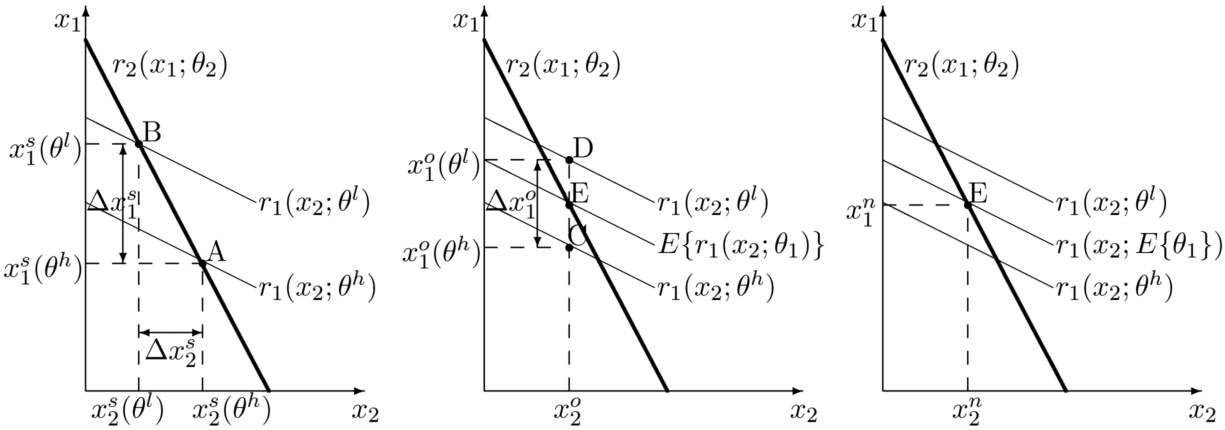

Fig. 1(a): regime (s) Fig. 1(b): regime (o) Fig. 1(c): regime (n)

Figure 1: Equilibrium output levels

Second, in the information-concealment regime (o) firm 1 learns perfectly its cost and keeps this information secret from its competitor (i.e., $\delta_1 = 1$, $\rho_1 = 0$). Then, firm 1 adjusts its output level to its productivity while firm 2 can only base its output decision on the average productivity of 1. That is, firm 2 plays a best response against the expected best response of firm 1, $E\{r_1(x_2; \theta_1)\}$. This gives equilibrium output x_2^o for firm 2, which corresponds to point E in Fig. 1(b). In turn, firm 1 plays a best response against x_2^o, which is $x_1^o(\theta^h)$ if $\theta_1 = \theta^h$ (corresponding to point C), and $x_1^o(\theta^l)$ if $\theta_1 = \theta^l$ (i.e., point D). Fig. 1(b) shows that the dispersion of firm 1's output in regime (o) is smaller than in regime (s), i.e., $\Delta x_1^o \equiv x_1^o(\theta^l) - x_1^o(\theta^h) < \Delta x_1^s$. The greater dispersion in regime (s) is due to the fact that the output adjustments of firm 2 augment the adjustments of firm 1 towards its information. The distortion of equilibrium output (4) also captures this.

Finally, in the no-information regime (n) firm 1 does not learn its cost and there is no information to transmit ($\delta_1 = 0$). Uninformed firms base their output choices on the average technology of firm 1. This gives firm 1 the best response $r_1(x_2; E\{\theta_1\})$, and yields the equilibrium in point E in Fig. 1(c). In this case, there is a single output level for firm 1 (i.e., $\Delta x_1^n = 0$).

The profit function of a firm (6) is convex in the firm's output level. Hence, firms 1 and 2 prefer regime (s) to regime (o) since the dispersions of their outputs are larger in the former regime (Proposition 4), i.e., $\Delta x_1^s > \Delta x_1^o$ and $\Delta x_2^s > 0 = \Delta x_2^o$. For the same token, Propositions 2 and 3(ii) are captured in this example, by comparing the increase in firm 1's profits from the no-information regime to either the regime (s) or regime (o). These increases in profits are also related to the increase in output dispersion. Clearly, information acquisition gives more dispersed outputs for the firms (Proposition 2), i.e., $\Delta x_1^r > 0 = \Delta x_1^n$ and $\Delta x_2^r \geq 0 = \Delta x_2^n$ for $r \in \{s, o\}$. Moreover, firm 1's profit increases more (and consequently the firm has a bigger incentive to invest in acquiring information) when it moves from regime (n) to regime (s) than when it moves from regime (n) to regime (o), i.e., $\Delta x_1^s - \Delta x_1^n > \Delta x_1^o - \Delta x_1^n$, as Proposition 3(ii) shows in general.

5 Expected Consumer Surplus

Using the definition of the surplus v in (2) for a bundle of outputs (x_1, x_2), we denote the expected consumer surplus for exogenously given information-acquisition levels as follows:

$$\begin{aligned} V(\delta_i, \delta_j; \rho_i, \rho_j) &\equiv E_{s_i,m_i}\left\{E_{s_j,m_j}\left[v\left(x_1^*(s_1; m_1, m_2), x_2^*(s_2; m_2, m_1)\right)\right]\right\} \\ &= \frac{1}{2}E_{s_i,m_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i; m_i, m_j) + x_j^*(s_j; m_j, m_i)\right)^2\right]\right\} \\ &\quad -(1-\beta)E_{s_i,m_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i; m_i, m_j)x_j^*(s_j; m_j, m_i)\right]\right\} \qquad (9) \end{aligned}$$

Below we analyze the effects of information sharing and acquisition on this expected surplus.

5.1 Consumer Surplus Properties

The next proposition establishes a basic property of the consumer surplus in our framework.

Proposition 5 *Surplus $V(\delta_i, \delta_j; \rho_i, \rho_j)$ is decreasing in ρ_k and increasing in δ_k for any $k \in \{i, j\}$.*

These surplus results are consequences of the quantity-adjustment effect, and the preference-for-variety effect (e.g., Kühn and Vives (1995)). Below we explain and illustrate the results in greater detail by means of the binary example.

Illustration of the Surplus Results. We return to the binary example of section 4.4. In our example, where goods are homogenous, only the first term of (9) matters. As a consequence, the consumer surplus is increasing in the dispersion of the total industry output since it is a convex function of $x_1^* + x_2^*$. At industry output X', the consumer surplus is simply the area under the demand curve, $P(X) \equiv \alpha - X$, between the price $P(X')$ and the intercept.

Figure 2 illustrates Proposition 5 by comparing the surpluses in regimes (s), (o) and (n). For regime $r \in \{s, o\}$ and cost state $k \in \{l, h\}$, we denote the industry output as $X_k^r \equiv x_1^r(\theta^k) + x_2^r(\theta^k)$, and its dispersion as $\Delta X^r \equiv X_l^r - X_h^r$. For regime (n), the industry output is $X^n \equiv x_1^n + x_2^n$.

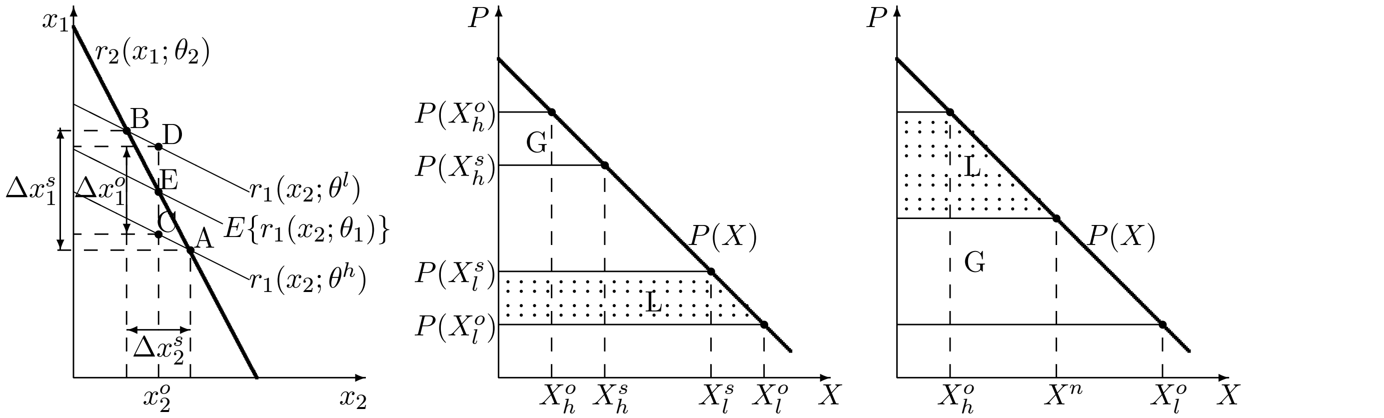

Fig. 2(a): Output dispersion   Fig. 2(b): Regime (s) vs. (o)   Fig. 2(c): Regime (o) vs. (n)

Figure 2: Effects of information sharing and acquisition on consumer surplus

For the first part of Proposition 5, we compare regime (o) with (s). Recall that in those regimes firm 1 learns its cost perfectly (i.e., $\delta_1 = 1$). Figure 2(a) combines Fig. 1(a)-(b), and it illustrates that the dispersion in industry output is lower in regime (s) than in regime (o), since $\Delta X^s = \Delta x_1^s - \Delta x_2^s < \Delta x_1^o = \Delta X^o$. In words, output adjustment by firm 2 in the information-sharing regime, countervails firm 1's adjustment, which creates lower variability of industry outputs than in the concealing regime. Fig. 2(b) illustrates how a lower dispersion of output in the regime (s) leads to a lower consumer surplus than in regime (o). The areas L and G represent respectively the loss (when $\theta_1 = \theta^l$) and the gain (when $\theta_1 = \theta^h$) between regime (s) and (o). The costs θ^l and θ^h are equally likely, and therefore the loss L and gain G receive equal weights from consumers. Hence, on average consumers are worse off in regime (s), since area L is larger than area G.

We illustrate the second part of Proposition 5 by comparing regimes (o) and (n). The industry output is dispersed in regime (o), whereas there is no dispersion in the no information regime (n). In Figure 2(c), the area G (L) illustrates the surplus gain (loss) that results from the relatively higher (lower) industry output in regime (o) for the cost $\theta_1 = \theta^l$ (resp., $\theta_1 = \theta^h$). As G is larger than L, and the two costs are equally likely, regime (o) yields a greater expected surplus than regime (n). The surplus comparison of the regimes (s) and (n) is analogous. These effects, related to the first term of (9), capture the quantity-adjustment effect (Kühn and Vives (1995)).

Notice that the second term of (9) is related to covariance of the firms' output levels and conflicts with the effects illustrated in the example and the statement of Proposition 5. Sharing information and acquiring information both reduce the covariance of firms output which increases consumer surplus. This is called the preference-for-variety effect (Kühn and Vives (1995)). The proof of Proposition 5 shows that the quantity-adjustment effect due to the first term of (9) dominates this second effect.

5.2 Consumer Surplus Trade-off

The effect of information sharing on the consumer surplus can be decomposed as follows:

$$\frac{dV(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{d\rho_i} = \frac{\partial V(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{\partial \delta_i} \cdot \frac{\partial \delta_i^*}{\partial \rho_i} + \frac{\partial V(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{\partial \delta_j} \cdot \frac{\partial \delta_j^*}{\partial \rho_i} + \frac{\partial V(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{\partial \rho_i}. \quad (10)$$

This decomposition yields an interesting trade-off. On the one hand, information sharing has a negative direct effect on the consumer surplus, as we show in Proposition 5. The last term of (10) captures this effect (i.e., $\partial V/\partial \rho_i < 0$). Therefore, if the precision were exogenously given, then sharing information should be prohibited. On the other hand, information sharing has a positive indirect effect on the consumer surplus. It increases the incentives to invest in information acquisition (i.e., $\partial \delta_i^*/\partial \rho_i > 0$ as Proposition 3(ii) shows). Higher investments increase the expected consumer surplus (i.e., $\partial V/\partial \delta_i > 0$ by Proposition 5). The first term of (10) captures this positive, indirect effect. The second term of (10) is zero, since $\partial \delta_j^*/\partial \rho_i = 0$ by Proposition 3(i).

Hence, when the signal's precision is not exogenous, but determined endogenously by information acquisition investments, the antitrust authority's choice (between allowing and disallowing information sharing) should depend on the trade-off between these two conflicting effects. In fact, it is possible that the second effect outweighs the first effect, as we illustrate below.

Illustration of the Trade-off: Information Sharing May Increase Consumer Surplus. In the example of section 4.4, the expected consumer surpluses under information sharing ($\rho_1 = 1$) and concealment ($\rho_1 = 0$) are, respectively:

$$\begin{aligned} V(\delta_1; 1) &= \frac{1}{2}\left(\delta_1 E_{\theta_1}\left\{[x_1^*(\theta_1; \theta_1, \theta_2) + x_2^*(\theta_2; \theta_2, \theta_1)]^2\right\} + (1-\delta_1)\,[x_1^*(\varnothing; \varnothing, \theta_2) + x_1^*(\theta_2; \theta_2, \varnothing)]^2\right) \\ V(\delta_1; 0) &= \frac{1}{2}\left(\delta_1 E_{\theta_1}\left\{[x_1^*(\theta_1; \varnothing, \theta_2) + x_2^*(\theta_2; \theta_2, \varnothing)]^2\right\} + (1-\delta_1)\,[x_1^*(\varnothing; \varnothing, \theta_2) + x_1^*(\theta_2; \theta_2, \varnothing)]^2\right) \end{aligned}$$

We illustrate these surpluses by means of Figure 3. The figure illustrates that information sharing decreases the surplus (i.e., $V(\delta_1; 1) \leq V(\delta_1; 0)$ for any δ_1), and information acquisition increases the surplus (i.e., $V(1; \rho_1) > V(0; \rho_1)$ for any ρ_1), as Proposition 5 shows in general.

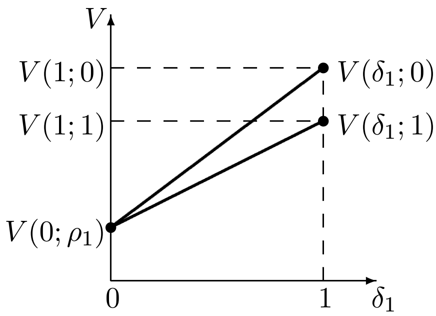

Figure 3: Trade-off for consumers

We can use Figure 3 also to illustrate the trade-off between the direct and indirect effects from information sharing. Whereas the direct effect of information sharing generates a lower surplus for given investments in information acquisition than concealment (i.e., $V(\delta_1;1) \leq V(\delta_1;0)$ as illustrated for $\delta_1 = 1$), the indirect effect favors information sharing. In our example, the indirect effect dominates under the following condition.

PROPOSITION 6 *Consider the binary example of section 4.4 with the cost of acquiring information* $c(0) = 0$ *and* $c(1) = \lambda$. *Then* $V(\delta_1^*(1);1) > V(\delta_1^*(0);0)$, *if* $\frac{(\theta^h-\theta^l)^2}{16} < \lambda < \frac{(\theta^h-\theta^l)^2}{9}$.

When λ satisfies the condition of Proposition 6, firm 1 acquires information only if it is allowed to share information, i.e., $\delta_1^*(1) = 1 > 0 = \delta_1^*(0)$. This favors information sharing, since the more information firm 1 acquires, the larger the surplus (i.e., $V(1;1) > V(0;1)$). The indirect effect dominates, since $V(1;1) > V(0;1) = V(0;0)$. Hence, for intermediate costs of information acquisition, the antitrust authority prefers to allow information sharing between competing firms.

For the remaining parameter values in the binary example, the surplus-maximizing information-sharing choice is as follows:

$$\overline{\rho_1} = \begin{cases} 0, & \text{if } 0 \leq \lambda < \frac{(\theta^h-\theta^l)^2}{16}, \\ 1, & \text{otherwise.} \end{cases} \tag{11}$$

In the binary example, the optimal probability of sharing information is increasing in the cost of information acquisition, λ.[21] In particular, if λ is lower than $\frac{(\theta^h-\theta^l)^2}{16}$, firm 1 always acquires information, and then the direct effect implies that $V(1;1) < V(1;0)$. In other words, for low costs of information acquisition, the indirect effect of information sharing is absent, and the antitrust authority prefers to prohibit information sharing. If λ is larger than $\frac{(\theta^h-\theta^l)^2}{9}$, the firm acquires

[21] In Appendix B, we show that the antitrust authority's optimal policy in the binary example does not change if we allow both firms to acquire and share information, if goods are differentiated, and if there are more than two firms in the industry. That is, the optimal policy in this example does not depend the degree of product differentiation and the number of firms. It only depends on the cost of information acquisition.

information neither with information sharing nor without it, and then $V(0;1) = V(0;0)$, i.e., the authority is indifferent and she may as well allow firm 1 to share its information.

6 Extensions

We extend our analysis in four directions. First, we apply our framework to an issue of data security. Second, we allow for more than two firms. Third, we analyze competition in prices (Bertrand competition) instead of outputs. Finally, we discuss the effects of introducing correlation between the firms' costs.

6.1 Protection of Proprietary Information

Our model considers firms that send public messages about their unit costs. Here we analyze a related model in which firms choose the intensity with which they protect their proprietary cost information. That is, firm i chooses the probability ρ_i with which information about its signal leaks out to the competitor for $i = 1, 2$. With probability $1 - \rho_i$, the firm's signal remains private.

For $0 < \rho_i < 1$, firm i does not know whether its competitor is informed about the firm's signal, s_i, since the firm does not observe whether the information leaked out or not. Hence, if firm i receives signal s_i and message $m_j \in \{s_j, \varnothing\}$ from its competitor, then its first-order condition from profit-maximization becomes (for $i, j = 1, 2$ and $i \neq j$):

$$x_i(s_i, m_j) = \frac{1}{2}\left(\alpha - E\{\theta_i|s_i\} - \beta\left[\rho_i E_{s_j}\{x_j(s_j, s_i)|m_j\} + (1-\rho_i)E_{s_j}\{x_j(s_j, \varnothing)|m_j\}\right]\right) \quad (12)$$

Since $x_i(s_i, \varnothing) = E_{s_j}\{x_i(s_i, s_j)|\varnothing\}$ in equilibrium, firm i's equilibrium output equals:

$$x_i^p(s_i, m_j) = \frac{1}{4-\beta^2}\left((2-\beta)\alpha - 2\overline{\theta_i} + \beta\overline{\theta_j}\right) - \frac{2\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]}{4-\beta^2\rho_i} + \frac{\beta\left[E\{\theta_j|m_j\} - \overline{\theta_j}\right]}{4-\beta^2\rho_j} \quad (13)$$

for $i, j = 1, 2$ and $i \neq j$. This equilibrium output depends on the information leakage probabilities ρ_i and ρ_j, whereas the output (4) does not. An increase of a firm's information leakage probability makes output adjustments by the competitor more likely. This yields a greater dispersion of the firms' equilibrium outputs. Also information acquisition increases a firm's output dispersion. By contrast, the average equilibrium output is constant in the information-acquisition and protection choices, and it equals (5). Hence, these choices only have an effect on the output dispersion.

The first-order condition (12) gives he following equilibrium profit (for $i, j = 1, 2$ and $i \neq j$):

$$\Pi_i^p(\delta_i, \delta_j; \rho_i, \rho_j) \equiv \rho_j E_{s_i} \left\{ E_{s_j} \left[x_i^p(s_i, s_j)^2 \right] \right\} + (1 - \rho_j) E_{s_i} \left\{ x_i^p(s_i, \varnothing)^2 \right\} - c(\delta_i) \tag{14}$$

As before, the expected profit is convex in the firm's outputs, and therefore it is increasing in the outputs' dispersion. We show in Appendix B that these basic features give all the qualitative results from Propositions 2-5.

6.2 Oligopoly

Our model assumes that there is competition between only two firms. This is without loss of generality, since an oligopoly model yields qualitatively identical results.

In a model with N firms and goods, with $N \geq 2$, the representative consumer's gross surplus from consuming $(x_1, .., x_N)$ is:

$$u(x_1, .., x_N) \equiv \alpha \sum_{\ell=1}^{N} x_\ell - \frac{1}{2} \left(\sum_{\ell=1}^{N} x_\ell \right)^2 + \frac{1}{2}(1 - \beta) \sum_{\ell=1}^{N} x_\ell \sum_{k \neq \ell} x_k. \tag{15}$$

As before, the inverse demand function for good i is linear: $P_i(x_i, x_{-i}) = \alpha - x_i - \beta \sum_{j \neq i} x_j$, where $x_{-i} \equiv (x_1, .., x_{i-1}, x_{i+1}, .., x_N)$. Firm i's profit of producing output x_i is simply $\pi_i(x_i, x_{-i}; \theta_i) = [P_i(x_i, x_{-i}) - \theta_i] x_i$ for $i = 1, .., N$. For any combination of messages $m_1, .., m_N$, firm i with signal s_i maximizes its expected profit, which yields the following equilibrium output level of firm i (for $i, j = 1, .., N$ with $j \neq i$):

$$\begin{aligned} x_i^*(s_i; m_i, m_{-i}) \;\; = \;\; & \frac{1}{[2 + (N - 1)\beta](2 - \beta)} \Bigg((2 - \beta)\alpha - [2 + (N - 2)\beta] E\{\theta_i | s_i\} + \beta \sum_{j \neq i} E\{\theta_j | m_j\} \\ & + (N - 1) \frac{\beta^2}{2} \left[E\{\theta_i | s_i\} - E\{\theta_i | m_i\} \right] \Bigg), \end{aligned} \tag{16}$$

where $E\{\theta_i | m_i\} = E_{s_i}\{E(\theta_i | s_i) | m_i\}$ and $m_{-i} \equiv (m_1, .., m_{i-1}, m_{i+1}, .., m_N)$. As before, information concealment creates a distortion, as captured by the last term of (16). This distortion dampens the sensitivity of the equilibrium outputs to the precision of information, which gives supermodular expected profits, and equilibrium strategies as in section 4 (see Appendix B).

For the analysis of the consumer surplus, we distinguish again the quantity-adjustment effect from the preference-for-variety effect. This gives essentially the same trade-off for consumers as we described in the duopoly model. In Appendix B we show formally that the same qualitative results emerge with more than two firms.

6.3 Bertrand Competition

We briefly consider the model where firms choose prices, $p_i \geq 0$ for $i = 1, 2$ (Bertrand competition), instead of output levels. The system of inverse demand functions gives the following direct demand function: $D_i(p_i, p_j) = \frac{1}{1-\beta^2}\Big((1-\beta)\alpha + \beta p_j - p_i\Big)$ for $i, j = 1, 2$ and $i \neq j$. For given prices (p_i, p_j) and cost θ_i, firm i's profit equals $\pi_i(p_i, p_j; \theta_i) \equiv (p_i - \theta_i)D_i(p_i, p_j)$.

6.3.1 Equilibrium Choices Each firm chooses its price on the basis of its own information, s_i, and the information received from its competitor, $m_j \in \{s_j, \varnothing\}$. We adopt the same notation for conditional and unconditional expectations as before without making explicit the dependence of s_i on δ_i. For any combination of messages m_i and m_j, firm i with signal s_i sets the following equilibrium price (for $i, j = 1, 2$ with $i \neq j$):

$$p_i^*(s_i; m_i, m_j) = \frac{1}{4-\beta^2}\left((2+\beta)(1-\beta)\alpha + 2E\{\theta_i|s_i\} + \beta E\{\theta_j|m_j\} - \frac{\beta^2}{2}\left[E\{\theta_i|s_i\} - E\{\theta_i|m_i\}\right]\right) \tag{17}$$

where $E\{\theta_i|m_i\} = E_{s_i}\{E(\theta_i|s_i)|m_i\}$. In equilibrium, the expected profit of firm i is:

$$\Pi_i^b(\delta_i, \delta_j; \rho_i, \rho_j) \equiv \frac{1}{1-\beta^2}E_{s_i,m_i}\left\{E_{s_j,m_j}\left[(p_i^*(s_i; m_i, m_j) - E\{\theta_i|s_i\})^2\right]\right\} - c(\delta_i) \tag{18}$$

These equilibrium profits determine the firm's incentive to acquire and share information. In particular, we can show that the following properties hold (see Appendix B for formal proofs). First, the expected product market profit of a firm is weakly increasing in the firms' investments in information acquisition, as in Proposition 2. Second, a firm's incentive to acquire information depends as follows on the information-sharing choices of the firms.

PROPOSITION 7 *Suppose that firms compete in prices in the last stage. For any (ρ_1, ρ_2), there exists an equilibrium in the information-acquisition stage $(\delta_1^b(\rho_1, \rho_2), \delta_2^b(\rho_2, \rho_1))$. For $i, j = 1, 2$ with $i \neq j$, firm i's equilibrium information-acquisition investment, $\delta_i^b(\rho_i, \rho_j)$, is: (i) independent of the competitor's information-sharing choice, ρ_j, and (ii) decreasing in the own choice, ρ_i.*

Part (i) is identical to Proposition 3(i). Interestingly, Proposition 7(ii) is the reverse result of Proposition 3(ii). Information sharing reduces a price setter's incentive to acquire information, whereas it enhances the information acquisition incentive of a quantity setter.[22]

Also the incentives to share information for an exogenously given precision are different for Bertrand competitors, as the following proposition shows.

PROPOSITION 8 *Suppose that firms compete in prices in the last stage. (i) The expected profit* $\Pi_i^b(\delta_i, \delta_j; \rho_i, \rho_j)$ *is decreasing in* ρ_i *and increasing in* ρ_j *for all* (δ_i, δ_j)*. Furthermore, the expected industry profit* $\sum_{\ell=1}^{2} \Pi_\ell^b(\delta_\ell, \delta_k; \rho_\ell, \rho_k)$ *is decreasing in* ρ_i *for all* (δ_i, δ_j)*, and* $i, k = 1, 2$ *with* $k \neq \ell$*. (ii) Firm i's unique equilibrium information-sharing strategy is to conceal all its information,* $\rho_i^b = 0$*. Moreover, if firms coordinate their information-sharing choices to maximize the anticipated industry profits* $\sum_{\ell=1}^{2} \Pi_\ell^b(\delta_\ell^b, \delta_k^b; \rho_\ell, \rho_k)$*, then they choose to conceal their information.*

The first result confirms Gal-Or (1986) who shows that information concealment is a dominant strategy for Bertrand duopolists. Further, it is intuitive that firms are better off by receiving information from their competitor. The effect of information sharing on the industry profits follows from the trade-off between the loss from sending information and the gain from receiving information. We show in Appendix B that the negative effect of information sharing dominates.

6.3.2 Equilibrium Profits Proposition 8(ii) holds, since the indirect effects from information sharing reinforce the direct effects of Proposition 8(i). First, the effect of a firm's information sharing on the firm's own profit is uniquely determined by the direct effect (i.e., $\partial \Pi_i^b / \partial \rho_i \leq 0$). As before, the indirect effect of information sharing is absent since the firm's information-sharing choice has no effect on the competitor's investment in information acquisition (Proposition 7(i)). Second, the indirect effect on the industry profit is non-positive for the following reasons. Information sharing by the competitor reduces his investment in information acquisition ($\partial \delta_j^b / \partial \rho_j \leq 0$ as in Proposition 7(ii)), and this reduced investment reduces the firm's profit ($\partial \Pi_i^b / \partial \delta_j \geq 0$ as in Proposition 2). In other words, the qualitative profit results do not change by endogenizing the information precision. Recall that this observation also holds for Cournot competitors.

6.3.3 Consumer Surplus The qualitative properties of the expected consumer surplus in equilibrium are identical to those in Proposition 5 (see Appendix B). This gives the following

[22]Formally, the profit Π_i^b is submodular in (δ_i, ρ_i), i.e., for $\rho_i > \rho_i'$, the difference $\Pi_i^b(\delta_i, \delta_j; \rho_i, \rho_j) - \Pi_i^b(\delta_i, \delta_j; \rho_i', \rho_j)$ is weakly decreasing in δ_i for all δ_j and ρ_j. By contrast, the Cournot profit Π_i is supermodular in (δ_i, ρ_i).

overall effect of information sharing on the expected consumer surplus. For a given precision of the firms' signals, information sharing decreases the expected consumer surplus (Sakai and Yamato (1989)). This is a direct effect of information sharing. Moreover, information sharing reduces the firms' investments in acquiring information (Proposition 7(ii)), which reduces the expected surplus even further. In other words, the indirect effect reinforces the direct effect of information sharing on the surplus when firms compete in prices. That is, submodularity of Bertrand duopolists' profits reverses the indirect effect of information sharing on the expected consumer surplus, and the previous trade-off ceases to exist.

6.4 Correlated Costs

We have analyzed an independent private value framework. In this framework, information acquisition creates an indirect effect of information sharing on the consumer surplus. This gives a trade-off between a negative direct effect and positive indirect effect. Now we briefly discuss the effects of introducing cost correlation.

Analyzing a model of imperfect positive correlation is complex, for the reasons mentioned below. However, it is tractable and illuminating to analyze a setting in which firms have perfectly correlated costs. In such a situation, information sharing also yields a trade-off between a direct and indirect effect on the consumer surplus. However, as Figure 4 illustrates, the directions of both effects are reversed.

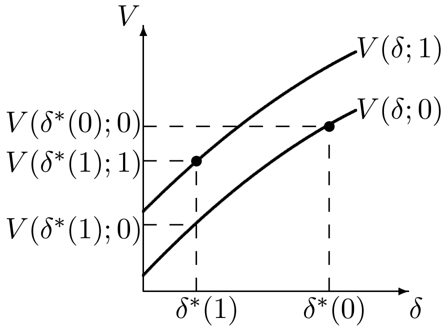

Figure 4: Perfect positive correlation

Vives (1984) shows that the direct effect of information sharing on consumer surplus is positive (i.e., $V(\delta;1) > V(\delta;0)$ as Figure 4 illustrates for $\delta = \delta^*(1)$). With perfect correlation, the dispersion of the industry output is larger when both firms adjusts their output levels to a common cost shock (similarly to a common demand shock). Information sharing also creates an indirect effect, as in the model with independent costs. Vives (1984) also shows that, similarly to present model, the more accurate is the firms' information, the larger is the consumer surplus (i.e., $V(\delta;1)$ and $V(\delta;0)$ are increasing in δ). As before, information sharing gives the firms a greater incentive to acquire information, since the output adjustments of the competitor increases a firm's own output dispersion. However, the more correlated are the firms' costs, the less important is this effect. In addition, cost correlation creates a free-riding problem, since firms may use the information of their competitors to learn about their own cost. Jansen (2008) shows that with perfectly correlated costs the free-rider effect can dominate, and sharing information about a common cost parameter leads to lower information acquisition investments ($\delta^*(1) < \delta^*(0)$ as in Figure 4). Therefore, information sharing has a negative indirect effect on the consumer surplus.[23]

In other words, the introduction of perfect cost correlation reverses the direction of both the direct and indirect effects compared with the independent-private-value setting. This make the task of analyzing an imperfect correlation framework very difficult, since it is likely that the signs of the direct and indirect effect are going to depend not only on the degree of cost correlation but also on the information structure that we use to set up the model.

7 Conclusion

We have shown that the incentives for acquiring information are larger when Cournot oligopolists are allowed to share information. A higher information-acquisition investment increases the consumer surplus. These observations have important implications for an antitrust authority's choice between allowing and disallowing information sharing. Whereas conventional wisdom predicts that information sharing reduces consumer surplus, our observations predict a surplus increase from information sharing. Overall, the trade-off between the positive and negative effects of information sharing can make the consumer surplus larger when firms are allowed to share information.

In the paper we used the expected consumer surplus as welfare measure. This enables us to distinguish the effects on firms from the effects on consumers. A more general welfare measure would be a weighed sum of consumer surplus and producer surplus (i.e., industry profits). The

[23]In fact, it can be shown that in the model corresponding to the binary example, the indirect effect can dominate the direct effect of information sharing, as Figure 4 illustrates.

adoption of such a general welfare function would not change the main conclusion of the paper, since information sharing gives a positive indirect effect on the producer surplus too. Information sharing gives higher information acquisition investments than information concealment (Proposition 3(ii)). The higher investment under information sharing increases the industry profit gain from information sharing (Proposition 2(ii)). In other words, the indirect effects of information sharing on consumers and producers are aligned, and favor information sharing.

Finally, we want to stress that we have undertaken the analysis using general information structures and new information orderings based on dispersion measures. This methodological approach allows us to show that our results and the results of previous literature crucially depend on the convexity of consumers' and firms' objective functions over output, as well as on the effect of information on the dispersion of equilibrium output.

In spite of this enhanced generality of our analysis, there remains some scope for further generalization. For example, we have assumed that a firm chooses the probability with which its shares information. This can be seen as a special case of a general model in which a firm chooses the precision of the message that it sends to its competitor. The analysis of such a general model would require analyzing the cumulative effect of two distortions of a firm's information. First, information acquisition determines the precision of a firm's signal. Second, information sharing affects the precision of a message about an imprecise signal. Whereas the analysis of such a model goes beyond the scope of this paper, we can analyze information sharing in a more general way if we abstract from information acquisition. In particular, Appendix B considers a model in which each firm privately observes its cost. Then, each firm determines its sharing policy by choosing among information structures ordered according to Lehmann informativeness criterion. We show that in this set-up, firms send messages of maximal precision in equilibrium.[24] In Appendix B we show this formally. An analysis which incorporates both general information acquisition and information sharing technologies awaits future research.

[24] A fim's expected profit depends on the covariance between the firm's cost information and the firm's expected cost conditional on the noisy signal about the cost. We need the more restrictive precision concept from Lehmann (1988) for firms' messages in order to get an unambiguous (positive) effect of the message precision on this covariance.

A Appendix

We make repeated use of the following result.

Lemma 1 *If δ ranks signals according to Integral Precision, then the variance of $E_i[\theta|S_i^\delta]$ is increasing in δ.*

Proof of Lemma 1: The variance of $E_i[\theta|S_i^\delta]$ is equal to: $\text{Var}(E_i[\theta|S_i^\delta]) = E\{(E_i[\theta|S_i^\delta] - \overline{\theta}_i)^2\}$. Given that $(E_i[\theta|S_i^\delta] - \overline{\theta}_i)^2$ is a convex function of $E_i[\theta|S_i^\delta]$, the result is a direct implication of the definitions of the convex order and integral precision. ■

By Lemma 1, $E\{E_i[\theta|S_i^\delta]^2\}$ is increasing in δ too, since $\text{Var}(E_i[\theta|S_i^\delta]) = E\{E_i[\theta|S_i^\delta]^2\} - \overline{\theta}_i^2$.

For the proofs of Propositions 2-4 it is convenient to rewrite the expected profit (6). First, by using the definition of $x_i^*(s_i; m_i, m_j)$ in (4), we can rewrite $E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i;\varnothing,m_j)^2]\right\}$ as follows:

$$E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i;\varnothing,m_j)^2]\right\} = E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j) + \frac{\beta^2}{2(4-\beta^2)}\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]\right)^2\right]\right\}$$

$$\begin{aligned}
&= E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)^2\right]\right\} \\
&\quad + \frac{\beta^2}{2(4-\beta^2)}E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]E_{s_j,m_j}\left[2x_i^*(s_i;s_i,m_j) + \frac{\beta^2}{2(4-\beta^2)}\left(E\{\theta_i|s_i\} - \overline{\theta_i}\right)\right]\right\} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)^2\right]\right\} \\
&\quad + \frac{\beta^2}{4(4-\beta^2)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]\left[4(2-\beta)\alpha + 4\beta\overline{\theta_j} - \beta^2\overline{\theta_i} - (8-\beta^2)E\{\theta_i|s_i\}\right]\right\} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)^2\right]\right\} - \frac{\beta^2(8-\beta^2)}{4(4-\beta^2)^2}\left(E_{s_i}\left\{E\left[\theta_i|s_i\right]^2\right\} - \overline{\theta_i}^2\right). \qquad (19)
\end{aligned}$$

In the last simplification, we use the property that $E_{s_i}\left\{E\left[\theta_i|s_i\right] - \overline{\theta_i}\right\} = 0$, and any constant multiplied by $E_{s_i}\left\{E\left[\theta_i|s_i\right] - \overline{\theta_i}\right\}$ also equals 0. Second, we rewrite $E_{s_i,m_i}\{x_i^*(s_i;m_i,\varnothing)^2\}$ as follows:

$$E_{s_i,m_i}\{x_i^*(s_i;m_i,\varnothing)^2\} = E_{s_i,m_i}\left\{E_{s_j}\left[\left(x_i^*(s_i;m_i,s_j) - \frac{\beta}{4-\beta^2}\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]\right)^2\right]\right\}$$

$$\begin{aligned}
&= E_{s_i,m_i}\left\{E_{s_j}[x_i^*(s_i;m_i,s_j)^2]\right\} \\
&\quad - \frac{\beta}{4-\beta^2}E_{s_j}\left\{\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]E_{s_i,m_i}\left[2x_i^*(s_i;m_i,s_j) - \frac{\beta}{4-\beta^2}\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]\right]\right\} \\
&= E_{s_i,m_i}\left\{E_{s_j}[x_i^*(s_i;m_i,s_j)^2]\right\} \\
&\quad - \frac{\beta}{(4-\beta^2)^2}E_{s_j}\left\{\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]\left(2(2-\beta)\alpha - 4\overline{\theta_i} + \beta\left[E\{\theta_j|s_j\} + \overline{\theta_j}\right]\right)\right\} \\
&= E_{s_i,m_i}\left\{E_{s_j}[x_i^*(s_i;m_i,s_j)^2]\right\} - \left(\frac{\beta}{4-\beta^2}\right)^2\left(E_{s_j}\left\{E\left[\theta_j|s_j\right]^2\right\} - \overline{\theta_j}^2\right). \qquad (20)
\end{aligned}$$

As before, in the last two simplifications, we use the property that $E_{s_j}\left\{E\left[\theta_j|s_j\right]-\overline{\theta_j}\right\}=0$ for any j. Using (19) and (20), the expected profit (6) simplifies as follows:

$$\begin{aligned}
\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j) &= \rho_i E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i;s_i,m_j)^2]\right\}+(1-\rho_i)E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i;\varnothing,m_j)^2]\right\}-c(\delta_i)\\
&= E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i;\varnothing,m_j)^2]\right\}+\rho_i\frac{\beta^2(8-\beta^2)}{4(4-\beta^2)^2}\mathrm{Var}(E_i[\theta|S_i^\delta])-c(\delta_i)\\
&= \rho_j E_{s_i}\left\{E_{s_j}[x_i^*(s_i;\varnothing,s_j)^2]\right\}+(1-\rho_j)E_{s_i}\left\{x_i^*(s_i;\varnothing,\varnothing)^2\right\}\\
&\quad +\rho_i\frac{\beta^2(8-\beta^2)}{4(4-\beta^2)^2}\mathrm{Var}(E_i[\theta|S_i^\delta])-c(\delta_i)\\
&= E_{s_i}\left\{x_i^*(s_i;\varnothing,\varnothing)^2\right\}+\rho_j\left(\frac{\beta}{4-\beta^2}\right)^2\mathrm{Var}(E_j[\theta|S_j^\delta])\\
&\quad +\rho_i\frac{\beta^2\left(8-\beta^2\right)}{4\left(4-\beta^2\right)^2}\mathrm{Var}(E_i[\theta|S_i^\delta])-c(\delta_i). \qquad (21)
\end{aligned}$$

Proof of Proposition 2:

Using (4), we can rewrite the first term of (21) as follows:

$$\begin{aligned}
E_{s_i}\left\{x_i^*(s_i;\varnothing,\varnothing)^2\right\} &= \frac{1}{\left(4-\beta^2\right)^2}E_{s_i}\left\{\left((2-\beta)\alpha-2\overline{\theta_i}+\beta\overline{\theta_j}-\frac{1}{2}\left(4-\beta^2\right)\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]\right)^2\right\}\\
&= \left(\frac{(2-\beta)\alpha-2\overline{\theta_i}+\beta\overline{\theta_j}}{4-\beta^2}\right)^2+\frac{1}{4}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}. \qquad (22)
\end{aligned}$$

In the last simplification, we use the property that $E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]\right\}=0$.

(i) Lemma 1 implies that (22) is increasing in δ_i. Further, the second term of (21) is independent of δ_i, while the third term is increasing in δ_i. Hence, $\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)+c(\delta_i)$ is increasing in δ_i.

(ii) The second term of (21) is (weakly) increasing in δ_j by Lemma 1. The remaining terms are independent of δ_j (see (22) for the first term). Hence, Π_i is weakly increasing in δ_j. ■

Proof of Proposition 3:

Only the first, third and last terms of (21) depend on δ_i, whereas they do not depend on δ_j, i.e., $\partial^2\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)/(\partial\delta_i\partial\delta_j)=0$. Hence, firm i's optimal information acquisition investment is independent of the competitor's investment, δ_j, and this optimal investment always exists.

(i) We want to show that $\partial^2\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)/(\partial\delta_i\partial\rho_j)=0$. It follows from (21) that:

$$\frac{\partial\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)}{\partial\rho_j}=\left(\frac{\beta}{4-\beta^2}\right)^2\left(E_{s_j}\left\{E\left[\theta_j|s_j\right]^2\right\}-\overline{\theta_j}^2\right).$$

As $\partial\Pi_i/\partial\rho_j$ is independent of δ_i, we have $\frac{\partial^2\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)}{\partial\delta_i\partial\rho_j}=0$ which concludes the proof.

(ii) For any $\rho_i,\rho_i'\in[0,1]$, expression (21) gives:

$$\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)-\Pi_i(\delta_i,\delta_j;\rho_i',\rho_j)=(\rho_i-\rho_i')\frac{\beta^2(8-\beta^2)}{4(4-\beta^2)^2}\mathrm{Var}(E_i[\theta|S_i^\delta]).$$

For $\rho_i>\rho_i'$, this expression is increasing in δ_i by Lemma 1, i.e., $\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)$ is supermodular in (δ_i,ρ_i). By Theorem 4 of Milgrom and Shanon (1994), this implies that $\delta_i^*(\rho_i,\rho_j)$ is increasing in ρ_i. ■

Proof of Proposition 4:

(i) It follows directly from (21) that:

$$\frac{\partial \Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)}{\partial \rho_i} = \frac{\beta^2(8-\beta^2)}{4(4-\beta^2)^2}\mathrm{Var}(E_i[\theta|S_i^\delta]) \geq 0, \text{ and}$$

$$\frac{\partial \Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)}{\partial \rho_j} = \left(\frac{\beta}{4-\beta^2}\right)^2 \mathrm{Var}(E_j[\theta|S_j^\delta]) \geq 0.$$

(ii) This part follows from the argument in the text after Proposition 4. ■

Proof of Proposition 5:

The expected consumer surplus can be rewritten as follows (for $i,j=1,2$ and $i\neq j$):

$$V(\cdot) = \rho_i E_{s_i}\left\{E_{s_j,m_j}\left[\frac{1}{2}\left(x_i^*(s_i;s_i,m_j)+x_j^*(s_j;m_j,s_i)\right)^2 - (1-\beta)x_i^*(s_i;s_i,m_j)x_j^*(s_j;m_j,s_i)\right]\right\}$$
$$+(1-\rho_i)E_{s_i}\left\{E_{s_j,m_j}\left[\frac{1}{2}\left(x_i^*(s_i;\varnothing,m_j)+x_j^*(s_j;m_j,\varnothing)\right)^2 - (1-\beta)x_i^*(s_i;\varnothing,m_j)x_j^*(s_j;m_j,\varnothing)\right]\right\}$$

Differentiating with respect to ρ_i gives (for $i,j=1,2$ and $i\neq j$):

$$\begin{aligned}\frac{\partial V}{\partial \rho_i} &= \frac{1}{2}E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j)+x_j^*(s_j;m_j,s_i)\right)^2 - \left(x_i^*(s_i;\varnothing,m_j)+x_j^*(s_j;m_j,\varnothing)\right)^2\right]\right\}\\ &\quad -(1-\beta)E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)x_j^*(s_j;m_j,s_i) - x_i^*(s_i;\varnothing,m_j)x_j^*(s_j;m_j,\varnothing)\right]\right\} \qquad (23)\end{aligned}$$

The first line of this expression can be simplified by using the following for $m_j\in\{s_j,\varnothing\}$:

$$\begin{aligned}&E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;\varnothing,m_j)+x_j^*(s_j;m_j,\varnothing)\right)^2\right]\right\}\\
&= E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j)+x_j^*(s_j;m_j,s_i) - \frac{\beta(2-\beta)}{2(4-\beta^2)}\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]\right)^2\right]\right\}\\
&= E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j)+x_j^*(s_j;m_j,s_i)\right)^2\right]\right\}\\
&\quad -\frac{\beta}{2(2+\beta)}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]E_{s_j,m_j}\left[2\left(x_i^*(s_i;s_i,m_j)+x_j^*(s_j;m_j,s_i)\right) - \frac{\beta\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{2(2+\beta)}\right]\right\}\\
&= E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j)+x_j^*(s_j;m_j,s_i)\right)^2\right]\right\} + \frac{\beta(4+\beta)}{4(2+\beta)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\} \qquad (24)\end{aligned}$$

The second line of (23) can be simplified by using the following for $m_j\in\{s_j,\varnothing\}$:

$$\begin{aligned}&E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;\varnothing,m_j)x_j^*(s_j;m_j,\varnothing)\right]\right\}\\
&= E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j)+\frac{\beta^2\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{2(4-\beta^2)}\right)\left(x_j^*(s_j;m_j,s_i)-\frac{\beta\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{4-\beta^2}\right)\right]\right\}\\
&= E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)x_j^*(s_j;m_j,s_i)\right]\right\} + E_{s_i}\left\{\frac{\beta^2\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{2(4-\beta^2)}E_{s_j,m_j}\left[x_j^*(s_j;m_j,s_i)\right]\right\}\\
&\quad -E_{s_i}\left\{\frac{\beta\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{4-\beta^2}E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)\right]\right\} - \frac{\beta^3}{2(4-\beta^2)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}\\
&= E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)x_j^*(s_j;m_j,s_i)\right]\right\} + \frac{2\beta}{(4-\beta^2)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\} \qquad (25)\end{aligned}$$

Substitution of (24) and (25) for $m_j \in \{s_j, \varnothing\}$ in (23) gives:

$$\begin{aligned}
\frac{\partial V}{\partial \rho_i} &= -\frac{1}{2}\cdot\frac{\beta(4+\beta)}{4(2+\beta)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}+(1-\beta)\frac{2\beta}{(4-\beta^2)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}\\
&= \frac{-\beta}{2(4-\beta^2)^2}\left[(4+\beta)\left(1-\frac{\beta}{2}\right)^2-4(1-\beta)\right]E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}\\
&= \frac{-\beta^2}{2(4-\beta^2)^2}\left(1+\frac{\beta^2}{4}\right)E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}<0
\end{aligned}$$

To prove that the expected surplus is increasing in δ_i, it is sufficient to show that all terms of V are increasing in δ_i. First, we show the first term of V is increasing in δ_i by rewriting its first component as follows:

$$\begin{aligned}
&\tfrac{1}{2}E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j)+x_j^*(s_j;m_j,s_i)\right)^2\right]\right\}\\
&\quad=\frac{1}{2}\left(\frac{2-\beta}{4-\beta^2}\right)^2E_{s_i}\left\{E_{s_j,m_j}\left[\left(2\alpha-E\{\theta_i|s_i\}-E\{\theta_j|s_j\}-\frac{\beta}{2}\left[E\{\theta_j|s_j\}-E\{\theta_j|m_j\}\right]\right)^2\right]\right\}\\
&\quad=\frac{1}{2(2+\beta)^2}E_{s_i}\left\{E_{s_j,m_j}\left[\left(2\alpha-E\{\theta_j|s_j\}-\frac{\beta}{2}\left[E\{\theta_j|s_j\}-E\{\theta_j|m_j\}\right]\right)^2\right.\right.\\
&\qquad\left.\left.-2\left(2\alpha-E\{\theta_j|s_j\}-\frac{\beta}{2}\left[E\{\theta_j|s_j\}-E\{\theta_j|m_j\}\right]\right)E\{\theta_i|s_i\}+E\{\theta_i|s_i\}^2\right]\right\}
\end{aligned}$$

Notice that only the last term depends on δ_i (i.e., $E_{s_i}\{E\left[\theta_i|s_i\right]^2\}$), and is increasing in δ_i. This immediately implies that $\frac{1}{2}E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j)+x_j^*(s_j;m_j,s_i)\right)^2\right]\right\}$ is increasing in δ_i. The remaining component of the first term equals:

$$\begin{aligned}
&(1-\beta)E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)x_j^*(s_j;m_j,s_i)\right]\right\}\\
&= \frac{1-\beta}{(4-\beta^2)^2}E_{s_i}\left\{E_{s_j,m_j}\left[\left((2-\beta)\alpha-2E\{\theta_i|s_i\}+\beta E\{\theta_j|m_j\}\right)\right.\right.\\
&\qquad\left.\left.*\left((2-\beta)\alpha-2E\{\theta_j|s_j\}+\beta E\{\theta_i|s_i\}+\frac{\beta^2}{2}\left[E\{\theta_j|s_j\}-E\{\theta_j|m_j\}\right]\right)\right]\right\}\\
&= \frac{1-\beta}{(4-\beta^2)^2}E_{s_i}\left\{E_{s_j,m_j}\left[\left((2-\beta)\alpha-2E\{\theta_i|s_i\}+\beta E\{\theta_j|m_j\}\right)\right.\right.\\
&\qquad\left.\left.*\left((2-\beta)\alpha-2E\{\theta_j|s_j\}+\frac{\beta^2}{2}\left[E\{\theta_j|s_j\}-E\{\theta_j|m_j\}\right]\right)\right]\right\}\\
&\quad+\frac{\beta(1-\beta)}{(4-\beta^2)^2}E_{s_i}\left\{E\{\theta_i|s_i\}E_{s_j,m_j}\left[(2-\beta)\alpha+\beta E\{\theta_j|m_j\}\right]-2E\{\theta_i|s_i\}^2\right\}
\end{aligned}$$

Again, only the last term depends on δ_i (i.e., $E_{s_i}\{E\left[\theta_i|s_i\right]^2\}$), and is increasing in δ_i. This immediately implies that $(1-\beta)E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)x_j^*(s_j;m_j,s_i)\right]\right\}$ is decreasing in δ_i. Subtracting the latter component from the former component immediately implies that the first term of V is increasing in δ_i. It is straightforward to show that the second term is also increasing in δ_i, by using the decompositions (24) and (25) in combination with the observation that the first term of V is increasing in δ_i. This proves that $\partial V/\partial \delta_i > 0$. ■

Proof of Proposition 6:
We first derive the firm's profit-maximizing information-acquisition investment. In our binary example, the expected equilibrium profit (6) reduces to:

$$\begin{aligned}\Pi_1(\delta_1;\rho_1) &= \delta_1\left[\rho_1 E_{\theta_1}\left\{x_1^*(\theta_1;\theta_1,\theta_2)^2\right\} + (1-\rho_1)E_{\theta_1}\left\{x_1^*(\theta_1;\varnothing,\theta_2)^2\right\}\right] \\ &\quad +(1-\delta_1)x_1^*(\varnothing;\varnothing,\theta_2)^2 - \lambda\delta_1 \\ &= x_1^*(\varnothing;\varnothing,\theta_2)^2 + \delta_1\rho_1\left[E_{\theta_1}\left\{x_1^*(\theta_1;\theta_1,\theta_2)^2 - x_1^*(\varnothing;\varnothing,\theta_2)^2\right\}\right] \\ &\quad +\delta_1(1-\rho_1)\left[E_{\theta_1}\left\{x_1^*(\theta_1;\varnothing,\theta_2)^2 - x_1^*(\varnothing;\varnothing,\theta_2)^2\right\}\right] - \lambda\delta_1 \\ &= x_1^*(\varnothing;\varnothing,\theta_2)^2 + \delta_1\left(\rho_1\frac{(\theta^h-\theta^l)^2}{9} + (1-\rho_1)\frac{(\theta^h-\theta^l)^2}{16} - \lambda\right)\end{aligned}$$

The first term is constant in δ_1. Hence, firm 1's equilibrium information acquisition choice is:

$$\delta_1^*(\rho_1) = \begin{cases} 1, & \text{if } \lambda \le \left(\rho_1\frac{4}{9} + (1-\rho_1)\frac{1}{4}\right)\frac{(\theta^h-\theta^l)^2}{4} \\ 0, & \text{otherwise}\end{cases} \tag{26}$$

If $\frac{(\theta^h-\theta^l)^2}{16} < \lambda < \frac{(\theta^h-\theta^l)^2}{9}$, then an information-sharing firm acquires information whereas an information-concealing firm acquires no information, i.e., $\delta_1^*(1) = 1 > 0 = \delta_1^*(0)$. Therefore, in this case, information sharing gives a higher expected surplus:

$$\begin{aligned}V(\delta_1^*(1);1) &= \frac{1}{2}E_{\theta_1}\left\{\left[x_1^*(\theta_1;\theta_1,\theta_2) + x_2^*(\theta_2,\theta_2,\theta_1)\right]^2\right\} \\ &> \frac{1}{2}\left[x_1^*(\varnothing;\varnothing,\theta_2) + x_2^*(\theta_2,\theta_2,\varnothing)\right]^2 = V(\delta_1^*(0);0). \quad ■\end{aligned}$$

B Supplementary Appendix: Extensions

Here we derive the results for the extensions of the model. First, we solve the game in which firms choose the intensity of protecting their private information. Second, we extend the results to an oligopoly with N risk-neutral firms that compete in quantities of differentiated goods (with $N \geq 2$). Third, we analyze competition in prices. Fourth, we analyze a firm's incentives to add noise to its cost message. Finally, we present the basic algebra for the extended binary example where both firms can acquire and share information.

B.1 Protection of Proprietary Information

Suppose that each firm chooses the intensity with which it protects its proprietary cost information. In particular, firm i's information about its signal leaks out to the competitor with probability ρ_i, while the firm's privacy is protected with probability $1-\rho_i$ for $i = 1,2$ and $0 \leq \rho_i \leq 1$. Whether or not information leaks out to a competitor is not observable to a firm. That is, each sender only learns which message it sends (i.e., for sender i the realization of the stochastic variable, which gives firm i's signal s_i with probability ρ_i, and the uninformative message with probability $1-\rho_i$) after the firms have chosen their output levels. That is, at the product market stage, firm i observes s_i and m_j, but not m_i. Firm i's expected profit at the product market stage equals:

$$\Pi_i^p(x_i, x_j; s_i, m_j) \equiv$$
$$E_{\theta_i}\left\{ E_{s_j}\left[\rho_i \pi_i(x_i(s_i, m_j), x_j(s_j, s_i); \theta_i) + (1-\rho_i)\pi_i(x_i(s_i, m_j), x_j(s_j, \varnothing); \theta_i) \middle| m_j\right] \middle| s_i\right\}$$

For any s_i and $m_j \in \{s_j, \varnothing\}$, profit-maximization by firm i with respect to the firm's output gives the first-order condition (12) for $i,j = 1,2$ and $i \neq j$. Notice that the equilibrium outputs are such that $x_i(s_i, \varnothing) = E_{s_j}\{x_i(s_i, s_j)|\varnothing\}$. Using this observation and some basic algebra, gives us (13) as the solution to the system of first-order conditions (for $i,j = 1,2$ and $i \neq j$). Using the first-order conditions, gives the following expected equilibrium profit for firm i (with $i,j = 1,2$ and $i \neq j$):

$$\begin{aligned}
\Pi_i^p(\delta_i, \delta_j; \rho_i, \rho_j) &\equiv \rho_j E_{s_i}\left\{E_{s_j}\left[x_i^p(s_i, s_j)^2\right]\right\} + (1-\rho_j)E_{s_i}\left\{x_i^p(s_i, \varnothing)^2\right\} - c(\delta_i) \\
&= \frac{1}{(4-\beta^2)^2}\left((2-\beta)\alpha - 2\overline{\theta_i} + \beta\overline{\theta_j}\right)^2 + \frac{4\text{Var}(E_i[\theta|S_i^\delta])}{(4-\beta^2\rho_i)^2} + \rho_j\frac{\beta^2\text{Var}(E_j[\theta|S_j^\delta])}{(4-\beta^2\rho_j)^2} \\
&\quad -c(\delta_i) \qquad (27)
\end{aligned}$$

The first term of equation (27) is constant. The remaining terms give the following properties.

First, firm i's expected equilibrium profit is increasing in the firms' leakage probabilities (i.e., $\partial\Pi_i^p/\partial\rho_\ell \geq 0$ for all $i, \ell = 1,2$) as in Proposition 4(i). In particular, for any $\rho_i, \rho_i' \in [0,1]$ we get:

$$\Pi_i^p(\delta_i, \delta_j; \rho_i, \rho_j) - \Pi_i^p(\delta_i, \delta_j; \rho_i', \rho_j)$$
$$= \left(\frac{1}{(4-\beta^2\rho_i)^2} - \frac{1}{(4-\beta^2\rho_i')^2}\right)4\text{Var}(E_i[\theta|S_i^\delta])$$

$$
\begin{aligned}
&= \left(\frac{1}{4-\beta^2\rho_i} - \frac{1}{4-\beta^2\rho_i'}\right)\left(\frac{1}{4-\beta^2\rho_i} + \frac{1}{4-\beta^2\rho_i'}\right) 4\mathrm{Var}(E_i[\theta|S_i^\delta]) \\
&= \left(\rho_i - \rho_i'\right) \frac{4\beta^2\mathrm{Var}(E_i[\theta|S_i^\delta])}{(4-\beta^2\rho_i)(4-\beta^2\rho_i')}\left(\frac{1}{4-\beta^2\rho_i} + \frac{1}{4-\beta^2\rho_i'}\right) \qquad (28)
\end{aligned}
$$

This expression is positive if and only if $\rho_i > \rho_i'$. That is, Π_i^p is increasing in ρ_i. Similarly, the third term of (27) is increasing in ρ_j, and it is the only term that depends on ρ_j.

Second, the qualitative results on the relationship between information acquisition and information leakage do not differ from those in Proposition 3. For $\rho_i > \rho_i'$, the profit difference in (28) is increasing in δ_i by Lemma 1. In other words, the expected profit $\Pi_i^p(\delta_i, \delta_j; \rho_i, \rho_j)$ is supermodular in (δ_i, ρ_i). Hence, by Theorem 4 of Milgrom and Shanon (1994), we conclude that firm i's equilibrium information acquisition investment, δ_i^p, is increasing in the firm's information leakage choice, ρ_i, as in Proposition 3(ii). As before, firm i's equilibrium information acquisition investment, δ_i^p, is independent of the competitor's information leakage choice, ρ_j. This follows from differentiating (27) with respect to ρ_j:

$$
\frac{\partial \Pi_i^p(\delta_i, \delta_j; \rho_i, \rho_j)}{\partial \rho_j} = \frac{\partial}{\partial \rho_j}\left(\frac{\rho_j}{(4-\beta^2\rho_j)^2}\right)\beta^2\mathrm{Var}(E_j[\theta|S_j^\delta])
$$

and observing that $\partial\Pi_i^p/\partial\rho_j$ is independent of δ_i. Hence, we have $\partial^2\Pi_i^p(\delta_i, \delta_j; \rho_i, \rho_j)/(\partial\delta_i\partial\rho_j) = 0$ which implies that δ_i^p is independent of ρ_j (for $i, j = 1, 2$ and $i \neq j$) as in Proposition 3(i).

Third, by using $\partial\delta_i^p/\partial\rho_j = 0$, the first-order condition $\partial\Pi_i^p(\delta_i^p, \delta_j^p; \rho_i, \rho_j)/\partial\delta_i = 0$, and $\partial\Pi_i^p/\partial\rho_i \geq 0$ in (7) for Π_i^p, we obtain that no protection of information is a dominant strategy for a firm (i.e., $\rho_i^p = 1$ in equilibrium for $i = 1, 2$), as in Proposition 4(ii).

Finally, we show that our observations on the expected consumer surplus in Proposition 5 remain valid. The expected consumer surplus equals:

$$
\begin{aligned}
V^p(\delta_i, \delta_j; \rho_i, \rho_j) \equiv\ & \frac{1}{2}E_{s_i,m_i}\left\{E_{s_j,m_j}\left[\left(x_i^p(s_i,m_j) + x_j^p(s_j,m_i)\right)^2\right]\right\} \\
& -(1-\beta)E_{s_i,m_i}\left\{E_{s_j,m_j}\left[x_i^p(s_i,m_j)x_j^p(s_j,m_i)\right]\right\} \\
=\ & \frac{1}{2}E_{s_i}\left\{E_{s_j,m_j}\left[\rho_i\left(x_i^p(s_i,m_j) + x_j^p(s_j,s_i)\right)^2 + (1-\rho_i)\left(x_i^p(s_i,m_j) + x_j^p(s_j,\varnothing)\right)^2\right]\right\} \\
& -(1-\beta)E_{s_i}\left\{E_{s_j,m_j}\left[\rho_i x_i^p(s_i,m_j)x_j^p(s_j,s_i) + (1-\rho_i)x_i^p(s_i,m_j)x_j^p(s_j,\varnothing)\right]\right\}
\end{aligned}
$$

Definition (5), the substitution of the equilibrium outputs (13), and the independence of the firms' information give the following:

$$
\begin{aligned}
& E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^p(s_i,m_j) + x_j^p(s_j,s_i)\right)^2\right]\right\} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[\left(\overline{x_i}^* + \overline{x_j}^* - \frac{2\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]}{4-\beta^2\rho_j} + \frac{\beta\left[E\{\theta_j|m_j\} - \overline{\theta_j}\right]}{4-\beta^2\rho_j} - \frac{(2-\beta)\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]}{4-\beta^2\rho_i}\right)^2\right]\right\}
\end{aligned}
$$

$$= \quad (\overline{x_i}^* + \overline{x_j}^*)^2 + E_{s_j,m_j}\left\{\left(\frac{2\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]}{4-\beta^2\rho_j} - \frac{\beta\left[E\{\theta_j|m_j\} - \overline{\theta_j}\right]}{4-\beta^2\rho_j}\right)^2\right\}$$

$$+\left(\frac{2-\beta}{4-\beta^2\rho_i}\right)^2 E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]^2\right\}$$

$$E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^p(s_i,m_j) + x_j^p(s_j,\varnothing)\right)^2\right]\right\}$$

$$= \quad E_{s_i}\left\{E_{s_j,m_j}\left[\left(\overline{x_i}^* + \overline{x_j}^* - \frac{2\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]}{4-\beta^2\rho_j} + \frac{\beta\left[E\{\theta_j|m_j\} - \overline{\theta_j}\right]}{4-\beta^2\rho_j} - \frac{2\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]}{4-\beta^2\rho_i}\right)^2\right]\right\}$$

$$= \quad (\overline{x_i}^* + \overline{x_j}^*)^2 + E_{s_j,m_j}\left\{\left(\frac{2\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]}{4-\beta^2\rho_j} - \frac{\beta\left[E\{\theta_j|m_j\} - \overline{\theta_j}\right]}{4-\beta^2\rho_j}\right)^2\right\} + \frac{4E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]^2\right\}}{(4-\beta^2\rho_i)^2}$$

$$E_{s_i}\left\{E_{s_j,m_j}\left[x_i^p(s_i,m_j)x_j^p(s_j,s_i)\right]\right\}$$

$$= \quad E_{s_i}\left\{E_{s_j,m_j}\left[\left(\overline{x_i}^* + \frac{\beta\left[E\{\theta_j|m_j\} - \overline{\theta_j}\right]}{4-\beta^2\rho_j} - \frac{2\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]}{4-\beta^2\rho_i}\right)\right.\right.$$

$$\left.\left.* \left(\overline{x_j}^* - \frac{2\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]}{4-\beta^2\rho_j} + \frac{\beta\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]}{4-\beta^2\rho_i}\right)\right]\right\}$$

$$= \quad \overline{x_i}^*\overline{x_j}^* - E_{s_j,m_j}\left\{\frac{2\beta\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]\left[E\{\theta_j|m_j\} - \overline{\theta_j}\right]}{(4-\beta^2\rho_j)^2}\right\} - \frac{2\beta E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]^2\right\}}{(4-\beta^2\rho_i)^2}$$

$$E_{s_i}\left\{E_{s_j,m_j}\left[x_i^p(s_i,m_j)x_j^p(s_j,\varnothing)\right]\right\}$$

$$= \quad E_{s_i}\left\{E_{s_j,m_j}\left[\left(\overline{x_i}^* + \frac{\beta\left[E\{\theta_j|m_j\} - \overline{\theta_j}\right]}{4-\beta^2\rho_j} - \frac{2\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]}{4-\beta^2\rho_i}\right)\left(\overline{x_j}^* - \frac{2\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]}{4-\beta^2\rho_j}\right)\right]\right\}$$

$$= \quad \overline{x_i}^*\overline{x_j}^* - E_{s_j,m_j}\left\{\frac{2\beta\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]\left[E\{\theta_j|m_j\} - \overline{\theta_j}\right]}{(4-\beta^2\rho_j)^2}\right\}$$

By using these expressions, we can rewrite the expected consumer surplus V^p as follows:

$$V^p(\delta_i,\delta_j;\rho_i,\rho_j) \quad = \quad \frac{1}{2}(\overline{x_i}^* + \overline{x_j}^*)^2 + \frac{1}{2}E_{s_j,m_j}\left\{\left(\frac{2\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]}{4-\beta^2\rho_j} - \frac{\beta\left[E\{\theta_j|m_j\} - \overline{\theta_j}\right]}{4-\beta^2\rho_j}\right)^2\right\}$$

$$+\frac{1}{2}\left[\rho_i(2-\beta)^2 + (1-\rho_i)4\right]\frac{E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]^2\right\}}{(4-\beta^2\rho_i)^2}$$

$$-(1-\beta)\overline{x_i}^*\overline{x_j}^* + (1-\beta)E_{s_j,m_j}\left\{\frac{2\beta\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]\left[E\{\theta_j|m_j\} - \overline{\theta_j}\right]}{(4-\beta^2\rho_j)^2}\right\}$$

$$+(1-\beta)\rho_i\frac{2\beta E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]^2\right\}}{(4-\beta^2\rho_i)^2}$$

$$
\begin{aligned}
= \; & \frac{1}{2}\left(\overline{x_i}^* + \overline{x_j}^*\right)^2 + \frac{1}{2}E_{s_j,m_j}\left\{\left(\frac{2\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]}{4-\beta^2\rho_j} - \frac{\beta\left[E\{\theta_j|m_j\} - \overline{\theta_j}\right]}{4-\beta^2\rho_j}\right)^2\right\} \\
& -(1-\beta)\overline{x_i}^*\overline{x_j}^* + (1-\beta)E_{s_j,m_j}\left\{\frac{2\beta\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]\left[E\{\theta_j|m_j\} - \overline{\theta_j}\right]}{\left(4-\beta^2\rho_j\right)^2}\right\} \\
& + \frac{\frac{1}{2}\left(4-\rho_i 3\beta^2\right)}{\left(4-\beta^2\rho_i\right)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]^2\right\}
\end{aligned}
$$

Only the last term of this expression depends on ρ_i and δ_i. First, V^p is decreasing in ρ_i, since $\frac{1}{2}\left(4-\rho_i 3\beta^2\right) / \left(4-\beta^2\rho_i\right)^2$ is decreasing in ρ_i, and $E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]^2\right\} > 0$ is constant in ρ_i. Second, V^p is increasing in the precision δ_i, since $E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]^2\right\}$ is increasing in δ_i, and $\frac{1}{2}\left(4-\rho_i 3\beta^2\right) / \left(4-\beta^2\rho_i\right)^2 > 0$.

B.2 Cournot Oligopoly

First, for the profit results, it is convenient to rewrite $E_{s_i}\left\{E_{s_{-i},m_{-i}}[x_i^*(s_i;\varnothing,m_{-i})^2]\right\}$ as follows by using (16):

$$
\begin{aligned}
& E_{s_i}\left\{E_{s_{-i},m_{-i}}[x_i^*(s_i;\varnothing,m_{-i})^2]\right\} \\
& \quad = E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[\left(x_i^*(s_i;s_i,m_{-i}) + \frac{(N-1)\beta^2\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]}{2[2+(N-1)\beta](2-\beta)}\right)^2\right]\right\} \\
& \quad = E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[x_i^*(s_i;s_i,m_{-i})^2\right]\right\} + \frac{(N-1)\beta^2}{2[2+(N-1)\beta](2-\beta)}E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]\right. \\
& \qquad\qquad \left. * E_{s_{-i},m_{-i}}\left[2x_i^*(s_i;s_i,m_{-i}) + \frac{(N-1)\beta^2\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]}{2[2+(N-1)\beta](2-\beta)}\right]\right\} \\
& \quad = E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[x_i^*(s_i;s_i,m_{-i})^2\right]\right\} + \frac{(N-1)\beta^2}{4[2+(N-1)\beta]^2(2-\beta)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]\right. \\
& \qquad \left. * \left[4(2-\beta)\alpha + 4\beta\sum_{j\neq i}\overline{\theta_j} - (N-1)\beta^2\overline{\theta_i} - \left(4[2+(N-2)\beta] - (N-1)\beta^2\right)E\{\theta_i|s_i\}\right]\right\} \\
& \quad = E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[x_i^*(s_i;s_i,m_{-i})^2\right]\right\} \\
& \qquad - \frac{(N-1)\beta^2\left[4(2-\beta) + (N-1)\beta(4-\beta)\right]}{4[2+(N-1)\beta]^2(2-\beta)^2}\left(E_{s_i}\left\{E\left[\theta_i|s_i\right]^2\right\} - \overline{\theta_i}^2\right). \qquad (29)
\end{aligned}
$$

In the last simplification, we use that $E_{s_i}\left\{E\left[\theta_i|s_i\right] - \overline{\theta_i}\right\} = 0$. Then, any constant multiplied by $E_{s_i}\left\{E\left[\theta_i|s_i\right] - \overline{\theta_i}\right\}$ also equals 0. Second, we rewrite $E_{s_i,m_i}\{E_{s_{-i},m_{-i}}[x_i^*(s_i;m_i,\varnothing,m_{-ij})^2]\}$, by using (16), as:

$$
\begin{aligned}
& E_{s_i,m_i}\{E_{s_{-i},m_{-i}}[x_i^*(s_i;m_i,\varnothing,m_{-ij})^2]\} \\
& \quad = E_{s_i,m_i}\left\{E_{s_{-i},m_{-i}}\left[\left(x_i^*(s_i;m_i,s_j,m_{-ij}) - \frac{\beta\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]}{[2+(N-1)\beta](2-\beta)}\right)^2\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
&= E_{s_i,m_i}\left\{E_{s_{-i},m_{-i}}[x_i^*(s_i;m_i,s_j,m_{-ij})^2]\right\} \\
&\quad - \frac{\beta}{[2+(N-1)\beta]^2(2-\beta)^2}E_{s_j}\Bigg\{\left[E\{\theta_j|s_j\}-\overline{\theta_j}\right]E_{s_{-j},m_{-j}}\Bigg[\Bigg(2(2-\beta)\alpha-2[2+(N-2)\beta]E\{\theta_i|s_i\} \\
&\quad +2\beta\sum\nolimits_{h\neq i,j}E\{\theta_h|m_h\}\ +2(N-1)\frac{\beta^2}{2}\left[E\{\theta_i|s_i\}-E\{\theta_i|m_i\}\right]+\beta\left[E\{\theta_j|s_j\}+\overline{\theta_j}\right]\Bigg)\Bigg]\Bigg\} \\
&= E_{s_i,m_i}\left\{E_{s_{-i},m_{-i}}[x_i^*(s_i;m_i,s_j,m_{-ij})^2]\right\} \\
&\quad - \frac{2\beta}{[2+(N-1)\beta]^2(2-\beta)^2}E_{s_j}\Bigg\{\left[E\{\theta_j|s_j\}-\overline{\theta_j}\right]\Bigg((2-\beta)\alpha-[2+(N-2)\beta]\overline{\theta_i} \\
&\qquad +\beta\sum\nolimits_{h\neq i,j}\overline{\theta_h}\ +\frac{\beta}{2}\left[E\{\theta_j|s_j\}+\overline{\theta_j}\right]\Bigg)\Bigg\} \\
&= E_{s_i,m_i}\left\{E_{s_{-i},m_{-i}}[x_i^*(s_i;m_i,s_j,m_{-ij})^2]\right\}-\frac{\beta^2}{[2+(N-1)\beta]^2(2-\beta)^2}\left(E_{s_j}\left\{E[\theta_j|s_j]^2\right\}-\overline{\theta_j}^2\right). \qquad (30)
\end{aligned}
$$

In the last two simplifications, we use the property $E_{s_h}\left\{\left[E\{\theta_h|s_h\}-\overline{\theta_h}\right]\right\}=0$ for any $h=1,..,N$. Using (29) and (30), we can rewrite the expected profit Π_i as follows:

$$
\begin{aligned}
&\Pi_i(\delta_i,\delta_{-i};\rho_i,\rho_{-i}) \\
&\quad= \rho_i E_{s_i}\left\{E_{s_{-i},m_{-i}}[x_i^*(s_i;s_i,m_{-i})^2]\right\}+(1-\rho_i)E_{s_i}\left\{E_{s_{-i},m_{-i}}[x_i^*(s_i;\varnothing,m_{-i})^2]\right\}-c(\delta_i) \\
&\quad= E_{s_i}\left\{E_{s_{-i},m_{-i}}[x_i^*(s_i;\varnothing,m_{-i})^2]\right\}-c(\delta_i) \\
&\qquad +\rho_i\frac{(N-1)\beta^2\left[4(2-\beta)+(N-1)\beta(4-\beta)\right]}{4[2+(N-1)\beta]^2(2-\beta)^2}\mathrm{Var}(E_i[\theta|S_i^\delta]) \\
&\quad= \rho_j E_{s_i}\left\{E_{s_j}E_{s_{-ij},m_{-ij}}[x_i^*(s_i;\varnothing,s_j,m_{-ij})^2]\right\} \\
&\qquad +(1-\rho_j)E_{s_i}\left\{E_{s_{-ij},m_{-ij}}\left[x_i^*(s_i;\varnothing,\varnothing,m_{-ij})^2\right]\right\}-c(\delta_i) \\
&\qquad +\rho_i\frac{(N-1)\beta^2\left[4(2-\beta)+(N-1)\beta(4-\beta)\right]}{4[2+(N-1)\beta]^2(2-\beta)^2}\mathrm{Var}(E_i[\theta|S_i^\delta]) \\
&\quad= E_{s_i}\left\{E_{s_{-ij},m_{-ij}}\left[x_i^*(s_i;\varnothing,\varnothing,m_{-ij})^2\right]\right\}+\rho_j\frac{\beta^2}{[2+(N-1)\beta]^2(2-\beta)^2}\mathrm{Var}(E_j[\theta|S_j^\delta]) \\
&\qquad +\rho_i\frac{(N-1)\beta^2\left[4(2-\beta)+(N-1)\beta(4-\beta)\right]}{4[2+(N-1)\beta]^2(2-\beta)^2}\mathrm{Var}(E_i[\theta|S_i^\delta])-c(\delta_i) \\
&\quad= ... = E_{s_i}\left\{x_i^*(s_i;\varnothing,..,\varnothing)^2\right\}+\frac{\beta^2}{[2+(N-1)\beta]^2(2-\beta)^2}\sum_{j\neq i}\rho_j\mathrm{Var}(E_j[\theta|S_j^\delta]) \\
&\qquad\quad +\rho_i\frac{(N-1)\beta^2\left[4(2-\beta)+(N-1)\beta(4-\beta)\right]}{4[2+(N-1)\beta]^2(2-\beta)^2}\mathrm{Var}(E_i[\theta|S_i^\delta])-c(\delta_i) \qquad (31)
\end{aligned}
$$

Proof of Proposition 2 for oligopoly:
Using (16), we can rewrite the first term of (31) as follows:

$$
\begin{aligned}
E_{s_i}\left\{x_i^*(s_i;\varnothing,..,\varnothing)^2\right\} &= \left(\frac{1}{[2+(N-1)\beta]\,(2-\beta)}\right)^2 E_{s_i}\Bigg\{\Bigg((2-\beta)\alpha-[2+(N-2)\beta]\,\overline{\theta_i} \\
&\quad +\beta\sum_{j\neq i}\overline{\theta_j}\ -\frac{1}{2}\left[2+(N-1)\beta\right](2-\beta)\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]\Bigg)^2\Bigg\}
\end{aligned}
$$

$$= \left(\frac{(2-\beta)\alpha - [2+(N-2)\beta]\,\overline{\theta_i} + \beta \sum_{j\neq i} \overline{\theta_j}}{[2+(N-1)\beta]\,(2-\beta)} \right)^2 + \frac{1}{4} E_{s_i} \left\{ \left[E\{\theta_i|s_i\} - \overline{\theta_i} \right]^2 \right\}.$$

In the last simplification, we use the property that $E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]\right\} = 0$. Lemma 1 implies that $E_{s_i}\left\{x_i^*(s_i;\varnothing,..,\varnothing)^2\right\}$ is increasing in δ_i and independent of δ_j.

(i) The first and third terms of (31) are increasing in δ_i, while the second term is independent of δ_i. Hence, $\Pi_i(\delta_i, \delta_j; \rho_i, \rho_j) + c(\delta_i)$ is increasing in δ_i.

(ii) The second term of (31) is (weakly) increasing in δ_j by Lemma 1. The remaining terms are independent of δ_j. Hence, Π_i is weakly increasing in δ_j. ■

Proof of Proposition 3 for oligopoly:

Only the first, third and last terms of (31) depend on δ_i, whereas they do not depend on δ_j, i.e., $\partial^2\Pi_i(\delta_i, \delta_{-i}; \rho_i, \rho_{-i})/(\partial\delta_i\partial\delta_j) = 0$ for $j \neq i$. Hence, firm i's optimal information acquisition investment is independent of a competitor's investment, and this optimal investment always exists.

(i) We want to show that $\partial^2\Pi_i(\delta_i, \delta_{-i}; \rho_i, \rho_{-i})/(\partial\delta_i\partial\rho_j) = 0$ for any $j \neq i$. It follows from (31) that (for any $j \neq i$):

$$\frac{\partial\Pi_i(\delta_i, \delta_{-i}; \rho_i, \rho_{-i})}{\partial\rho_j} = \left(\frac{\beta}{[2+(N-1)\beta](2-\beta)} \right)^2 \left(E_{s_j}\left\{ E\left[\theta_j|s_j\right]^2 \right\} - \overline{\theta_j}^2 \right).$$

As $\partial\Pi_i/\partial\rho_j$ is independent of δ_i, this gives $\partial^2\Pi_i(\delta_i, \delta_{-i}; \rho_i, \rho_{-i})/(\partial\delta_i\partial\rho_j) = 0$ for any $j \neq i$.

(ii) For any $\rho_i, \rho_i' \in [0,1]$, expression (31) gives:

$$\Pi_i(\delta_i, \delta_{-i}; \rho_i, \rho_{-i}) - \Pi_i(\delta_i, \delta_{-i}; \rho_i', \rho_{-i}) = (\rho_i - \rho_i') \frac{(N-1)\beta^2\,[4(2-\beta) + (N-1)\beta(4-\beta)]}{4[2+(N-1)\beta]^2(2-\beta)^2} \mathrm{Var}(E_i[\theta|S_i^\delta]).$$

For $\rho_i > \rho_i'$, this expression is increasing in δ_i by Lemma 1. By Theorem 4 of Milgrom and Shanon (1994), supermodularity of $\Pi_i(\delta_i, \delta_j; \rho_i, \rho_j)$ in (δ_i, ρ_i) implies that $\delta_i^*(\rho_i)$ is increasing in ρ_i. ■

Proof of Proposition 4 for oligopoly:

(i) It follows directly from (21) that (for any $k \neq i$.):

$$\begin{aligned} \frac{\partial\Pi_i(\delta_i, \delta_{-i}; \rho_i, \rho_{-i})}{\partial\rho_i} &= \frac{(N-1)\beta^2\,[4(2-\beta) + (N-1)\beta(4-\beta)]}{4[2+(N-1)\beta]^2(2-\beta)^2} \mathrm{Var}(E_i[\theta|S_i^\delta]) \geq 0, \text{ and} \\ \frac{\partial\Pi_i(\delta_i, \delta_{-i}; \rho_i, \rho_{-i})}{\partial\rho_k} &= \left(\frac{\beta}{[2+(N-1)\beta](2-\beta)} \right)^2 \mathrm{Var}(E_k[\theta|S_k^\delta]) \geq 0 \end{aligned}$$

(ii) The proof of this part is analogous to the proof of Proposition 4(ii). ■

Proof of Proposition 5 for oligopoly:

We obtain the representative consumer's net surplus $v(x_1, .., x_N)$ by subtracting the consumer's expenditures from the gross surplus $u(x_1, .., x_N)$ in (15):

$$v(x_1, .., x_N) \equiv u(x_1, .., x_N) - \sum_{i=1}^{N} P_i(x_i, x_{-i})x_i = \frac{1}{2} \left[\left(\sum_{\ell=1}^{N} x_\ell \right)^2 - (1-\beta) \sum_{\ell=1}^{N} x_\ell \sum_{k\neq\ell} x_k \right]. \quad (32)$$

After substitution of the equilibrium output levels in the expected consumer surplus (32), we obtain the following (by slightly abusing notation):

$$\begin{aligned} V(\boldsymbol{\delta},\boldsymbol{\rho}) &\equiv E_{s_i,m_i}\left\{E_{s_{-i},m_{-i}}\left[v\left(x_1^*(s_1;m_1,m_{-1}),..,x_N^*(s_N;m_N,m_{-N})\right)\right]\right\} \\ &= \frac{1}{2}E_{s_i,m_i}\Bigg\{E_{s_{-i},m_{-i}}\Bigg[\Bigg(x_i^*(s_i;m_i,m_{-i})+\sum_{j\neq i}x_j^*(s_j;m_{-i},m_i)\Bigg)^2 \\ &\qquad -(1-\beta)x_i^*(s_i;m_i,m_{-i})\sum_{j\neq i}x_j^*(s_j;m_{-i},m_i) \\ &\qquad -(1-\beta)\sum_{j\neq i}x_j^*(s_j;m_{-i},m_i)\Bigg(x_i^*(s_i;m_i,m_{-i})+\sum_{h\neq i,j}x_h^*(s_h;m_{-i},m_i)\Bigg)\Bigg]\Bigg\} \end{aligned} \tag{33}$$

The expected consumer surplus (33) can be rewritten as follows (for $i,j,h=1,..,N$):

$$\begin{aligned} V(\cdot) &= \rho_i\frac{1}{2}E_{s_i}\Bigg\{E_{s_{-i},m_{-i}}\Bigg[\Bigg(x_i^*(s_i;s_i,m_{-i})+\sum_{j\neq i}x_j^*(s_j;m_{-i},s_i)\Bigg)^2 \\ &\qquad -(1-\beta)x_i^*(s_i;s_i,m_{-i})\sum_{j\neq i}x_j^*(s_j;m_{-i},s_i) \\ &\qquad -(1-\beta)\sum_{j\neq i}x_j^*(s_j;m_{-i},s_i)\Bigg(x_i^*(s_i;s_i,m_{-i})+\sum_{h\neq i,j}x_h^*(s_h;m_{-i},s_i)\Bigg)\Bigg]\Bigg\} \\ &\quad +(1-\rho_i)\frac{1}{2}E_{s_i}\Bigg\{E_{s_{-i},m_{-i}}\Bigg[\Bigg(x_i^*(s_i;\varnothing,m_{-i})+\sum_{j\neq i}x_j^*(s_j;m_{-i},\varnothing)\Bigg)^2 \\ &\qquad -(1-\beta)x_i^*(s_i;\varnothing,m_{-i})\sum_{j\neq i}x_j^*(s_j;m_{-i},\varnothing) \\ &\qquad -(1-\beta)\sum_{j\neq i}x_j^*(s_j;m_{-i},\varnothing)\Bigg(x_i^*(s_i;\varnothing,m_{-i})+\sum_{h\neq i,j}x_h^*(s_h;m_{-i},\varnothing)\Bigg)\Bigg]\Bigg\} \end{aligned}$$

Differentiating with respect to ρ_i gives (for $i,j=1,..,N$ and $i\neq j$):

$$\begin{aligned} \frac{\partial V}{\partial\rho_i} &= \frac{1}{2}E_{s_i}\Bigg\{E_{s_{-i},m_{-i}}\Bigg[\Bigg(x_i^*(s_i;s_i,m_{-i})+\sum_{j\neq i}x_j^*(s_j;m_{-i},s_i)\Bigg)^2 \\ &\qquad -\Bigg(x_i^*(s_i;\varnothing,m_{-i})+\sum_{j\neq i}x_j^*(s_j;m_{-i},\varnothing)\Bigg)^2 \\ &\quad -(1-\beta)\Bigg(x_i^*(s_i;s_i,m_{-i})\sum_{j\neq i}x_j^*(s_j;m_{-i},s_i)-x_i^*(s_i;\varnothing,m_{-i})\sum_{j\neq i}x_j^*(s_j;m_j,\varnothing)\Bigg) \end{aligned}$$

$$- (1-\beta)\left(\sum_{j\neq i} x_j^*(s_j;m_{-i},s_i)\left(x_i^*(s_i;s_i,m_{-i}) + \sum_{h\neq i,j} x_h^*(s_h;m_{-i},s_i)\right)\right.$$

$$\left.\left.\left. - \sum_{j\neq i} x_j^*(s_j;m_{-i},\varnothing)\left(x_i^*(s_i;\varnothing,m_{-i}) + \sum_{h\neq i,j} x_h^*(s_h;m_{-i},\varnothing)\right)\right)\right]\right\} \quad (34)$$

The first two lines of this expression can be simplified by using the following:

$$E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[\left(x_i^*(s_i;\varnothing,m_{-i}) + \textstyle\sum_{j\neq i} x_j^*(s_j;m_{-i},\varnothing)\right)^2\right]\right\}$$

$$\begin{aligned}
&= E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[\left(x_i^*(s_i;s_i,m_{-i}) + \sum_{j\neq i} x_j^*(s_j;m_{-i},s_i) - \frac{(N-1)\beta\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]}{2[2+(N-1)\beta]}\right)^2\right]\right\} \\
&= E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[\left(x_i^*(s_i;s_i,m_{-i}) + \textstyle\sum_{j\neq i} x_j^*(s_j;m_{-i},s_i)\right)^2\right]\right\} \\
&\quad - \frac{(N-1)\beta}{2[2+(N-1)\beta]} E_{s_i}\Big\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right] \\
&\quad\quad * E_{s_{-i},m_{-i}}\left[2\left(x_i^*(s_i;s_i,m_{-i}) + \textstyle\sum_{j\neq i} x_j^*(s_j;m_{-i},s_i)\right) - \frac{(N-1)\beta\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]}{2[2+(N-1)\beta]}\right]\Big\} \\
&= E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[\left(x_i^*(s_i;s_i,m_{-i}) + \textstyle\sum_{j\neq i} x_j^*(s_j;m_{-i},s_i)\right)^2\right]\right\} \\
&\quad + \frac{(N-1)\beta[4+(N-1)\beta]}{4[2+(N-1)\beta]^2} E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]^2\right\} \qquad (35)
\end{aligned}$$

The third line of (34) can be simplified by using the following:

$$E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[x_i^*(s_i;\varnothing,m_{-i}) \textstyle\sum_{j\neq i} x_j^*(s_j;m_{-i},\varnothing)\right]\right\}$$

$$\begin{aligned}
&= E_{s_i}\Big\{E_{s_{-i},m_{-i}}\Big[\left(x_i^*(s_i;s_i,m_{-i}) + \frac{(N-1)\beta^2\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]}{2(2-\beta)[2+(N-1)\beta]}\right) \\
&\quad\quad * \left(\textstyle\sum_{j\neq i} x_j^*(s_j;m_{-i},s_i) - \frac{(N-1)\beta\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]}{(2-\beta)[2+(N-1)\beta]}\right)\Big]\Big\} \\
&= E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[x_i^*(s_i;s_i,m_{-i}) \textstyle\sum_{j\neq i} x_j^*(s_j;m_{-i},s_i)\right]\right\} \\
&\quad + E_{s_i}\left\{\frac{(N-1)\beta\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]}{(2-\beta)[2+(N-1)\beta]} E_{s_{-i},m_{-i}}\left[\frac{\beta}{2}\textstyle\sum_{j\neq i} x_j^*(s_j;m_{-i},\varnothing) - x_i^*(s_i;s_i,m_{-i})\right]\right\} \\
&= E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[x_i^*(s_i;s_i,m_{-i}) \textstyle\sum_{j\neq i} x_j^*(s_j;m_{-i},s_i)\right]\right\} \\
&\quad + \frac{(N-1)\beta\,[2+(N-2)\beta]}{(2-\beta)^2[2+(N-1)\beta]^2} E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]^2\right\} \qquad (36)
\end{aligned}$$

Finally, the last two lines of (34) can be simplified by using the following:

$$E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[x_j^*(s_j;m_{-i},\varnothing)\left(x_i^*(s_i;\varnothing,m_{-i})+\sum_{h\neq i,j}x_h^*(s_h;m_{-i},\varnothing)\right)\right]\right\}$$

$$\begin{aligned}
&= E_{s_i}\Bigg\{E_{s_{-i},m_{-i}}\Bigg[\left(x_j^*(s_j;m_{-i},s_i)-\frac{\beta\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{(2-\beta)[2+(N-1)\beta]}\right) \\
&\quad *\left(x_i^*(s_i;s_i,m_{-i})+\sum_{h\neq i,j}x_h^*(s_h;m_{-i},s_i)+\frac{\left[(N-1)\frac{\beta}{2}-(N-2)\right]\beta\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{(2-\beta)[2+(N-1)\beta]}\right)\Bigg]\Bigg\} \\
&= E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[x_j^*(s_j;m_{-i},s_i)\left(x_i^*(s_i;s_i,m_{-i})+\sum_{h\neq i,j}x_h^*(s_h;m_{-i},s_i)\right)\right]\right\} \\
&\quad +E_{s_i}\Bigg\{\frac{\beta\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{(2-\beta)[2+(N-1)\beta]}E_{s_{-i},m_{-i}}\Bigg[\left((N-1)\frac{\beta}{2}-(N-2)\right)x_j^*(s_j;m_{-i},s_i) \\
&\qquad -x_i^*(s_i;s_i,m_{-i})-\frac{(N-1)\frac{\beta^2}{2}\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{(2-\beta)[2+(N-1)\beta]}-\sum_{h\neq i,j}x_h^*(s_h;m_{-i},\varnothing)\Bigg]\Bigg\} \\
&= E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[x_j^*(s_j;m_{-i},s_i)\left(x_i^*(s_i;s_i,m_{-i})+\sum_{h\neq i,j}x_h^*(s_h;m_{-i},s_i)\right)\right]\right\} \\
&\quad +\frac{2\beta}{(2-\beta)^2[2+(N-1)\beta]^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\} \qquad (37)
\end{aligned}$$

Substitution of (35), (36) and (37) in (34) gives:

$$\begin{aligned}
\frac{\partial V}{\partial\rho_i} &= -\frac{1}{2}\Bigg(\frac{(N-1)\beta[4+(N-1)\beta]}{4[2+(N-1)\beta]^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\} \\
&\qquad -(1-\beta)\frac{(N-1)\beta\,[2+(N-2)\beta]}{(2-\beta)^2[2+(N-1)\beta]^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\} \\
&\qquad -(1-\beta)\frac{2(N-1)\beta}{(2-\beta)^2[2+(N-1)\beta]^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}\Bigg) \\
&= \frac{-(N-1)\beta E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}}{2(2-\beta)^2[2+(N-1)\beta]^2}\left[\left(1-\frac{\beta}{2}\right)^2[4+(N-1)\beta]-(1-\beta)\,[4+(N-2)\beta]\right] \\
&= \frac{-(N-1)\beta^2\left[1+\frac{1}{4}(N-1)\beta^2\right]}{2(2-\beta)^2[2+(N-1)\beta]^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}<0.
\end{aligned}$$

To prove that the expected surplus is increasing in δ_i, it is sufficient to show that all terms of V are increasing in δ_i. First, we show the first term of V is increasing in δ_i by rewriting it as follows:

$$\begin{aligned}
&E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[\left(x_i^*(s_i;s_i,m_{-i})+\sum_{j\neq i}x_j^*(s_j;m_{-i},s_i)\right)^2\right]\right\} \\
&\quad = \frac{1}{[2+(N-1)\beta]^2}E_{s_i}\Bigg\{E_{s_{-i},m_{-i}}\Bigg[\Bigg(N\alpha-E\{\theta_i|s_i\}-\sum_{j\neq i}E\{\theta_j|s_j\} \\
&\qquad -(N-1)\frac{\beta^2}{2}\sum_{j\neq i}[E\{\theta_j|s_j\}-E\{\theta_j|m_j\}]\Bigg)^2\Bigg]\Bigg\}
\end{aligned}$$

$$= \frac{1}{[2+(N-1)\beta]^2} E_{s_i} \left\{ E_{s_{-i},m_{-i}} \left[\left(N\alpha - \sum_{j\neq i} \left(E\{\theta_j|s_j\} - (N-1)\frac{\beta^2}{2} \left[E\{\theta_j|s_j\} - E\{\theta_j|m_j\} \right] \right) \right)^2 \right.\right.$$

$$\left.\left. -2 \left(N\alpha - \sum_{j\neq i} \left(E\{\theta_j|s_j\} - (N-1)\frac{\beta^2}{2} \left[E\{\theta_j|s_j\} - E\{\theta_j|m_j\} \right] \right) \right) E\{\theta_i|s_i\} + E\{\theta_i|s_i\}^2 \right] \right\}$$

Notice that only the last term depends on δ_i (i.e., $E_{s_i}\{E\left[\theta_i|s_i\right]^2\}$), and is increasing in δ_i. This immediately implies that $\frac{1}{2}E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[\left(x_i^*(s_i;s_i,m_{-i}) + \sum_{j\neq i} x_j^*(s_j;m_{-i},s_i)\right)^2\right]\right\}$ is increasing in δ_i. Similarly, it is easy to show that the second and third terms of V are increasing in δ_i. It is straightforward to show that the remaining terms are also increasing in δ_i, by using the decompositions (35), (36) and (37) in combination with the observation that the first three terms of V are increasing in δ_i. This proves that $\partial V/\partial\delta_i > 0$. ■

B.3 Bertrand Competition

Before we derive results with Bertrand competition, it is convenient to rewrite the expected profit (18). We define the equilibrium price-cost margin as $P_i^*(s_i, m_i, m_j) \equiv p_i^*(s_i; m_i, m_j) - E\{\theta_i|s_i\}$, with the equilibrium price p_i^* as in (17). First, we rewrite $E_{s_i}\left\{E_{s_j,m_j}[P_i^*(s_i;\varnothing,m_j)^2]\right\}$ as follows:

$$E_{s_i}\left\{E_{s_j,m_j}[P_i^*(s_i;\varnothing,m_j)^2]\right\} = E_{s_i}\left\{E_{s_j,m_j}\left[\left(P_i^*(s_i;s_i,m_j) - \frac{\beta^2}{2(4-\beta^2)}\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]\right)^2\right]\right\}$$

$$\begin{aligned}
&= E_{s_i}\left\{E_{s_j,m_j}\left[P_i^*(s_i;s_i,m_j)^2\right]\right\} \\
&\quad -\frac{\beta^2}{2(4-\beta^2)}E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]E_{s_j,m_j}\left[2P_i^*(s_i;s_i,m_j) - \frac{\beta^2}{2(4-\beta^2)}\left(E\{\theta_i|s_i\} - \overline{\theta_i}\right)\right]\right\} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[P_i^*(s_i;s_i,m_j)^2\right]\right\} \\
&\quad -\frac{\beta^2}{4(4-\beta^2)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]\left[4(2+\beta)(1-\beta)\alpha + 4\beta\overline{\theta_j} + \beta^2\overline{\theta_i} - (8-3\beta^2)E\{\theta_i|s_i\}\right]\right\} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[P_i^*(s_i;s_i,m_j)^2\right]\right\} + \frac{\beta^2(8-3\beta^2)}{4(4-\beta^2)^2}\left(E_{s_i}\left\{E\left[\theta_i|s_i\right]^2\right\} - \overline{\theta_i}^2\right) \qquad (38)
\end{aligned}$$

In this last simplification, we use the property that $E_{s_i}\left\{E_{s_j,m_j}[E\{\theta_i|s_i\} - \overline{\theta_i}]\right\} = 0$. Second, by following essentially the same steps as in (20), we rewrite $E_{s_i,m_i}\left\{P_i^*(s_i;m_i,\varnothing)^2\right\}$ as follows:

$$E_{s_i,m_i}\{P_i^*(s_i;m_i,\varnothing)^2\} = E_{s_i,m_i}\left\{E_{s_j}[P_i^*(s_i;m_i,s_j)^2]\right\} - \left(\frac{\beta}{4-\beta^2}\right)^2\left(E_{s_j}\left\{E\left[\theta_j|s_j\right]^2\right\} - \overline{\theta_j}^2\right) \qquad (39)$$

Using (38) and (39), the expected profit (18) simplifies as follows:

$$\Pi_i^b(\delta_i,\delta_j;\rho_i,\rho_j) = \frac{1}{1-\beta^2}E_{s_i}\left\{E_{s_j,m_j}\left[\rho_i P_i^*(s_i;s_i,m_j)^2 + (1-\rho_i)P_i^*(s_i;\varnothing,m_j)^2\right]\right\} - c(\delta_i)$$

$$
\begin{aligned}
&= \frac{E_{s_i}\left\{E_{s_j,m_j}[P_i^*(s_i;\varnothing,m_j)^2]\right\}}{1-\beta^2} - \rho_i\frac{\beta^2(8-3\beta^2)}{4(4-\beta^2)^2(1-\beta^2)}\mathrm{Var}(E_i[\theta|S_i^\delta]) - c(\delta_i) \\
&= \rho_j\frac{E_{s_i}\left\{E_{s_j}[P_i^*(s_i;\varnothing,s_j)^2]\right\}}{1-\beta^2} + (1-\rho_j)\frac{E_{s_i}\left\{P_i^*(s_i;\varnothing,\varnothing)^2\right\}}{1-\beta^2} \\
&\quad -\rho_i\frac{\beta^2(8-3\beta^2)}{4(4-\beta^2)^2(1-\beta^2)}\mathrm{Var}(E_i[\theta|S_i^\delta]) - c(\delta_i) \\
&= \frac{E_{s_i}\left\{P_i^*(s_i;\varnothing,\varnothing)^2\right\}}{1-\beta^2} + \rho_j\left(\frac{\beta}{4-\beta^2}\right)^2\frac{1}{1-\beta^2}\mathrm{Var}(E_j[\theta|S_j^\delta]) \\
&\quad -\rho_i\frac{\beta^2\left(8-3\beta^2\right)}{4\left(4-\beta^2\right)^2\left(1-\beta^2\right)}\mathrm{Var}(E_i[\theta|S_i^\delta]) - c(\delta_i). \qquad (40)
\end{aligned}
$$

Finally, by using (17), we can rewrite the first term of (40) as follows:

$$
\begin{aligned}
E_{s_i}\left\{P_i^*(s_i;\varnothing,\varnothing)^2\right\} &= E_{s_i}\left\{\left(\frac{(2+\beta)(1-\beta)\alpha-(2-\beta^2)\overline{\theta_i}+\beta\overline{\theta_j}}{4-\beta^2} - \frac{1}{2}\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]\right)^2\right\} \\
&= \left(\frac{(2+\beta)(1-\beta)\alpha-(2-\beta^2)\overline{\theta_i}+\beta\overline{\theta_j}}{4-\beta^2}\right)^2 + \frac{1}{4}\mathrm{Var}(E_i[\theta|S_i^\delta]). \qquad (41)
\end{aligned}
$$

In the last simplification, we use the property that $E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]\right\}=0$.

Proof of Proposition 2 with Bertrand competition:

(i) Recall that Lemma 1 gives that $\mathrm{Var}(E_i[\theta|S_i^\delta])$ is increasing in δ_i. By substituting (41) in (40), we obtain the following:

$$
\begin{aligned}
\Pi_i^b(\delta_i,\delta_j;\rho_i,\rho_j)+c(\delta_i) &= \frac{\left[(2+\beta)(1-\beta)\alpha-(2-\beta^2)\overline{\theta_i}+\beta\overline{\theta_j}\right]^2}{\left(4-\beta^2\right)^2\left(1-\beta^2\right)} + \rho_j\left(\frac{\beta}{4-\beta^2}\right)^2\frac{\mathrm{Var}(E_j[\theta|S_j^\delta])}{1-\beta^2} \\
&\quad + \frac{1}{4(1-\beta^2)}\left(1-\rho_i\frac{\beta^2\left(8-3\beta^2\right)}{\left(4-\beta^2\right)^2}\right)\mathrm{Var}(E_i[\theta|S_i^\delta])
\end{aligned}
$$

Clearly, only the last term depends on δ_i, and it is increasing in δ_i, since

$$
1-\rho_i\frac{\beta^2\left(8-3\beta^2\right)}{\left(4-\beta^2\right)^2} \geq 1-\frac{\beta^2\left(8-3\beta^2\right)}{\left(4-\beta^2\right)^2} = \frac{2\left(8-5\beta^2+2\beta^4\right)}{\left(4-\beta^2\right)^2} > 0
$$

This implies that $\Pi_i^b(\delta_i,\delta_j;\rho_i,\rho_j)+c(\delta_i)$ is weakly increasing in δ_i.
(ii) The second term of (40) is (weakly) increasing in δ_j by Lemma 1. The remaining terms are independent of δ_j. Hence, Π_i^b is weakly increasing in δ_j. ■

Proof of Proposition 7:

Only the first, third and last terms of (40) depend on δ_i, whereas they do not depend on δ_j, i.e., $\partial^2\Pi_i^b(\delta_i,\delta_j;\rho_i,\rho_j)/(\partial\delta_i\partial\delta_j)=0$. Hence, firm i's optimal information acquisition investment is independent of the competitor's investment, δ_j, and this optimal investment always exists.
(i) We want to show that $\partial^2\Pi_i^b(\delta_i,\delta_j;\rho_i,\rho_j)/(\partial\delta_i\partial\rho_j)=0$. It follows from (40) that:

$$
\frac{\partial\Pi_i^b(\delta_i,\delta_j;\rho_i,\rho_j)}{\partial\rho_j} = \left(\frac{\beta}{4-\beta^2}\right)^2\frac{1}{1-\beta^2}\mathrm{Var}(E_j[\theta|S_j^\delta]).
$$

As $\partial\Pi_i^b/\partial\rho_j$ is independent of δ_i, we have $\frac{\partial^2\Pi_i^b(\delta_i,\delta_j;\rho_i,\rho_j)}{\partial\delta_i\partial\rho_j} = 0$ which concludes the proof.
(ii) First, we show that $\Pi_i^b(\delta_i, \delta_j; \rho_i, \rho_j)$ is submodular in (δ_i, ρ_i). For any $\rho_i, \rho_i' \in [0, 1]$, equation (40) gives:

$$\Pi_i^b(\delta_i, \delta_j; \rho_i, \rho_j) - \Pi_i^b(\delta_i, \delta_j; \rho_i', \rho_j) = -(\rho_i - \rho_i')\frac{\beta^2 \left(8 - 3\beta^2\right)}{4\left(4 - \beta^2\right)^2 \left(1 - \beta^2\right)}\text{Var}(E_i[\theta|S_i^\delta])$$

For $\rho_i > \rho_i'$, this expression is decreasing in δ_i by Lemma 1 (see Appendix A). Second, result (ii) follows from Theorem 4 of Milgrom and Shanon (1994) and the submodularity of Π_i^b. ■

Proof of Proposition 8:
(i) It follows directly from (40) that:

$$\begin{aligned}\frac{\partial\Pi_i^b(\delta_i, \delta_j; \rho_i, \rho_j)}{\partial\rho_i} &= -\left(\frac{\beta}{4 - \beta^2}\right)^2 \frac{1}{1 - \beta^2} \cdot \frac{8 - 3\beta^2}{4}\text{Var}(E_i[\theta|S_i^\delta]) \leq 0, \text{ and} \\ \frac{\partial\Pi_j^b(\delta_j, \delta_i; \rho_j, \rho_i)}{\partial\rho_i} &= \left(\frac{\beta}{4 - \beta^2}\right)^2 \frac{1}{1 - \beta^2}\text{Var}(E_i[\theta|S_i^\delta]) \geq 0.\end{aligned}$$

For the effect of ρ_i on the industry profit, we add up these two expressions, which gives:

$$\frac{\partial\left(\Pi_i^b + \Pi_j^b\right)}{\partial\rho_i} = -\left(\frac{\beta}{4 - \beta^2}\right)^2 \frac{1}{1 - \beta^2} \cdot \frac{4 - 3\beta^2}{4}\text{Var}(E_i[\theta|S_i^\delta]) \leq 0.$$

Hence, both firm i's individual profit, and the industry profits are weakly decreasing in ρ_i.
(ii) The proof of this part follows immediately from the argument in section 6.3.2. ■

Proof of Proposition 5 with Bertrand competition:
The proof is analogous to the original proof (with Cournot competition). If the firms compete in prices, firm i's equilibrium output level relates as follows to the equilibrium price-cost margin:

$$x_i^b(s_i; m_i, m_j) = \frac{p_i^*(s_i; m_i, m_j) - E\{\theta_i|s_i\}}{1 - \beta^2} \tag{42}$$

Differentiating the expected surplus V with respect to ρ_i gives (23) for $i, j = 1, 2$ and $i \neq j$. The first line of this expression can be simplified by using (17) and (42) for $m_j \in \{s_j, \varnothing\}$:

$$\begin{aligned}&E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^b(s_i;\varnothing, m_j) + x_j^b(s_j; m_j, \varnothing)\right)^2\right]\right\} \\ &= E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^b(s_i; s_i, m_j) + x_j^b(s_j; m_j, s_i) - \frac{\beta(2+\beta)}{2(4-\beta^2)(1-\beta^2)}\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]\right)^2\right]\right\} \\ &= E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^b(s_i; s_i, m_j) + x_j^b(s_j; m_j, s_i)\right)^2\right]\right\} \\ &\quad -\frac{\beta}{2(2-\beta)(1-\beta^2)}E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]E_{s_j,m_j}\left[2\left(x_i^b(s_i; s_i, m_j) + x_j^b(s_j; m_j, s_i)\right) - \frac{\beta\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]}{2(2-\beta)(1-\beta^2)}\right]\right\} \\ &= E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^b(s_i; s_i, m_j) + x_j^b(s_j; m_j, s_i)\right)^2\right]\right\} + \frac{\beta(8-2\beta-3\beta^2)}{4(2-\beta)(4-\beta^2)(1-\beta^2)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]^2\right\}\end{aligned} \tag{43}$$

The second line of (23) can be simplified by using (17) and (42) for $m_j \in \{s_j, \varnothing\}$:

$$
\begin{aligned}
&E_{s_i}\left\{E_{s_j,m_j}\left[x_i^b(s_i;\varnothing,m_j)x_j^b(s_j;m_j,\varnothing)\right]\right\} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^b(s_i;s_i,m_j) - \frac{\beta^2\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{2(4-\beta^2)(1-\beta^2)}\right)\left(x_j^b(s_j;m_j,s_i) - \frac{\beta\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{(4-\beta^2)(1-\beta^2)}\right)\right]\right\} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[x_i^b(s_i;s_i,m_j)x_j^b(s_j;m_j,s_i)\right]\right\} - E_{s_i}\left\{\frac{\beta^2\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{2(4-\beta^2)(1-\beta^2)}E_{s_j,m_j}\left[x_j^b(s_j;m_j,s_i)\right]\right\} \\
&\quad -E_{s_i}\left\{\frac{\beta\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{(4-\beta^2)(1-\beta^2)}E_{s_j,m_j}\left[x_i^b(s_i;s_i,m_j)\right]\right\} + \frac{\beta^3 E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}}{2(4-\beta^2)^2(1-\beta^2)} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[x_i^b(s_i;s_i,m_j)x_j^b(s_j;m_j,s_i)\right]\right\} + \frac{\beta(2-\beta^2)}{(4-\beta^2)^2(1-\beta^2)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\} \qquad (44)
\end{aligned}
$$

Substitution of (43) and (44) for $m_j \in \{s_j, \varnothing\}$ in (23) gives:

$$
\begin{aligned}
\frac{\partial V}{\partial \rho_i} &= -\left(\frac{\beta(8-2\beta-3\beta^2)}{8(2-\beta)(4-\beta^2)(1-\beta^2)^2} - (1-\beta)\frac{\beta(2-\beta^2)}{(4-\beta^2)^2(1-\beta^2)^2}\right)E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\} \\
&= \frac{-\beta}{8(4-\beta^2)^2(1-\beta^2)^2}\left((2+\beta)(8-2\beta-3\beta^2) - 8(1-\beta)(2-\beta^2)\right)E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\} \\
&= \frac{-\beta^2(20-11\beta^2)}{8(4-\beta^2)^2(1-\beta^2)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\} < 0.
\end{aligned}
$$

To prove that the expected surplus is increasing in δ_i, it is sufficient to show that all terms of V are increasing in δ_i. First, we show the first term of V is increasing in δ_i by rewriting its first component as follows (by using (17) and (42)):

$$
\begin{aligned}
&\tfrac{1}{2}E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^b(s_i;s_i,m_j)+x_j^b(s_j;m_j,s_i)\right)^2\right]\right\} \\
&= \frac{1}{2}\frac{(2+\beta)^2}{(4-\beta^2)^2(1-\beta^2)^2}E_{s_i}\left\{E_{s_j,m_j}\left[\left((1-\beta)\left[2\alpha - E\{\theta_i|s_i\} - E\{\theta_j|s_j\}\right]\right.\right.\right. \\
&\qquad\qquad \left.\left.\left. -\frac{\beta}{2}\left[E\{\theta_j|s_j\} - E\{\theta_j|m_j\}\right]\right)^2\right]\right\} \\
&= \frac{1}{2(2-\beta)^2(1-\beta^2)^2}E_{s_i}\left\{E_{s_j,m_j}\left[\left((1-\beta)\left[2\alpha - E\{\theta_j|s_j\}\right] - \frac{\beta}{2}\left[E\{\theta_j|s_j\} - E\{\theta_j|m_j\}\right]\right)^2\right.\right. \\
&\quad \left.\left. -2\left((1-\beta)\left[2\alpha - E\{\theta_j|s_j\}\right] - \frac{\beta}{2}\left[E\{\theta_j|s_j\} - E\{\theta_j|m_j\}\right]\right)(1-\beta)E\{\theta_i|s_i\} + (1-\beta)^2 E\{\theta_i|s_i\}^2\right]\right\}
\end{aligned}
$$

Notice that only the last term depends on δ_i (i.e., $E_{s_i}\{E\left[\theta_i|s_i\right]^2\}$), and is increasing in δ_i. This immediately implies that $\frac{1}{2}E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^b(s_i;s_i,m_j)+x_j^b(s_j;m_j,s_i)\right)^2\right]\right\}$ is increasing in δ_i. By using (17) and (42), the remaining component of the first term equals can be written as:

$$(1-\beta)E_{s_i}\left\{E_{s_{j,m_j}}\left[x_i^b(s_i;s_i,m_j)x_j^b(s_j;m_j,s_i)\right]\right\}$$

$$= \frac{1-\beta}{(4-\beta^2)^2(1-\beta^2)^2}E_{s_i}\Big\{E_{s_j,m_j}\Big[\Big((2+\beta)(1-\beta)\alpha-(2-\beta^2)E\{\theta_i|s_i\}+\beta E\{\theta_j|m_j\}\Big) * \Big((2+\beta)(1-\beta)\alpha-(2-\beta^2)E\{\theta_j|s_j\}+\beta E\{\theta_i|s_i\}-\frac{\beta}{2}\left[E\{\theta_j|s_j\}-E\{\theta_j|m_j\}\right]\Big)\Big]\Big\}$$

$$= \frac{1-\beta}{(4-\beta^2)^2(1-\beta^2)^2}E_{s_i}\Big\{E_{s_j,m_j}\Big[\Big((2+\beta)(1-\beta)\alpha-(2-\beta^2)E\{\theta_i|s_i\}+\beta E\{\theta_j|m_j\}\Big) * \Big((2+\beta)(1-\beta)\alpha-(2-\beta^2)E\{\theta_j|s_j\}-\frac{\beta}{2}\left[E\{\theta_j|s_j\}-E\{\theta_j|m_j\}\right]\Big)\Big]\Big\}$$
$$+ \frac{\beta(1-\beta)}{(4-\beta^2)^2(1-\beta^2)^2}E_{s_i}\Big\{E_{s_j,m_j}\Big[E\{\theta_i|s_i\}\Big((2+\beta)(1-\beta)\alpha+\beta E\{\theta_j|m_j\}\Big)-(2-\beta^2)E\{\theta_i|s_i\}^2\Big]\Big\}$$

Again, only the last term depends on δ_i (i.e., $E_{s_i}\{E\left[\theta_i|s_i\right]^2\}$), and this makes the second component of the first term increasing in δ_i. We can obtain similar results for the second term of V. ■

B.4 Noisy Messages

For simplicity, we assume that each firm receives a perfect signal about its cost (i.e., firm i learns its cost θ_i for $i = 1, 2$). Therefore, the information acquisition stage of the game (stage 4) is no longer relevant here. We modify stage 2 of the game by letting each firm send a noisy message about its cost θ_i to its competitor. In stage 2, each firm chooses the message precision, μ_i for firm i, whereas each firm chose the probability of information sharing in the original specification in Section 2.3. That is, firm i sends to its competitor some message m_i, which is the realization of a random variable with precision μ_i. Clearly, the firms' messages are independently distributed, since the costs (θ_1, θ_2) are independently distributed.

At the product market stage, firm i observes the realizations of the noisy messages, (m_i, m_j), in addition to the firm's own cost θ_i. This gives the following first-order condition for firm i's output choice (for $i, j = 1, 2$ and $i \neq j$):

$$x_i(\theta_i) = \frac{1}{2}\Big(\alpha-\theta_i-\beta E\{x_j(\theta_j)|m_j\}\Big)$$

Solving for the equilibrium output levels gives (for $i, j = 1, 2$ and $i \neq j$):

$$x_i^o(\theta_i;m_i,m_j) = \frac{1}{4-\beta^2}\left((2-\beta)\alpha-2\theta_i+\beta E\{\theta_j|m_j\}+\frac{\beta^2}{2}\left[\theta_i-E\{\theta_i|m_i\}\right]\right)$$

Firm i's expected profit can be rewritten as follows:

$$\begin{aligned}
\Pi_i^o(\mu_i,\mu_j) &\equiv E_{\theta_i,\mu_i}\left\{E_{\mu_j}[x_i^o(s_i;m_i,m_j)^2]\right\} \\
&= E_{\theta_i,\mu_i}\left\{E_{\mu_j}\left[\left(x_i^o(\theta_i;m_i,\overline{\theta_j})+\frac{\beta}{4-\beta^2}\left[E\{\theta_j|m_j\}-\overline{\theta_j}\right]\right)^2\right]\right\} \\
&= E_{\theta_i,\mu_i}\left\{x_i^o(\theta_i;m_i,\overline{\theta_j})^2\right\}+\left(\frac{\beta}{4-\beta^2}\right)^2E_{\mu_j}\left\{\left[E\{\theta_j|m_j\}-\overline{\theta_j}\right]^2\right\}
\end{aligned}$$

$$
\begin{aligned}
&= E_{\theta_i,\mu_i}\left\{\frac{1}{(4-\beta^2)^2}\left((2-\beta)\alpha-\frac{4-\beta^2}{2}\theta_i+\beta\overline{\theta_j}-\frac{\beta^2}{2}E\{\theta_i|m_i\}\right)^2\right\} \\
&\quad+\left(\frac{\beta}{4-\beta^2}\right)^2 E_{\mu_j}\left\{\left[E\{\theta_j|m_j\}-\overline{\theta_j}\right]^2\right\} \\
&= \frac{1}{(4-\beta^2)^2}E_{\theta_i,\mu_i}\left\{\left((2-\beta)\alpha-2\overline{\theta_i}+\beta\overline{\theta_j}-\frac{4-\beta^2}{2}\left(\theta_i-\overline{\theta_i}\right)-\frac{\beta^2}{2}\left[E\{\theta_i|m_i\}-\overline{\theta_i}\right]\right)^2\right\} \\
&\quad+\left(\frac{\beta}{4-\beta^2}\right)^2 E_{\mu_j}\left\{\left[E\{\theta_j|m_j\}-\overline{\theta_j}\right]^2\right\} \\
&= \frac{1}{(4-\beta^2)^2}\left((2-\beta)\alpha-2\overline{\theta_i}+\beta\overline{\theta_j}\right)^2+\left(\frac{\beta}{4-\beta^2}\right)^2 E_{\mu_j}\left\{\left[E\{\theta_j|m_j\}-\overline{\theta_j}\right]^2\right\} \\
&\quad+\frac{1}{4\left(4-\beta^2\right)^2}E_{\theta_i,\mu_i}\left\{\left(\left(4-\beta^2\right)\left(\theta_i-\overline{\theta_i}\right)+\beta^2\left[E\{\theta_i|m_i\}-\overline{\theta_i}\right]\right)^2\right\} \qquad (45)
\end{aligned}
$$

The first term of (45) is constant in the precisions of firms' messages (μ_i, μ_j). The second term of (45) is proportional to the variance of firm j's conditionally expected cost, $E_{\mu_j}\left\{\left[E\{\theta_j|m_j\}-\overline{\theta_j}\right]^2\right\}$. This variance is increasing in the precision of firm j's message, μ_j. We can decompose the last term of (45) as follows:

$$
\begin{aligned}
E_{\theta_i,\mu_i}\left\{\left(\left(4-\beta^2\right)\left(\theta_i-\overline{\theta_i}\right)+\beta^2\left[E\{\theta_i|m_i\}-\overline{\theta_i}\right]\right)^2\right\} &= \left(4-\beta^2\right)^2 E_{\theta_i}\left\{\left(\theta_i-\overline{\theta_i}\right)^2\right\} \qquad (46) \\
+\,2\beta^2\left(4-\beta^2\right)E_{\theta_i,\mu_i}\left\{\left(\theta_i-\overline{\theta_i}\right)\left[E\{\theta_i|m_i\}-\overline{\theta_i}\right]\right\}&+\beta^4 E_{\theta_i,\mu_i}\left\{\left[E\{\theta_i|m_i\}-\overline{\theta_i}\right]^2\right\}
\end{aligned}
$$

The first term of (46) is the variance of the firm's cost. This variance does not depend on the precision of the firm's message. The last term term of (46) is the variance of the firm's conditionally expected cost (i.e., conditional on the noisy signal). This variance is increasing in the precision of firm i's message (μ_i). Finally, the second term of (46) is proportional to the covariance between the firm's cost and the firm's expected cost conditional on the noisy signal. It remains to be shown that this covariance is increasing in the precision of firm i's message.[25]

We analyze the effect of message precision μ_i on the covariance $E_{\theta_i,\mu_i}\{\left(\theta_i-\overline{\theta_i}\right)\left[E\{\theta_i|m_i\}-\overline{\theta_i}\right]\}$ under the assumption that a higher precision μ_i makes firm i's message more Lehmann informative. Although the Lehmann concept of informativeness is more restrictive than integral precision (Ganuza and Penalva (2010)), it still includes many commonly used information models, such as the linear experiment, and the normal experiment. The Lehmann informativeness criterion requires that the signal and the state of the world are ordered according to the stochastically increasing order, which is a dependence order.

Definition 3 (Lehmann (1988)) *Let $\theta \in \Theta$ be the state of the world with a marginal distribution G, and Let $s_1 \in S_1, s_2 \in S_2$ be two signals with marginal distributions F_1 and F_2. If S_1 is more Lehmann informative than S_2 regarding Θ then S_1 is more stochastically increasing in Θ than S_2 is in Θ (i.e., $\{S_1,\Theta\} \succeq_{SI} \{S_2,\Theta\}$), which implies that $F_1^{-1}(F_2(s_2|\theta)|\theta)$ is increasing in θ for all s_2.*

[25] In fact, for our purposes, it would be sufficient to show that the covariance is maximized when the firm's message is perfectly informative (i.e., $\mu_i = \infty$). This is straightforward.

Without loss of generality, we can assume that the marginal distributions of the two signals coincide $F_1 = F_2$.[26] Under this additional assumption, Khaledi and Kochar (2005, p. 359-360) make the following observation:

Lemma 2 (Khaledi and Kochar (2005)) *If $F_1 = F_2$ and S_1 is more stochastically increasing in Θ than S_2 is in Θ (i.e., $\{S_1, \Theta\} \succeq_{SI} \{S_2, \Theta\}$), then the signals are ordered by Pearson's correlation coefficient, namely, $cor\{S_1, \Theta\} \geq cor\{S_2, \Theta\}$, where $cor\{S, \Theta\} \equiv \frac{cov\{S,\Theta\}}{Var\{S\}\, Var\{\Theta\}}$.*

This observation implies that $cov\{S_1, \Theta\} \geq cov\{S_2, \Theta\}$, if $F_1 = F_2$ and $\{S_1, \Theta\} \succeq_{SI} \{S_2, \Theta\}$, since $\text{Var}\{S_1\} \geq \text{Var}\{S_2\}$. Hence, we obtain that the covariance $E_{\theta_i,\mu_i}\{\left(\theta_i - \overline{\theta_i}\right)\left[E\{\theta_i|m_i\} - \overline{\theta_i}\right]\}$ is increasing in μ_i, if μ_i orders the message according to the Lehmann concept of informativeness. (Although this reduces the scope of the result somewhat, it remains consistent with our notion of informativeness since signals ordered according to Lehmann informativeness are also ordered according to integral precision. However, the reverse is not true.)

B.5 Extended Binary Example

Here we extend the binary example of section 4.4 by allowing both firms to acquire and share information, and by allowing for product differentiation goods ($0 < \beta \leq 1$ as in the general model). As in section 4.4, we consider a Cournot duopoly with risk-neutrality. Nature draws firm i's unit cost θ_i from the set $\{\theta^l, \theta^h\}$ with equal probability and sends a private signal to firm i for $i = 1, 2$:

$$S_i^\delta = \begin{cases} \theta_i & \text{with probability } \delta_i \\ \varnothing & \text{with probability } 1 - \delta_i. \end{cases}$$

The information sharing policies and information acquisition strategies of the firms are binary, i.e. $\delta_i, \rho_i \in \{0, 1\}$ for $i = 1, 2$.

First, we solve the product market stage. For any combination of messages $m_i, m_j \in \{\theta^l, \theta^h, \varnothing\}$, firm i with signal $s_i \in \{\theta^l, \theta^h, \varnothing\}$ chooses the output (4) for $i, j = 1, 2$ with $i \neq j$.

Second, we analyze the incentives to acquire information. The expected profit of firm i can be written as follows:

$$\begin{aligned} \Pi_i(\delta_i, \delta_j; \rho_i, \rho_j) \;\equiv\; & \rho_i \delta_i E_{\theta_i}\left\{E_{s_j,m_j}[x_i^*(\theta_i; \theta_i, m_j)^2]\right\} + (1-\rho_i)\delta_i E_{\theta_i}\left\{E_{s_j,m_j}[x_i^*(\theta_i; \varnothing, m_j)^2]\right\} \\ & +(1-\delta_i)E_{s_j,m_j}[x_i^*(\varnothing; \varnothing, m_j)^2] - \lambda\delta_i \\ = \; & E_{s_j,m_j}[x_i^*(\varnothing; \varnothing, m_j)^2] + \delta_i\left(\rho_i E_{\theta_i}\left\{E_{s_j,m_j}[x_i^*(\theta_i; \theta_i, m_j)^2 - x_i^*(\varnothing; \varnothing, m_j)^2]\right\}\right. \\ & \left. + (1-\rho_i)E_{\theta_i}\left\{E_{s_j,m_j}[x_i^*(\theta_i; \varnothing, m_j)^2 - x_i^*(\varnothing; \varnothing, m_j)^2]\right\} - \lambda\right) \end{aligned} \tag{47}$$

Notice that the first term of (47) does not depend on (δ_i, ρ_i). The second term of (47), can be simplified by using (4) to obtain the following:

$$\begin{aligned} E_{\theta_i}\left\{E_{s_j,m_j}[x_i^*(\theta_i; \theta_i, m_j)^2]\right\} &= E_{\theta_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(\varnothing; \varnothing, m_j) - \frac{2}{4-\beta^2}\left[\theta_i - \overline{\theta_i}\right]\right)^2\right]\right\} \\ &= E_{s_j,m_j}\left[x_i^*(\varnothing; \varnothing, m_j)^2\right] + \frac{4}{(4-\beta^2)^2}\text{Var}(\theta_i) \end{aligned}$$

[26] We can obtain this for two arbitrary signals by using the integral transformation (Ganuza and Penalva (2010)).

Similarly, for the third term of (47), we use (4) to obtain the following:

$$\begin{aligned} E_{\theta_i}\left\{E_{s_j,m_j}[x_i^*(\theta_i;\varnothing,m_j)^2]\right\} &= E_{\theta_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(\varnothing;\varnothing,m_j)-\frac{1}{2}\left[\theta_i-\overline{\theta_i}\right]\right)^2\right]\right\} \\ &= E_{s_j,m_j}\left[x_i^*(\varnothing;\varnothing,m_j)^2\right]+\frac{1}{4}\mathrm{Var}(\theta_i) \end{aligned}$$

These derivations (in combination with the observation that $\mathrm{Var}(\theta_i)=(\theta^h-\theta^l)^2/4$) enable us to write firm i's expected profit as follows:

$$\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)=E_{s_j,m_j}[x_i^*(\varnothing;\varnothing,m_j)^2]+\delta_i\left(\left[\rho_i\frac{4}{(4-\beta^2)^2}+(1-\rho_i)\frac{1}{4}\right]\frac{(\theta^h-\theta^l)^2}{4}-\lambda\right) \quad (48)$$

Again, notice that the first term of this expression does not depend on (δ_i,ρ_i), whereas the second term does not depend on (δ_j,ρ_j). Hence, the firm's optimal information acquisition investment does not depend on the competitor's information sharing choice as Proposition 3(i) shows in general. The second term of (48) determines the equilibrium investment in information acquisition:

$$\delta_i^*(\rho_i)=\begin{cases} 1, & \text{if } \left[\rho_i\frac{4}{(4-\beta^2)^2}+(1-\rho_i)\frac{1}{4}\right]\frac{(\theta^h-\theta^l)^2}{4}\geq\lambda, \\ 0, & \text{otherwise.}\end{cases} \quad (49)$$

In other words, the trade-off between the marginal revenue and cost of information acquisition determines the equilibrium investment level. The marginal revenue from information acquisition, $\left[\rho_i\frac{4}{(4-\beta^2)^2}+(1-\rho_i)\frac{1}{4}\right]\frac{(\theta^h-\theta^l)^2}{4}$, is increasing in the information-sharing variable ρ_i. Hence, the equilibrium information acquisition investment is increasing in information sharing, i.e., $d\delta_i^*/d\rho_i\geq 0$ as we show in Proposition 3(ii).

Information sharing has the following effects on the expected profit (48). The first term is constant in ρ_i whereas the second term is increasing in ρ_i. Hence, the expected profit is increasing in the firm's information-sharing variable (i.e., $\partial\Pi_i/\partial\rho_i\geq 0$ as Proposition 4(i) confirms). This implies that firm i shares its information in equilibrium. Only the first term of (48) depends on ρ_j, and it can be written as follows:

$$\begin{aligned} E_{s_j,m_j}\left\{x_i^*(\varnothing;\varnothing,m_j)^2\right\} &= \delta_j\rho_jE_{\theta_j}\left\{x_i^*(\varnothing;\varnothing,\theta_j)^2\right\}+(1-\delta_j\rho_j)x_i^*(\varnothing;\varnothing,\varnothing)^2 \\ &= x_i^*(\varnothing;\varnothing,\varnothing)^2+\delta_j\rho_j\left[E_{\theta_j}\left\{x_i^*(\varnothing;\varnothing,\theta_j)^2\right\}-x_i^*(\varnothing;\varnothing,\varnothing)^2\right] \\ &= \left(\overline{x_i}^*\right)^2+\delta_j\rho_j\left[E_{\theta_j}\left\{\left(x_i^*(\varnothing;\varnothing,\varnothing)+\frac{\beta}{4-\beta^2}\left[\theta_j-\overline{\theta_j}\right]\right)^2\right\}-x_i^*(\varnothing;\varnothing,\varnothing)^2\right] \\ &= \left(\overline{x_i}^*\right)^2+\delta_j\rho_j\left(\frac{\beta}{4-\beta^2}\right)^2\mathrm{Var}(\theta_j) \end{aligned}$$

After substituting this expression into the expected profit function (48), we obtain the following:

$$\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)=\left(\overline{x_i}^*\right)^2+\delta_i\left(\left[\rho_i\frac{4}{(4-\beta^2)^2}+(1-\rho_i)\frac{1}{4}\right]\frac{(\theta^h-\theta^l)^2}{4}-\lambda\right)+\delta_j\rho_j\left(\frac{\beta}{4-\beta^2}\right)^2\frac{(\theta^h-\theta^l)^2}{4}$$

Clearly, firm i's expected profit is non-decreasing in the competitor's information-sharing variable (i.e., $\partial\Pi_i/\partial\rho_j\geq 0$ as in Proposition 4(i)).

Finally, we consider the expected consumer surplus in our extended example. By using essentially the same decomposition as in (47), we can write the expected consumer surplus as follows:

$$
\begin{aligned}
&V(\delta_i, \delta_j; \rho_i, \rho_j) \\
&= \rho_i E_{s_i}\left\{E_{s_j,m_j}\left[\frac{1}{2}\left(x_i^*(s_i; s_i, m_j) + x_j^*(s_j; m_j, s_i)\right)^2 - (1-\beta)x_i^*(s_i; s_i, m_j)x_j^*(s_j; m_j, s_i)\right]\right\} \\
&\quad +(1-\rho_i)E_{s_i}\left\{E_{s_j,m_j}\left[\frac{1}{2}\left(x_i^*(s_i; \varnothing, m_j) + x_j^*(s_j; m_j, \varnothing)\right)^2 - (1-\beta)x_i^*(s_i; \varnothing, m_j)x_j^*(s_j; m_j, \varnothing)\right]\right\} \\
&= E_{s_j,m_j}\left\{\frac{1}{2}\left(x_i^*(\varnothing; \varnothing, m_j) + x_j^*(s_j; m_j, \varnothing)\right)^2 - (1-\beta)x_i^*(\varnothing; \varnothing, m_j)x_j^*(s_j; m_j, \varnothing)\right\} \\
&\quad +\rho_i\delta_i\frac{1}{2}E_{\theta_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(\theta_i; \theta_i, m_j) + x_j^*(s_j; m_j, \theta_i)\right)^2 - \left(x_i^*(\varnothing; \varnothing, m_j) + x_j^*(s_j; m_j, \varnothing)\right)^2\right]\right\} \\
&\quad -\rho_i\delta_i(1-\beta)E_{\theta_i}\left\{E_{s_j,m_j}\left[x_i^*(\theta_i; \theta_i, m_j)x_j^*(s_j; m_j, \theta_i) - x_i^*(\varnothing; \varnothing, m_j)x_j^*(s_j; m_j, \varnothing)\right]\right\} \\
&\quad +(1-\rho_i)\delta_i\frac{1}{2}E_{\theta_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(\theta_i; \varnothing, m_j) + x_j^*(s_j; m_j, \varnothing)\right)^2 - \left(x_i^*(\varnothing; \varnothing, m_j) + x_j^*(s_j; m_j, \varnothing)\right)^2\right]\right\} \\
&\quad -(1-\rho_i)\delta_i(1-\beta)E_{\theta_i}\left\{E_{s_j,m_j}\left[x_i^*(\theta_i; \varnothing, m_j)x_j^*(s_j; m_j, \varnothing) - x_i^*(\varnothing; \varnothing, m_j)x_j^*(s_j; m_j, \varnothing)\right]\right\}
\end{aligned}
$$

where

$$
\begin{aligned}
&E_{\theta_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(\theta_i; \theta_i, m_j) + x_j^*(s_j; m_j, \theta_i)\right)^2\right]\right\} \\
&\qquad = E_{\theta_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(\varnothing; \varnothing, m_j) + x_j^*(s_j; m_j, \varnothing) - \frac{1}{2+\beta}\left[\theta_i - \overline{\theta_i}\right]\right)^2\right]\right\} \\
&\qquad = E_{s_j,m_j}\left\{\left(x_i^*(\varnothing; \varnothing, m_j) + x_j^*(s_j; m_j, \varnothing)\right)^2\right\} + \left(\frac{1}{2+\beta}\right)^2 \mathrm{Var}(\theta_i),
\end{aligned}
$$

$$
\begin{aligned}
&E_{\theta_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(\theta_i; \varnothing, m_j) + x_j^*(s_j; m_j, \varnothing)\right)^2\right]\right\} \\
&\qquad = E_{\theta_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(\varnothing; \varnothing, m_j) + x_j^*(s_j; m_j, \varnothing) - \frac{1}{2}\left[\theta_i - \overline{\theta_i}\right]\right)^2\right]\right\} \\
&\qquad = E_{s_j,m_j}\left\{\left(x_i^*(\varnothing; \varnothing, m_j) + x_j^*(s_j; m_j, \varnothing)\right)^2\right\} + \frac{1}{4}\mathrm{Var}(\theta_i),
\end{aligned}
$$

$$
\begin{aligned}
&E_{\theta_i}\left\{E_{s_j,m_j}\left[x_i^*(\theta_i; \theta_i, m_j)x_j^*(s_j; m_j, \theta_i)\right]\right\} \\
&\qquad = E_{\theta_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(\varnothing; \varnothing, m_j) - \frac{2\left[\theta_i - \overline{\theta_i}\right]}{4-\beta^2}\right)\left(x_j^*(s_j; m_j, \varnothing) + \frac{\beta\left[\theta_i - \overline{\theta_i}\right]}{4-\beta^2}\right)\right]\right\} \\
&\qquad = E_{s_j,m_j}\left\{x_i^*(\varnothing; \varnothing, m_j)x_j^*(s_j; m_j, \varnothing)\right\} - \frac{2\beta}{(4-\beta^2)^2}\mathrm{Var}(\theta_i),
\end{aligned}
$$

and $E_{\theta_i}\left\{E_{s_j,m_j}\left[x_i^*(\theta_i; \varnothing, m_j)x_j^*(s_j; m_j, \varnothing)\right]\right\} = E_{s_j,m_j}\left\{x_i^*(\varnothing; \varnothing, m_j)x_j^*(s_j; m_j, \varnothing)\right\}$. Using these equations, simplifies the expected surplus as follows:

$$
\begin{aligned}
V(\delta_i, \delta_j; \rho_i, \rho_j) &= E_{s_j,m_j}\left\{\frac{1}{2}\left(x_i^*(\varnothing; \varnothing, m_j) + x_j^*(s_j; m_j, \varnothing)\right)^2 - (1-\beta)x_i^*(\varnothing; \varnothing, m_j)x_j^*(s_j; m_j, \varnothing)\right\} \\
&\quad +\frac{1}{2}\delta_i\left[\rho_i\frac{4-3\beta^2}{(4-\beta^2)^2} + (1-\rho_i)\frac{1}{4}\right]\mathrm{Var}(\theta_i)
\end{aligned}
$$

By using an analogous decomposition for firm j's output levels (and by recalling that $\text{Var}(\theta_i) = (\theta^h - \theta^l)^2/4$ in our example), we can reduce the expected consumer surplus to the following:

$$\begin{aligned} V(\delta_i, \delta_j; \rho_i, \rho_j) &= \frac{1}{2}\left(x_i^*(\varnothing; \varnothing, \varnothing) + x_j^*(\varnothing; \varnothing, \varnothing)\right)^2 - (1-\beta)x_i^*(\varnothing; \varnothing, \varnothing)x_j^*(\varnothing; \varnothing, \varnothing) \\ &\quad + \frac{1}{2}\frac{(\theta^h - \theta^l)^2}{4}\sum_{\ell=1}^{2}\delta_\ell\left[\rho_\ell\frac{4-3\beta^2}{(4-\beta^2)^2} + (1-\rho_\ell)\frac{1}{4}\right] \end{aligned} \tag{50}$$

The first two terms of this expression are constant in (δ_i, ρ_i). The last term for $\ell = i$ is increasing in firm i's information acquisition investment (i.e., $\partial V/\partial \delta_i > 0$), and it is decreasing in firm i's information sharing choice (i.e., $\partial V/\partial \rho_i \leq 0$) for $i = 1, 2$. This confirms our general finding in Proposition 5.

Finally, we derive the antitrust authority's optimal policy on information sharing from equations (49) and (50). The surplus-maximizing policy in our example $\overline{\rho_i}$ equals $\overline{\rho_1}$ in (11) for $i = 1, 2$. First, if λ is lower than $\frac{(\theta^h-\theta^l)^2}{16}$, then the firms always acquire information, and the antitrust authority prefers to prohibit information sharing, since $V(1,1;1,1) < V(1,1;0,0)$. Second, if $\frac{(\theta^h-\theta^l)^2}{16} < \lambda < \frac{(\theta^h-\theta^l)^2}{(4-\beta^2)^2}$, then firms acquire information only if they are allowed to share information, i.e., $\delta_i^*(1) = 1 > 0 = \delta_i^*(0)$ for $i = 1, 2$. This favors information sharing and the antitrust authority prefers to allow information sharing between competing firms, since $V(1,1;1,1) > V(0,0;1,1) = V(0,0;0,0)$. Finally, if λ is larger than $\frac{(\theta^h-\theta^l)^2}{(4-\beta^2)^2}$, the firms acquire information neither with information sharing nor without it, and then the authority is indifferent, since $V(0,0;1,1) = V(0,0;0,0)$. In this case, information sharing may as well be allowed.

It is straightforward to show that the policy (11) remains optimal if we extend the example to an oligopoly with N firms (for $N \geq 2$). For any combination of messages $m_1, ..., m_N \in \{\theta^l, \theta^h, \varnothing\}$, firm i with signal $s_i \in \{\theta^l, \theta^h, \varnothing\}$ chooses the output (16) for $i = 1, ..., N$. Then by following similar steps as above, we can rewrite firm i's expected equilibrium profit as follows:

$$\begin{aligned} \Pi_i(\delta_i, \delta_{-i}; \rho_i, \rho_{-i}) &= E_{s_{-i}, m_{-i}}\left\{x_i^*(\varnothing; \varnothing, m_{-i})^2\right\} \\ &\quad + \delta_i\left(\left[\rho_i\left(\frac{2+(N-2)\beta}{[2+(N-1)\beta](2-\beta)}\right)^2 + (1-\rho_i)\frac{1}{4}\right]\frac{(\theta^h-\theta^l)^2}{4} - \lambda\right) \end{aligned}$$

This yields the following equilibrium investment in information acquisition (for $i = 1, ..., N$):

$$\delta_i^*(\rho_i) = \begin{cases} 1, & \text{if } \left[\rho_i\left(\frac{2+(N-2)\beta}{[2+(N-1)\beta](2-\beta)}\right)^2 + (1-\rho_i)\frac{1}{4}\right]\frac{(\theta^h-\theta^l)^2}{4} \geq \lambda, \\ 0, & \text{otherwise.} \end{cases}$$

Again, the optimal policy $\overline{\rho_i}$ equals (11) for $i = 1, ..., N$. First, if $\lambda < \frac{(\theta^h-\theta^l)^2}{16}$, then $\delta_i^*(\rho_i) = 1$ for all ρ_i and i, and consumers prefer information concealment, since $V(1, ..., 1; 0, ..., 0) > V(1, ..., 1; 1, ..., 1)$. Second, if $\frac{(\theta^h-\theta^l)^2}{16} < \lambda < \left(\frac{[2+(N-2)\beta](\theta^h-\theta^l)}{2[2+(N-1)\beta](2-\beta)}\right)^2$, then $\delta_i^*(1) = 1 > 0 = \delta_i^*(0)$ for $i = 1, ..., N$. In this case, consumers prefer information sharing, since $V(1, ..., 1; 1, ..., 1) > V(0, ..., 0; 1, ..., 1) = V(0, ..., 0; 0, ..., 0)$. Finally, if $\lambda > \left(\frac{[2+(N-2)\beta](\theta^h-\theta^l)}{2[2+(N-1)\beta](2-\beta)}\right)^2$, then $\delta_i^*(\rho_i) = 0$ for all ρ_i and i, and consumers are indifferent, since $V(0, ..., 0; 1, ..., 1) = V(0, ..., 0; 0, ..., 0)$.

Notice that the optimal policy (11) is increasing in the cost of information acquisition, λ, and it is independent of the degree of product substitutability, β, and the number of firms, N.

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