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THE PENALTY-DUEL AND INSTITUTIONAL DESIGN: IS THERE A NEESKENS-EFFECT?

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The Penalty-Duel and Institutional Design: Is there a Neeskens-Effect?

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"Angsthasen!"

Harald "Toni" Schumacher in a conversation with us about penalty kickers, who shoot to the middle of the goal.

I. Introduction: Penalty taking and game theory

In soccer, penalty kicks and shootouts are taken from twelve yards (= 10, 9728 meters) out from goal, with only the goalkeeper between the penalty taker and the goal. Penalty kicks were first introduced in Ireland in the 1891-92 season in order to punish a foul within the penalty area. Penalty shootouts were introduced in 1970 to determine who progresses after a tied match. Since then, penalty taking determined the outcome of numerous soccer games and tournaments, including, for instance, the FIFA World Cup finals 1994 between Brazil and Italy, and 2006 between Italy and France. In this paper, we ask what strategies in penalty-taking can be considered 'clever' or 'rational.' We focus in particular on strategies that involve shooting to the middle of the goal.

Game theory analyzes and predicts behavior when people interact with each other. Taking the framework of the interaction – the game – as given, game theory offers solutions. A *game* includes the rules of interaction, the strategies available to the "players," and the payoffs (or 'utilities') each player assigns to all possible outcomes of the game. A solution (equilibrium) is a prediction or recommendation which strategy to choose, assuming that all players wish to maximize their given payoffs. A *game form* is the description of an interactive decision situation without the specification of particular payoffs for the players involved. The idea of the game form is to describe, analyse and evaluate (e.g. in terms of efficiency) equilibrium behavior for any kind of preferences players of this game may reasonably have. A game form can be identified with an *institution* within which players have to act. From this perspective, game theory focusses on behavior arising from a given set of institutional rules and asks how institutions affect behavior. The answer, as we will argue, may depend on the *shared* perceptions of the interactive decision situation by the players.

Regarding penalty taking, the institutional rules are pretty clear, even though they may change over time.[1] The strategies available to the players are, however, less obvious, and in

[1] According to the official FIFA laws (Fédération Internationale de Football Association), "A penalty kick is awarded against a team that commits one of the ten offences for which a direct free kick is awarded, inside its own penalty area and while the ball is in play. A goal may be scored directly from a penalty kick. Additional time is allowed for a penalty kick to be taken at the end of each half or at the end of periods of extra time. […] The ball is placed on the penalty mark. The player taking the penalty kick is properly identified. The defending goalkeeper remains on his goal line, facing the kicker, between the goalposts until the ball has been kicked."; see http://www.fifa.com/en/laws/Laws14_01.htm for this and more rules about penalty kicks. Penalty shootouts are generally governed by similar rules.

fact we will discuss the impact of different possible sets of strategies – and hence different game forms – on *behavior* in this paper. We will also argue that the payoffs assigned to the outcomes of penalty taking, goal or no goal, are more subtle than has previously been assumed. Players may not only want to maximize respectively minimize the probability of a goal, but there are also indications that players have preferences over the strategies that can be chosen.

II. A (too) simple game theoretic model

Let us begin with the simplest possible game theoretic modelling of what might be called the "penalty-duel." A penalty-duel involves two players, the kicker and the goalkeeper.[2] The interests of the players are perfectly opposing; success of the kicker implies failure of the goalkeeper, and the other way round. More precisely, the conflict structure is such that the goalkeeper wants to coordinate his action with the one of the kicker, while the kicker aims at discoordination of actions. We also assume that the two players must move simultaneously, implying that players are not able to react to the movements of the opponent. Indeed, many observers argue that neither does the speed of the ball allow goalkeepers to react to the ball's course (e.g., Palacios-Huerta 2003) nor does a goalkeeper wish to move early in order to avoid signalling to the kicker about his intentions. Furthermore, Chiappori et al. (2002), among others, provide empirical evidence suggesting that moves of the players in penalty taking are simultaneous ones.[3] Summing up, from a simple game theoretic perspective, a penalty-duel can be seen as a simultaneous, two-person game with strictly opposing preferences.

What are the strategies available to the players? A simple model assumes that the kicker (*K*) has only two options; he can either choose to kick to the left side (**L**) or to the right side (**R**); the option of shooting to the middle is ignored here for the moment (as it is the case in large parts of the literature). Accordingly, the goalkeeper (*G*) can either jump to the left or to the right side. (Here and in the following, left and right is defined from the kicker's perspective.) The following table then shows the 'payoffs' from the four potential pairs of actions.

[2] Here and in the following, we abstract away from the possibilities of rebounds.
[3] We will briefly come back to the simultaneity assumption in the conclusions.

G \ *K*		y	1-y
		L	**R**
x	**L**	1,0	0,1
1-x	**R**	0,1	1,0

Game I: *A simple penalty game.*

If both players choose the same side (the top left or the bottom right corner of the table), the goalkeeper succeeded in coordination and manages to save the penalty. In this case, the kicker’s payoff is zero and the goalkeeper’s payoff is normalized to one. In the cases, when the goalkeeper fails to jump to the side chosen by the kicker, who succeeded in discoordinating actions, there is a goal and the goalkeeper’s payoff is zero while the kicker’s payoff is normalized to one.[4] Because interests are perfectly opposed, payoffs always sum up to the same amount, regardless of the strategies being chosen. Normalizing this amount to one gives the payoffs a probability interpretation; kickers aim to maximize the probability of scoring, while goalkeepers aim to minimize this probability.

What is a rational strategy in the penalty-duel, as described in Game I? A good strategy should be a best response to the opponent’s strategy. When all players choose a best response to the other players’ strategies, this set of strategies constitutes what is called a (Nash) *equilibrium* in game theory. In equilibrium, nobody has a unilateral incentive to deviate from his own strategy. Off equilibrium, however, there is at least one player who can improve upon his outcome (given the utilities assigned to each outcome) by switching the strategy. So, any advice to the players that does not constitute an equilibrium would be bad advice for at least one player, because if all other players follow their parts of the advice, it would be better for some player to do differently than he was advised. From another perspective, any prediction that is not equilibrium would be problematic, because such prediction would be based on the assumption that at least one player does not play in line with his interests. Finally, an equilibrium can often be interpreted as a potential stable point of a dynamic adjustment process in which players search for better strategies. Summing up,

[4] For simplicity, we abstract away from the possibility that the kicker misses the goal.

equilibrium strategies are not only rational strategies but also natural candidates for predictions of behavior.

Games like the penalty-duel, in which payoffs always sum up to the same constant, are called *constant-sum games* and are well-understood from the very beginning of game theory. A constant-sum game is equivalent to a zero-sum game, because the value of the constant is arbitrary; it measures utility, which can be freely normalized. Von Neumann (1928) and von Neumann and Morgenstern (1944) developed a solution theory for such games based on the notion of a maxminimizer. A unique feature of this class of games is, that any (pure stratgy) Nash equilibrium consists of strategies, which are maxminimizers for the players. These strategies require players to react to the "worst case" possible; i.e. always to believe that the opponent takes the least preferred (or most harmful) action. Indeed, in games with strictly opposing interests, this is a reasonable belief. Not all games considered in this paper are constant-sum, but all are strictly competitive. *Strictly competitive games* have the property that an advantageous change in behavior for one player is automatically disadvantegous to the other one. Such games are the ordinal equivalent of zero-sum games.

It can easily be seen that the penalty-duel has no equilibrium in so-called *pure strategies*. If the goalkeeper chooses **L**, the kicker's best response is **R**. But if the kicker chooses **R**, the goalkeeper's best response is **R** etc. No pair of pure strategies, one for the kicker and one for the goalkeeper, constitutes an equilibrium. However, the game theoretic notion of a strategy is quite general, and in particular it includes "mixed" strategies that are *probability distributions* over actions such as **R** and **L**. When mixing, players choose their actions randomly. In our simple penalty game, there is a unique equilibrium in mixed strategies, which gives an equal chance of winning to each player. The probabilities of choosing **L** respectively **R** for the kicker and the goalkeeper, respectively, in Game I are:

$$(x, 1-x) = (0.5, 0.5)$$
$$(y, 1-y) = (0.5, 0.5).$$

Clearly, when following these strategies, each player chooses a best response against the opponent's strategy. The resulting equilibrium payoffs for the goalkeeper and the kicker, respectively, are calculated as the expected payoffs given the equilibrium strategies and can be interpreted as probability of success. For Game I, these payoffs are given by:

$$\Pi_G = \Pi_K = 0.5$$

for the goalkeeper and the kicker, respectively.

So, in penalty-duel, each player must randomize. Randomization makes sure that there is no recognizable pattern in the behavior of a player that could be exploited by the opponent to his detriment (and his own advantage). How does this state come about? The *Fundamental Lemma* (see e.g. Osborne 2004, Chapter 4) characterizes mixed strategy equilibria of finite games as follows:

> A mixed strategy profile is a (mixed strategy) Nash equilibrium, if and only if, for each player, the expected payoff to every action to which a player`s strategy assigns positive probability is the same (and at least the expected payoff to any action which is assigned zero probability).

So, any player’s expected payoff in equilibrium is her expected payoff to any of his actions that he uses with positive probability. A player is thus indifferent between these actions; more precisely, he is *made* indifferent by the opponent´s behavior, to which all these pure strategies are best replies. And this aim in turn determines the choice of probabilities by the opponent.

Randomization does not imply, however, that a player actually flips a coin, he may still choose a pure strategy (which need not be arrived at as the realization of a coin flip). But his mixed strategy represents a consistent way to reason about the opponent`s behavior (who may have several best replies against this strategy).[5]

We finally note that, mainly because of the random element, penalty taking has been criticized as an unsatisfactory way to decide a soccer game. Therefore, various alternatives have been proposed, including golden and silver goal methods, replaying a match that has ended in a tie, winner determination according to the number of shots on goal, number of corner kicks or other measures (see http://en.wikipedia.org/wiki/Penalty_shootout_(football)). However, before 1970, drawn games between national teams were decided by another random move: the toss of a coin. Random decisions are in fact quite often used in modern societies, not only in sports to break ties, but also in social life to resolve conflicts in the allocation of public housing and scarce medical resources, the awarding of oil drilling leases, admission to

[5] For empirical studies of mixed equilibrium play in soccer see Chiappori et al. (2002), Palacios-Huerta (2003), Moschine (2004), and Sonnabend (2006), and for a corresponding study in tennis see Walker and Wooders (2001). A survey of empirical and experimental studies on mixed-equilibrium play related to penalty taking in soccer is provided by Sonnabend (2006).

educational institutions, professional athletic drafts, tax auditing, as well as military drafts and jury selection (Elster 1989). Recent experimental evidence demonstrates that fair random lotteries are often acceptable to all participants, even those, who lose the lotteries (Bolton et al. 2005, Bolton and Ockenfels 2006). In this sense, the random element in equilibrium play appears to be less problematic than it seems at first glance. However, later in this paper, we will develop a more realistic model, in which the players' skills affect the probability of success, so that the outcome is still influenced but not entirely determined by randomness.

III. Stability and Institutional Design

The institutional context is critical to game-theoretic analysis. Optimal strategies depend on the rules of the game, on the assumed payoff structure, and on behavioral standards as reflected by the strategies available to the players. Game theoretic equilibrium identifies mutually stable strategies within a given institution, but pure equilibrium analysis typically does not address the stability of the institutions themselves.[6] For instance, the rules for penalty kicks have been changed repeatedly since 1891, indicating that earlier proposals were not stable. Initially, the goalkeeper was permitted to move up to six yards away from his goal-line to defend a penalty, while later he was not allowed to leave the goal line. Then, he was not allowed to move on the goal line before the kick was executed, until 1997 when moving before the kick became allowed.[7]

In this section, we want to emphasize a more subtle evolution of institutional design through *behavioral innovation*, which affects the set of strategies available to the players *within* the rules of soccer as specified by the governing bodies. In particular, we emphasize that the simple game studied above, which essentially views the penalty duel between goalkeeper and kicker as the choice of a corner of the goal, is not stable against the introduction of a new strategy or behavior of the kicker, namely, opting for the the middle of the goal. If the goalkeeper insists on opting for corners exclusively, this would guarantee success for the kicker choosing middle, **M.** Consequently, the goalkeeper is forced to consider strategy **M** as well. The analysis of the resulting 3x3-game confirms that the kicker *gains* from the introduction of **M** *in equilibrium*:

[6] Güth and Ockenfels (2003 and forthcoming) study the mutual stability of institutions and behavioral patterns in evolutionary models, where institutions and behavior are allowed to affect each other through co-evolution.
[7] See http://www.fifa.com/en/news/index/0,1464,22688,00.html?articleid=22688 for some historical fact about the evolution of the penalty law.

G \ K		y_1 **L**	y_2 **M**	y_3 **R**
x_1	**L**	1,0	0,1	0,1
x_2	**M**	0,1	1,0	0,1
x_3	**R**	0,1	0,1	1,0

Game II: *A penalty game, which includes the middle strategy.*

Equilibrium strategies are now given by

$$(x_1, x_2, x_3,) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$$
$$(y_1, y_2, y_3,) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

Note that the kicker, who succeeds in 6 of the 9 possible instances in discoordinating actions, now has a probability of success, which is twice the one of the goalkeeper, who (as before) only succeeds on the diagonal in coordinating actions. The reason is that equilibrium strategies make the realization of all 9 instances equally likely. This *reduces* the expected payoff of the goalkeeper from 1/2 in the previous game to 1/3 and *increases* the payoff of the kicker from 1/2 to 2/3:

$$\Pi_G = \frac{1}{3}$$
$$\Pi_K = \frac{2}{3}$$

We credit Johan Neeskens, a Dutch midfielder playing for Ajax Amsterdam and FC Barcelona in the 1970s, with this clever innovation, which gradually changed the *perception* of the essence of the penalty game by kicker *and* goalkeeper from a 2x2 to a 3x3 game. E.g., the German site of Wikipedia mentions that ever since the 1974 World Cup Final, Neeskens is credited by soccer commentaries as the inventor of the *Neeskens-variant* ("Neeskens-Variante") of taking a penalty: shooting straight to the middle, while the goalkeeper dives for a corner (http://de.wikipedia.org/wiki/Johan_Neeskens). In this game the referee awarded a penalty to the Netherlands against Germany in the second minute of the game, which Neeskens had the nerve to take in precisely this way – while Sepp Maier, the German goal-keeper, opted for his right corner. During his long and distinguished career Neeskens was runner-up at the World Cups 1974 (in Germany) and 1978 (in Argentina), when in each case his Dutch side lost the final to the host country. He won the European Club Championship 1971-1973 with Ajax Amsterdam and the European Cup Winners Cup in 1979 with FC Barcelona (against Fortuna Düsseldorf (!) in a most dramatic match, which ended 4:3 after extra time). In 1981 he appeared in his last of 49 matches for the Netherlands.

As mentioned by wikipedia the double prominence of the occasion, a penalty in the second minute of World Cup Final, propelled the *Neeskens-variant* to international prominence and recognition as a valid and serious option of taking a penalty. It is not known to us, whether there ever was a penalty at a similarly important or internationally prominent occasion, which was (intentionally?) taken this way before. However, there was another – and now in Germany equally historical – penalty taken this way at an almost equally important and prominent occasion two years later: the last penalty of the penalty shoot-out that decided the Final of the European Championships in 1976 between Germany and Tchechoslovakia. It was taken by Antonin Panenka, who delicately chipped the ball right into the middle of the goal mouth – while Sepp Maier opted for his left corner. Panenka undoubtedly did so intentionally. He later said: "I knew long before that I would take the penalty this way." His own goalkeeper warned him the night before the final, that – if the occasion should arise – doing so would be "too arrogant." Panenka himself was aware of this and the risk that belonged to it: "If Maier would have stayed put, they would have sent me to the factories for the next 25 years. The communists would have accused me of ridiculing their system" (Martens, 2003). Alas, it went well to the fame of Panenka (and the Neeskens-variant).

These observations lead us to the following claim: *before* 1974 (and the Neeskens-penalty) the "institution" penalty-duel was a standard of behavior, according to which kickers chose **L** or **R** and goalkeepers did likewise. That is, a 2x2-game form gives an accurate description of the penalty situation. *After* 1976 (and the Panenka-penalty) the "institution" penalty-duel was perceived as a 3x3 game form constrained by certain behavioral rules.

In order to check for the empirical validity of our claim, we tried to get adequate data from penalty kicks and shootouts before Neeskens'and after Panenka's goal. However, although we spoke to Bundesliga and national goalkeepers, journalists, TV stations, professional sports data providers, many scientists, and others, we could not trace detailed data that include information about the players' strategies and sufficient data from penalties before 1974.[8] What we have, however, are the following summary statistics of Bundesliga penalties from 1963-1990 (Ressource: Bundesliga Datenbank, Ismaning).

Year	Total penalties	Goals in penalties
1990	83	62
1989	70	53
1988	72	50
1987	83	67
1986	110	86
1985	111	92
1984	100	71
1983	113	86
1982	86	66
1981	116	92
1980	94	77
1979	94	59
1978	88	70
1977	102	83
1976	79	59
1975	106	74
1974	126	91
1973	98	76
1972	114	83
1971	102	71
1970	72	58
1969	71	46
1968	72	49
1967	90	66
1966	81	53
1965	93	66
1964	72	48
1963	63	41
Sum	2561	1895

Scoring rate over time

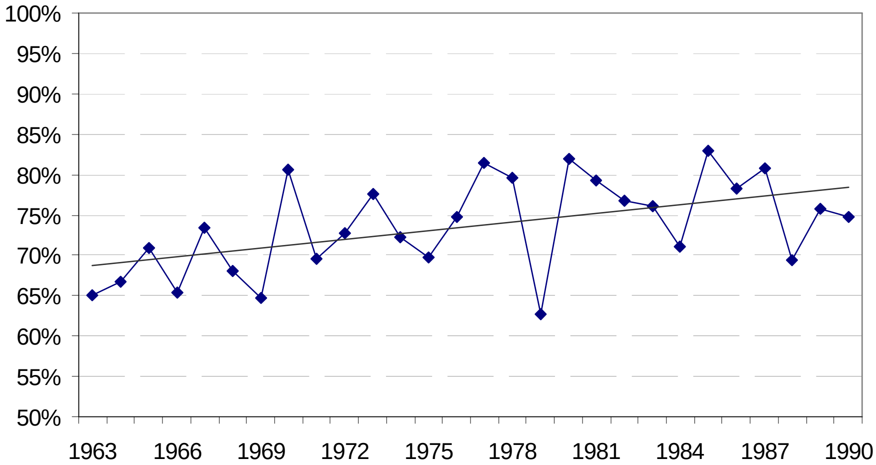

Figure 1: *Bundesliga penalties 1963-1990*

Figure 1 shows that there are between 63 (1963) and 126 (1974) penalties per season in the German Bundesliga. The overall success rate of penalty kicks is 74%, ranging from 62% (1963) to 82% (1985). The average success rate from 1963 to 1973 is 69%, and the average success rate 1977 – 1987 is 77%. Applying an exact Mann Whitney U test, this difference is significant at the 1% level (two-sided). The same test yields a two-sided p = 0.013 if we compare the 1963-1974 with 1975-1990.

[8] Toni Schumacher lent us his famous note book, in which he documented penalty taking strategies for his own use. See Appendix I for a sample page. However, the data turned out not to be helpful for our purposes, because there are not enough data for statistical analysis, especially before Neeskens' penalty kick in 1974. Sepp Maier told us that he is a "practitioner" and not a "theorist" and that therefore he was never interested in statistics, and that he never systematically collected data on penalty kicks.

The graph in Figure 1 suggests, however, that the difference is at least partly due a general trend (the straight line shows the result of a simple linear regression); in fact, statistics cannot reveal a structural break in the mid seventies, which would be an indicator that Neeskens' innovation caused a significant advantage for the kicker. However, it might also be that Neeskens' innovation diffused only slowly over time, so that our data would be too rough to detect supportive evidence for our claim. An ultimate test must be left to further research based on more detailed data about shot and jump directions before and after 1974.

*IV. Opting for alternative **M:** Why is the middle relatively unattractive to players?*

While there is anecdotic evidence, there is no statistically unambiguous evidence in our data that Neeskens' innovation significantly changed the way penalty kicks were executed. Part of the reason might be that the middle is a less attractive strategy than the corners to both, kickers and goalkeepers: Based on data from 459 penalties from French and Italian League 1997 – 2000, Chiappori et al. (2002) found that

- the kicker chooses middle less often than either corner,
- the goalkeeper chooses middle less often than either corner, and
- the goalkeeper chooses middle less often than kicker (in fact, goalkeepers almost never stay in the middle).[9]

As a consequence, kicking to the middle on average has the highest probability of scoring (81.0% as compared to 70.1% for the right corner and 76.7% for the left corner; see Table 4 in Chiappori et al. 2002). Obviously, these observations are not in line with our simple model of Game II, which suggests that the middle should be as attractive as each of the corners. They also appear inconsistent with the Fundamental Lemma described above: in equilibrium, the expected payoff to every action to which a player's strategy assigns positive probability should be the same.

Chiappori et al. (2002) explain some of these effects with heterogeneity with respect to the players' capabilities to score when choosing one of the three strategies, left, right and middle. For instance, it is typically easier for a kicker to score when kicking to the converse side of his strong foot. This heterogeneity can explain why certain strategies are more often chosen than others by certain kickers. (In fact, goalkeepers seem to take into account kickers'

[9] Other studies, including the work by Palacios-Huerta (2003), do not take shots to the middle as a possible strategy into account of the analyses.

past behavior, which supports the view that heterogeneity drives some of the results.) But it cannot explain the superior average scoring rate for the middle strategy. So we suspect that heterogeneity is only part of the explanation.

In order to find out more about why the middle seems a rather unattractive strategy despite the strategic advantage of using it, we asked Harald “Toni” Schumacher and Hans-Jörg Butt about their strategies in penalties.[10] Toni Schumacher is considered one of the world’s best goalkeepers during the 1980s. He was capped 76 times for Germany, and he was a winner of the European Championship 1980 and World Cup finalist in 1982 and 1986 with the German team. He won national titles in the Bundesliga with 1. FC Köln and Borussia Dortmund. Hans-Jörg Butt is an active goalkeeper playing in the Bundesliga for Bayer Leverkusen, and he was elected three times to the German national team. In 2002 he was runner-up with his team Bayer Leverkusen in the European Champions League, the German Bundesliga and the German Cup competition. He is most remarkable – and unique – among German goal keepers, because he is not only considered a ‘penalty killer` - Wikipedia notes that he on average saves 7 out of 10 penalties -, but at the same time one of the most successful penalty takers. He currently (November 2006) has 26 goals from the penalty spot to his credit, more than any other goalkeeper ever in the Bundesliga. So, if anyone knows how to put himself into the opponent`s shoes, it is him.

Both goalkeepers confirm that there is heterogeneity among kickers, though the classifications they use are quite different. Toni Schumacher mentioned that there are 4 types of kickers distinguishable according to whether they shoot with the left or right foot, and whether the shot is powerful or more technically demanding. A powerful shot from a right-footer, for instance, would most likely go to the left corner. Hans-Jörg Butt, on the other hand, classifies kickers according to whether they try to react to goalkeepers, or whether they choose the corner independent of the movements of the goalkeeper. So, there is heterogeneity – implying that we should not observe equal probabilities for choosing the actions right, middle and left. At the same time, however, both goalkeepers suggested that the unattractiveness of the middle-strategy is also driven by an asymmetric *payoff* structure, which has not been considered in earlier work. Toni Schumacher, for instance, noted that he *never* just stayed in the middle. Asked why, he responded that this would be “against my honour.” When we remarked that, knowing this, a kicker’s best response would be to shoot in the middle, he answered that a kicker, who shoots in the middle, “does not deserve to kick a penalty against me,” and that this would be “a different game” (sic!). While Hans-Jörg Butt

[10] Our meeting with Harald Anton Schumacher, commonly known as Toni Schumacher, took place on 21 August 2006 in Cologne, and the meeting with Hans-Jörg Butt on 31 August 2006 in Leverkusen.

also pointed out that there are technical difficulties with shooting to the middle (“schippen”), he reasoned that it is a larger “disaster” for the kicker when a shot to the middle is saved (recall Panenka`s statement!), than it is a disaster for the goalkeeper when he does not save a shot to the middle. A kicker, who gets caught in the middle is a “fool” (in German: “Depp”). Gary Lineker of England can attest to this, because he was unfortunate enough to try to copy Panenka in a match England versus Brasil in 1991: his fine, slow chip to the middle of the goal mouth found the literally unmoved Brasilian goalkeeper waiting for it on the line. It truly looked ridiculous (and became part of soccer folklore, see e.g. Martens, 2003).

Both views indicate that the payoffs associated with corner respectively middle actions need to be treated asymmetrically. The following variation of Game II incorporates Butt`s argument, the new game has an asymmetric payoff payoff structure:

G \ *K*		y_1 **L**	y_2 **M**	y_3 **R**
x_1	**L**	1,0	0,1	0,1
x_2	**M**	0,1	1+b,-b	0,1
x_3	**R**	0,1	0,1	1,0

Game III: *A penalty game with modified payoffs.*

Note that game III, too, is a constant-sum game; i.e., players again will play there maxmin-strategies in equilibrium. The following table gives equilibrium strategies for different values of the parameter *b* > 0:

	$b=0$	$b=\frac{1}{2}$	$b=1$	$b=\infty$
x^*	$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$	$\left(\frac{3}{8},\frac{1}{4},\frac{3}{8}\right)$	$\left(\frac{2}{5},\frac{1}{5},\frac{2}{5}\right)$	$\left(\frac{1}{2},0,\frac{1}{2}\right)$
y^*	$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$	$\left(\frac{3}{8},\frac{1}{4},\frac{3}{8}\right)$	$\left(\frac{2}{5},\frac{1}{5},\frac{2}{5}\right)$	$\left(\frac{1}{2},0,\frac{1}{2}\right)$

Table 1: *Equilibrium strategies for Game III.*

The general formulae for the unique mixed strategy equilibrium of the game and the equilibrium payoffs are given by

$$x^* = (x_1, x_2, x_3) = ((1+b)/(3+2b), 1/(3+2b), (1+b)/(3+2b))$$
$$y^* = (y_1, y_2, y_3) = ((1+b)/(3+2b), 1/(3+2b),(1+b)/(3+2b)),$$

and the equilibrium payoffs are :

$$\Pi_G = (1+b)/(3+2b)$$
$$\Pi_K = (2+b)/(3+2b).$$

The equilibrium is symmetric, **M** is used less and less often with increasing *b* in favour of **L** and **R**, and the kicker gets a higher payoff than the goalkeeper for all *b*. The extreme values for *b* yield our Games I and II: *b* = 0 obviously results in Game II, while – interestingly – *b* going to infinity yields *equilibrium play* of Game I (which is a 2x2-game) in this 3x3-game. The unique mixed strategy equilibrium in this limiting case is no more completely mixed. It only mixes over the corner strategies **L** and **R**, which are the only strategies in Game I. Butt's argument therefore accounts for the different treatment of corner and middle options by the two players. However, it cannot account for the different treatment of the middle option by the two players, which we will address later.

Alternatively, consider yet another variation, Game IV. Suppose that there is some uncertainty as to whether a penalty is actually saved when both, kicker and goalkeeper choose the same corner. This uncertainty is represented by t in [0,1], where t represents the capability of the kicker; i.e. $t = 1$ implies that the kicker always scores when kicking to one of the corners, even when the goalkeeper jumps to the same side. Kicking to the middle will not score when the goalkeeper chooses to remain in the middle, however. Accordingly, $t = 0$ implies that the kicker never scores when the goalkeeper jumps to the same side, because, say, the kicker cannot shoot sufficiently powerful and accurate (as was implicitly assumed in the Games I-III). In this sense, t represents the quality of the kicker.

G \ K		y_1 **L**	y_2 **M**	y_3 **R**
x_1	**L**	1-t,t	0,1	0,1
x_2	**M**	0,1	1,0	0,1
x_3	**R**	0,1	0,1	1-t,t

Game IV: *An equivalent penalty game with modified payoffs.*

Dependent on t equilibrium strategies and payoffs are given by

$$x^* = (x_1, x_2, x_3) = (1/(3-t), (1-t)/(3-t), 1/(3-t))$$
$$y^* = (y_1, y_2, y_3) = (1/(3-t), (1-t)/(3-t), 1/(3-t))$$

$$\Pi_G = (1-t)/(3-t)$$
$$\Pi_K = 2/(3-t)$$

By inspection of the formulae, and perhaps somewhat surprisingly, this story yields the *same* prediction for equilibrium behavior as Game III before: Game II now corresponds to the extreme value $t = 0$, and *equilibrium play* of Game I follows for $t = 1$. But note that the equilibrium payoffs in the latter case are now 0 (for the goalkeeper) and 1 (for the kicker). Perfect kickers ($t = 1$) never miss, if they aim for a corner (irrespective of the goalkeepers action). So Game IV, too, may account for the relative unattractiveness of the strategy **M**. An outside observer could not say, whether observed behavior (frequency of actions by each player) results from equilibrium play of Game III with parameter b or Game IV with parameter $t = b/(1 + b)$ as both games determine equilibrium identically (!) under this condition.

Which description of the game, Game III or Game IV, and which corresponding interpretation is the right one? People may differ in their opinions. However, as it turns out, both games and both interpretations are, in fact, equivalent: the payoffs in Game IV can be obtained by an affine transformation of the payoffs in Game III. Because payoffs in our games can be arbitrarily normalized, affine payoff transformations do not change predicted strategies or outcomes. More specifically, affine equivalent games have the same (mixed) strategy equilibria (see e.g. Leininger et al. 1988, or Ritzberger 2002, chapter 5). In Appendix II we give an explicit transformation of Game III into Game IV by purely affine operations.

Once realized, this should not be too surprising to the reader. The more reliable a kicker can score by shooting to one of the corners, the greater the *relative* disaster from being unsuccessful when shooting to the middle, and the larger the payoff to the goalkeeper to have succeeded against a high-quality kicker. In the end, both payoff presentations tell the same story from two different angles. In equilibrium then the *same* behavior is required to level *relative* advantages of pure strategies in a mixed strategy equilibrium according to the Fundamental Lemma (see above).

	G \ K	y **L**	1-y **R**
x	**L**	1-t,t	0,1
1-x	**R**	0,1	1-t,t

Game V: *Another penalty game with modified payoffs.*

Game IV, too, may be regarded as an institutionally changed version of a 2x2 game: comparing the 3x3 Game IV with the 2x2 Game V yields a slightly different, but related story for the unattractiveness of the middle strategy. Game V is identical to Game IV with the exception that the middle strategy is not present. That is, by comparing the equilibrium outcomes of Game V with the outcomes of Game IV, we can again measure the effect of having the middle strategy available.

In equilibrium of Game V, the probability of choosing **L** or **R** is

$$x^* = y^* = ½$$

for both players (and all t ε [0,1]). The scoring probabilities (equilibrium payoffs) for goalkeeper and kicker, respectively, are:

$$\Pi_G = (1-t)/2$$
$$\Pi_K = (1 + t)/2$$

Observe that the kicker's payoff ranges from 1/2 for $t = 0$ to 1 for $t = 1$.

Compare now payoffs in equilibrium of Games IV and V: since $2/(3-t) - (1+t)/2 = (1-t)^2/(6-2t) > 0$ for all t ε [0,1), it follows that the introduction of **M** to the stratgy sets of the penalty game works in favour of the kicker.Yet, a high quality kicker (t large) only stands to profit marginally from the availability of the middle strategy and hence uses it rarely when indeed it is available. For instance, comparing scoring probabilities to the kicker in both games, we obtain:

- For $t = 0$ the scoring probability increases from 1/2 to 2/3.
- For $t = ½$ the scoring probability increases from 0.80 to 0.87.
- For $t = 0{,}80$ the scoring probability increases from 0.90 to below 0.91.
- For $t = 1$ the scoring probability in both cases is 1.

Assume now that kickers have private information about their quality t. Then, each shot reveals information about the payoffs involved, and thus about the kicker's quality. In particular, a shot to the middle decreases the estimation of a kicker's t. (This might mirror Toni Schumacher's statement that considering shots to the middle would involve a "different game.")[11] For this reason, kickers may hesitate to choose the middle.

[11] One testable hypothesis from this might be that amateur soccer players, with smaller t-values, shoot more often to the middle than professionals.

V. Explaining the players' different uses of ***M***

So far, all equilibria of all games have been symmetric in the sense that kickers and goalkeepers opt for **M** with the *same* probability. However, Chiappori et al. (2002) remarked that goalkeepers very rarely chose **M** compared to kickers. In their data, only 11 out of 459 goalkeepers remained in the middle, while 79 out of 459 kickers chose to kick to the middle (see their Table 3). This observation appears to be in line with the statements of both, Toni Schumacher and Hans-Jörg Butt, who confirmed that staying in the middle is a very unlikely strategy for goalkeepers. Schumacher's further elaboration, that a shot to the middle, whether successful or not, discredits a kicker with dishonourable or "un(sports)manly" behavior, takes the argument one step further: the following model captures this additional effect without increasing the number of parameters compared to our Games III and IV. It reflects the idea that not only missing in the middle is bad for the kicker, but also *scoring* to the middle yields smaller payoffs to the kicker than scoring by shooting to one of the corners. This yields the following *non*-constant-sum game, which nevertheless is strictly competitive:

	G \ *K*	y_1 **L**	y_2 **M**	y_3 **R**
x_1	**L**	1,0	0,b	0,1
x_2	**M**	0,1	1+b,-b	0,1
x_3	**R**	0,1	0,b	1,0

Game VI: *A non-constant-sum penalty game.*

The following table gives equilibrium strategies for various values of the parameter *b*:

	b = 1	*b = 0.8*	*b = 0.6*	*b = 0.5*
*x**	$\left(\frac{2}{5}, \frac{1}{5}, \frac{2}{5}\right)$	(0.43, 0.14, 0.43)	(0.47, 0.06, 0.47)	$\left(\frac{1}{2}, 0, \frac{1}{2}\right)$
*y**	$\left(\frac{2}{5}, \frac{1}{5}, \frac{2}{5}\right)$	(0.39, 0.22, 0.39)	(0.38, 0.24, 0.38)	$\left(\frac{3}{8}, \frac{1}{4}, \frac{3}{8}\right)$

Table 2: *Equilibrium strategies for Game VI.*

The formula for equilibrium strategies reads:

$$(x_1, x_2, x_3,) = \left(\frac{1+b}{4b+1}, \frac{2b-1}{4b+1}, \frac{1+b}{4b+1}\right)$$
$$(y_1, y_2, y_3,) = \left(\frac{1+b}{3+2b}, \frac{1}{3+2b}, \frac{1+b}{3+2b}\right)$$

For the relevant parameter range 0.5 < *b* < 1, both players use **M** less often than **L** or **R**, and the goalkeeper less often than the kicker. At the extreme value *b* = 1 (scoring to the middle is as good as scoring to a corner) both players use middle with the same probability. A decrease in *b*; i.e. making scoring to the middle less attractive than scoring to a corner, leads to *less* frequent use of **M** by the goalkeeper and *more* frequent use by the kicker**.** These – at first sight somewhat counterintuitive consequences – are once more easily understood by the content of the Fundamental Lemma about the strategic interaction in mixed strategy equilibrium: the decreased attractiveness of **M** to the kicker has to be offset by the goalkeeper with a corresponding decrease in attractiveness of **L** and **R** for the kicker in order to restore indifference for the kicker between all options. The goalkeeper does so by choosing **L** and **R** *more* often at the expense of choosing **M.** This more than offsets the lost attractiveness of **M** to the kicker, who now opts for **M** *more* often. At the other extreme (*b* = ½) this argument drives the goalkeeper to *always* choosing **L** or **R** (and hence *never* going for **M**). Because of the relative unattractiveness of scoring to the middle, the kicker uses **M** nevertheless only with probability ¼.

Still, for all b ε (0.5, 1) it is true, that the kicker's payoff in equilibrium exceeds the goalkeeper's payoff:

$$\Pi_G = (4b^2+5b+1)/[(4b+1)(3+2b)]$$
$$\Pi_K = (4b^2+8b+1)/[(4b+1)(3+2b)]$$

More importantly, the scoring probability is larger than ½, and varies between 15/24 = 0.625 for b = ½ and 16/25 = 0.64 for b =1.

This simple one-parameter model captures the essential features of the data, in particular with respect to the strategy **M**, and confirms Neeskens' intuition that establishing the middle alternative as a valid option for the kicker results in an *advantage* for the kicker. It increases the probability of scoring *in equilibrium*, when both players behave optimally according to the recommendations of game theory.

VI. Conclusions

Our paper demonstrates, along with previous work in sports economics, that game theory is a powerful tool to analyze and describe strategic behavior in the penalty-duel. At the same time, we emphasize that game theoretic advice and predictions depend critically on the exact characterization of not only the underlying game pattern but also the perceptions of this by the players. In more sociological terms, a penalty-duel interlocks the *roles* of kicker and goalkeeper in an interactive decision situation. Thus, from an analytical perspective, the relevant penalty-duel institutions do not only include the objective rules of the game determined by the FIFA but also the players' expectations about how the different roles are supposed to perform. Identifying such institutions can be difficult. Players may change their behavior and hence the way institutions are perceived over time, they may even differ in their perceptions of the game being played at a given time; and the perceptions may differ between insiders, who are actually playing the game, and outsiders, who are trying to understand behavior.

In game theory, a player's task is to find an equilibrium strategy taking the game as given. However, understanding the (equilibrium) interaction is only part of the art of playing games. Neeskens and Panenka demonstrated that going beyond shared perceptions of how to play the game (i.e. fulfil the roles of the institution), and devising new, innovative strategies may yield a competitive edge. They possibly changed the way penalty-kicks are perceived in a permanent way.[12] Their innovation seems to account for and answer to the growing competitiveness of professional sports – and in particular soccer – due to increased commercialization. While game theory may capture the consequences of institutional change created by innovators such as Neeskens and Panenka (as we demonstrate in this paper), it cannot easily promote creativity in institutional design, because it must start with a description of all available strategies.[13]

Another conclusion from our study is that even if players perceive institutions differently, they might in fact play the same game, with perfectly equivalent incentive structures. So, different perceptions do not necessarily exclude standard game theoretic analyses. However, game theoretic analysis must take the perceptions of the players serious. We demonstrate how a number of empirical observations and statements by goalkeepers (and a kicker) about the attractiveness of the middle in penalty taking can be accounted for by an equilibrium analysis of the penalty duel with modified payoffs. Kickers and goalkeepers do not only have preferences on the outcome of the game, goal or not goal, but also on the way the goal is shot or saved, which has to do with the perception of their roles. Although new behavior in a role may become legitimate, it need not be perceived as acceptable as the traditional one. More specifically, our study strongly suggests that a shot or even goal to the middle is evaluated differently than a shot or goal to a corner by both, goalkeepers and kickers. This has not been observed before.

[12] Of course, breaking out of conventions and (re)shaping the way institutions are perceived can also be a clever strategy with long-lasting effects in economic and social environments.

[13] However, there is a growing literature on economic design challenges; see, e.g., Roth (2002), Kittsteiner and Ockenfels (2006) and Ockenfels et al. (2006) for selective reviews. Also, there appears to be more room for institutional design in penalty taking. Toni Schumacher and Hans-Jörg Butt mentioned many other strategies that add to the institutional complexity and that are neither considered in our game models in this paper, nor by any other game theoretic approaches to penalty taking that we are aware of. Toni Schumacher, for instance, mentioned that a couple of 'signals' from the kicker helped him to save penalties, such as the line of sight after the kicker has put the ball on the penalty spot, or the position of the foot of the supporting leg. Both goalkeepers also noted that they tried to make the kicker insecure. Toni Schumacher (Hans Jörg Butt) mentioned, for instance, that he would lean towards the corner which he did (not) choose, or that he sometimes offered the kicker a bet. Moreover, Hans Jörg Butt gave an intriguing account of his strategies as a penalty taker and goalkeeper, which suggests that a penalty duel could be modelled as a waiting game. It is exactly these kinds of subtleties that seem to make the duel interesting.

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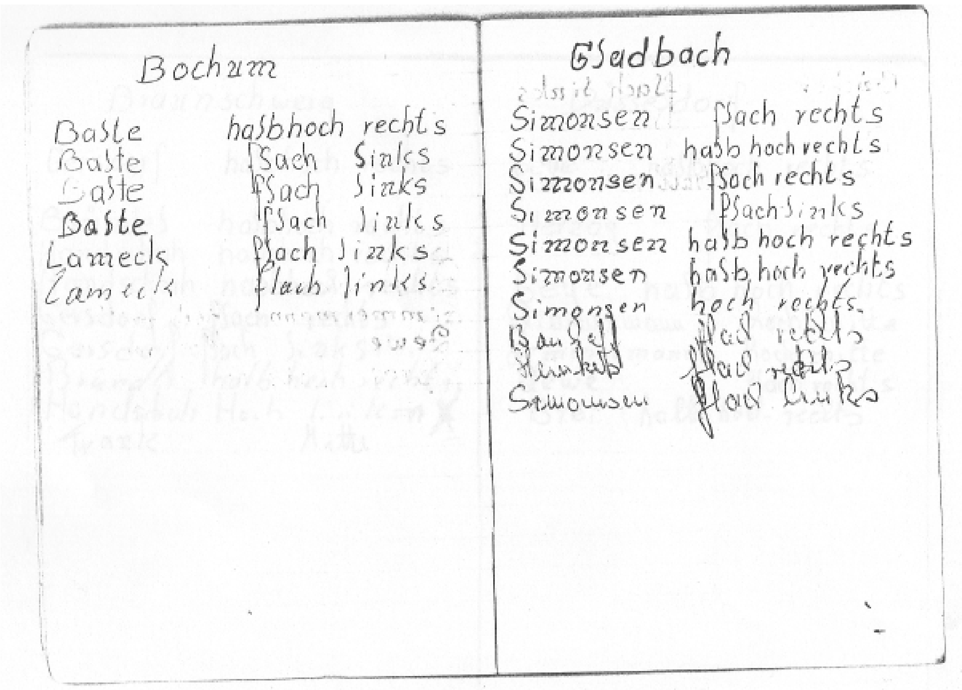
Appendix I : Excerpt from Toni Schumacher´s notes



Appendix II: Transformation of Game III into Game IV

G \ K	L	M	R
L	1,0	0,1	0,1
M	0,1	1+b,-b	0,1
R	0,1	0,1	1,0

Game III

set

b = $\frac{t}{1-t}$

G \ K	L	M	R
L	1,0	0,1	0,1
M	0,1	$\frac{1}{1-t}, \frac{-t}{1-t}$	0,1
R	0,1	0,1	1,0

Multiplying player G's payoffs with the constant c_1=(1-t) yields:

G \ K	L	M	R
L	1-t,0	0,1	0,1
M	0,1	$1,\frac{-t}{1-t}$	0,1
R	0,1	0,1	1-t,0

Game III (1)

add constant $c_2 = \frac{t}{1-t}$ to all of player K´s payoffs <u>given</u> M of player G

G \ K	L	M	R
L	1-t,0	0,1	0,1
M	$0, \frac{1}{1-t}$	1,0	$0, \frac{1}{1-t}$
R	0,1	0,1	1-t,0

Game III (2)

Next, multiply player K´s payoffs with the constant c_3 = (1-t):

G \ K	L	M	R
L	1-t,0	0,1-t	0,1-t
M	0,1	1,0	0,1
R	0,1-t	0,1-t	1-t,0

Game III (3)

and finally add c_4= t to the payoffs of player K <u>given</u> L of G and <u>given</u> R of G

G \ K	L	M	R
L	1-t,t	0,1	0,1
M	0,1	1,0	0,1
R	0,1	0,1	1-t,t

Game IV

Game III has the same equilibria as Game III (1).
Game III (1) has the same equilibria as Game III (2).
Game III (2) has the same equilibria as Game III (3).
Game III (3) has the same equilibria as Game IV.