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SHORT-TERM PRICE RIGIDITY IN AN ENDOGENOUS GROWTH MODEL: NON-SUPERNEUTRALITY AND A NON-VERTICAL LONG-TERM PHILLIPS-CURVE

PETER FUNK
BETTINA KROMEN

Department of Economics
University of Cologne
Albertus-Magnus-Platz
D-50923 Köln
Germany

http://www.wiso.uni-koeln.de

Short-term price rigidity in an endogenous growth model: Non-Superneutrality and a non-vertical long-term Phillips-curve*

Peter Funk[†] and Bettina Kromen[‡]

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Abstract

This model analyses the interaction between inflation and the long-run levels of employment and output growth in a Schumpeterian growth model with quality improving innovations under nominal price rigidity. At the unique REE steady state equilibrium, both employment and growth are hump-shaped functions of money growth peaking at positive inflation rates. This is due to four effects of money growth under rigidity: *Erosion* of its *relative price* through inflation and the optimal *initial mark-up* set in anticipation of this influence a firm's profits. *Dispersion* in relative prices causes inefficient production while the change in the *average mark-up* influences aggregate demand.
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1 Introduction

In this paper, we analyse the effects of *short-term* New-Keynesian price setting frictions on *long-run* economic development in a Schumpeterian model of innovation-driven growth. In particular, we analyse the effects of money growth and inflation on the steady state values of the level of employment and the growth rate of output in this setting. The permanent presence of short-lived price rigidity allows inflation to influence these long-run variables.

*Corresponding author: Peter Funk, Staatswissenschaftliches Seminar, Universität zu Köln, Albertus-Magnus-Platz, 50923 Köln, Germany. Tel.: +49/221/4704496, Fax: +49/221/4705143, Email: Peter.Funk@uni-koeln.de

[†]University of Cologne

[‡]University of Cologne

Due to price rigidity inflation distorts relative prices, which in turn have a level effect on the allocation of the economy's resources and the efficiency with which they are used. These level effects influence the real wage, which in our model determines labour supply and employment. Given that long-run growth is but repeated short-run growth determined by the incentive to innovate in the R&D-sector, the distortion of relative prices caused by price rigidity also influences the long-run growth rate both directly and indirectly through its effect on employment. Thus given price rigidity, there is a non-trivial relationship between the money growth rate on the one hand and employment and output growth on the other hand, which also implies that monetary policy has some scope to influence long-run outcomes.

In the past decades, these above-mentioned long-run relationships have received rather limited attention in the literature, as the existence of a vertical long-run Phillips curve was widely accepted since the seminal papers of Friedman [1968] and Phelps [1967]. While the surge of the New Neoclassical Synthesis literature in the 1990s lead to renewed interest in the analysis of money's relation to the real economy, this was limited to the short-run effects of money: These effects were analysed by studying the behaviour of the linearised economy around a zero inflation steady state - abstracting from inflation in the long run was deemed an innocuous assumption given the conviction that money did not matter in the long run. Only recently have attempts been made to understand the consequences of positive steady state inflation in this Dynamic General Equilibrium (DGE) framework.[1]

If money growth and inflation were believed irrelevant for the determination of the long-run levels of output and employment, there was no reason to believe they would have a non-negligible impact on the long-run growth rate of output. The bulk of the new endogenous growth literature that originated in the 1990s abstracted completely from monetary aspects. More recently, a small literature has developed that investigates the effect of inflation on the rate of economic growth.[2]

At the same time, a number of recent empirical studies investigate the (non-)superneutrality of money[3] with regard to the level of employment and output or the economic growth rate, respectively. The former studies try to find evidence for a non-vertical long-run Phillips-curve for the US or Europe. The evidence is mixed: While in some cases a vertical long-run Phillips curve cannot be rejected, some evidence points to a favourable long-run trade-off between inflation and unemployment.[4] At the same time, a limited number of

[1] See Ascari [1998, 2004], Devereux and Yetman [2002] and Graham and Snower [2004].

[2] A recent survey paper is Gillman and Kejak [2005a]. We discuss this literature in a companion paper (Funk/Kromen [2005]), that presents a related model with a detailed analysis of the long-run relationship between money growth and economic growth.

[3] Money is said to be superneutral if the growth rate of money supply does not affect real outcomes.

[4] Relevant recent studies are mostly based on two influential papers by Watson and King [1994,1997].

recent studies reports a positive slope of the long-run inflation-unemployment relationship.[5] In contrast, in the literature studying the empirical relationship between inflation and economic growth,[6] a consensus seems to have emerged recently that at least high rates of inflation are detrimental to growth.[7] Evidence of this negative relationship has been found in a host of studies making use mainly of panel regression models.[8] Several studies find a linear negative relationship where a 10 percentage point increase in the monetary growth rate decreases economic growth by 0.2-0.3 percentage points.[9] Most recent studies furthermore find evidence of non-linearity in the data. Specifically, there may be a threshold value below which the inflation growth is not significantly positive. E.g., Khan and Senhadji [2001] report that the inflation-growth relationship is weakly but significantly positive for industrialised countries below an inflation threshold of 1%.[10,11]

Summing up, there is indeed considerable evidence for the non-superneutrality of money in the data.

For recent studies using European data see, e.g. Koustas and Serletis [2003] and Karanassou, Sala and Snower [2003]. A quantification of the negative effect is given by Karanassou, Sala and Snower [2003]: A "10 percent increase in long-run money growth (equal to long-run inflation) is associated with a 3.18 percentage point fall in the EU unemployment rate." Setterfield and Leblond [2003] use data for the US.

[5] Beyer and Farmer [2002] and Russell and Banerjee [2006] find evidence of a significantly positive long-run relationship between inflation and unemployment in the US. In the latter paper, the effect is quantified as implying that "an increase in inflation of around 5 percentage points [...] would be associated with an increase in unemployment in the long-run of about 1 1/2 percentage points" (p. 14).

[6] Temple [2000] contains a survey of recent empirical contributions as well as a discussion of the methodological difficulties involved. Summaries of empirical investigations are also given in Ghosh and Phillips [1998], Bruno and Easterly [1998], Ragan [1998] and Briault [1995].

[7] Earlier studies, using mostly cross-country data, found no significant correlation between monetary variables and economic growth. The cross-country study of McCandless and Weber [1995] is a good example and contains further references. More recently, Judson and Orphanides [1999] find no significant relation in cross-country data but a negative relation when panel data are used. In a time–series setup, Geweke [1986] finds support for the superneutrality hypothesis using a century of annual U.S. data.

[8] Cf. e.g. to Barro [1996], Judson and Orphanides [1999], Gylfason and Herbertsson [2001] and Gillman, Harris and Mátyás [2004]. As Temple [2000] points out, the results of those studies that average data over 10 or 15 years rather than over five years or less may be interpreted with more confidence as reflecting the medium or long-term effects of inflation. There is some evidence that the relationship is stronger in higher frequency data, see e.g. Ghosh and Phillips [1998].

[9] Barro [1996] reports this using 10-year averages of data for over 100 countries spanning the period 1960-1990. Similar estimates are reported by Fischer [1993] and Motley [1998] who uses cross-sectional data, though.

[10] When both industrialised and developing countries are included, the threshold is at an inflation rate of 11% and the relationship below this value is insignificantly positive. Sarel [1996], Gylfason and Herbertsson [2001], Judson and Orphanides [1999] and Burdekin et al [2004] also report threshold values.

[11] Further, the negative relationship seems to be of a convex form, so that the marginal cost of inflation decreases in the inflation rate, see, e.g. Gylfason and Herbertsson [2001] and Gillman, Harris and Mátyás [2004] or Ghosh and Phillips [1998].

Our aim in this paper is to analyse these long-term effects of money in a framework that combines elements from the two mentioned strands of theoretical literature dealing with monetary effects: Whereas the New Neoclassical synthesis (NNS) introduces Keynesian elements such as price rigidity into the standard *Real Business Cycle* model, we introduce a Keynesian friction into the standard *Schumpeterian growth* model to analyse the long-run relationship between inflation, employment and growth. There are to our knowledge no papers investigating whether the sustained presence of short-term frictions has an impact on these long-term relationships in a growth model.[12]

The reader might at first be surprised by our joint analysis of an economy's long-run performance and nominal price rigidity, which is more frequently integrated into short-run business cycle models. Yet in spite of the different time dimension of these elements, a significant effect of changes in the degree of price rigidity on long-run output growth is reasonably to be expected when taking into account the following points: Firstly, although individual prices are fixed for short periods of time only, price rigidity is a permanent feature of the economy such that under non-zero inflation, relative prices are consistently distorted in the short run. Since relative prices determine an economy's resource allocation, the latter is permanently affected by price rigidity in the short run. Finally, note that long-run growth is but repeated short-run growth stemming from agents' optimal choice between consumption and investment. This choice is certainly influenced by both relative prices and the short-run levels of other economic variables. Therefore, we must indeed expect an economy's growth rate and the levels of other variables in the long run to be influenced by short-run price rigidity.

Our approach to modelling economic growth follows the quality-ladder model of Aghion and Howitt [1992] and Grossman and Helpman [1991] in that growth is achieved through the improvement of intermediate good types that are imperfectly substitutable. Quality improvements are embedded in innovative intermediate goods the patents for which are sold to monopolistically competitive intermediate goods firms by successful R&D firms. Success in research arrives stochastically in the R&D sector. These intermediate goods and labour are inputs in the production of the economy's final good in a perfectly competitive final goods sector. Labour supply can be thought of as unionised and is introduced

[12] The paper from the aforementioned DGE literature that is closest to ours is Graham and Snower [2004]. In a standard DGE model with differentiated labour as an input in production and staggered wage setting à la Taylor [1980], they find that employment increases in inflation since inflation lowers the real wages set by monopolistic wage setters, comparable to our average mark-up effect. Output is a hump-shaped function of inflation in their model since at higher inflation rates, the effect of increased employment is dominated by the inefficient composition of the labour aggregate, which is close in spirit to our price dispersion effect. Since there is no long-run output growth in their model, they cannot analyse the interactions of inflation and employment with growth.

through an exogenous function that increases in the real wage in efficiency units. We introduce money into the model by assuming that households derive utility from holding cash balances, following Sidrauski [1967]. Prices and inflation matter in the model because in line with the recent DGE literature, we assume the existence of nominal price rigidity à la Calvo [1983] in the intermediate goods market. The change in relative prices caused by inflation under price rigidity influences demand for intermediate goods and both their and labour's productivity, which in turn affects both employment and the profits accruing to an innovator and hence, economic growth.

As a result, at steady state both **employment and the growth rate are hump-shaped functions of money growth** whose **peaks** are reached at money growth rates associated with **positive inflation rates**.

These non-linear relationships are due to four effects of an increase in money growth under price rigidity which we now very briefly discuss:

An increase in the absolute value of the money growth rate that raises the absolute value of inflation will under rigidity lead to an increase in *relative price dispersion* concerning intermediate goods. Given that these are imperfect substitutes, the distortion of quantities demanded resulting from price dispersion causes inefficient production, which in turn reduces labour's productivity and employment.

At the same time, the marginal productivity of labour and employment increase in the total amount of intermediate goods used in final good production. This amount in turn depends on the *average mark-up* charged by intermediate goods producers which is influenced by money growth through two channels: Firstly, an increase in money growth and inflation raises marginal cost while prices are fixed under rigidity, lowering effective mark-ups. Secondly, in anticipation of this effect, the initial mark-ups set by firms increase in inflation and the money growth rate, which tends to increase the average mark-up.

Taking account of these two effects, employment increases in money growth at low inflation rates but decreases in the money growth rate at high inflation rates, such that a monetary authority seeking to promote employment should be aware of the detrimental effects of high inflation on employment but prefer a policy of very moderate inflation to price stability.

Output growth in our model depends positively on the level of employment, so that the two above-discussed effects of money growth indirectly influence the growth rate, too. In addition, the incentive to innovate and the growth rate are more directly influenced by money growth's two influences on the relative price charged by an intermediate good firm: Given infrequent price adjustment, a firm's optimal *initial mark-up* increases in money growth and inflation because inflation later leads to *mark-up erosion* while its price is fixed. Therefore, the firm's mark-up and relative price are initially higher and later lower than optimal, with countervailing effects on demand. Taken together, suboptimal mark-up levels lower demand and profits relative to their value under price stability. Since the profits accruing to intermediate goods producers determine the incentive to engage in research activities to develop new intermediate goods, this inversely affects innovation-driven economic growth.

Given the resulting hump-shaped money-growth relationship, a monetary authority interested in fostering economic growth would also choose a money growth rate leading to moderate inflation.

Thus we find that the influence of short-term price rigidity is indeed not limited to the short-tun. Rather, it allows inflation to affect both the long-run level of employment and output and the growth rate of output in a way that is consistent with a non-linear long-run Phillips-curve facing monetary policy authorities.

A realistically calibrated numerical example is used to illustrate the results. The effect of money growth on economic growth is quantitatively in line with the results of the empirical literature.

The remainder of the paper is organised as follows: Section 2 presents the model, while Section 3 discusses the general equilibrium. Comparative statics and a calibrated example are presented in Section 5 and Section 6 concludes.

2 The model

2.1 Final good sector

In the perfectly competitive final goods sector, the economy's final good Y is produced using labour L and N varieties of differentiated intermediate goods. Following Dixit and Stiglitz [1977], the intermediate goods are combined according to a constant-elasticity-of-substitution aggregator:

$$Y(\tau) = AL(\tau)^{1/\alpha} \sum_{j=1}^{N} \left(q^{k_j(\tau)} x_j(\tau)\right)^{(\alpha-1)/\alpha} \tag{1}$$

where x_j is the amount of sector j intermediate good used, q^{k_j} is this type's productivity and we assume $\alpha > 1$.[13]

[13] We make sure that only the latest quality is available in each sector by assuming that parameters are such that the innovator's monopolistic mark-up makes production unprofitable for the incumbent. Given the steady state mark-up from (22), $q > \frac{\alpha}{\alpha-1} \frac{\rho+\beta-(\alpha-1)\psi}{\rho+\beta-\alpha\psi}$ is a sufficient condition. This condition is satisfied at the examined money growth rates in our calibrated examples.

The representative firm's profits are given by

$$\Pi^Y(\tau) = P(\tau)Y(\tau) - \sum_{j=1}^{N} p_j(\tau)x_j(\tau) - w(\tau)L(\tau) \tag{2}$$

where $P(\tau)$ is the final good price, $p_j(\tau)$ is the price charged for one unit of sector j intermediate good and $w(\tau)$ is the wage. The firm's optimal demand for labour and for intermediate good j, respectively, are given by

$$\frac{1}{\alpha}\frac{Y(\tau)}{L^d(\tau)} = \frac{w(\tau)}{P(\tau)} \tag{3}$$

and

$$x_j(\tau) = \left(\frac{p_j(\tau)}{P(\tau)}\right)^{-\alpha}\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha} L(\tau)\, q^{(\alpha-1)k_j} \tag{4}$$

Optimal demand for the type j intermediate good depends negatively on the type's relative price and positively on its productivity $q^{k_j(\alpha-1)}$ and on employment $L(\tau)$.

2.2 Intermediate goods sector

The firm that bought the patent for intermediate good j from the research firm that developed the innovation produces the intermediate good one for one with output:

$$x_j(\tau) = h_j(\tau) \tag{5}$$

where h_j is the quantity of output used for production. Given the linear production function, the development of marginal cost is given by the development of the economy's output price level $P(\tau)$.

An intermediate good producer's pricing problem The fact that the N intermediate goods are imperfect substitutes in final good production implies that intermediate goods producers act in an environment of monopolistic competition and allows them to choose an optimal price subject to the final good sector's demand function. Prices in the intermediate goods market can only be changed infrequently, where the modelling of price rigidity follows Calvo [1983] and Kimball [1995]: At any moment in time, a firm may only change its price if it receives a stochastic signal that is Poisson-distributed with parameter β. Also, any firm replacing the incumbent in sector j by entering the market with a new variety of intermediate good j may choose a price at the time of market entry. Whenever they have the opportunity to readjust prices, firms choose a price to maximise the expected present value of nominal profits obtained while their price is fixed, which is given by

$$E\left[V(p_j,\tau)\right] = \int_{\tau}^{\infty} \widetilde{B} e^{-\int_{\tau}^{s}\left[i(\theta)+\mu_{k_j}(\theta)+\beta\right]d\theta}\left[p_j - P(s)\right]x_j(s)\,ds \tag{6}$$

where $[p_j - P(s)]\, x_j\,(s)$ is the firm's profit at time s and $\widetilde{B}$ is a constant from the integration of the probability distribution of the price reset signal and i is the nominal interest rate. The term $e^{-\int_{\tau}^{s}\left[i(\theta)+\mu_{k_j}(\theta)+\beta\right]d\theta}$ is the discount factor which is adjusted for the probability of obsolescence facing the firm in two different ways: Firstly, $e^{-\int_{\tau}^{s}\beta d\theta}$ represents the probability of not receiving a price setting signal before time s in the future. Secondly, the research intensity μ_{k_j} in research firm j determines the intermediate firm's probability $e^{-\int_{\tau}^{s}\mu_{k_j}(\theta)d\theta}$ of not having being replaced by a successful innovator by time s. Since profits accruing after either of these two events occurs are irrelevant for the firm's pricing decision at time τ, discounting of future profits is the stronger, the higher β and μ_{k_j}.

It will be shown in Section (3.4) that the optimal price chosen by the firm at steady state depends crucially on the inflation rate, creating a channel for monetary policy to influence the real side of the economy in the model.

2.3 Patents and the R&D sector

There is free entry to the research and development sector where small firms try to improve existing intermediate goods. The parameter $\mu_{k_j}(\tau)$ of the Poisson process governing the probability of making an innovation that improves intermediate good j depends linearly on the amount of final good used, $z_j(\tau)$, for a given quality rung k_j (i.e., current position of sector j):

$$\mu_{k_j}(\tau) = \phi(k_j(\tau))z_j(\tau) \tag{7}$$

Sector j research firm's expected profit at time τ is given by the expected revenue $\mu_{k_j}(\tau)E\left[V_{k_j+1}(\tau)\,\middle|\,t_{k_j}=\tau\right]$, where $E\left[V_{k_j+1}(\tau)\,\middle|\,t_{k_j}=\tau\right]$ is the expected present value at market entry of all future profits accruing to a potential producer of the new intermediate good, as given in equation (24), minus the input cost $P(\tau)z_j(\tau)$.

There is free entry into the research sector, so firm j's expected profit is zero at every instant which using (7) implies that

$$\phi(k_j(\tau))E\left[V_{k_j+1}(\tau)\,\middle|\,t_{k_j}=\tau\right] - P(\tau) = 0 \tag{8}$$

holds for all active research firms.[14]

We choose a standard knife-edge specification for $\phi(k_j(\tau))$ that makes sure that the optimal research intensity μ can be constant and independent of a sector's position and which implies the existence of spillovers in research. Specifically, the lower the sector's quality level, the easier is making an innovation:

$$\phi(k_j(\tau)) = \frac{1}{\lambda}q^{-(\alpha-1)(k_j+1)} \tag{9}$$

[14] Note that the firm's value is $E\left(V_{k_j+1}(\tau)\,\middle|\,t_{k_j}=\tau\right)$ because it will produce the *next* quality, k_j+1 for the sector, which is about to be developed.

where $1/\lambda$ is the productivity of labour in research.

2.4 Public Sector

We choose a very parsimonious specification for the public sector:[15] The state does not levy taxes or issue bonds. Its only policy instrument is the money supply, $M^s(\tau)$ which is perfectly controlled by an independent central bank by setting the constant exogenous money growth rate ψ:

$$\frac{\dot{M^s}(\tau)}{M(\tau)} = \psi \tag{10}$$

All revenue from money creation is allocated to households in form of a lump-sum cash transfer, $T(\tau)$

$$\dot{M^s}(\tau) = T(\tau) \tag{11}$$

There is no government spending apart from the transfer of seigniorage to households.

2.5 Consumption and money demand

Consider a household representative of a continuum of infinitely lived households with mass one distributed uniformly on the interval $[0, 1]$. The representative household maximises the discounted present value of his lifetime utility flows, where $\rho > 0$ is the discount factor. We assume that the household derives utility both from consumption $c(\tau)$ of the economy's final good and from holding real balances $m(\tau) = \frac{M(\tau)}{P(\tau)}$. The latter is a standard assumption that can be justified by assuming that the household needs cash for transaction purposes.[16] A standard specification for households' utility is

$$U = \int_{s=0}^{\infty} e^{-\rho s} \frac{(c(s)^{1-\theta} m(s)^{\theta})^{1-\eta} - 1}{1-\eta} ds \tag{12}$$

where we assume $\eta \geq 1$, $\theta \in [0, 1)$ and abstract from population growth. The representative household maximises (12) subject to his budget constraint given labour:

$$\dot{v}(\tau) = \frac{w(\tau)}{P(\tau)} L(\tau) + \frac{T(\tau)}{P(\tau)} + r(\tau)v(\tau) - c(\tau) - [\pi(\tau) + r(\tau)]\, m(\tau) \tag{13}$$

where v is the real value of the household's monetary and non-monetary wealth, $\frac{w}{P}L$ is the household's real wage income from being employed $L \leq \overline{L}$ hours, $\frac{T}{P}$ is the real value of the transfer received from the government, and r is the real interest rate which is paid on the firms real holdings of shares in investment funds that finance R&D firms' activities.[17]

[15] Similar specifications are used in the related literature by, e.g. Gillman and Kejak [2005b], Chang [2002], Marquis and Reffett [1995], Orphanides and Solow [1990].

[16] Feenstra [1986] shows that our case of non-separable utility for consumption and real balances is equivalent to the explicit modelling of cash holdings' transaction cost reducing function.

[17] The household receives real interest payments of $r(\tau)$ on his non-monetary assets, $v(\tau) - m(\tau)$ while the value of real money holdings depreciates at rate $\pi(\tau)$, where $\pi(\tau)$ is the rate of inflation.

Solving the household's maximisation problem leads to the following first-order conditions:

$$\frac{\theta}{1-\theta}\frac{c(\tau)}{m(\tau)} = r(\tau) + \pi(\tau) \tag{14}$$

$$[\eta + \theta(1-\eta)]\frac{\dot{c}(\tau)}{c(\tau)} - \theta(1-\eta)\frac{\dot{m}(\tau)}{m(\tau)} = r(\tau) - \rho \tag{15}$$

Equation (14) is a static efficiency condition requiring that the ratio of marginal utilities from money holdings and consumption equal their cost ratio, where the opportunity cost of holding cash is the nominal interest rate $i(\tau) = r(\tau) + \pi(\tau)$. Equation (15) governs the utility-maximising allocation of the household's resources over time and will in steady state equilibrium reduce to the familiar Ramsey rule.

2.6 Labour supply

Labour supply is introduced in the simplest possible way as an *exogenously* given function $L^s(\widetilde{w})$ of real wages per efficiency unit $\widetilde{w}(\tau) = \frac{w(\tau)}{P(\tau)Q(\tau)}$, where $L^S(\widetilde{w})$ is strictly increasing in $\widetilde{w}$ from $L^s(0) = L^{min} > 0$ to $\overline{L} = \lim_{\widetilde{w}\to\infty} L^s(\widetilde{w})$.[18]

For the sake of concreteness we assume that

$$L^s(\widetilde{w}) = \overline{L}\left(1 - \frac{e^{-\sigma\widetilde{w}}}{2}\right) \tag{16}$$

where $\overline{L} > 0$ is the maximal employment (full employment), $L^{min} = \overline{L}/2$ and $\sigma > 0$ is a parameter reflecting the reactiveness of employment with respect to the wage per efficiency unit $\widetilde{w}$ where at time τ we have $\widetilde{w}(\tau) = w(\tau)/(P(\tau)Q(\tau))$. L^s will be constant in steady

[18] One way to think about $L^s(\widetilde{w})$ is to assume that it results from the utility maximisation of households with extremely separable preferences: The household's "worker" maximises a function $v(\widetilde{w}_\tau l_\tau, l_\tau)$ facing a trade-off between the disutility of too much work and bringing home high labour income $\widetilde{w}_\tau l_\tau$. The household's "shopper" receives $(\widetilde{w}_\tau l_\tau)_{\tau\geq 0}$ maximises (12) given $\{l_\tau\}_{\tau\geq 0}$ since he does not interfere with the "worker's" decision.

Assuming $v_1 > 0$, $v_{11} < 0$, $v_2 \gtreqless 0$ for $L \lesseqgtr L^{min}$ and $v_2 \to -\infty$ for $L \to \overline{L}$, the worker's choice of $L^s(\widetilde{w})$ has the desired form.

Another way to think about the inverse of $L^s(\widetilde{w})$ is to assume that wages $w(\tau)$ are set by a centralised labour union. The union's real wage claims per efficiency unit are moderated by a high level of unemployment- leading to a positive relation between wages and employment. This may either reflect the union's genuine interest in low unemployment together with its belief that a moderation of wage claims reduces unemployment or it may directly reflect the waning of the union's power to implement high wages when unemployment rises. Note that in the present setting control over nominal wages $w(\tau)$ in fact allows to control real wages per efficiency unit $\widetilde{w}$ and also that the union's belief of a negative short-run relation between $\widetilde{w}(\tau)$ (and $w(\tau)$) and employment is warranted.

Note that only the second interpretation allows us to discuss unemployment that is involuntary for the individual worker.

state equilibrium where $\widetilde{w}(\tau)$ is constant. The strength of labour supply's reaction with respect to the wage depends on the parameter σ.

3 Steady state equilibrium

We now analyse the model's general equilibrium restricting our attention to Rational Expectations steady state equilibria with constant output growth.

3.1 Households

From the household's static optimality condition (14), we have that the growth rates of consumption and real money holdings are equal at steady state equilibrium. Using this in the household's dynamic optimality condition and rearranging yields the familiar Ramsey rule:

$$\frac{\dot{c}(\tau)}{c(\tau)} = \frac{r-\rho}{\eta} \tag{17}$$

3.2 Money market equilibrium

The money market is in equilibrium when money demand equals supply, $M^s(\tau) = M^d(\tau)$. Equivalently, given the initial money stock owned by households $M(0)$, the growth rate of real money supply, $\frac{\dot{m}^s(\tau)}{m^s(\tau)} = \frac{\dot{M}^s(\tau)}{M^s(\tau)} - \frac{\dot{P}(\tau)}{P(\tau)} = \psi - \pi$, must equal the growth rate of demand for real balances $\frac{\dot{m}^d(\tau)}{m^d(\tau)}$. Using again that the household's desired growth rates for consumption $\dot{c}(\tau)/c(\tau)$ and money holdings are identical at steady state equilibrium,[19] and assuming that at steady state equilibrium the growth rate of consumption coincides with the growth rate of final good production γ,[20] we have $\gamma = \frac{\dot{c}(\tau)}{c(\tau)} = \frac{\dot{m}^d(\tau)}{m^d(\tau)}$.

Setting $\frac{\dot{m}^s(\tau)}{m^s(\tau)} = \frac{\dot{m}^d(\tau)}{m^d(\tau)}$, we have that the inflation rate at steady state is the output-growth adjusted money growth rate

$$\pi = \psi - \gamma \tag{18}$$

3.3 Behaviour of the aggregate quality index $Q(\tau)$ and the growth rate

We define the economy's aggregate technology index, $Q(\tau)$, as the weighted sum of the productivities $q^{k_j(\tau)}$ associated with each sector's intermediate good

$$Q(\tau) = \sum_{j=1}^{N} q^{(\alpha-1)k_j(\tau)} \tag{19}$$

[19] See the household's static optimality condition (14).

[20] See equation (26) in Section 3.5.

The expected growth rate of the quality index Q at time τ, $E\left[\gamma_Q(\tau)\right]$, can be found by aggregating over j the changes in sector $j's$ quality brought about by an innovation, weighted with the flow probability that an innovation will occur in sector j in the infinitesimal time interval beginning at τ. In steady state equilibrium, this probability will be constant and the same for all sectors, so we set $\mu_{k_j}(\tau) = \mu(\tau) = \mu$. Using the law of large numbers, the expected and actual growth rates of the quality coincide. Following these steps gives us

$$\gamma_Q = \left(q^{\alpha-1} - 1\right)\mu \tag{20}$$

Since at steady state, the growth rate of output γ equals the growth rate of the aggregate quality index,[21] we have

$$\gamma = \left(q^{\alpha-1} - 1\right)\mu \tag{21}$$

3.4 Equilibrium in the market for intermediate goods

We now derive the optimal mark-up chosen by readjusting firms in equilibrium. Together with the final sector's demand function (4), this allows us to derive the market value of an intermediate goods firm at market entry, which will determine the equilibrium patent price charged by successful R&D firms. We further use the optimal initial mark-up and equation (4) to find the quantity of intermediates produced in steady state equilibrium.

3.4.1 Optimal price at steady state equilibrium

We find the optimal price for an intermediate goods firm that may first choose or readjust its price by maximising the expected value of profits given in (6) with respect to the price p_j subject to the final good producing firms' demand function (4). Using that at steady state, the price level $P(\tau)$ grows at rate π, and that the research intensity μ is equal for all sectors and constant leads to the following expression for the optimal price at time τ:[22]

$$p^*(\tau) = \frac{\alpha}{\alpha - 1}\frac{r + \mu + \beta - (\alpha - 1)\pi}{r + \mu + \beta - \alpha\pi}P(\tau) \tag{22}$$

where r is the real interest rate.

The optimal price is a mark-up over marginal cost $P(\tau)$. When prices can be constantly readjusted ($\beta \to \infty$), the optimal mark-up reduces to it flex-price value $\alpha/(\alpha - 1)$ from static profit maximisation. Under price rigidity, the mark-up is higher (lower) than the optimal flex-price mark-up when the growth rate of marginal cost, the inflation rate π, is positive (negative). This higher (lower) mark-up is chosen by the firm in anticipation of the

[21] This follows from equation (27) in Section 3.5.

[22] The maximisation problem has a well-defined solution for $r + \mu + \beta - (\alpha - 1)\pi > 0$. Assumption (37) guarantees that this inequality holds in equilibrium.

fact that while its price is fixed, the firm's revenue per unit will be constant while unit cost grows at rate π- i.e., inflation (deflation) will lead to erosion (appreciation) of the firm's mark-up. The mark-up is chosen so as to offset this effect of inflation on the expected present value of profit per unit. Further, under inflation (deflation) the optimal mark-up ceteris paribus decreases (increases) in the real interest rate r, the research intensity associated with the probability of being replaced by a successful innovator μ and the flow probability of receiving a price resetting signal β. This is because an increase in any of these variables reduces the weight given to future profits relative to current ones, drawing the mark-up closer to the static optimum.

Given that at steady state the inflation rate π ceteris paribus increases in the growth rate of money supply, ψ,[23] we therefore have that the initial mark-up increases ceteris paribus with money growth, allowing it to influence real activity.

3.4.2 An intermediate good producer's market value at market entry

The market value $E\left(V_{k_j}(\tau)\left|t_{k_j}=\tau\right.\right)$ at the time of market entry t_{k_j} of a new intermediate goods firm j determines the value of the patent for the good developed in the R&D-sector. This market value is the expected present value at time τ of of all future profits of the firm, given that $t_{k_j}=\tau$:

$$E\left(V_{k_j}(\tau)\left|t_{k_j}=\tau\right.\right)=\widetilde{A}L\int_{\tau}^{\infty}e^{-(i+\mu)(s-\tau)}\left[p_j(s)-P(s)\right]\left(\frac{p_j(s)}{P(s)}\right)^{-\alpha}ds \tag{23}$$

with $\widetilde{A}=\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha}q^{(\alpha-1)k_j}$.[24]

In the absence of price rigidity when firms can constantly readjust their prices (i.e., $\beta\to\infty$), $p_j(s)=\alpha/(\alpha-1)P(s)$ so that the innovating firm's market value at market entry is given by

$$E\left(V_{k_j}(\tau)\left|t_{k_j}=\tau\right.\right)_{\beta\to\infty}=\frac{\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha}q^{(\alpha-1)k_j}\frac{1}{\alpha-1}P(\tau)\left(\frac{p^*_{flex}(\tau)}{P(\tau)}\right)^{-\alpha}L(\tau)}{r+\mu}$$

The real market value $E\left(V_{k_j}(\tau)\left|t_{k_j}=\tau\right.\right)_{\beta\to\infty}/P(\tau)$ can be interpreted as the properly discounted present value of an infinite stream of profits growing at a constant rate: The numerator of this term corresponds to the firm's instantaneous profit, while the obsolescence-adjusted discount rate is given in the denominator. Since the firm's profit growth rate is zero, the discount factor is $r+\mu-0$.[25] Further, the firm's value is proportional to the

[23] See section 3.2.

[24] Note that the wage adjusts freely to clear the labour market such that in equilibrium, employment in the final good sector equals the constant labour supply L at all times.

[25] Remember that the appropriate discount rate for an infinite stream of profits that grows at constant rate x is $d-x$ where d is the discount factor.

amount of labour L employed in final good production since intermediate goods' productivity increases in L and proportional to $P(\tau)$ since the price level determines both the firm's revenues and costs.

In the presence of Calvo-type price rigidity, deriving the firm's expected market value at market entry is rather complex since the consequences of the stochastic timing of future price changes for the firm's profits have to be accounted for. Going through a number of steps leads to the following equation:[26]

$$E\left(V_{k_j}(\tau)\left|t_{k_j}=\tau\right.\right)=\frac{\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha} q^{(\alpha-1)k_j}\frac{1}{\alpha-1}P(\tau)\left(\frac{p^*(\tau)}{P(\tau)}\right)^{-\alpha}}{(r+\mu)\frac{r+\mu+\beta-\alpha\pi}{r+\mu+\beta}}L(\tau) \tag{24}$$

Equation (24) differs from the flex-price market value in two respects: First, as seen in equation (22) in the previous section, with positive inflation the initial mark-up $p^*(\tau)/P(\tau)$ chosen by the firm under price rigidity is higher than the optimal mark-up under flexibility, $p^*_{flex}(\tau)/P(\tau)=\alpha/(\alpha-1)$. This reduces demand for the good (see equation (4)) and therefore, the firm's instantaneous profits. Secondly, the discount rate under flexibility $r+\mu$ is replaced by a compound discount rate where the *flex-price* discount rate is corrected with the factor $(r+\mu+\beta-\alpha\pi)/(r+\mu+\beta)$ that consists of the appropriate discount rates for a firm under price *rigidity* for periods where prices can be changed or are fixed, respectively. The discount rate for periods where prices are fixed decreases in inflation. This is because while prices are fixed, the new good's mark-up and relative price erode at rate $-\pi$, which by equation (4) leads to a growth rate $\alpha\pi$ of demand for the good. Given positive profits per unit,[27] the rising demand translates into a higher growth rate of the new intermediate firm's profits. Since the discount rate is the obsolescence-adjusted interest minus the profit growth rate, an increase in inflation thus reduces the discount rate for periods where prices are fixed and the compound discount rate.

An increase in the frequency of price adjustment, $\beta+\mu$ reduces the weight given to periods where prices cannot be changed and therefore reduces the necessary correction.[28]

3.4.3 Intermediate goods production in steady state equilibrium

The final good sector's demand for intermediate goods can now be found by using the final good sector's demand function (4) for good j and aggregating over all intermediate

[26] Derivation of the market value is described in more detail in Appendix 1.

[27] The firm's optimal price is chosen so as to offset the effect of money growth on the present value of the firm's profit *per unit*, see Appendix 1.

[28] Note that at $\pi<0$, the flex-price discount rate has to be corrected *upwards* for the negative growth rate of profits in periods where the mark-up appreciates. An increase in β here means that the correction term *rises* to reduce the extent of correction.

goods:[29]

$$X(\tau)=\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha}LQ(\tau)\sum_{j=1}^{N}\left(\frac{p_j(\tau)}{P(\tau)}\right)^{-\alpha}\frac{q^{(\alpha-1)k_j}}{Q(\tau)}$$

Aggregate demand for intermediate goods grows with the technology aggregate $Q(\tau)$ and depends negatively on the average relative price of intermediate goods. The average price effective at time τ can be written as a weighted average of those past optimal prices set by firms at the last (stochastic) point in time s where they could change their prices. The weights $f(s,\tau)$ therefore reflect to the probability that a price charged at time τ was set at time s without having since been changed:

$$\sum_{j=1}^{N}p_j(\tau)^{-\alpha}=\int_{-\infty}^{\tau}f(s,\tau)\left[p^*(s)\right]^{-\alpha}ds$$

In particular, they reflect the probability that an innovation was made *or* a price reset signal was received at time s, $(\mu+\beta)$, *and* that no such event took place between times s and τ, $e^{(\mu+\beta)(s-\tau)}$, so that $f(s,\tau)=(\mu+\beta)\,e^{(\mu+\beta)(s-\tau)}$. Using this and going through a number of steps, we have[30]

$$\left(\frac{X(\tau)}{Q(\tau)}\right)^{*}=\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha}L\left[\frac{p^*(\tau)}{P(\tau)}\left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{-\frac{1}{\alpha}}\right]^{-\alpha}\tag{25}$$

where the term $\frac{p^*(\tau)}{P(\tau)}\left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{-1/\alpha}$ is the average relative price, or equivalently, the *average mark-up*, which under flexible prices reduces to $\alpha/(\alpha-1)$. Since both components of the term depend on the inflation rate π, the average mark-up and hence, total demand for intermediate goods can be influenced by monetary policy.[31] The average mark-up increases in the optimal initial mark-up $p^*(\tau)/P(\tau)$ whose determinants were discussed in section 3.4.1. At the same time, the influence of past mark-ups on the average mark-up, which as explained above is a weighted average of the current and past values of the optimal mark-up, is captured in the term $\left[(\beta+\mu)/(\beta+\mu-\alpha\pi)\right]^{-1/\alpha}$: It implies that the average mark-up is lower (higher) than the current value under inflation (deflation) because past optimal mark-ups are lower (higher). The weight of past mark-ups decreases in the frequency of price adjustments $\beta+\mu$: The more frequent the arrival of the price setting signal or the higher the frequency of market entry with new prices, the closer the average mark-up to its current value.

[29] Note that due to the linear production function (5), the total production of intermediate goods equals both the final goods sector's demand for intermediate goods and the intermediate goods sector's demand for the final good as an input.

[30] For more details on the derivation of equation (25), see Appendix 2.

[31] Details will be discussed in Section 3.6.1.

Since $p^*(\tau)/P(\tau)$ is a constant at steady state equilibrium, $X(\tau)$ grows at the same rate as $Q(\tau)$. Bearing in mind that intermediate goods are produced one to one with output, $X(\tau)$ is also the intermediate sector's total demand for output.

3.5 Equilibrium in the final good market

Market equilibrium For the final good market to be in equilibrium, households' consumption must equal the difference between total final good production $Y(\tau)$ and the sum of the demands for final good by the intermediate goods and research sectors, which are $X(\tau)$ and $Z(\tau)$, respectively. In efficiency units, this is

$$\left(\frac{c(\tau)}{Q(\tau)}\right)^* = \left(\frac{Y(\tau)}{Q(\tau)}\right)^* - \left(\frac{X(\tau)}{Q(\tau)}\right)^* - \left(\frac{Z(\tau)}{Q(\tau)}\right)^* \tag{26}$$

Having already determined $(X(\tau)/Q(\tau))^*$ in the last section, we now turn to the steady state value of final good production in efficiency units, $(Y(\tau)/Q(\tau))^*$.[32]

Final good production in steady state equilibrium Now that we know both the final good sector's demand function for intermediate goods (4) and the optimal price chosen by intermediate goods producers (22), we can insert those equations into the final good production function to find that total production is

$$Y(\tau) = AL(\tau)^{\frac{1}{\alpha}} \sum_{j=1}^{N} \left[q^{\alpha k_j(\tau)} \left(\frac{p_j(\tau)}{P(\tau)}\right)^{-\alpha} \left(\frac{\alpha-1}{\alpha}A\right)^{\alpha} L(\tau) \right]^{\frac{\alpha-1}{\alpha}}$$

Going through similar steps as in the derivation of total intermediate good production (25) and some additional steps, this can be rewritten as[33]

$$\left(\frac{Y(\tau)}{Q(\tau)}\right)^* = AL(\tau)^{\frac{1}{\alpha}} \left(\frac{X(\tau)}{Q(\tau)}\right)^{\frac{\alpha-1}{\alpha}} \frac{\frac{\beta+\mu}{\beta+\mu-(\alpha-1)\pi}}{\left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{\frac{\alpha-1}{\alpha}}} \tag{27}$$

where the total amount of intermediate goods produced $X(\tau)/Q(\tau)$ is given in equation (25). Note that since $X(\tau)/Q(\tau)$ is constant at steady state equilibrium, $Y(\tau)$ grows at the same rate as $Q(\tau)$.

Output production in equation (27) is the product of two terms: The term $AL(\tau)^{\frac{1}{\alpha}}[X(\tau)/Q(\tau)]^{\frac{\alpha-1}{\alpha}}$ shows production when a total of $X(\tau)/Q(\tau)$ quality-weighted intermediate goods is employed efficiently. In contrast, the term $\left(\frac{\beta+\mu}{\beta+\mu-(\alpha-1)\pi}\right)/\left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{\frac{\alpha-1}{\alpha}} \leq 1$ represents the production inefficiency caused by *price dispersion* under price rigidity: When inflation, the growth rate of marginal cost, is zero, all intermediate goods prices are equal in spite of price rigidity because the optimal price does not change over time. Given equation (4),

[32] The value $(Z(\tau)/Q(\tau))^*$ will be determined in equation (31) of Section 3.7.

[33] See Appendix 3.

the goods are then demanded in (quality-weighted) equal amounts, which given the constant elasticity of substitution between individual quality-weighted intermediates in the Dixit-Stiglitz final good production function means production is efficient.[34] Any non-zero inflation rate in contrast implies that the optimal price changes over time so that there is dispersion in effective prices and demanded quantities of intermediates. The production inefficiency term $\left(\frac{\beta+\mu}{\beta+\mu-(\alpha-1)\pi}\right)/\left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{\frac{\alpha-1}{\alpha}}$ consists of the ratio of output actually produced with a given total amount of intermediate goods given current relative prices and output that could be produced with this input spread efficiently over the intermediate goods types.[35] Price dispersion and production inefficiency are the more pronounced, the higher the absolute value of the growth rate of optimal prices π, and the higher price rigidity, i.e. the lower $\beta+\mu$.

3.6 Labour market equilibrium given the innovation rate μ

By introducing the equilibrium amount of final goods produced (27) into the wage equation (3), we get the equilibrium real wage in efficiency units $\widetilde{w}(\tau)=w(\tau)/[P(\tau)Q(\tau)]$:

$$\widetilde{w}(\tau)=\frac{1}{\alpha}AL(\tau)^{\frac{1-\alpha}{\alpha}}\left(\frac{X(\tau)}{Q(\tau)}\right)^{\frac{\alpha-1}{\alpha}}\frac{\frac{\beta+\mu}{\beta+\mu-(\alpha-1)\pi}}{\left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{\frac{\alpha-1}{\alpha}}}$$

which inserting the equilibrium amount of intermediate goods produced X/Q from equation (25), inserting the optimal initial mark-up $p^*(\tau)/P(\tau)=\frac{\alpha}{\alpha-1}\frac{r+\mu+\beta-(\alpha-1)\pi}{r+\mu+\beta-\alpha\pi}$ from equation (22) and using the Euler equation (17), $\pi=\psi-\gamma$ and $\gamma=\overline{q}\mu$ can be rewritten as

$$\widetilde{w}=A_1\left[\frac{\rho+\beta+\mu(1+\alpha\overline{q})-(\alpha-1)\psi}{\rho+\beta+\mu[1+(1+\alpha)\overline{q}]-\alpha\psi}\left(\frac{\beta+\mu}{\beta+\mu-\alpha(\psi-\overline{q}\mu)}\right)^{-\frac{1}{\alpha}}\right]^{-(\alpha-1)}\frac{\frac{\beta+\mu}{\beta+\mu-(\alpha-1)(\psi-\overline{q}\mu)}}{\left(\frac{\beta+\mu}{\beta+\mu-\alpha(\psi-\overline{q}\mu)}\right)^{\frac{\alpha-1}{\alpha}}} \tag{28}$$

where $A_1=\frac{\left(\frac{\alpha}{\alpha-1}\right)^{-2(\alpha-1)}}{\alpha}A^{\alpha}$.

3.6.1 Properties of the real wage function

From equation (28), the steady state real wage in efficiency units is a function of the research intensity μ and of exogenous parameters, in particular of the money growth rate ψ and of the price rigidity parameter β:

$$\widetilde{w}(\mu,\psi,\beta) \tag{29}$$

[34] The term $\left(\frac{\beta+\mu}{\beta+\mu-(\alpha-1)\pi}\right)/\left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{\frac{\alpha-1}{\alpha}}$ reaches its maximum value, unity, for $\pi=0$.

[35] For details see Appendix 3.

We now discuss the properties of this function in some detail because given equation (16), equilibrium employment has qualitatively the same properties. By equation (3) the real wage is determined by output per unit of labour $Y(\tau) / [Q(\tau) L(\tau)]$. Thus any influence of parameters on the total input of intermediate goods and on the efficiency with which this amount is used affects the wage and employment. First note that since $Y(\tau) / [Q(\tau) L(\tau)]$ is independent of total employment, so is $\widetilde{w}$. This facilitates our analysis considerably.

Wage is a hump-shaped function of money growth ψ The two influences of money growth on the wage via the *average mark-up* and on *price dispersion* make $\widetilde{w}(\mu, \psi, \beta)$ a hump-shaped function of money growth. We discuss both influences in turn.

Price dispersion effect As explained in Section 3.5, any increase in the money growth rate that increases inflation (decreases deflation) raises (lowers) the absolute value of the growth rate of optimal prices and therefore raises (lowers) price dispersion and production inefficiency $\frac{\beta+\mu}{\beta+\mu-(\alpha-1)(\psi-\overline{q}\mu)}\left(\frac{\beta+\mu}{\beta+\mu-\alpha(\psi-\overline{q}\mu)}\right)^{-\frac{\alpha-1}{\alpha}}$ which in turn lowers (raises) the productivity of labour and the wage.[36]

Average mark-up effect As explained in the last part of Section 3.4.3, total demand for intermediate goods according to equation (25) depends negatively on the average mark-up charged by intermediate goods firms, $\frac{p^*(\tau)}{P(\tau)}\left(\frac{\beta+\mu}{\beta+\mu-(\alpha-1)\pi}\right)^{-\frac{1}{\alpha-1}}$ which is altered by an increase in money growth in two ways: Firstly, as discussed in section 3.4.1, an intermediate good firm's optimal *initial mark-up* $p^*(\tau)/P(\tau)$ increases in ψ since the growth rate of marginal cost π ceteris paribus rises in ψ, accelerating (slowing down) the future mark-up erosion (appreciation) under inflation (deflation). At the same time, the weight of past mark-ups in the average mark-up, $\{(\beta+\mu) / [\beta+\mu-(\alpha-1)\pi]\}^{-\frac{1}{\alpha-1}}$, decreases in ψ since the higher inflation (the smaller deflation), the lower are past mark-ups relative to the current one and thus the smaller the average mark-up relative to the current one.

Net effect of money growth on employment:

Lemma 1 *The wage $w(\mu, \psi, \beta)$ is a hump-shaped function of the money growth rate ψ with a maximum at $\psi_1 > 0$ where ψ_1 is given in Appendix 4. At this unique maximum, the inflation rate $\pi(\psi_1)$ is strictly positive.*

The proof to the lemma can be found in Appendix 4.

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[36] $\frac{\partial\left[\left(\frac{\beta+\mu}{\beta+\mu-(\alpha-1)\pi}\right)/\left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{\frac{\alpha-1}{\alpha}}\right]}{\partial\psi} = \frac{-(\alpha-1)\pi\frac{\beta+\mu}{\beta+\mu-(\alpha-1)\pi}}{\left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{\frac{\alpha-1}{\alpha}}[\beta+\mu-(\alpha-1)\pi](\beta+\mu-\alpha\pi)} \lesseqgtr 0$ as $\pi \gtreqless 0$.

Thus holding constant the research intensity, the wage is a hump-shaped function of the money growth rate with its peak at a money growth rate associated with a positive inflation rate.

An increase in price rigidity can increase the wage The effects of an increase in β, i.e. decreased price rigidity, are very similar to the effect of a decrease in the *absolute value* of the inflation rate in that the fact that prices can be readjusted more frequently reduces the strength of both price dispersion and the average mark-up effect.[37]

Lemma 2 *The wage and employment increase in the price flexibility parameter β under deflation. Under small positive inflation rates, wage and employment decrease in β.*

Intuitively, an increase in price rigidity raises employment under moderate inflation since under these circumstances, an increase in (the absolute value of) money growth and inflation increases employment. Since an increase in price flexibility mitigates the effects of money growth, it reduces employment.[38]

Wage is approximately unchanged by the research intensity μ The effects of an increase in μ on the wage are qualitatively nearly identical to the effects of an increase in β since an increase in the innovation frequency reduces price rigidity, as does an increase in β.[39] Yet since the frequency of innovation μ is small compared to the frequency of Calvo-price adjustments β, its contribution to the degree of price flexibility $\beta + \mu$ is small. Therefore, the elasticities of price dispersion, the initial mark-up and the deviation of the average mark-up from the initial mark-up with respect to μ are very small. In fact, all the aforementioned elasticities with respect to μ go to zero for $\mu / (\mu + \beta) \to 0$ which holds approximately for all reasonable calibrations so that the effects of an increase in μ on the wage are quantitatively negligible.[40]

[37] An increase in β draws the initial mark-up closer to the static optimum as the weight put on future profits decreases and lowers the weight of past mark-ups in the average average mark-up since effective prices were on average set more recently. See Section 3.4for details.

[38] It is intuitive that for the same reasons under high inflation rates where an increase in the money growth rate reduces employment, an increase in the price flexibility parameter β raises employment. While we do not prove this analytically, it is confirmed by all our numerical examples.

[39] The effects of increases in μ and β are perfectly identical regarding price dispersion and the deviation of the average mark-up from the initial mark-up. In contrast, the initial mark-up decreases in μ not only due to the latter's influence on the degree of price flexibility $\beta + \mu$ but also via its indirect influence via the growth rate γ that raises the real interest rate r and lowers the inflation rate π. See Section 3.4.1 for a description of the effects of r and π on the initial mark-up. Yet these indirect influences are not important numerically since the elasticity of the initial mark-up with respect to μ vanishes for $\mu / (\mu + \beta) \to 0$.

[40] E. g., in the baseline case of our leading example, $\mu / (\mu + \beta) = 0.007$.

3.6.2 Equilibrium employment $L(\mu, \beta, \psi)$

Given that labour supply from equation (16) increases monotonically in $\widetilde{w}$, the employment function (30) preserves the above-discussed properties of the wage function (29).

$$L(\mu, \beta, \psi) = L^s[\widetilde{w}(\mu, \psi, \beta)] \tag{30}$$

In particular, employment given the innovation rate μ is a hump-shaped function of money growth peaking at a value of ψ associated with a positive inflation rate, may be increased by an increase in rigidity under small positive inflation rates and is approximately invariant to the innovation rate μ.

3.7 Research market equilibrium given employment L

3.7.1 Equilibrium in the market for patents

The prospect of positive profits in intermediate goods production leads to buyers' competition in the market for patents in the course of which the price is bidden up to the market value of the new firm using the patent, (24). Given that research firms charge exactly this price, all new patents will be bought and the market for patents clears.

3.7.2 The R&D sector's demand for the final good at steady state equilibrium

The research sector's demand for the final good is found by rearranging (7), inserting $\phi(k_j(\tau))$ as defined in equation (9) and aggregating over all research firms. In efficiency units, this yields

$$\left(\frac{Z(\tau)}{Q(\tau)}\right)^* = \lambda\mu q^{\alpha-1} \tag{31}$$

The constant steady state equilibrium demand $\left(\frac{Z(\tau)}{Q(\tau)}\right)^*$ depends on the value of the equilibrium research intensity μ that we determine next.

3.7.3 Equilibrium research intensity

Using a new firm's expected market value $E\left(V_{k_j+1}(\tau)\,|t_{k_j} = \tau\right)$ given in equation (24) and $\phi(k_j(\tau))$ from equation (9) in the zero profit condition (8) gives us an equation determining the equilibrium research intensity μ which makes current research firms indifferent with regard to the amount of research input used:

$$\frac{L}{\lambda}\frac{\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha}\frac{1}{\alpha-1}P(\tau)\left(\frac{p^*(\tau)}{P(\tau)}\right)^{-\alpha}}{(r+\mu)\frac{r+\mu+\beta-\alpha\pi}{r+\mu+\beta}} = P(\tau) \tag{32}$$

Note that consistent with the assumption first made in section 3.3, the resulting steady state research intensity μ is the same for all research firms regardless of their sector's current position on the quality ladder.

Further using the optimal initial mark-up $\frac{\alpha}{\alpha-1}\frac{r+\mu+\beta-(\alpha-1)\pi}{r+\mu+\beta-\alpha\pi}$ from equation (22), the Euler equation (17), the equation relating economic growth to research intensity (21), using that equilibrium in the money market implies $\pi=\psi-\gamma$ and rearranging, we get an equation in μ, employment L and the model's parameters:

$$\frac{L}{\lambda}\left(\frac{\alpha}{\alpha-1}\right)^{-\alpha}A^{\alpha}\frac{1}{\alpha-1}\left(\frac{\alpha}{\alpha-1}\frac{\rho+\beta-(\alpha-1)\psi+(\overline{\eta}-\overline{q})\mu}{\rho+\beta-\alpha\psi+\overline{\eta}\mu}\right)^{-\alpha}=\left[\rho+(\eta\overline{q}+1)\mu\right]\frac{\rho+\beta-\alpha\psi+\overline{\eta}\mu}{\rho+\beta+(\eta\overline{q}+1)\mu} \tag{33}$$

where $\overline{q}=\left(q^{\alpha-1}-1\right)>0$, $\overline{\eta}=[(\eta+\alpha)\overline{q}+1]$ and $\overline{\eta}>\overline{q}$.

Both sides of the equation show the dependence of the optimal research intensity μ on the new firm's value: The LHS of equation (33) shows the instantaneous profits for a firm entering the market with a new patent as a function of the research intensity while the RHS represents the compound discount rate for this firm's future profit streams as a function of μ. Figure 1 depicts the LHS and the RHS of this equation.

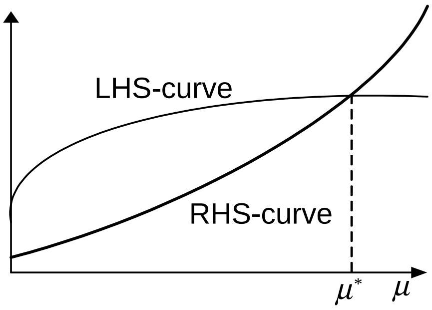

Figure 1: Partial equilibrium research intensity μ given employment $\overline{L}$

The solution to equation (33) is a function of employment, money growth and rigidity:

$$\mu(L,\psi,\beta) \tag{34}$$

Lemma 3 *Under conditions (35)-(37), there is a unique steady state equilibrium research intensity* $\mu(L,\psi,\beta)$ *for any* $L\in\left(\frac{1}{2}\overline{L},\overline{L}\right)$.[41]

$$\frac{1}{2}\frac{\overline{L}}{\lambda} > \frac{\rho^{\frac{\rho+\beta-\alpha\psi}{\rho+\beta}}}{\left(\frac{\alpha}{\alpha-1}\right)^{-2\alpha}A^{\alpha}\frac{1}{\alpha-1}}\left(\frac{\rho+\beta-(\alpha-1)\psi}{\rho+\beta-\alpha\psi}\right)^{\alpha} \tag{35}$$

$$-\frac{\overline{q}}{\eta\overline{q}+1}(\rho+\beta) < \psi<\frac{1}{\alpha}\frac{(\rho+\beta)\left[2(\eta\overline{q}+1)+(\alpha-1)\overline{q}\right]}{\left(2+\frac{\alpha-1}{\alpha}\right)(\eta\overline{q}+1)+2(\alpha-1)\overline{q}} \tag{36}$$

$$\psi < \frac{1}{\alpha}\beta \tag{37}$$

where $\overline{q}=q^{\alpha-1}-1$. Condition (35) ensures that $\lim_{\mu\to0}LHS>\lim_{\mu\to0}RHS$ in equation (33). It implies that the efficiency weighted labour force cannot be too small. For $\beta<\infty$,

[41] All proofs can be found in Appendix 4.

conditions (36) and (37) are jointly sufficient for the LHS of equation (33) to be concave in μ, while condition (37) and the first inequality in condition (36) are sufficient to ensure that the RHS of the equation is convex in μ as depicted in fig. 1. Condition (36) can always be satisfied since the term to the very left is negative while the expression on the right hand side is positive. Conditions (36) and (37) imply that for any given β, there exist a lower and an upper bound on the growth rate of money supply ψ compatible with steady state equilibrium.

All conditions are easily satisfied in all our numerical examples.[42]

Intuition For intuition concerning the form of the LHS-curve, first note that in the case without price rigidity ($\beta \to \infty$), the LHS of equation (33), which represents the instantaneous real profit associated with the production of the new good, simplifies to the constant $\frac{L}{\lambda}\left(\frac{\alpha}{\alpha-1}\right)^{-2\alpha} A^{\alpha}\frac{1}{\alpha-1}$. For $\beta < \infty$, the curve has a positive slope in μ since as discussed in Section 3.4.1 the forward-looking initial mark-up chosen by firms under price rigidity decreases in μ.[43] Since demand for the new firm's good is inversely related to its mark-up and relative price, an increase in μ increases the instantaneous profits associated with its invention, hence the positive slope of the LHS-curve.

The RHS of equation (33) represents the compound discount rate applicable to the new firm's profits. For $\beta \to \infty$, the discount rate reduces to $r + \mu$ which increases linearly in μ since the probability of being replaced increases. Under price rigidity ($\beta < \infty$), this effect of of an increase in μ is reinforced through an increase in the correction factor $(\rho + \beta - \alpha\psi + \overline{\eta}\mu) / [\rho + \beta + (\eta\overline{q} + 1)\mu]$.[44]

[42] In the leading example we introduce in Section 5, condition 35 implies $\frac{\overline{L}}{\lambda} > 2.28$ while we choose $\frac{\overline{L}}{\lambda} = 4.725$. Condition 37 is less restrictive than condition 36 which implies $-1.12 < \psi < 0.14$. The upper bound, which corresponds to an inflation rate of $\pi = 12.5\%$, does not restrict our analysis of innovation-driven growth unduly.

[43] Note that in addition to the direct effect on the mark-up of an increase in the probability of being replaced by an innovator, an increase in μ has several indirect effects on the mark-up through its proportionality to the output growth rate γ and through the latter's effect on the interest rate $r = \rho + \eta\gamma$ and on the inflation rate $\pi = \psi - \gamma$ (see Section 3.4.1 for an analysis of the influence of r and π on the mark-up). For $\pi > 0$, the net indirect effect is negative and thus reinforces the direct effect of an increase in μ.

At $\pi < 0$, the rigidity-caused part of the initial mark-up is *smaller* than unity because the mark-up will appreciate under deflation. An increase in μ further decreases the initial mark-up due to the indirect effect that $\pi = \psi - \gamma$ becomes even more negative, such that the future growth rates of revenues and costs diverge even further.

[44] This implies that the extent of correction decreases (increases) at $\pi > 0$ ($\pi < 0$) where the correction factor is smaller (bigger) than unity: The main reason is that through its proportionality to γ, an increase in μ lowers inflation (increases deflation) $\pi = \psi - \gamma$, thereby lowering (increasing) the positive (negative) profit growth rate in periods where the erosion (appreciation) of the firm's mark-up through inflation (deflation) leads to an increase (decrease) in demand for the good. Thus the deviation of the profit growth

Note that given assumptions (35)-(37) and concavity of the LHS-curve, the slope at the steady state equilibrium of the LHS-curve is smaller than that of the RHS-curve. Intuitively, the increase in the discount rate caused by an increase in μ is bigger than the associated increase in instantaneous profits implying that expected profit from an innovation decreases in μ, as in the model without money.

We now discuss the properties of the research intensity function (34) with the help of figure 1.

3.7.4 Standard scale effect of employment L on the innovation rate μ

Lemma 4 *The innovation rate $\mu(L, \psi, \beta)$ increases monotonically in L.*

An increase in L raises instantaneous profits, shifting up the LHS-curve in figure 1. Given that the RHS-curve is unaffected by the change in L and given the curves' shapes, the increase in $\overline{L}$ results in a higher partial equilibrium innovation rate. The positive scale effect on growth of an increase in employment is a is a well-known feature of the underlying real growth model. In general equilibrium, this will allow for additional influences of exogenous parameters on the growth rate through their influence on employment.

3.7.5 Innovation rate μ depends negatively on absolute value of inflation $\pi = \psi - \gamma$ under price rigidity

Using equation (33), we note first that it is the presence of price rigidity that allows money to have an impact on $\mu(L, \psi, \beta)$:

Lemma 5 *In the limiting case without rigidities, money is superneutral:* $\lim_{\beta \to \infty} \frac{\partial \mu(L,\psi,\beta)}{\partial \psi} = 0.$

Intuitively, when prices are perfectly flexible, relative prices and mark-ups are independent of inflation, so that demand and hence, a research firm's profits are unaffected by money growth.

In contrast, for $\beta < \infty$, the money growth rate ψ has two clear-cut countervailing effects on the innovation rate $\mu(L, \psi, \beta)$ which operate through money growth's influence on the firm's mark-up and relative price:

Negative initial mark-up effect of money growth under price rigidity As explained in Section 3.4, an increase in ψ that raises the growth rate of marginal cost π raises the initial mark-up and relative price chosen by an intermediate good firm under price rigidity. The increase in the relative price lowers demand for the firm's good and

rate from its flex-price value that that requires correction decreases (increases) in μ.

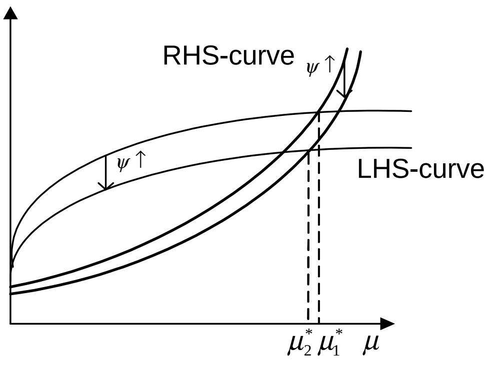

Figure 2: Effects on the partial equilibrium research intensity given employment $\mu\left(\psi, \overline{L}, \beta\right)$ of an increase in ψ when inflation is positive.

hence, its instantaneous profit which in turn determines the incentive to innovate.[45,46] In figure 2, the increase in ψ causes a downward shift of the LHS-curve which ceteris paribus reduces the innovation rate $\mu(L, \psi, \beta)$.

Positive mark-up erosion effect of money growth under price rigidity The RHS of equation (33) is the new firm's compound discount rate. As first discussed in Section 3.4.2, the compound discount rate decreases in the inflation rate which determines the rate of demand- and profit-raising mark-up erosion.[47] Since an increase in the money growth rate ψ ceteris paribus raises inflation, it therefore ceteris paribus raises the incentive to innovate and the innovation rate $\mu(L, \psi, \beta)$ via a decrease in the compound discount rate.

Graphically, the increase in ψ causes a downward shift in the RHS-curve in figure 2, which ceteris paribus raises μ.

Net effect of money growth on economic growth depends on whether inflation is positive or negative. The negative price dispersion effect of an increase in money growth ψ shifts the LHS-curve of equation (33) downward while the positive mark-up erosion effect shifts the RHS-curve downward. Which effect is stronger, i.e. the sign of the net effect of money growth on $\mu(L, \psi, \beta)$ depends on whether inflation is positive or negative:

Lemma 6 *An increase in the steady state money growth rate ψ decreases (increases) the*

[45] $\frac{\partial LHS}{\partial \psi} = -\frac{\alpha LHS[(\eta\overline{q}+1)\mu+(\rho+\beta)]}{(\rho+\beta-\alpha\psi+\overline{\eta}\mu)[\rho+\beta-(\alpha-1)\psi+(\overline{\eta}-\overline{q})\mu]} < 0$ given condition (37) and $\overline{\eta} - \overline{q} = [\eta + (\alpha - 1)]\left(q^{\alpha-1} - 1\right) + 1 > 0$.

[46] Note that inflation only has an effect on profits through its influence on demand since the initial mark-up under price rigidity is optimally chosen by the firm to offset the direct effect of the changing mark-up on the firm's profit *per unit*.

[47] Remember that the discount rate is the obsolescence-adjusted interest minus the profit growth rate.

innovation rate $\mu(L, \psi, \beta)$ *when inflation is positive (negative).*

This is intuitive since both the discussed effects describe the impact on a firm's effective mark-up of a restriction on its price setting . This restriction which leads to suboptimal mark-ups cannot make the firm better off. Now while at $\pi = 0$, rigidity is ineffective since marginal cost is constant over time so that firms have no desire to readjust prices, for any departure from price stability, price rigidity becomes binding. At $\pi < 0$, an increase in ψ moves inflation closer to $\pi = 0$, reducing the distortion of the firm's mark-up and therefore increasing profits and the incentive to innovate which determines the growth rate. In contrast, at $\pi > 0$, an increase in ψ raises inflation and thus exacerbates the effects of rigidity, reducing profits and economic growth.

3.7.6 Innovation rate μ depends negatively on price rigidity $1/\beta$

Lemma 7 *An increase in the level of rigidity (i.e., decrease in* β*) decreases the innovation rate* $\mu(L, \psi, \beta)$ *for* $\beta < \infty$*.*

Analogously to the discussion of β's effect on the wage in Section 3.6.1, an increase in the frequency of price adjustments β has qualitatively the same effects on μ as a reduction in the inflation rate π: It reduces the need to have a forward-looking initial mark-up, reducing the *initial mark-up* effect of money growth by drawing the initial mark-up chosen closer to the static optimum. Graphically, an increase in β shifts the LHS-curve upward (downward) in figure 1 when inflation is positive (negative), which ceteris paribus decreases (increases) the partial equilibrium innovation rate.

At the same time, an increase in the frequency of price adjustment via β reduces the *mark-up erosion* effect of money growth since it shifts more weight to the discount rate for periods when prices are flexible, reducing the weight of the correction factor. Graphically, an increase in β shifts the RHS-curve upward (downward) in figure 1 when inflation is positive (negative), ceteris paribus increasing (decreasing) μ.

The intuition for the negative effect of rigidity regardless of whether inflation is positive or negative is closely connected to the intuition concerning the effect of ψ: An intermediate good producer's profit is affected by rigidity only through the latter's effect on the firm's optimal and effective price. If changing prices infrequently were a profit-maximising strategy, the firm would have chosen this pricing strategy under flexibility, so there is no scope for price rigidity to increase the return to R&D.

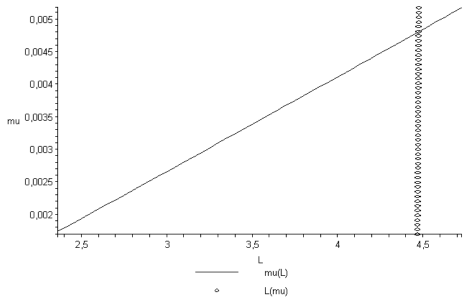

Figure 3: Existence and uniqueness of general steady state equilibrium research intensity mu in our leading example

4 Existence and uniqueness of the steady state equilibrium

Any solution to the equation:

$$\mu\left[L\left(\mu,\psi,\beta\right),\psi,\beta\right]=\mu \tag{38}$$

is a steady state equilibrium innovation rate and $L^{*}\left(\psi,\beta\right):=L\left[\mu\left(L,\psi,\beta\right),\psi,\beta\right]$ is the corresponding equilibrium employment level.

Remark 1 *Existence of a solution to equation (38) follows from the facts that $\mu\left(L,\psi,\beta\right)$ increases monotonically in L for any $L>\frac{1}{2}\overline{L}$ and $L\left(\mu\right)>\frac{1}{2}\overline{L}$ is defined for each μ and is continuous in μ.*

In the leading example to be presented in the next section, the steady state equilibrium is unique. More generally, we can say the following:

Remark 2 *If the maximum feasible μ in the economy, $\mu^{max}=\mu\left(\overline{L}\right)$ is sufficiently small, then there is a unique steady state equilibrium.*

To get an intuition for this result remember that we showed in Section 3.7.4 that for any given $L>\frac{1}{2}\overline{L}$, the innovation rate is unique and increases monotonically in L. Further, we explained that the wage and employment are approximately invariant to changes in μ when μ is small in relation to $\beta+\mu$. So sufficiently small in this context means that the maximum feasible innovation rate μ^{max} must be small in relation to the frequency of price adjustment β, which is the case for all plausible economies. The $L\left(\mu\right)$-curve is then approximately linear and crosses the $\mu\left(L\right)$-curve once. Figure 3 illustrates this for our leading example.

5 Comparative statics: Employment level and economic growth rate in General equilibrium

In this section, we discuss the comparative static properties of the output growth rate, which is proportional to the innovation rate, and the level of employment in steady state equilibrium. These properties are determined by four effects we have already discussed: The *Initial Mark-up effect*, the *Mark-up erosion effect*, the *Average mark-up effect* and the *Price dispersion effect*. Using a calibrated example, we will in particular discuss which monetary policies would be chosen by monetary authorities interested in promoting employment and economic growth, respectively, and analyse the effect of price rigidity on growth and employment.

For our leading example, we have chosen the following calibration:

parameter	value	parameter	value
q	1.2	ρ	0.015
α	10	σ	150
β	2.5	$\overline{L}$	4.7250
η	2		

The calibration is chosen to yield realistic and empirically plausible values for the economy's endogenous variables at a baseline money growth rate $\psi = 0.055$ per annum that was the average US M1 growth rate between 1979 and 2004.[48] At this baseline money growth rate, the rate of economic growth is 2% while the unemployment rate is 5.3%. The mark-up chosen by firms amounts to 12.9%, while the average period during which prices are fixed is 0.40 years or 4.8 months.[49]

5.1 Inflation and employment: Monetary policy for promoting employment

First, note that the superneutrality result remains unchanged:

Proposition 8 $\lim_{\beta\to\infty} \frac{dL}{d\psi} = \lim_{\beta\to\infty} \frac{d\mu}{d\psi} = \lim_{\beta\to\infty} \frac{d\gamma}{d\psi} = 0.$

The intuition remains unchanged: With perfectly flexible prices, inflation has no influence on relative prices or average mark-ups and therefore does not influence real variables.

Regarding the effect of money growth on employment in the presence of price rigidity ($\beta < \infty$), we first present the following proposition:

[48] We calculated the average growth rate of the monetary aggregate M1 in the US between 1979 and 2004 based on data from www.federalreserve.gov/releases/h6/hist/.

[49] Both values are well in line with empirical estimates, see Basu and Fernald [1995, 1997] and Bils and Klenow [2004], respectively.

Proposition 9 *Starting from an equilibrium with price stability, an increase in the money growth rate ψ increases employment.*

Taking into consideration the indirect effect of money growth on employment via the research intensity as well as the direct effects, starting from an equilibrium with price stability, an increase in the money growth rate ψ lowers the average mark-up, so that output per labour unit, the wage and employment increase in ψ. Thus a monetary policy entailing moderate inflation is preferable to price stability for a monetary authority that wants to increase employment.

To analyse further the shape of the function $L\left[\mu\left(\psi, L, \beta\right), \psi, \beta\right]$, note that the total derivative of employment with respect to the money growth rate can be written as $\frac{dL}{d\psi} = \frac{L}{\psi}\left[\varepsilon_{L,\psi} + \varepsilon_{L,\mu} * \varepsilon_{\mu,\psi}\right]$ where $\varepsilon_{x,y}$ is the partial elasticity of x with respect to y. As argued in Section 3.6.1, the elasticity of employment with respect to the innovation rate μ vanishes for $\mu/\left(\mu+\beta\right) \rightarrow 0$ and is indeed very small for all sensible calibrations since the contribution of the innovation rate to the degree of price flexibility $\beta + \mu$ is small.[50] Thus, $\varepsilon_{L,\mu} * \varepsilon_{\mu,\psi}$ is very small and the the indirect effect of money growth on employment is negligible. We then have that Lemma (1) holds in general equilibrium:

Corollary 10 *For sufficiently small $\varepsilon_{L,\mu} * \varepsilon_{\mu,\psi}$, employment is a hump-shaped function of money growth with a maximum at a money growth rate $\psi_2 > 0$ associated with a positive inflation rate $\pi\left(\psi_2\right) > 0$.*

Figure 4 reflects this result for our leading example: The solid line depicts the function $L\left(\psi\right)$ in general equilibrium. The pointed line, which shows only the partial equilibrium effect of ψ on employment L given μ, is virtually indistinguishable from the solid line for negative and small ψ. For bigger values of ψ, the figure shows that the indirect effect through the research intensity reinforces the direct effect of money growth on employment, yet to a quantitatively small degree.

Thus through an increase in money growth starting from small rates of inflation, the monetary authority is successful in lowering the average mark-up in spite of the fact that intermediate good firms raise their initial mark-up in anticipation of price rigidity. This means monetary policy in this range successfully raises aggregate demand for intermediate goods which is inefficiently low due to monopolistic competition. The positive effect on aggregate via a lower average mark-up dominates money growth's negative effect on

[50] At the same time, the point elasticity of the research intensity μ with respect to the money growth rate ψ also has to be small in all realistic examples- remember that given empirical estimtes, a large discrete increase in the money growth rate from 1 percentage point to 10 percentage points should lower growth by not significantly more than a quarter percentage point.

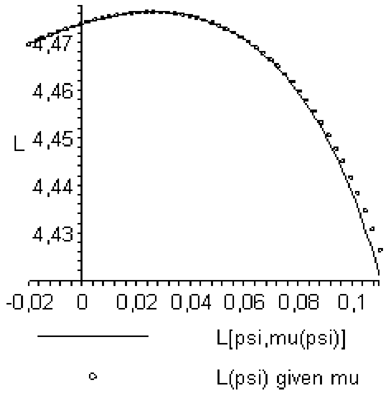

Figure 4: Partial and total effect of money growth on employment

production efficiency at low levels of inflation, leading to a higher real wage and higher equilibrium employment than under price stability.

There is thus a range of money growth rates that entails a *Phillips-Curve*-trade-off for the monetary authority: Higher employment is only to be had at the price of higher inflation. In our leading example, this is true for money growth rates up to 2.60% or equivalently, positive inflation rates of up to 0.57%. In the range of money growth rates between 2.60% and 3.15% (inflation rate of 1.12%), employment again declines in money growth but is still higher than under price stability. Yet the effect is quantitatively small: At its maximum, employment is less than 0.01% higher than in the case of flexible prices. At the same time, the effect of an increase in money growth from $\psi = 0.01$ to $\psi = 0.1$ is quite sizeable: It increases the unemployment rate by over 0.8 percentage points from 5.28% to 6.08%.

Therefore monetary policy aimed at fostering employment should feature a moderate inflation rate, while high inflation should be avoided since it significantly reduces employment.

5.2 Inflation and economic growth: Monetary policy for promoting growth

Remember that in Section 3.7.5 holding constant employment, we found a hump-shaped relationship between the innovation rate and money growth or inflation, respectively. The innovation rate peaked at an inflation rate of $\pi = 0$. Our subsequent analysis of how the additional influence of money growth on the wage and employment changes these results shows that while the hump-shaped relationship persists, the best policy for a monetary authority interested in promoting economic growth features a positive rate of inflation.

Regarding the hump-shaped relationship between money growth and economic growth, which by equation (21) is a linear function of the innovation rate, we have the following proposition:

Proposition 11 *For small values of the money growth rate ψ, the economic growth rate γ increases in ψ, for large values of ψ, the growth rate γ decreases in ψ.*

Thus the qualitative relation is similar in partial equilibrium and general equilibrium. Yet there is one qualitative difference:

Proposition 12 *The economic growth rate reaches its maximum at a positive rate of inflation.*

As shown in Section 3.7.5, *holding constant employment*, the incentive to innovate is reduced by nonzero inflation because an intermediate good producer's profits decreases due to the restriction on his price setting imposed by price rigidity: In anticipation of price rigidity, the initial mark-up chosen is higher than the static optimum. During the firm's life time, inflation erodes the mark-up. Consequently, the mark-up generically does not correspond to the optimal one, lowering profits and thus, the patent price.

In general equilibrium, these effects are still present. Yet at the same time, lemma 4 shows that the innovation rate increases monotonically in employment. As seen in Section 3.6.1, at zero inflation and small positive inflation rates the wage and employment increase in the money growth rate ψ. At small positive rates of inflation, the positive indirect effect of an increase in ψ on growth via employment is stronger than the negative direct effect so that the incentive to innovate and the growth rate increase in the money growth rate.

Yet as the money growth rate ψ continues to rise, distortions increase and the wage begins to fall in ψ, which adds to the mark-up distorting effects of positive money growth in causing a fall in the economic growth rate.

Figure (5) shows the economic growth rate as a function of money growth in our leading example.[51]

The economic growth rate is maximised at $\psi = 0.021$, which corresponds to the positive inflation rate $\pi = 0.07\%$. At this inflation rate, the economic growth rate is 2.03% compared to 2.0% in the baseline case. While this effect is rather small, the effect of inflation on growth is more drastic at inflation rates that are further away from the maximum: When the money growth rate increases from $\psi = 0.01$ to $\psi = 0.1$, the growth

[51] Note that the negative part of the inflation-growth relationship is concave. This is not a contradiction to the empirical result that the marginal cost of inflation should decrease at very high inflation: Our model says nothing about the inflation-growth relationship at these high inflation rate since given conditions (36) and (37), our analysis is limited to moderate inflation rates.

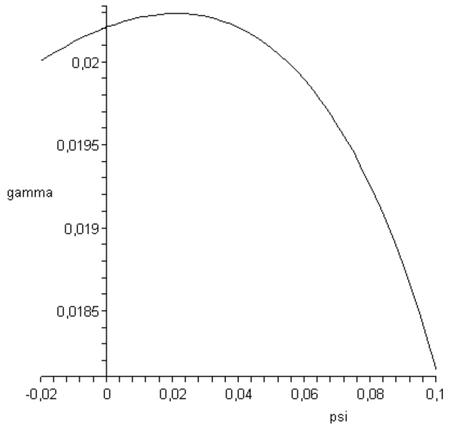

Figure 5: Economic growth rate as a function of the money growth rate

rate decreases by 0.21 percentage point, which corresponds closely to empirical estimates mentioned in the introduction.

Thus while a monetary authority that wants to promote growth should allow for moderate inflation rather than aim at price stability, it should also be aware of the growth depressing effects of high inflation.

5.3 Limited trade-off for monetary policy between employment and growth

In the preceding two sections, we examined which monetary policy would be optimal for a monetary authority interested in promoting either employment or economic growth. We found that some inflation raises the wage and employment. Through the effect on employment, a small positive inflation rate also fosters economic growth. At the same time, too much inflation proved to reduce both employment and inflation.[52]

There is no strong conflict between promoting growth and raising employment for the central bank: Given perfect information about the central union's policy and given any preference structure involving the goals of employment and economic growth, the monetary authority will always choose a money growth rate from the range $\psi \in \left(\psi_\gamma, \psi_L\right)$ where ψ_γ (ψ_L) maximises economic growth (employment). Within this range, an increase in ψ always increases employment and lowers economic growth.[53] The trade-off is limited in our

[52] This implies that a long-run version of *Okun's law*, according to which an increase in economic growth is always accompanied by a decrease in the unemployment rate, holds in our model for most money growth rates.

[53] This follows from the fact that the inequality $\psi_\gamma < \psi_L$ always holds. To get intuition for this fact, remember that the total effect of an increase in money growth on economic growth comprises the sum of non-employment related effects and the employment related effect, where we know that the former are negative at positive rates of inflation. Thus, the total effect of ψ on γ $(d\gamma/d\psi)$ is always smaller than the

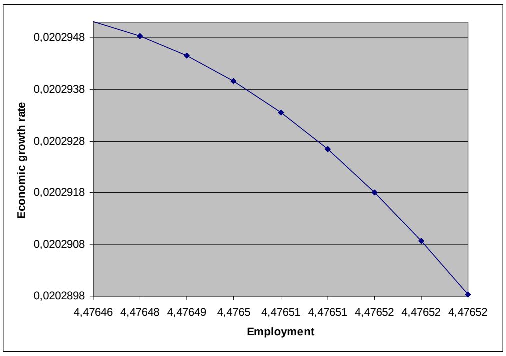

Figure 6: Trade-off between employment and growth

calibrated examples where ψ_γ and ψ_L are very close. Figure 6 illustrates the trade-off for our leading example, where the range of money growth rates involved is $\psi \in (0.021, 0.026)$ which corresponds to inflation rates between 0.07% and 0.57%.

5.4 Comparative statics regarding the level of price rigidity

Proposition 13 *At sufficiently low levels of positive inflation, employment and economic growth are higher under price rigidity than in a world without price rigidity.*

Thus unlike in partial equilibrium with constant employment where the innovation rate growth increased in β whenever $\pi \neq 0$, here price rigidity is not universally bad for innovation and economic growth. In spite of the distortions it entails, the very presence of price rigidity allows the monetary authority to implement a policy that through the lowering of the average mark-up raises employment and with it, the economic growth rate beyond its level in a world without rigidities.

6 Conclusion

Studies investigating the empirical relationship between social and private returns to R&D find convincing evidence that the investment in R&D in a decentralised economy is lower than socially optimal,[54] which implies that the growth rate is too low, too. From that perspective, a monetary policy authority should choose the money growth rate that maximises economic growth.

In our model, a small positive rate of inflation is desirable both from a growth and an employment perspective. This stems from inflation's influence on price dispersion and

employment related effect at at $\pi > 0$. Therefore, at the money growth rate that maximises employment $(dL/d\psi = 0)$, $d\gamma/d\psi < 0$ so $\gamma(\psi)$ reaches its maximum at a smaller ψ.

[54] See Jones and Williams [1998].

the average mark-up. Are these effects empirically relevant? The answer seems to be yes: There are several studies investigating the relationship between inflation and mark-ups,[55] which find evidence of a negative influence of inflation on mark-ups. At the same time, higher inflation also seems also to be associated with more price dispersion empirically.[56]

Thus we believe that the mechanisms analysed are indeed relevant in the context of a non-vertical long-run Phillips curve. At the same time, our analysis shows that the short-run and the long-run development of the economy as well as the effects of inflation on growth and employment are closely related. Short run frictions matter for long-run behaviour. These interrelations merit further empirical and theoretical investigation.

7 Appendix

Appendix 1 ***A new intermediate good producing firm's market value***

The firm's market value is the discounted sum of profits from future periods s where the profits are weighted due to two independent sources of uncertainty: The first weight is given by the probability $e^{-\mu(s-\tau)}$ of not having been replaced by time s. The second source of uncertainty is given by the firm's price in period s: The price charged can be any past optimal price $p^*(\theta)$ with $\theta \in (\tau, s)$ depending on when the last reset signal for the price was received between τ and s. Thus, the price charged at time s can be represented as a weighted sum of the past optimal prices, where the weights are as follows: The flow probability that a signal to reset prices was received in period θ is β. With probability $e^{-\beta(s-\theta)}$, no signal was received between θ and s.[57] As these two events are independent, the probability of having last reset one's price due to a price reset signal at $\theta \in (\tau, s)$ is $\beta e^{-\beta(s-\theta)}$. Additionally, if no reset signal has been received up to period s, the firm's price will continue to be $p^*(\tau)$, which has probability $(1 - \int_\tau^s \beta e^{-\beta(s-\theta)} d\theta)$. Since the processes for innovations and reset signals are independent, the joint probability of the described events takes on a multiplicative form:

$$E\left(V_{k_j}(\tau)\left|t_{k_j}=\tau\right.\right) = \widetilde{A}\int_\tau^\infty e^{-\chi(s-\tau)}\left[\int_\tau^s \beta e^{-\beta(s-\theta)} p^*(\theta)^{1-\alpha}\, d\theta + (1-\int_\tau^s \beta e^{-\beta(s-\theta)} d\theta) p^*(\tau)^{1-\alpha}\right] ds$$
$$-\widetilde{A}w(\tau)\int_\tau^\infty e^{-(\chi-\omega)(s-\tau)}\left[\int_\tau^s \beta e^{-\beta(s-\theta)} p^*(\theta)^{-\alpha}\, d\theta + (1-\int_\tau^s \beta e^{-\beta(s-\theta)} d\theta) p^*(\tau)^{-\alpha}\right] ds$$

[55] E. g., Benabou [1992], Banerjee and Russell [2005] and Banerjee, Mizen and Russell [2006]. More references can be found in the last mentioned paper.

[56] Parks [1978] is a seminal paper in this literature. For recent contributions see Banerjee, Mizen and Russell [2006] and the references therein.

[57] Here, we have been able to definitize the constant $\widetilde{B} = 1$ since we know that the probability of receiving two or more signals at time θ is negligible, such that $B(\theta) = 0$.

where $\widetilde{A}=\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha}q^{(\alpha-1)k_j}L$ and $\chi=i+\mu-\alpha\pi$. Using that the optimal price grows with the growth rate of marginal cost, π, at steady state equilibrium and simplifying yields

$$\begin{aligned}E\left(V_{k_j}(\tau)\left|t_{k_j}=\tau\right.\right)=&\widetilde{A}P(\tau)^{\alpha}\int_{\tau}^{\infty}e^{-\chi(s-\tau)}\left[p^*(s)^{1-\alpha}\int_{\tau}^{s}\beta e^{-[\beta-(\alpha-1)\pi](s-\theta)}d\theta+e^{-\beta(s-\tau)}p^*(\tau)^{1-\alpha}\right]ds\\&-\widetilde{A}P(\tau)^{1+\alpha}\int_{\tau}^{\infty}e^{-(\chi-\pi)(s-\tau)}\left[p^*(s)^{-\alpha}\int_{\tau}^{s}\beta e^{-(\beta-\alpha\pi)(s-\theta)}d\theta+e^{-\beta(s-\tau)}p^*(\tau)^{-\alpha}\right]ds\end{aligned}$$

Solving the integrals associated with the probability of receiving a price resetting signal yields:

$$\begin{aligned}E\left(V_{k_j}\left|\tau\right.\right)=&\widetilde{A}P(\tau)^{\alpha}\int_{\tau}^{\infty}e^{-\chi(s-\tau)}\left[p^*(s)^{-(\alpha-1)}\frac{\beta\left(1-e^{-[\beta-(\alpha-1)\pi](s-\tau)}\right)}{\beta-(\alpha-1)\pi}+e^{-\beta(s-\tau)}p^*(\tau)^{-(\alpha-1)}\right]ds\\&-\widetilde{A}P(\tau)^{(\alpha+1)}\int_{\tau}^{\infty}e^{-(\chi-\pi)(s-\tau)}\left[p^*(s)^{-\alpha}\frac{\beta\left(1-e^{-(\beta-\alpha\pi)(s-\tau)}\right)}{\beta-\alpha\pi}+e^{-\beta(s-\tau)}p^*(\tau)^{-\alpha}\right]ds\end{aligned}$$

where $E\left(V_{k_j}\left|\tau\right.\right)$ is a shorthand form for $E\left(V_{k_j}(\tau)\left|t_{k_j}=\tau\right.\right)$. Calculating the value of the integrals and rearranging yields

$$\begin{aligned}E\left(V_{k_j}\left|\tau\right.\right)=&\frac{\widetilde{A}p^*(\tau)\left(\frac{p^*(\tau)}{P(\tau)}\right)^{-\alpha}}{\beta-(\alpha-1)\pi}\left[\frac{\beta}{\chi+(\alpha-1)\pi}-\frac{(\alpha-1)\pi}{\chi+\beta}\right]\\&-\frac{\widetilde{A}\left(\frac{p^*(\tau)}{P(\tau)}\right)^{-\alpha}P(\tau)}{\beta-\alpha\pi}\left[\frac{\beta}{\chi-\pi+\alpha\pi}-\frac{\alpha\pi}{\chi+\beta-\pi}\right]\end{aligned}$$

Multiplying out the terms in square brackets, we have

$$E\left(V_{k_j}\left|\tau\right.\right)=\widetilde{A}\left(\frac{p^*(\tau)}{P(\tau)}\right)^{-\alpha}\frac{\chi+\beta+(\alpha-1)\pi}{\chi+(\alpha-1)\pi}\left[\frac{p^*(\tau)}{\chi+\beta}-\frac{P(\tau)}{\chi+\beta-\pi}\right]$$

The average size of profits per unit sold is $\frac{p^*(\tau)}{\chi+\beta}-\frac{P(\tau)}{\chi+\beta-\pi}$. The denominators reflect the different growth rates of revenues $(\alpha\pi+\gamma)$ and costs $(\alpha\pi+\gamma+\psi)$. This difference in growth rates is taken account of in the endogenous choice of optimal price by the firm: The initial mark-up $\frac{\alpha}{\alpha-1}\frac{\chi+\beta}{\chi+\beta-\pi}$ is chosen such that the present value of revenues is identical to what it would have been if revenues had grown at the same constant rate as costs. Using the equation for the optimal price $p^*(\tau)$ (22) and reinserting $\chi=i+\mu-\alpha\pi$ and $\widetilde{A}=\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha}q^{(\alpha-1)k_j}L$ we have equation (24) in the main text.

Appendix 2 *Total intermediate good production*

The total production of intermediate goods at time τ can be rewritten as

$$X(\tau)=\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha}L(\tau)\,Q(\tau)\,P(\tau)^{\alpha}\sum_{k=1}^{k^{\max}}d_k(\tau)\frac{q^{(\alpha-1)k}}{Q(\tau)}\sum_{\{j|k_j=k\}}\left(p_{k_j}(\tau)\right)^{-\alpha}$$

where p_{k_j} is the price of sector j that is at quality rung k and $d_k(\tau)$ is the number of sectors at quality rung k at time τ.

Following (Benhabib, Schmitt-Grohé and Uribe [2001a] and [2001b], Leith and Wren-Lewis [2000] and Wolman [1999]), the average price effective at time τ can be expressed as a weighted average of past optimal prices, where the weights $\widetilde{f}(s,\tau)$ refer to the probability that a price valid at time τ has not been changed since time s. Further, the timing of innovations is independent of a sector's position on the quality ladder q^{k_j}, such that the structure of prices for a given q^k is the same as the structure for all sectors. Thus we have

$$X(\tau)=\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha}L(\tau)\,Q(\tau)\,P(\tau)^{\alpha}\int_{-\infty}^{\tau}\widetilde{f}(s,\tau)\,p^{*}(s)^{-\alpha}ds$$

The weights $\widetilde{f}(s,\tau)=(\mu+\beta)\,e^{-(\mu+\beta)(\tau-s)}$ represent the probability that no price resetting signal was received *and* no innovation in sector j made between times τ and s *and* either a reset signal occurred *or* a new firm entered the market with a new price in sector j at time s.[58] Using these and steady growth of p^* at rate π, we have

$$X(\tau)=\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha}L(\tau)\left[\frac{p^{*}(\tau)}{P(\tau)}\right]^{-\alpha}Q(\tau)\int_{-\infty}^{\tau}(\beta+\mu)\,e^{-(\beta+\mu-\alpha\pi)(\tau-s)}ds$$

Solving the integral which converges for $\beta>\alpha\psi$ leads to (25) in the main text.

Appendix 3 *Total final good production*

Total final good production can be rewritten as

$$Y(\tau)=A^{\alpha}\left(\frac{\alpha-1}{\alpha}\right)^{\alpha-1}L(\tau)\,P(\tau)^{\alpha-1}Q(\tau)\sum_{j=1}^{N}\frac{q^{(\alpha-1)k_j(\tau)}}{Q(\tau)}p_j(\tau)^{-(\alpha-1)}$$

As in Appendix 2, the average intermediate good price effective at τ can be expressed as a weighted average of past optimal prices with the weights $\widetilde{f}(s,\tau)$ defined in Appendix 2.

$$Y(\tau)=A^{\alpha}\left(\frac{\alpha-1}{\alpha}\right)^{\alpha-1}L(\tau)\,P(\tau)^{\alpha-1}Q(\tau)\int_{-\infty}^{\tau}p^{*}(s)^{-(\alpha-1)}\,\widetilde{f}(s,\tau)\,ds$$

Inserting $\widetilde{f}(s,\tau)=(\mu+\beta)\,e^{-(\mu+\beta)(\tau-s)}$ and using that the optimal price grows at rate π in equilibrium, we can calculate the integral's value which gives

$$\frac{Y(\tau)}{Q(\tau)}=\left(\frac{\alpha-1}{\alpha}\right)^{\alpha-1}A^{\alpha}L(\tau)\left[\frac{p^{*}(\tau)}{P(\tau)}\left(\frac{\beta+\mu}{\beta+\mu-(\alpha-1)\pi}\right)^{-\frac{1}{\alpha-1}}\right]^{-(\alpha-1)}$$

Note that convergence of the integral is ensured by assumption (37).

[58] Note that the two Poisson processes governing innovations and the occurrence of price adjustment sigals are stochastically independent so that that the joint probability is the product of the individual probabilities.

Next, we want to rewrite Y/Q as a product of efficient production with a given X/Q and a term that describes production inefficiency due to price dispersion. To find the maximum amount of final goods Y^{eff} that can be produced with a given amount X of intermediate goods when X is distributed efficiently among the intermediate good types x_j, we solve the following problem:

$$\max_{x_j} Y + \varpi \left[X - \sum_j x_j \right]$$

subject to the final good production function (1). The first order condition for x_j can be rewritten as

$$\left(\frac{\frac{\alpha-1}{\alpha} AL^{1/\alpha} q^{k_j \frac{\alpha-1}{\alpha}}}{\varpi} \right)^{\alpha} = x_j$$

Aggregating over the intermediate good types and solving for ϖ gives

$$\frac{\alpha-1}{\alpha} AL^{1/\alpha} \left(\frac{X}{Q} \right)^{-1/\alpha} = \varpi$$

Reinserting this into the first order condition gives

$$q^{k_j(\alpha-1)} \left(\frac{X}{Q} \right) = x_j$$

which can in turn be reinserted in the final good production function (1), yielding

$$Y^{eff}(\tau) = AL\,(\tau)^{1/\alpha} \left(\frac{X(\tau)}{Q(\tau)} \right)^{\frac{\alpha-1}{\alpha}} Q(\tau)$$

Now multiplying and dividing actual total final good production (27) by Y^{eff}, replacing $X(\tau)/Q(\tau)$ in the denominator with the amount of intermediate goods actually used (25) and simplifying, we have equation (27) in the text.

Appendix 4 *Proofs of propositions, lemmata and corollaries (3)-(12)*

Proof of lemma 1. The derivative with respect to the money growth rate ψ of the function $\widetilde{w}(\psi)$ from equation (28) for a given innovation rate μ is $\partial\widetilde{w}/\partial\psi = \frac{(\alpha-1)\widetilde{w}}{\beta+\mu-(\alpha-1)\pi} \left\{ -\frac{r+\beta+\mu}{r+\beta+\mu-\alpha\pi} \frac{\beta+\mu-(\alpha-1)\pi}{r+\beta+\mu-(\alpha-1)\pi} + 1 \right\}$. Examining the nulls of the derivative shows that the the function has extrema at

$\psi_{1/2} = \frac{1}{2}\frac{1}{\alpha-1} \left\{ [r+\beta+\mu+2(\alpha-1)\gamma] \mp (\beta+\mu+r)^{1/2} \left(\beta+\mu+\frac{4-3\alpha}{\alpha}r\right)^{1/2} \right\}$ associated with

$\pi_{1/2} = \frac{1}{2}\frac{1}{\alpha-1} (\beta+\mu+r)^{1/2} \left\{ (\beta+\mu+r)^{1/2} \mp \left(\beta+\mu+\frac{4-3\alpha}{\alpha}r\right)^{1/2} \right\}$

with $\pi(\psi_2) > \pi(\psi_1) > 0$.

Examining the second derivative at ψ_1 and ψ_2 shows that $\widetilde{w}(\psi)$ has a maximum (minimum) at ψ_1 (ψ_2) because

$$\left.\frac{\partial^2 \frac{w(\tau)}{P(\tau)Q(\tau)}}{\partial\psi^2}\right|_{\psi=\psi_1} = \frac{-4\alpha\left[\beta+\mu+\frac{4-3\alpha}{\alpha}r\right]^{1/2}(\beta+\mu+r)^{-1/2}(\alpha-1)\widetilde{w}}{\left\{(\beta+\mu+r)^{1/2}+\left[\beta+\mu+\frac{4-3\alpha}{\alpha}r\right]^{1/2}\right\}^2\left\{\frac{1}{2}\frac{\alpha-2}{\alpha-1}(\beta+\mu+r)^{1/2}+\frac{1}{2}\frac{\alpha}{\alpha-1}\left[\beta+\mu+\frac{4-3\alpha}{\alpha}r\right]^{1/2}\right\}^2} < 0$$

and

$$\left.\frac{\partial^2 \frac{w(\tau)}{P(\tau)Q(\tau)}}{\partial\psi^2}\right|_{\psi=\psi_2} = \frac{+4\alpha(\beta+\mu+r)^{-1/2}\left[\beta+\mu+\frac{4-3\alpha}{\alpha}r\right]^{1/2}(\alpha-1)\widetilde{w}}{\left\{\frac{1}{2}\frac{\alpha-2}{\alpha-1}(\beta+\mu+r)^{1/2}-\frac{1}{2}\frac{\alpha}{\alpha-1}\left[\beta+\mu+\frac{4-3\alpha}{\alpha}r\right]^{1/2}\right\}^2\left\{(\beta+\mu+r)^{1/2}-\left[\beta+\mu+\frac{4-3\alpha}{\alpha}r\right]^{1/2}\right\}^2} > 0.$$

Further, we find that $\psi_2 > \frac{1}{\alpha-1}\beta$, which is the maximum admissible money growth rate from condition (37), since $\frac{1}{2}\frac{1}{\alpha-1}\left\{[r+\beta+\mu+2(\alpha-1)\gamma]+(\beta+\mu+r)^{1/2}\left(\beta+\mu+\frac{4-3\alpha}{\alpha}r\right)^{1/2}\right\} > \frac{1}{\alpha-1}\beta$. Therefore, $\widetilde{w}(\psi)$ increases (decreases) in ψ for all admissible money growth rates ψ with $\psi < \psi_1$ ($\psi > \psi_1$).

The inflation rate $\pi(\psi_1) = \frac{1}{2}\frac{1}{\alpha-1}(\beta+\mu+r)^{1/2}\left\{(\beta+\mu+r)^{1/2}-\left(\beta+\mu+\frac{4-3\alpha}{\alpha}r\right)^{1/2}\right\}$ is positive since $\alpha > 1$ ensures that $1 > (4-3\alpha)/\alpha$. ■

Proof of lemma 2. Straightforward calculus shows that $\frac{\partial\widetilde{w}}{\partial\beta} = \frac{(\alpha-1)\widetilde{w}\pi}{(\rho+\beta-\alpha\psi+\overline{\eta}\mu)[\rho+\beta-(\alpha-1)\psi+(\overline{\eta}-\overline{q})\mu]} * \left[1-\frac{r+\beta+\mu-\alpha\pi}{\beta+\mu}\frac{r+\beta+\mu-(\alpha-1)\pi}{\beta+\mu-(\alpha-1)\pi}\right]$. The term in square brackets is always negative under deflation, while the fraction in front of the square brackets is negative (positive) for $\pi < 0$ ($\pi > 0$), so that $\frac{\partial\widetilde{w}}{\partial\beta} > 0$ for $\psi < \psi_0$ with $\pi(\psi_0) = 0$. A very strict sufficient condition for the term in square brackets to be positive is $r > \alpha\pi$, for which $\rho/\alpha > \psi$ is again a sufficient condition so that $\frac{\partial\widetilde{w}}{\partial\beta} > 0$ holds for $\rho/\alpha > \psi > \psi_0$. ■

Proof of lemma 3. Conditions (36) and (37) are jointly sufficient for the LHS of equation (33) to be concave in μ: $\frac{\partial^2 LHS}{\partial\mu^2} = \frac{C_1[(\overline{\eta}-\alpha\overline{q})\psi+\overline{q}(\rho+\beta)]\{(1+\alpha)[(\eta\overline{q}+1)\psi+\overline{q}(\rho+\beta)]-2\overline{\eta}[\rho+\beta-(\alpha-1)\psi+[(\eta+\alpha-1)\overline{q}+1]\mu]\}}{(\rho+\beta-\alpha\psi+\overline{\eta}\mu)^3[\rho+\beta-(\alpha-1)\psi+(\overline{\eta}-\overline{q})\mu]\left(\frac{\rho+\beta-(\alpha-1)\psi+(\overline{\eta}-\overline{q})\mu}{\rho+\beta-\alpha\psi+\overline{\eta}\mu}\right)^{1+\alpha}}$ where $C_1 = \alpha\frac{L}{\lambda}A^\alpha\frac{1}{\alpha-1}\left(\frac{\alpha-1}{\alpha}\right)^{2\alpha}$, $\overline{q} = q^{\alpha-1}-1 > 0$ and $\overline{\eta} = [(\eta+\alpha)\overline{q}+1]$. Given that $\overline{\eta}-\alpha\overline{q} = \eta\overline{q}+1 > 0$ and $\beta > \alpha\psi$, $\frac{\partial^2 LHS}{\partial\mu^2} < 0$ when condition (36) holds. Also, $\frac{\partial^2 RHS}{\partial\mu^2} = \frac{2\alpha\beta(\eta\overline{q}+1)[(\overline{\eta}-\alpha\overline{q})\psi+\overline{q}(\rho+\beta)]}{[\rho+\beta+(\eta\overline{q}+1)\mu]^3} > 0$ when condition (37) and the first inequality in condition (36) hold, so these conditions are sufficient to ensure that the RHS of the equation is convex in μ. With condition (35) we make sure that the value for $\mu \to 0$ of the LHS of equation (33) is larger than that of the RHS. Further, note that the RHS of equation (33) goes to infinity as $\mu \to \infty$ ($\lim\limits_{\mu\to\infty}\frac{\rho+\left[\eta\left(q^{\alpha-1}-1\right)+1\right]\mu}{\rho+\beta+[\eta(q^{\alpha-1}-1)+1]\mu}(\rho+\beta-\alpha\psi+\overline{\eta}\mu) = \infty$ since $\overline{\eta} > 0$) while the limit of the LHS is bounded ($\lim\limits_{\mu\to\infty} LHS = \frac{L}{\lambda}A^\alpha\frac{1}{\alpha-1}\left(\frac{\alpha-1}{\alpha}\right)^{2\alpha}\left(\frac{\overline{\eta}-\left(q^{\alpha-1}-1\right)}{\overline{\eta}}\right)^{-\alpha} < \infty$). Thus the two functions have a unique intersection with $\mu > 0$. ■

Proof of lemma 5. For $\beta \to \infty$, the zero profit condition (33) reduces to $\frac{L}{\lambda}\left(\frac{\alpha}{\alpha-1}\right)^{-2\alpha}A^\alpha\frac{1}{\alpha-1} = [\rho+(\eta\overline{q}+1)\mu]$ so that the growth rate of money ψ has no influence on the equilibrium research intensity μ. Since by equation (21) $\gamma = \left(q^{\alpha-1}-1\right)\mu$, the economy's real growth rate γ is independent of ψ, too. ■

Proof of lemma 6. Consider equation (33). First refer to figure 1 to see that given assumptions (35)-(37) and concavity of the LHS-curve, the latter's slope is always smaller than that of the RHS-curve at the equilibrium ($\frac{\partial LHS}{\partial \mu} - \frac{\partial RHS}{\partial \mu} < 0$). Further, $\frac{\partial LHS}{\partial \psi} - \frac{\partial RHS}{\partial \psi} = -\frac{\alpha(\alpha-1)LHS(\psi-\overline{q}\mu)}{(\rho+\beta-\alpha\psi+\overline{\eta}\mu)[\rho+\beta-(\alpha-1)\psi+(\overline{\eta}-\overline{q})\mu]}$. From assumption (37) we have that $\beta > \alpha\psi$. Further, $\overline{\eta} = [(\eta+\alpha)\,\overline{q}+1] > \overline{q}$ and from equations (21) and (18) we have $\psi-\overline{q}\mu = \pi$, so that $\frac{\partial LHS}{\partial \psi} - \frac{\partial RHS}{\partial \psi} \lesseqgtr 0$ for $\pi \gtreqless 0$. Thus we have that $\frac{d\mu}{d\psi} = -\frac{\frac{\partial LHS}{\partial \psi} - \frac{\partial RHS}{\partial \psi}}{\frac{\partial LHS}{\partial \mu} - \frac{\partial RHS}{\partial \mu}} \lesseqgtr 0$ for $\pi \gtreqless 0$. Further, since $\gamma = \left(q^{\alpha-1}-1\right)\mu$, $\frac{d\gamma}{d\psi} = \left(q^{\alpha-1}-1\right)\frac{d\mu}{d\psi} \lesseqgtr 0$ for $\pi \gtreqless 0$. ■

Proof of lemma 7.

From equation (33), $\frac{d\mu}{d\beta} = -\frac{\frac{\partial LHS}{\partial \beta} - \frac{\partial RHS}{\partial \beta}}{\frac{\partial LHS}{\partial \mu} - \frac{\partial RHS}{\partial \mu}} > 0$ since $\frac{\partial LHS}{\partial \beta} - \frac{\partial RHS}{\partial \beta} = LHS\frac{(\alpha-1)\alpha(\psi-\overline{q}\mu)^2\{\rho+\beta-(\alpha-1)\psi+(\overline{\eta}-\overline{q})\mu\}^{-1}}{(\rho+\beta-\alpha\psi+\overline{\eta}\mu)\{\rho+\beta+(\eta\overline{q}+1)\mu\}} > 0$ is positive for $\psi-\overline{q}\mu = \pi \neq 0$ as $\beta > \alpha\psi$ given assumption (37) and $\overline{\eta} = [(\eta+\alpha)\,\overline{q}+1] > \overline{q}$ and $\frac{\partial LHS}{\partial \mu} - \frac{\partial RHS}{\partial \mu} < 0$ from the proof of lemma 6. ■

Proof of proposition 8. From equation (28), for $\beta \to \infty$ the wage $w^{flex} = \frac{1}{\alpha}A^{\alpha}\left(\frac{\alpha}{\alpha-1}\right)^{-2(\alpha-1)}$ is constant so employment is independent of the money growth rate ψ. The equation determining the research intensity (38) reduces to $\frac{L\left(w^{flex}\right)}{\lambda}\left[\overline{\alpha}\left(\frac{\alpha}{\alpha-1}\right)^{-\alpha}\right] = (\rho+\overline{\eta}_2\mu)$ which is independent of ψ, too. ■

Proof of proposition 9. We analyse the influence at $\pi = 0$ of ψ on $L^* = L\left\{\widetilde{w}\left[\mu^*(\psi)\right],\psi\right\}$ (equation (30)). Here μ^* is the solution to equation (33) where L has been replaced by the endogenous $L(\mu,\psi,\beta)$ from equation (30). We then have that $\frac{dL^*}{d\psi} = \frac{\partial L^*}{\partial \psi} + \frac{\partial L^*}{\partial \mu}\frac{\partial \mu^*(\psi)}{\partial \psi} = \frac{\partial L^*}{\partial \widetilde{w}}\frac{\partial \widetilde{w}}{\partial \psi} + \frac{\partial L^*}{\partial \widetilde{w}}\frac{\partial \widetilde{w}}{\partial \mu^*}\frac{\partial \mu^*(\psi)}{\partial \psi}$. From equation (16), $\frac{\partial L}{\partial \widetilde{w}} > 0$ for all values of $\widetilde{w}$. Further, taking the derivatives of the wage in equation (28), we find that $\left.\frac{d\widetilde{w}}{d\psi}\right|_{\pi=0} = \frac{(\alpha-1)C_1 r}{\left(\frac{\alpha}{\alpha-1}\right)^{\alpha-1}(\beta+\mu)(r+\beta+\mu)} > 0$ with ($C_1 = \frac{1}{\alpha}\left(\frac{\alpha-1}{\alpha}\right)^{\alpha-1}A^{\alpha}$) and $\left.\frac{d\widetilde{w}}{d\mu}\right|_{\pi=0} = \frac{-rC_1(\alpha-1)\overline{q}}{\left(\frac{\alpha}{\alpha-1}\right)^{\alpha-1}(r+\beta+\mu)(\beta+\mu)} = -\overline{q}\left.\frac{d\widetilde{w}}{d\psi}\right|_{\pi=0} < 0$. Finally, $\left.\frac{\partial \mu^*(\psi)}{\partial \psi}\right|_{\pi=0} = \frac{\left.\frac{\partial L(\mu,\psi,\beta)}{\partial \psi}\right|_{\pi=0}\frac{1}{\lambda}\left(\frac{\alpha}{\alpha-1}\right)^{-2\alpha}A^{\alpha}\frac{1}{\alpha-1}}{-\left\{-(\eta\overline{q}+1)+\frac{1}{\lambda}\left(\frac{\alpha}{\alpha-1}\right)^{-2\alpha}A^{\alpha}\frac{1}{\alpha-1}\left.\frac{\partial L(\mu,\psi,\beta)}{\partial \mu}\right|_{\pi=0}\right\}}$ from equation (33) with endogenous $L(\mu,\psi,\beta)$. So $\left.\frac{dL^*}{d\psi}\right|_{\pi=0} = \left.\frac{\partial L}{\partial \widetilde{w}}\right|_{\pi=0}\left\{\left.\frac{\partial \widetilde{w}}{\partial \psi}\right|_{\pi=0} + \left.\frac{\partial \widetilde{w}}{\partial \mu}\right|_{\pi=0}\frac{\left.\frac{\partial L(\mu,\psi,\beta)}{\partial \psi}\right|_{\pi=0}\frac{1}{\lambda}\left(\frac{\alpha}{\alpha-1}\right)^{-2\alpha}A^{\alpha}\frac{1}{\alpha-1}}{-\left\{-(\eta\overline{q}+1)+\frac{1}{\lambda}\left(\frac{\alpha}{\alpha-1}\right)^{-2\alpha}A^{\alpha}\frac{1}{\alpha-1}\left.\frac{\partial L(\mu,\psi,\beta)}{\partial \mu}\right|_{\pi=0}\right\}}\right\}$ which using that $\left.\frac{\partial \widetilde{w}}{\partial \mu}\right|_{\pi=0} = -\overline{q}\left.\frac{d\widetilde{w}}{d\psi}\right|_{\pi=0}$ can be rewritten as $\left.\frac{dL^*}{d\psi}\right|_{\pi=0} = \left.\frac{\partial L}{\partial \widetilde{w}}\right|_{\pi=0} * \left.\frac{\partial \widetilde{w}}{\partial \psi}\right|_{\pi=0}\left\{1-\overline{q}\frac{-\frac{1}{\overline{q}}\left.\frac{\partial L(\mu,\psi,\beta)}{\partial \mu}\right|_{\pi=0}\frac{1}{\lambda}\left(\frac{\alpha}{\alpha-1}\right)^{-2\alpha}A^{\alpha}\frac{1}{\alpha-1}}{(\eta\overline{q}+1)-\frac{1}{\lambda}\left(\frac{\alpha}{\alpha-1}\right)^{-2\alpha}A^{\alpha}\frac{1}{\alpha-1}\left.\frac{\partial L(\mu,\psi,\beta)}{\partial \mu}\right|_{\pi=0}}\right\} > 0$ since the term in curly brackets is positive given $\left.\frac{\partial L(\mu,\psi,\beta)}{\partial \mu}\right|_{\pi=0} < 0$. ■

Proof of proposition 11. We know from the proof of lemma 1 that $\partial\widetilde{w}/\partial\psi > 0$ for all admissible $\psi < \psi_1$ with $\pi(\psi_1) > 0$ and $\partial\widetilde{w}/\partial\psi < 0$ for all $\psi > \psi_1$ that are compatible with the uniqueness condition (37) Since employment L increases monotonically in $\widetilde{w}$ by equation (16) , the same applies to employment as a function of money growth ψ. From the proof of lemma 6, we further have that *given* employment $\frac{\partial \mu}{\partial \psi} \gtreqless 0$ for $\psi \lesseqgtr \psi_0$ with $\pi(\psi_0) = 0$. Hence we have that $d\mu\left[L(\psi),\psi\right]/d\psi > 0$ for all $\psi \leq \psi_0$, since here, both the direct and effect of money growth and its indirect effect via employment on economic growth are positive, and $d\mu \left[L\left(\psi\right), \psi \right] / d\psi < 0$ for all $\psi \geq \psi_1$, since here both effects are negative. This completes the proof. ■

Proof of proposition 12. From the proof of lemma 6, we know that at $\pi = 0$ ($\pi < 0$), the non-employment-related effects of money growth ψ on economic growth γ are zero (positive). At the same time, from the proof of lemma 1 $\frac{\partial L}{\partial \psi} > 0$ for $\psi < \psi_1$ with $\pi\left(\psi_1\right) > 0$. Therefore, the $d\gamma \left[L^* \left(\psi, \beta\right), \psi, \beta \right] / d\psi > 0$ for $\pi \leq 0$ and the maximum growth rate is reached at a $\pi > 0$. ■

Proof of proposition 13. The proposition follows from the facts that first, at $\beta < \infty$ and $\pi = 0$ we have $d\gamma / d\psi > 0$ and $dL/d\psi > 0$ and second, the real outcomes of the models with price rigidity and with flexibility are identical at $\pi = 0$, which can be seen by letting $\beta \rightarrow \infty$ or setting $\pi = 0$, respectively, in equations (28) and (38). ■

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