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Abstract

The introduction of unemployment insurance is usually thought to increase welfare if workers are sufficiently risk averse. We analyse the effects of introducing mandatory unemployment insurance in the shirking model. Surprisingly, we find that introducing unemployment insurance reduces welfare irrespective of the degree of risk aversion.

JEL-class.: J0, J3, H1

Key Words: Efficiency Wages, Shirking, Unemployment Insurance
1 Introduction

The standard justification for the introduction of unemployment insurance is based on the idea that workers are risk averse. For instance, Holmlund (1998) uses a search model to show that positive unemployment benefits are optimal if workers are risk averse.\(^1\) It is the purpose of this paper to show that, surprisingly, this result does not carry over to another standard labour market model, the shirking model, originally developed by Shapiro and Stiglitz (1984).\(^2\) The reason is that a high degree of risk aversion does not only increase the benefits from unemployment compensation but also the costs in terms of negative incentives. If unemployment insurance makes unemployed workers much better off, it also reduces the incentives to provide effort. In order to preserve effort, unemployment must increase. The higher the benefit from the introduction of unemployment compensation, the higher the increase in unemployment required to preserve effort. Our analysis shows that the second effect dominates the first. To our knowledge, although the role of unemployment insurance in the shirking model did receive some attention in the debate following the publication of Shapiro and Stiglitz (1984),\(^3\) the fundamental issue of whether or not mandatory unemployment insurance is desirable if risk aversion is introduced into this model has not been investigated.\(^4\)

\(^1\)For similar conclusions in search models see also Mortensen (1983) and Acemoglu and Shimer (1999).

\(^2\)Shapiro and Stiglitz (1984), page 441, claim but do not show explicitly that "Clearly the social optimum involves [a mandatory minimum benefit level of] \(\bar{w} > 0\) if risk aversion is great enough." As the following analysis shows, this claim is not correct.

\(^3\)See for example Bull (1985) and Carmichael (1985).

\(^4\)The impact of unemployment insurance on employment and wages in shirking models with risk neutral workers has, for example, been analysed by Chatterji and Sparks (1991)
A limiting assumption of the original Shapiro-Stiglitz-model is that the effort standard is given exogenously. One may ask whether our result is an artefact of this assumption. We therefore extend the model by endogenizing the effort level following Rasmussen (2002). It turns out that our main result, the negative welfare effect of mandatory unemployment insurance does not change. The paper is set up as follows. In section 2, we analyse the effects of introducing unemployment insurance in the original Shapiro-Stiglitz-model, the only difference being that we consider risk averse, rather than risk neutral workers. We distinguish between the short and the long run by allowing entry and exit of firms only in the long run. In section 3, we endogenize the effort level. Section 4 concludes.

2 The shirking model with risk averse workers

Assume that there are $N$ identical risk averse workers in the economy. The instantaneous utility of an employed worker is $u(w) - g(e)$ where $u(w)$ is increasing and strictly concave in wage income per period $w$ and $g(e)$ is increasing and convex in the level of effort $e$ and $g(0) = 0$. Workers choose

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5 As Goerke (2002) points out, the outcome of an efficiency wage model consistently differs quite strongly in the short-run and the long-run perspective. Therefore, we also follow the extension of the Shapiro-Stiglitz-model by Albrecht and Vroman (1996) and allow for entry and exit of firms to investigate the long-run welfare effects of unemployment insurance.

6 The convexity assumption will assure that the first order condition of the firms’ profit maximisation problem describes a maximum even in the case of a linear production technology and risk neutrality of workers. In the original version of the model by Shapiro and Stiglitz (1984), instantaneous utility is specified with $g(e) = e$, so that $g''(e) = 0$. The
between two effort levels: they either take the high value \( e = \bar{e} > 0 \) or the low one \( e = 0 \). The instantaneous utility of an unemployed worker is \( u(\bar{w}) \), where \( \bar{w} \) is a transfer the unemployed worker receives from his last employer.\(^7\) Each worker maximizes the discounted value of expected lifetime utility. The discount rate is denoted by \( r \).

There is an exogenous job destruction rate \( b \) which may cause workers to be dismissed irrespective of the effort level they choose. In addition, a worker who spends zero effort is detected and dismissed for shirking with probability \( q \).

The expected lifetime utility of an employed worker who spends effort \( (V^e) \) is given by the equation

\[
rv^e = u(w) - g(\bar{e}) + b(V^u - V^e). \tag{1}
\]

where \( V^u \) is the expected lifetime utility of an unemployed worker. For an employed worker who spends zero effort, we have

\[
rv^n = u(w) + (b + q)(V^u - V^n). \tag{2}
\]

Workers will choose the high effort level if and only if \( V^e \geq V^n \). Each firm and each worker takes the utility of the unemployed as given. Combining (1)

\[^7\text{We follow Shapiro and Stiglitz (1984) by assuming that firms directly pay benefits to workers. Assuming that benefits are financed by a tax per employee paid by firms would not change the results. This issue will be discussed further below.}\]
and (2) yields the high effort condition

\[ u(w) - g(\tilde{e}) - r V^u - g(\tilde{e}) \frac{r + b}{q} = 0. \] (3)

There is a given number of \( M \) identical risk neutral firms in the economy. They produce a homogenous good \( y \) using effective labour as the only factor of production. Each employed worker provides \( e \) units of effective labour per unit of time. If all workers in a firm choose the high effort level \( \tilde{e} \), the input in terms of effective labour is \( L = n\tilde{e} \), where \( n \) is the number of employed workers. The production function is \( F(L) \), with \( F_L > 0 \) and \( F_{LL} < 0 \). Firms are price takers in the output market and the price of \( y \) is given and normalized to unity.

The unemployment insurance system consists of unemployment benefits \( w\bar{w} \) which firms have to pay to each dismissed worker during the time the worker is unemployed. The expected cost per dismissed worker \( \bar{W} \) is given by

\[ r\bar{W} = \bar{w} - a\bar{W}, \]

where \( a \) is the probability of exit from unemployment.\(^8\) The profit of a firm

\(^8\)Note that the considered unemployment insurance system is completely experience-rated. Alternatively, \( \bar{W} \) can be interpreted as a mandatory severance payment to dismissed employees. If the unemployment contributions a firm has to pay depend on the number of workers dismissed by the firm, as assumed here (following Shapiro and Stiglitz (1984)), the question arises whether the threat of a firm to dismiss workers caught shirking is credible. Since all workers are homogeneous, the firm knows that workers replacing those who are dismissed are of the same type. Thus, if there are layoff costs, the firm will be better off by keeping workers although they are caught shirking. In our analysis, this is not problematic because the results of the model would not change if we assumed that firms had to pay unemployment contributions in the form of a tax per employee, independent of the number of workers the firm lays off. In this case, the tax which balances the budget of the unemployment insurance system would be equal to \( b\bar{W} \). An efficiency wage model with heterogenous workers, which allows to study layoff costs, is developed in Fath and
per unit of time is
\[ \pi = F(L) - wn - bn\bar{W} - k \]

where \( k \) represents a fixed cost of running a firm. The first order condition
for the firm's maximization problem is
\[ \frac{\partial \pi}{\partial n} = F_L(L)\bar{e} - w - b\bar{W} = 0. \] \((4)\)

The labour market equilibrium is characterized as follows. The expected
lifetime utility of an unemployed worker is given by
\[ rV^u = u(\bar{w}) + a(V^e - V^u) \] \((5)\)

where \( a \) is the probability of exit from unemployment. In a steady state,
the inflow of workers to the pool of unemployed, \( bnM \), equals the number of
workers leaving unemployment, \( a(N - Mn) \). This implies
\[ a(n, M) = \frac{bnM}{N - Mn}. \] \((6)\)

Using (3), (5) and (6), we can now derive the aggregate high effort condition,
which is given by
\[ u(w) - g(\bar{e}) - u(\bar{w}) - g(\bar{e}) \frac{r + a(n, M) + b}{q} = 0. \] \((7)\)

Fuest (2005).
Using (6) in (4) yields

\[ F_L(L)\tilde{\epsilon} - w - \frac{\tilde{w}b}{a(n, M) + r} = 0. \tag{8} \]

Equations (7) and (8) determine the steady state values of \( n \) and \( w \), for given values of \( \tilde{w} \). Standard comparative static analysis leads to the unsurprising result that an increase in \( \tilde{w} \), departing from \( \tilde{w} = 0 \) reduces employment:

\[ \frac{d(Mn)}{d\tilde{w}} = \frac{M}{\Delta} \left( u'(\tilde{w}) + u'(w) \frac{b}{a + r} \right) < 0 \tag{9} \]

where

\[ \Delta = u'(w)\tilde{\epsilon}^2 F_{LL}(L) - \frac{g(\tilde{e}) (a + b) M}{q} \frac{(a + b) M}{N - Mn} < 0. \]

The effect on the wage rate is ambiguous

\[ \frac{dw}{d\tilde{w}} = \frac{1}{\Delta} \left( u'(\tilde{w})\tilde{\epsilon}^2 F_{LL}(L) + \frac{g(\tilde{e})}{q} \frac{bM}{(N - nM)} \frac{(a + b)}{(a + r)} \right) \leq 0. \tag{10} \]

If the production technology converges to linearity \( (F_{LL}(L) \to 0) \), firms make zero profits and an increase in unemployment benefits can only be financed by a wage reduction. If there are profits, the cost of providing unemployment benefits is partly shifted to profit income. The possible wage increase arises from the aggregate high effort condition in (7). According to that, a rise in unemployment compensation pushes efficiency wages upwards to sustain incentives to work. If the marginal utility of income of the unemployed is sufficiently high (consider \( u'(\tilde{w}) \to \infty \) as an extreme case), the positive incentive effect dominates the negative labour cost effect on wages.
2.1 Short-run welfare effects of introducing mandatory unemployment insurance

In a laissez faire equilibrium, firms would not pay unemployment benefits to workers. But would a mandatory introduction of transfers to the unemployed increase welfare? Let us first examine the impact on the expected utilities of employed and unemployed workers. Using (1) and (5) - (7), they are given by

\[ V^e = \frac{1}{r} \left( u(w) - g(\bar{e}) \frac{q + b}{q} \right) \]

\[ V^u = V^e - g(\bar{e}) \frac{q}{q} \]

which shows that in equilibrium a constant gap \( g(\bar{e}) \frac{q}{q} \) between the two utility values guarantees workers’ provision of effort. Obviously, expected utility of both groups will rise, if the introduction of unemployment insurance leads to an increase in equilibrium wages. But at the same time there arises a negative employment effect which expands the group of the unemployed who face lower expected utility. To account for the negative employment effect, we use the following utilitarian welfare function as a criterion for social welfare \( W \) of workers\(^9\)

\[ W = MnV^e + (N - Mn)V^u. \]

\(^9\)The qualitative results do not change if we include profits into the welfare analysis by assuming that workers and firm owners are the same individuals and the income per worker also contains profit income \( \frac{\text{P}}{N} \).
Using (11), (12) and their derivatives, the welfare effect of a change in $\bar{w}$, departing from $\bar{w} = 0$, can be written as:

$$
\frac{dW}{d\bar{w}} = \frac{g(\bar{e})}{q} \frac{d(Mn)}{d\bar{w}} + N \frac{u'(w)}{r} \frac{dw}{d\bar{w}}.
$$

(14)

Note again that, since $\frac{d(Mn)}{d\bar{w}} < 0$, (14) implies that a positive welfare effect of increasing $\bar{w}$ can only emerge if the wage rate increases, which requires $F_{LL}(L) < 0$ (see eq. (10)). Substituting (9) and (10) into (14) and making some rearrangements yields

$$
\frac{dW}{d\bar{w}} = \frac{1}{\Delta} [u'(\bar{w}) \left( M \frac{g(\bar{e})}{q} + u'(w)e^2 F_{LL}(L) \frac{N}{r} \right) \\
+ u'(w) M \frac{g(\bar{e})}{q} \frac{b}{r+a} \left( 1 + \frac{(a+b)N}{r(N-nM)} \right)].
$$

This implies that a necessary condition for a positive welfare effect is

$$
M \frac{g(\bar{e})}{q} + \frac{N}{r} u'(w)e^2 F_{LL}(L) < 0
$$

(15)

which, obviously, may be violated for any degree of workers’ risk aversion.

It is easy to check that, if the marginal utility of income of the unemployed approaches infinity ($u'(\bar{w}) \to \infty$), (15) is necessary and sufficient for a positive welfare effect. If (15) holds, workers may benefit from the introduction of unemployment benefits. But this is only possible if $F_{LL}(L) < 0$, which implies that there are positive profits in equilibrium. In this case, workers may benefit from the introduction of unemployment insurance because the cost of financing the benefits is at least partly shifted to firms, not because
workers are risk averse.\textsuperscript{10}

These findings may be summarized in the following

**Proposition 1** *In the short run, a marginal increase in mandatory unemployment benefits, departing from the laissez faire equilibrium with $\bar{w} = 0$,

1. reduces the welfare of workers, irrespective of the degree of risk aversion, if $F_{LL}(L) < 0$ and

2. reduces the welfare of workers, if $M_{q}^{\frac{\xi}{\eta}} + \frac{N}{r} u'(w) \epsilon^2 F_{LL}(L) \geq 0$ and $F_{LL}(L) < 0$.

\subsection*{2.2 Long-run welfare effects of introducing mandatory unemployment insurance}

In the long-run perspective, firms may enter or leave the market so that the equilibrium will be characterized by a zero profit condition:

\begin{equation}
\pi = F(L) - wn - bn \frac{\bar{w}}{a(n, M) + r} - k = 0. \tag{16}
\end{equation}

(16) together with (4) implies that average employment costs $w + \frac{b\bar{w}}{a+r}$ remain constant in equilibrium. Wages will therefore decrease with increasing unemployment benefits:

\begin{equation}
\frac{d\pi}{d\pi} = \frac{\pi_{\bar{w}}}{\pi_{w}} = \frac{-b}{a + r} < 0 \tag{17}
\end{equation}

\textsuperscript{10}If there are positive profits, social welfare would have to take into account the utility of firm owners. It is straightforward to show that profits never increase in response to an increase in benefits, so that the negative welfare effects of increasing benefits are only exacerbated if profits are taken into account.
The reason is that there are no profits which could finance higher efficiency wages after unemployment insurance is introduced. The wage reduction is induced by the exit of firms which reduces employment in the economy as a whole. High effort can be maintained despite increasing benefits and decreasing wages because some firms exit the market and the rate of unemployment increases.

The welfare effect is stated in

**Proposition 2** A marginal increase in mandatory unemployment benefits, departing from the laissez faire equilibrium with \( \bar{w} = 0 \), unambiguously reduces the welfare of workers in the long run, irrespective of the degree of risk aversion.

**Proof.** See the appendix.

As a consequence of lower employment and wages, both expected utilities and welfare as presented in (11) - (13) unambiguously decrease. This result is in line with proposition 1.1 and can be ascribed to the zero profit condition.

3 The shirking model with endogenous effort

As has become clear in the previous analysis, the change in the efficiency wage plays an important role for the welfare effects of unemployment insurance. Obviously, the results are limited by the fact that the required effort standard of firms is exogenously given. Demanding a higher performance level respectively enhancing the productivity of the employees may be a possibility for the firms to balance higher employment costs. In this chapter we
therefore investigate the implications of endogenising effort for the effects of unemployment insurance. We stick to the long run version of the model, i.e. we allow for entry and exit of firms. We will extend the model of the previous section in the following way: Like in Rasmussen (2000), a representative firm is now able to choose the effort standard \( \bar{e} \). This choice influences the wage setting of the firm via the high effort condition in (3). If (3) describes the implicit function \( \omega(w, \bar{e}) \), the change in the efficiency wage induced by a given change in the effort standard, \( w_e \), is given by

\[
 w_e \equiv \frac{\omega_e}{\omega_w} = \frac{g'(\bar{e}) (r + b + q)}{qu'(w)} > 0.
\]

(18)

The first order conditions of the firm now are given by (4) and

\[
 \frac{\partial \pi}{\partial \bar{e}} = nF_L(L) - nw_e = 0
\]

(19)

where the firm takes into account the required changes in \( w \) according to (18). From (4) and (19) one can derive a modified version of the Solow condition of efficiency wages which implies that firms maximise profits by equalising the variable costs per efficiency unit and the marginal cost of effort

\[
 w + b\frac{\bar{e}}{a+r} = w_e.
\]

(20)

According to equation (20) a single firm will raise the effort standard to balance an increase in unemployment benefits taking into account the necessary wage increase.

In the long run, the equilibrium values of \( w, \bar{e}, n \) and \( M \) for any given
level of \( \bar{w} \) are identified by (4), (7), (16) and (19).

### 3.1 Mandatory unemployment insurance

The introduction of mandatory unemployment insurance now has the following effects:

**Proposition 3** If the effort standard is set by firms, a marginal increase in mandatory unemployment benefits, departing from the laissez faire equilibrium with \( \bar{w} = 0 \), unambiguously reduces employment and increases the effort standard in the long run. Wages will decline.

**Proof.** See the appendix.

For given wages and effort, an increase in \( \bar{w} \) increases the cost per worker faced by firms, and profits become negative. In order to restore zero profits, effort must increase and/or wages must decline. This is only compatible with the no shirking condition if unemployment increases. and firms leave the market. The number of operating firms and, hence, the level of employment will decline until the zero profit condition is satisfied.

### 3.2 Welfare effects of introducing mandatory unemployment insurance

How does the introduction of mandatory unemployment insurance now affect welfare? Overall welfare is again given by \( W = MnV^e + (N - Mn)V^u \). As can easily be checked by differentiating (11) and (12), expected utilities of both employed and unemployed workers decline with decreasing wages and
increasing effort. Moreover, if the effort level increases, the gap between the utilities of employed and unemployed workers $V^e - V^u = \frac{g(e)}{q}$ must widen. Finally, the overall welfare effect accounts for rising unemployment and is stated in

**Proposition 4** A marginal increase in mandatory unemployment benefits, departing from the laisser faire equilibrium with $\bar{w} = 0$ where the effort standard is endogenously determined, unambiguously reduces the welfare of workers in the long run.

**Proof.** See the appendix. ■

### 4 Conclusions

Risk aversion is commonly thought to be sufficient as a justification for the introduction of mandatory unemployment insurance. This paper shows that, in the shirking model as introduced by Shapiro and Stiglitz (1984) as well as in the extended version with endogenous effort, risk aversion increases not only the benefit of unemployment insurance but also the cost in the form of high unemployment. If a sufficient part of the burden of unemployment insurance can be shifted to profits, the introduction of unemployment insurance may increase the welfare of workers at the expense of firm owners. But in this case, the positive effect on the welfare of workers does not depend on the degree of risk aversion. In the long run, where profits are zero, however, the introduction of unemployment insurance unambiguously reduces worker welfare, irrespective of the degree of risk aversion. The reason is that a very high marginal utility of income of the unemployed implies that the welfare
gain from transfers to the unemployed is high, but the negative incentive effect of the transfer is also high. Therefore, the increase in unemployment required to maintain effort is also very high.

Appendix

Proof of proposition 2

Equations (7), (8) and (16) determine the steady state values of $w$, $n$, and $M$, for given values of $\bar{w}$. Total differentiation at $\bar{w} = 0$ yields

$$u'(w)dw - \frac{g(\bar{w})}{q} \frac{bN}{(N - Mn)^2} (MdN + ndM) - u'(\bar{w})d\bar{w} = 0 \quad (A.1)$$

$$-dw + \bar{w}^2 F_{d\bar{w}}dn - \frac{b}{a + r} d\bar{w} = 0 \quad (A.2)$$

$$-ndw - \frac{nb}{a + r} d\bar{w} = 0. \quad (A.3)$$

>From (21) we already derived $\frac{dw}{d\bar{w}} < 0$ in (17). From inserting (21) into (21) it follows that $\frac{dn}{d\bar{w}} = 0$. Equation (21) then shows that employment must decline with increasing unemployment benefits: $\frac{d(Mn)}{d\bar{w}} = n \frac{dM}{d\bar{w}} < 0$. The change in welfare is given by (14). Given that employment and wages decline, the welfare effect is unambiguously negative.

q.e.d.
Proof of proposition 3

Total differentiation of (4), (7), (16) and (19) with respect to \( w, \bar{e}, n, M \) and \( \bar{w} \) departing from \( \bar{w} = 0 \) yields

\[
\begin{align*}
\frac{d}{d\bar{w}} w'(w)dw - \frac{g'(\bar{e})}{q} (a + b + q + r) \frac{d\bar{e}}{d\bar{w}} &= 0 \\
-\frac{g(\bar{e})}{q} \frac{bN}{(N-Mn)^2} (Mdn + ndM) - u'(\bar{w})d\bar{w} &= 0 \quad (A.4)
\end{align*}
\]

\[
-\frac{d\bar{w}}{d\bar{e}} + (\bar{e}nF_{LL} + F_L) \frac{d\bar{e}}{d\bar{w}} + \bar{e}^2 F_{LL}dn - \frac{b}{a + r} d\bar{w} &= 0 \quad (A.5)
\]

\[
-\frac{n}{\bar{w}} \frac{\partial w_e}{\partial \bar{e}} dw + \left( n^2 F_{LL} - n \frac{\partial w_e}{\partial \bar{e}} \right) d\bar{w} + \bar{e}nF_{LL}dn = 0 \quad (A.6)
\]

\[
-ndw + nF_Ld\bar{e} - \frac{nb}{a + r} d\bar{w} = 0 \quad (A.7)
\]

where

\[
\frac{\partial w_e}{\partial \bar{e}} = w_e \frac{g''(\bar{e})}{g'(\bar{e})}; \quad \frac{\partial w_e}{\partial w} = -w_e \frac{u''(w)}{u'(w)}.
\]

Inserting (21) into (21) yields

\[
\bar{e}nF_{LL}d\bar{e} + \bar{e}^2 F_{LL}dn = 0 \quad (A.8)
\]

which implies \( \frac{dn}{de} = -\frac{n}{\bar{e}} < 0 \). Substituting (A.8) into (21) and rearranging yields \( \frac{dw}{de} < 0 \). The last result together with (21) implies \( \frac{de}{d\bar{w}} > 0 \) so that \( \frac{dn}{de} < 0 \) and \( \frac{dw}{d\bar{w}} < 0 \). Finally, the result that employment decreases with unemployment benefits can be derived by rearranging (21), which yields

\[
\frac{g(\bar{e})}{q} \frac{bN}{(N-Mn)^2} \left( M \frac{dn}{d\bar{w}} + n \frac{dM}{d\bar{w}} \right) = u'(w) \frac{dw}{d\bar{w}} - \frac{g'(\bar{e})}{q} (a + b + q + r) \frac{d\bar{e}}{d\bar{w}} - u'(\bar{w}) < 0
\]

q.e.d.
Proof of proposition 4

Welfare is given by

\[ W = MnV^e + (N - Mn)V^u. \]

Using (11) and (12), \( W \) can be written as

\[
W = NV^e - (N - Mn)\frac{g(\bar{e})}{q} \\
= \frac{N}{r} \left( u(w) - g(\bar{e})\frac{q + b}{q} \right) - (N - Mn)\frac{g(\bar{e})}{q}
\]

The welfare effect of a change in \( \bar{w} \), departing from \( \bar{w} = 0 \), is given by:

\[
\frac{dW}{d\bar{w}} = \frac{g(\bar{e})}{q} \frac{d(Mn)}{d\bar{w}} + N \frac{u'(w)}{r} \frac{dw}{d\bar{w}} - \frac{g'(\bar{e})}{qr} (r (N - Mn) + (q + b) N) \frac{d\bar{e}}{d\bar{w}} < 0.
\]

q.e.d.

References


