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COMPETITION AND GROWTH IN A VINTAGE KNOWLEDGE MODEL

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Abstract

This paper models the relationship between growth, technology-lifetime, entry, and competition in a vintage-knowledge model of endogenous growth and perfect competition. The model has a unique steady state REE equilibrium. Variations of R&D-efficiency lead to a negative relation between growth and vintage-lifetime and indicate a non-monotonic relation between growth and competition. A shift of population size and its growth rates have qualitatively different consequences here than in standard models. The extent of entry constitutes a buffer, neutralizing the effect of population size or population growth rates on per-capita income levels and growth rates.

Keywords: Endogenous Growth, Vintage-Model, Perfect Competition
JEL: D40, D90, O30, O40

1 Introduction

This paper models the interaction between productivity growth, active lifetime of technology generations, entry, and competition in a vintage-knowledge model of perfect competition and endogenous growth.

At unique steady-state rational expectation equilibrium, a variation of most exogenous parameters induces an opposite reaction of productivity growth and active technology lifetime. Variations of these parameters therefore induce a negative relation between productivity (and per capita income) growth on the one hand and the active lifetime of technology vintages on the other hand. This prediction is in accord with the observations of Habakkuk [1962], Williamson [1971], Pack [1986] or Hsieh [2001].

In contrast, there is no such clear cut relation between the intensity of competition and growth. In numerical examples the relation is either positive or non-monotonic, which is compatible with cross-sectional observations for instance by Nickel [1996], Blundell, Griffith and VanReenen [1999] (who find a positive empirical relation) and Aghion, Bloom, Blundell, Griffith and Howitt [2005] (who find a non-monotonic empirical relation).

Furthermore, a shift of population size and its growth rates have qualitatively different consequences here than in standard endogenous growth models: The extent of entry constitutes a perfect buffer, completely neutralizing the effect of mere size on the steady state growth rate or the steady state level of per capita income. This may help explain why, following Sala-I-Martin, Doppelhofer, and Miller [2004], neither population size nor population growth rate are "significantly related to growth". Nevertheless, in contrast to the prediction of semi-endogenous growth models, policy can raise the long-run per-capita growth rate by subsidizing R&D-activity. An increase of the population *growth rate* has no permanent effect on the per-capita income growth rate, a negative effect on the level of per-capita income, and a positive effect on the research intensity and its share in GDP. The present model allows to explain why past per-capita growth rates have not increased despite a strong observed increase in the resources spent on R&D (in contrast to the Romer-type endogenous growth models) without giving up the possibility of perpetual growth even when these resources and population no longer grow (in contrast to the Jones-type semi-endogenous growth models).

1.1 A vintage model of perfect competition and endogenous growth

There are only two levels of activity at the production-side of the economy:

1. the production of a homogenous final-consumption good by many small incumbent firms, each of which is characterized by a level of "knowledge-capital" determining its labor productivity and

2. the production of new knowledge-capital by many small reseach-labs (entrants) who can later use their knowledge-capital as final-good incumbents. Building on past knowledge each individual entrant has to produce his "own" knowledge-capital. The quality of the knowledge-capital (futur labor productivity in final-good production) depends in a deterministic way on the entrant's R&D-intensity.

There is perfect competition on both layers of the model:[1] Assuming small efficient scales for the individual final-good firm, competition on the goods market will naturally lead to **perfect competi-**

[1] The idea of a formal model of endogenous growth with perfect competition can already be found in Shell [1973]. The title "A competitive model in which inventive activity is financed from quasi-rents in advanced technology" of Section IV of Shell's article maps the road to endogenous growth taken in the present article. While Shell's model is partial equilibrium, Funk [1996] analyzes a general equilibrium model with perfect competition and endogenous growth and provides conditions on the productivity of research that ensure the persistence of endogenous growth. As in the present paper, these assumptions mimic those of Romer [1990], Grossman and Helpman [1991a,b] or Aghion and Howitt [1992]: there has to be the right amount of positive spill-overs from research onto the productivity of further research. Concerning the nature of competition and endogenous growth the present model is a representative consumer continuous-time version with rational expectations of the discrete time OLG model in Funk [1996]. In contrast to the present paper,

tion à la Arrow-Debreu, all firms (and consumers) will be assumed to be price takers. This does allow firms to make strictly positive short-run profits (quasi-rents) justifying costly entrance with new or rejuvenated technologies and hence is perfectly compatible with endogenous growth – provided that instantaneous entry to the most up to date technologies is not completely free and provided that – given past investment – individual firms' short-run technologies exhibit decreasing returns to scale.

An important feature of the present model is the endogenous determination of the active lifetime of individual technology vintages. An incumbent firm which has entered with a given technology in period t remains active with this technology-vintage as long as it remains profitable. The less firms enter in subsequent periods with new improved knowledge, the longer the period of activity of the technology-vintage t will be. As in other endogenous growth models the future short-run profits an entrant anticipates provide the necessary incentive for his research. In most monopolisitcally competitive endogenous growth models these profits arise due to a constant mark-up of prices over marginal costs. Typically this mark-up is fixed exogenously: Either by the elasticity of substitution of the innovator's brand with other brands or – closer to the present setting – by the fixed quality-adjusted productivity distance of the best to the second best firm (limit-pricing in case of a non-drastic vertical process or

new knowledge in Funk [1996] can be imitated free of cost by the marked after an exogenously fixed number of periods. The length of the period during which a technology-vintage can realize quasi-rents was thus not determined endogenously.

While in the present paper (as in Shell [1973], Funk [1996]) there are *many* small innovators in each period, there is a single small innovator per period, whose achievements can be copied free of cost after one period in Funk [1998] (and similarly in Funk [2002] or Funk and Vogel [2004]). Funk [1996,2002] also relate the perfectly competitive endogenous growth models to those of Schumpeter's early work (Schumpeter [1911] or "Schumpeter Mark I" in Scherer's [1992] terminology) as well as to the literature of induced change of the 60ies (Kennedy [1964], Samuelson [1965], Drandakis and Phelps [1966]). For instance it is argued that the perfectly competitive model of endogenous growth best captures Samuelson's idea of "Darwinian perfect competition" (Samuelson [1965], p. 351).

The present paper (and more so the aforementioned discrete time models with only one innovator, who can be copied after one period) is also related to the theory by Boldrin and Levine [1999, 2002, 2004] who advocate a more rigorous return to the competitive model. It is difficult not to agree with these authors that nobody has ever used an idea which was not embodied in some rival object or subject.

Hellwig and Irmen [2001] analyze a model with many small innovating firms similar to the discrete time model of Funk [1996]. New knowledge can already be freely copied after one period, so that the vintage of one technology generation is exogenously fixed to one. In Hellwig and Irmen [2001] each firm chooses the efficient scale of inputs ('capacity') of the next period's small final good technology generated by the innovation as well as the productivity at these scales. In contrast, in the present setting the efficient scale of the chosen individual technology is fixed exogenously.

In contrast to Shell [1973], Funk [1996,1998,2002], Hellwig and Irmen [2001] or Funk and Vogel [2004]), the active lifetime of individual technology vintages is determined endogenously here. Finally the present model is related to Wälde [2004], who uses a perfectly competitive model of endogenous growth to study the possibility and the determinants of cyclical behavior of R&D-investment. In contrast to the present paper – where the finite lifetime of technology vintages is determined endogenously – old knowledge (embodied in old machines) never becomes obsolete.

product innovation). Short-run profits in the present setting arise in a similar way as these constant mark-up profits due to limit-pricing. However, the limit-firm is not the second best firm but rather the endogenously determined oldest active incumbent. As a consequence

1. the mark-up for any given incumbent j is determined **endogenously** by the productivity-distance between j and the oldest active incumbent as well as by the active technology lifetime which determines *who* is the oldest active incumbent

2. the mark-up for any given incumbent j is **not constant**, because the productivity-distance between j and the currently oldest active incumbent shrinks in each period (until j becomes the oldest current firm himself before being driven out of business).

Although there is no scope for collusion and there are no stratigic consideration involved in the present perfectly competitive framework, the knowledge asymmetry between small incumbent firms makes it useful to talk about more intense or less intense (perfect) competition. This corresponds to colloquial usage of the term[2] and is also captured by standard measures of competition based on price-costs margins. I will therefore speak of high (low) intensity of competition when there are many (few) incumbent firms close to the current frontier of knowledge and use a standard measure of the intensity of competition which captures this idea.[3]

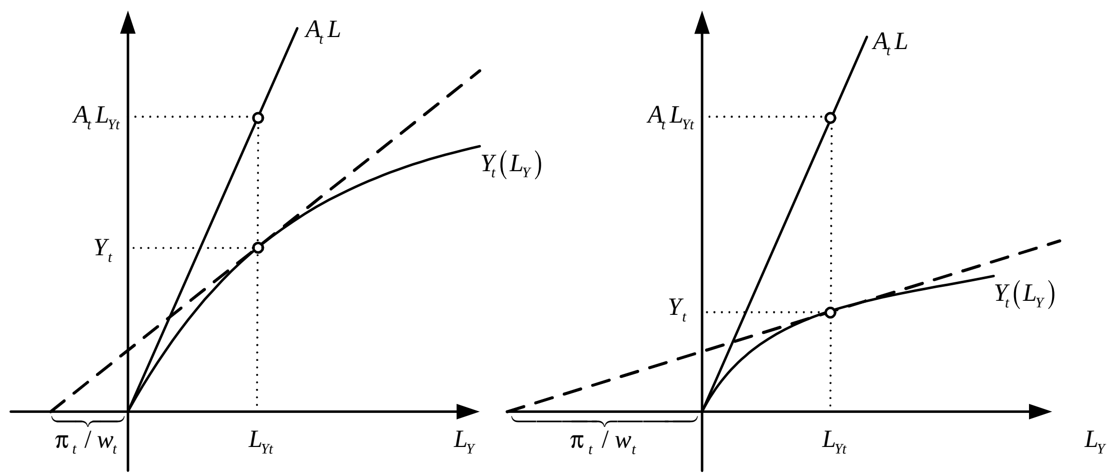
In the vintage-knowledge model the **intensity** of (perfect) competition thus depends on the distribution of incumbent firms over 'knowledge-capital' of different vintages, which in turn depends on how many firms have invested in knowledge-capital in the past. The more firms have access to recent and correspondingly more modern techniques, the more final output Y_t a given number of workers L_{Yt} can produce, the fiercer is competition and the smaller are profits at (short-run) equilibrium. The highest possible intensity of competition is attained when the (short-run) aggregate technology exhibits constant returns to scale and short-run profits are zero, which would be the case if there were free entry to the most advanced technical knowledge. I call this limit case – which does not occur at equilibrium – the case of **complete competition**. In other words, complete competition would be achieved in the limit where all knowledge would be available publicly and could be copied and applied without cost and without delay. Only this limit case would result in a linear-homogeneous short-run aggregate production-function. Correspondingly, only in this limit case, there would be no incentive to

[2] In a 100 m race, an exam at graduate school, a beauty or music contest one says that competition is intense when there are many almost equally fast, learned, beautiful or skilled contenders (even if strategic considerations play no role).

[3] To fix attention I will follow Aghion et al. [2005] and measure the intensity of competition by a weighted sum of individual Lerner-indices ([Price-MC]/Price), which in the present setting corresponds to an atomless version of $1 - \frac{1}{\#\text{ active firms}} \sum_{j \text{ active}} \frac{\text{profit}_j}{\text{output}_j}$. The simpler measure $1 - \frac{\text{total profit}}{\text{total output}}$ would yield similar steady state dependencies of competition on growth and technology lifetime.

spend costly resources for R&D to enter the market with new knowledge-capital, so that there would be no (endogenous) technical change (see Romer [1990]).

Figure 1.1 shows two examples of **short-run aggregate production functions** (as functions of labor input). They are strictly concave since reducing employment L_{Yt} drives out of business the least productive firms first. If many firms know the latest technology, the short-run aggregate production function is nearly linear on a fairly large domain. Vintage lifetimes are short, competition on the final good market is fierce, (perfectly competitive) profits are small. The current incentive for new firms to enter will be weak. If in contrast, relatively few firms use the latest technologies (have recently entered), then the short-run aggregate production function exhibits rather strong decreasing returns to scale. Competition is moderate, profits are high. The current incentive to enter the market with new knowledge-capital will be strong. See figure 1.1A and 1.1B for two economies with same current employment L_{Yt} and the same currently leading technology A_t but different degrees of competition. More firms use technologies that are almost as efficient as A_t in economy A than in economy B. Correspondingly production is higher in A, while (perfectly competitive) profits are higher in B.



A: strong competition, high output, small profits B: weak competition, low output, large profits

The perfectly competitive endogenous growth framework rests on two crucial assumption: First, an individual research lab passes on its research to only one final-output firm. While old knowledge can be cheaply copied, more own effort has to be invested to get close to the frontier of knowledge or even improve upon it. Technical change therefore comes along with a lot of duplication of research effort. Second, the individual final good firm has small efficient scales relative to aggregate resource supply. Similar restrictions both of the public replicability of individual knowledge as well as of the individual replicability of small efficient scales are typically required to justify the assumption of perfect competition in *any* general equilibrium framework (For a microeconomic foundation of the of the aggregate short-run production and short-run Walrasian competition in the present framework see for instance Novshek and Sonnenschein [1980]).

Relation to previous vintage-capital models of endogenous growth Many of the leading growth theorists of the 1960ies have described growth within models of vintage-capital and -knowledge. Among others they were interested in the relation between growth rates and the endogenous active lifespan of technology generations (see for instance Solow [1962], Phelps [1963], Solow, Tobin, von Weizsäcker, and Yaari [1966]). While these models assumed competitive markets as does the present paper, technical change was of course exogenous. Exogenous technical progress raises wages, reducing the short-run profits that can be realized by incumbent technologies of older vintages until they are scrapped. A similar "inflationary wage scheme" determines the active lifespan of technologies and the age structure of active technologies in the present paper, except that the rate of technical progress inflating wages is endogenous too.

Although the simultaneous activity of machines and technics of different ages is an obvious fact, only the latest technology vintage is active in most post-1960 (exogenous or endogenous) growth models. The reason for this narrowing of scope lies in the technical difficulties of vintage models. The present paper avoids most of these difficulties by restricting itself to steady state equilibria. Motivated by the omnipresence of heterogenous vintage structures in reality and the inconsistencies of non-vintage growth theory with aggregate data (see Cooley, Greenwood, and Yorukoglu [1994]) research in vintage models has recently been resumed (see for instance Benhabib and Rustichini [1991], Caballero and Hammour [1996], Bardhan and Priale [1996], Boucekkine, Germain, Licandro, and Magnus [1998], Boucekkine, del Rio, and Licandro [1999], Hsieh [2001]). A common issue of this literature is the possibility of endogenous fluctuations that arises naturally at non-steady state equilibria of vintage models. Boucekkine, Licandro, and Paul [1997] as well as Boucekkine, Licandro, Puch and Paul [2005] provide numerical methods to solve vintage models, which may allow to study the off steady-state equilibrium behavior of the present model.

1.2 Intuition and summary of results

Steady-state mechanics with constant population and constant aggregate research intensity: Positive mechanical relation between actve lifetime and growth; Negative mechanical relation between competition and growth . The present paper only studies steady-state equilibria. To get a first idea of the mechanical relation between active lifetime and growth and between competition and growth which will be formally described in Section 3 first consider Figure 1 depicting an economy with constant population L in which the number L_Y of workers in final-good production and the number L_A of workers in R&D ("researchers") are exogenously given such that the overall research intensity $l_A = L_A/L$, with $L = L_A + L_Y$ of the economy is constant. The L_A researchers are assigned to Λ research labs (or research firms), so that each individual research lab employs $h = L_A/\Lambda$ researchers. As each research lab is associated with exactly one final-good producer the number of research firms Λ also determines the extent of entry to final-good production by new ore rejuvenated firms. Consider a steady state with common and constant individual research intensity h. Obviously – given overall research intensity l_A – there is an immediate trade-off between high individual research intensity h and a high extent of entry $\Lambda = L_A/h$. To see how this translates

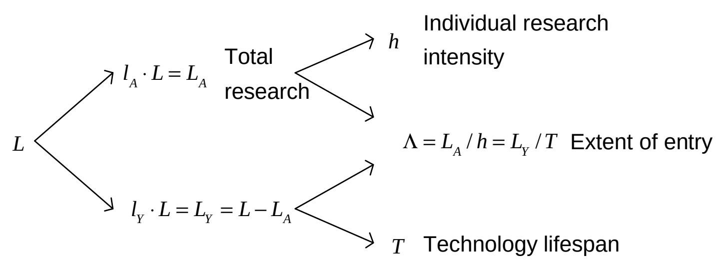


Figure 1: Research, extent of entry and technology-lifespan with constant population

into a trade-off between growth and competition consider what happens when h is raised. On the one hand, the common individual research intensity h of each individual research-lab uniquely determines the intensity of the steady state productivity growth rate $g = f(h)$. An increase of h therefore raises productivity growth g what in turn raises growth of per-capita income and consumption. On the other hand increasing h affects the intensity of competition (measured by the share $1 - \pi/Y$ of non-profit incomes from final-good production or by the Lerner-Index) through two channels:

(1) First, given total research L_A an increase of the individual research intensity h reduces the number $\Lambda = L_A/h$ of research labs and thus the extent of entry in each period. Since the total number L_Y of workers in the final-good sector as well as the optimal scale of input of each individual producer are given, a smaller number Λ of firms of each vintage will result in a larger lifetime T of each vintage (see Figure 1). This increases the ratio of the productivity of younger technology-vintages to the productivity of the oldest active vintage. Since the productivity of the oldest active vintage determines final-good sector wages, the increased active lifespan reduces the "wage inflation" inherent in the "obsolesence mechanism". As a result the short-run profits of active firms as well as the ratio π/Y of total profits to output (or a weighted sum of individual profit-shares) are increased by the increase of h and the corresponding reduction of Λ.

(2) Second, given the active lifetime T of technology vintages, the increasing productivity growth g (caused by an increase of h) raises the ratio of the productivity of any active vintage to the productivity of the oldest active vintage. This too increases the share of profits in total production.

Given L_A and L_Y, an increase of h thus increases the productivity growth rate g and reduces the intensity of competition by the two mechanism described in (1) and (2): Less entry means that for each knowledge-vintage fewer customers can be served by final-good producers with better technologies. The reduction of the wage inflation caused by "obsolescence mechanism" of creative destruction caused both directly by an increasing g as well as indirectly by the induced increase of T raises the ratio of (short run) profits to production.

Adding population growth The population size need not be constant for the presence of the described trade-off. Let L_t grow at the constant rate n and assume that both industrial employment L_{Yt} and research L_{At} grow proportionally so that as before the overall research intensity $l_A = L_{At}/L_t$ remains constant. At steady state the research intensity h of the individual research lab will still be constant, such that the number of research labs and the extent of entry Λ_t grows at the pace of populating growth and the per-capita extent of entry $\lambda = \Lambda_t/L_t$ remains constant. As before the immediate trade-off between h and λ translates into one between competition and growth. However, the steady state lifetime T of technology-vintages will now depend on the rate n of population growth. Without population growth total employment L_Y is $T\Lambda\kappa$, that is the number of active vintages T times the number Λ of firms per vintage times the number κ of workers per firm. The steady state lifetime $T = L_Y/\Lambda\kappa = (h/\kappa) \cdot (L_Y/L_A)$ thus is proportional to the individual research intensity. With positive population growth a slightly more complex but similar mechanical relation between T, h and L_{At}/L_{Yt} will hold: Given L_{At}/L_{Yt}, the active lifetime T increases with individual research intensity h as well as with the population growth rate n. The reason is simple: The faster population grows, the smaller the ratio of a given number of firms Λ_τ of any incumbent vintage $\tau < t$ to current employment L_t.

Equal marginal present values from increasing the induvidual R&D-intensity and increasing the extend of entry: Negative relation between active lifetime and growth Apart from these mechanical conditions the usual Euler-equation for optimal consumption growth has to be satisfied and there are two crucial equilibrium conditions from the research and production side of the economy: firstly individual research intensity h should maximize the present value of future quasi-rents generated by a research lab minus the cost or research and secondly these difference should be zero (free-entry to research). The first condition will require that the marginal present value (MPV) from increasing h equals the research wage and the second condition can be interpreted as requiring that

MPV from increasing the rate of entry λ equals the research wage. In Section 4 it will be shown that the difference between the MPV of λ and the MPV of h increases both in the productivity growth rate g and in the technology lifespan T. This provides a **negative** relation beween the g and T.

Existence and uniqueness of balanced growth equilibrium when the total research intensity is exogenous As has been explained, the mechanical steady state relations of Section 3 provide a positve relation between g and T. Equalizing the MPV from increasing h and λ in Section 4 yields a negative relation between g and T. Section 5 puts together the two relations between g and T and shows that they uniquely determine productivity and income growth rate, the individual research intensity, the active life-time of each technology-vintage as well as the extent of entry. An exogenous variation of total R&D-intensity, of a research-efficiency parameter, of impatience or of the population growth rate each effects g and T in opposite directions. An empirical dispersion of any of these parameters thus leads to a negative relation between growth and active lifetime of technology vintages.

Endogenous total research intensity. Adding the trade-off between consumption and R&D investment: Positive relation between active lifetime and growth Now assume that final-good production and research are no longer fixed exogenously. The two sectors use the same input L, which now will be endogenously allocated to industrial production and research. This adds the usual endogenous growth trade-off between current consumption (determined by L_Y) and total research L_A to the above trade-off. On the one hand there now is an additional variable that has to be determined endogenously (total R&D-intensity $l_A = L_A/L$). On the other hand there is an additional equilibrium condition requiring that research wages correspond to the wages in final-good production. This condition leads to a positve relation between lifetime T and growth g. The effect is similar to the positive growth effect of an increase of the mark-up over marginal costs in standard monopolistically competitive models: Consider Figure 1.1 on page 5: Larger T (given g) means lower intensity of competition, higher profits, higher incentives to innovate and higher g.

In Section 6 it is shown that this positive relation between g and T (“equal wages") together with the negative realtion of Section 4 (“equal MVP") provides a unique steady state equilibrium determining g and T independently from the "mechanical steady-state" relation of Section 3. The latter relation is used to determine the now endogenous total R&D-intensity. At this unique steady state equilibrium, the shares in total resources devoted to final-good production and to research as well as the (per capita) number of innovators and the degree to which innovations improve over incumbent technologies are constant. The research efficiency parameters as well as the degree of consumers' patience have the usual positive effect on steady state income growth rates. In contrast, a shift of population size and its growth rates have qualitatively different consequences here than in previous endogenous growth models. The following paragraphs therefore discuss this in more detail.

The effect of population size: Complete absence of scale effects For the case without population growth it is immediately seen from Figure 1 that an increase of L *may* remain without effect on the growth rate $g = f(h)$ and the lifespan T, if it neither affects the total research intensity l_A nor the individual research intensity h. This would be the case if at equilibrium the extent of entry Λ would vary proportionally with L. This is exactly what happens in the present framework. At the heart of the differences in the predictions of the effects of populations size lies the observation that the two crucial equilibrium conditions (optimal individual research intensity and free entry to research) do not (directly) depend on the total size of the economy. The individual research intensity and the expected new technology's lifetime satisfying these two conditions only depend on interest rates, on current research wages, and on future final-good sector wages. The reason for the independence from total population size are those assumptions which also justify the framework of perfect competition: first, the efficient scale size of the individual final-good producer does not depend on the size of the market, and second, each individual final-good producer has to perform its own additional research to reach or improve upon the current knowledge frontier.

In the model with one type of labor for both sectors, current research wages equal current final-good sector wages. At steady state the latter will be shown to grow as usual at the rate of productivity growth. Furthermore current wages are determined by the productivity of the oldest currently active technology vintage, which in turn only depends on productivity growth rate g and lifetime T (at steady state). Finally, the common individual research intensity determines steady state productivity growth g. Thus, given the interest rate, the two equilibrium condition for individual research firms can be reduced to two conditions only involving technology vintage-lifetime T and productivity growth g (which will be shown to have a unique solution). Since the two conditions are independent of the size of the economy, the growth rate too will be independent of the size of the economy!

As a consequence, an increase of total resources increases the steady state extent of entry without affecting the growth rate or per capita incomes. Thus, the extent of entry is higher in a large country than in a small country. Not so the growth rate or the per-capita income level. In other words, in the present paper the **extent of entry constitutes a perfect buffer, completely neutralizing the effect of mere size on the steady state growth rate or the steady state level of per capita income**.

This conclusion distinguishes the vintage-knowledge perfectly competitive model – to different degrees – from monopolistically competitive endogenous growth models. In an overview article on the recent growth theory Charles Jones notes that "virtually all idea-based growth models involve some kind of scale effect. [Jones 2003, p. 55]" and that " ...this result is not surprising given that these are idea-based growth models, but it is useful to recognize since many of the papers in this literature have titles that include the phrase 'growth without scale effects'... [p. 46]". Jones even holds that the presence of

> "... [S]cale effect is so inextricably tied to idea-based growth models that rejecting one is largely equivalent to rejecting the other [Jones 2003, p. 38]".

The present paper shows that this claim does not extend to the present setting. Growth in this paper is certainly driven by the accumulation of ideas. The essential feature leading to perpetual and endogenous growth is a sufficiently strong externality from the current stock of ideas onto the productivity of further research for new ideas, exactly as in previous endogenous growth models. Yet, endogenous growth in the present idea-based model is completely disentangled from size effects.

The effect of population growth: No growth effect, negative level effect, positive effect on research intensity. While the *level* of population has no effect on steady state income growth rates and levels, the same is not true for the population *growth rate*. As in the standard Solow-model an increase of population growth reduces the steady state level of per capita income.

As has been mentioned, given total research intensity $l_A = L_A/L$ and the growth rate of knowledge g, an increase of population growth increases the active lifetime T of a knowledge-capital vintage. This in turn has a negative effect on knowledge and income growth. Thus, if the total research intensity L_A/L is exogenously held constant as in Section 5, population growth has a negative effect on income growth. However, for the reasons sketched above, in the complete model of Section 6 with endogenous research intensity l_A, productivity and per-capita income growth rates are determined by the two individual research lab's equilibrium conditions. Thus total research intensity $l_A = L_A/L$ will endogenously rise after an increase of population growth, exactly neutralizing the negative direct effect of population growth on per capita income growth. Not only the level of population but also its growth rate has no growth effect! This prediction too contradicts those of most previous endogenous growth models.

It is also shown that despite the absence of scale effects, the present model is a model of endogenous growth not only in the sense that growth occurs due to profit-seeking R&D, but also in the sense that the growth rate can be affected by public policy. An R&D subsidy (financed by a lump-sum tax) increases the steady state income growth rate.

Constant growth rates despite increasing research intensity and increasing research-share The research intensity and hence also the share of incomes from research in total incomes increase with rising population growth rate without affecting per-capita income-growth. This result allows to reconcile two stylized facts within an endogenous growth model that can generate perpetually balanced growth without depending on perpetual population growth: the increasing aggregate research intensity observed in recent decades with the more or less constant productivity and per-capita income growth rates (See Sections 7 and 8).

The remaining sections are organized as follows. Section 2 introduces the vintage model of perfect competition and of endogenous growth and defines steady states. Section 3 discusses some basic properties of steady state equilibrium. Section 4 derives two essential equilibrium conditions. In Section 5 steady state equilibrium is first analyzed assuming an exogenously fixed total research intensity. Given the fraction of total resources dedicated to innovative investment, it remains to determine *how many* innovators enter (and how many firms upgrade their technology) and *how drastic* their innovations are. It is shown that there always exists a unique steady state equilibrium and comparative statics are studied. Assuming that workers can choose whether to work in the final-good sector or in the research sector, Section 6 adds the second endogenous growth dimension, usually modelled in endogenous growth models. Existence and uniqueness of steady state equilibrium are shown as well as all the aforementioned comparative static results: the absence of level and growth effects of the population size, the absence of a growth effect of population growth rate despite its positive effect on the research intensity, the negative level effect of population growth, and the potential growth effect of public policy. Sections 7 and 8 conclude by comparing the scale effects of the present model with those of previous endogenous growth models and with recent stylized facts.

2 The model

2.1 Final-good production

At period $t \in \Re_+$ a final-good producer of type $\tau \in (-\infty, t]$ is characterized by the number A_τ of output units he can produce with the efficient-scale amount of labor. A_τ is τ's knowledge-capital arising from past innovative investment which will be described below.[4] His type will later be identified with the time he first entered the market or last up-dated his knowledge-capital, which I call his vintage. In the basic version of the model I normalize his short-run technology such that he uses exactly one unit

[4] This 'knowledge-capital' can either be intangible abstract knowledge that has been generated in the past and can be used by τ (as in most endogenous growth theory) or it can be interpreted as physical capital owned by τ that has been produced in the past and that embodies a corresponding state of knowledge (as in Boldrin and Levine [2002]).

of labor (if he decides to be active).[5] At t the index of the most recent technology is $\tau = t$.

The total number (mass) of small final good-producers of type τ is Λ_τ. The final good sector at t is completely described by the range $[A_{-\infty}, A_t]$ of known technologies at t and the distribution $\{\Lambda_\tau\}_{\tau \in [-\infty, t]}$ of incumbents over this range. I denote by $\theta(t)$ the vintage of the oldest technology still in use at t, which needs of course to be determined endogenously. $T_t := t - \theta(t)$ then is the active lifetime of this technology.

There is perfect competition on the final good-market among all existing firms of all types at t, so that only the best and most recent technologies will be active (those of vintage $\theta(t)$ or younger). Total output at t is

$$Y_t = \int_{\theta(t)}^{t} \Lambda_\tau A_\tau d\tau$$

and total labor employed in the final-good sector is

$$L_{Yt} = \int_{\theta(t)}^{t} \Lambda_\tau d\tau.$$

The intensity of (perfect) competition on the final good market depends on the present distribution $\{\Lambda_\tau\}_{\tau \in [-\infty, t]}$ of incumbents over known technologies. If many firms know the latest technology (have recently entered, $T_t = t - \theta(t)$ small), the short-run aggregate production function (as a function of labor input) is nearly linear on a fairly large domain. Return to figure 1.1A and 1.1B which show two economies with same the L_{Yt} and the same leading technology A_t but different intensities of competition.

A microeconomic foundation of the aggregate short-run production function fitting into the present setting is given in Novshek and Sonnenschein [1980] in a Cournot-Walras framework. A similar foundation can be given in the Bertrand-Walras setting of Funk [1995].

2.2 Innovation

The knowledge-capital production function On the basis of already known technologies, research lab j can develop new technologies. The extent of the improvement over known technologies

[5] Note that the crucial assumption allowing a sound micro-foundation of the perfect competition assumed her is that individual firms' technologies have efficient scales at which the input amounts are small relative to the total supply of these inputs. This is also the essential assumption behind the absence of scale-effects. To simplify the exposition I normalize this efficient scale input to one unit of labor. Later I show that this is in fact a normalization which has no effect on the results. The simplest way to guarantee that an active individual producer will in fact employ *exactly* one unit of labor is to assume that an individual technology τ produces no output with less than one unit of labor and produces A_τ units of output with at least one unit of labor. Note that while this extreme form of decreasing returns to scale simplifies the exposition, it is neither necessary for the perfect competition assumed here (see in Novshek and Sonnenschein [1980]) nor for the absence of scale effects.

depends on the basis of knowledge accessible to research firm j and on the amount h_{jt} of labor employed by j. A research lab employing h_{jt} units of labor at t comes up (with certaninty) with a technology with productivity

$$A_{jt} = f(h_{jt}) \cdot \int_{-\infty}^{t} A_\tau d\tau, \tag{1}$$

where A_τ is the productivity of the leading technology at τ.

Thus, slightly deviating from the language used in much of the new growth literature, R&D firms do not 'create a number $\dot{A}_t$ of new designs' or products, but rather come up with new or known technologies for producing the final good output and parametrized by their productivity A_{jt}. Note that at equilibrium all research firms at t will chose the same number of researchers h_t and therefore generate the same technology $A_t = f(h_t) \int_{-\infty}^{t} A_\tau d\tau$. Hence at a steady state equilibrium (with constant h_t)

$$\dot{A}_t = f(h_t) \int_{-\infty}^{t} \dot{A}_\tau d\tau = f(h_t) A_t, \tag{2}$$

which is equivalent to the more standard expression for R&D production functions: If $h_t = h$ is constant, then the growth rate of labor productivity $g = \dot{A}_t / A_t = f(h)$ is constant as well. The scale elasticity of A_t in knowledge-capital production is therefore one, as in first generation and the 'generation 98' endogenous growth models.

To fix ideas I assume throughout that the knowledge-capital production function $f(h)$ is given by

$$f(h) = \left(\frac{a h^\alpha}{1 + a h^\alpha} \right)^\gamma, \tag{3}$$

with $a, \alpha, \gamma > 0$. The relevant features of f are that it is increasing ($f'(h) > 0$) with positive and declining elasticity: $\varepsilon(g) := f'(h)h/f(h) > 0$ and $\frac{d\varepsilon(g)}{dg} \leq 0$.[6] These features are indeed satisfied for the example function: For $h \geq 0$, f is increasing with $f(h) \in [0, 1)$. The elasticity $\varepsilon(g) := f'(h)h/f(h)$ at $h = f^{-1}(g)$ is $\varepsilon(g) = \gamma\alpha(1 - g^{1/\gamma})$. It decreases with rising g: $d\varepsilon(g)/dg = -\alpha g^{(1-\gamma)/\gamma} < 0$ if $g > 0$.

The objective of research firms Each individual research lab succeeds with its envisaged production of knowledge-capital. After innovation research lab j with knowledge-capital A_{jt} turns into a final good producer using the new know-ledge capital to produce A_{jt} units of output with one unit of

[6] For most results it would be sufficient to assume $f(h)$ increasing ($f'(h) > 0$) at a declining rate ($f'' < 0$), as is for instance satisfied by the function $f(h) = ah^\alpha$, with $a, \alpha > 0$ and $\alpha < 1$. While for a research production function like $f(h) = ah^\alpha$, the marginal productivity of research declines with increasing effort, the momentaneous productivity growth rate $f(h) = ah^\alpha$ resulting from the research of an individual research unit is in principle unbounded (provided research input where unbounded), which seems rather implausible. The elasticity of f with respect to h is constant $\alpha > 0$: A one percent increase of research h increase always result in a α percent increase of productivity growth $f(g)$. It seems to be more realistic to assume that the momentaneous productivity growth rate is bounded, so that for already high research intensity the elasticity of $f(h)$ declines with increasing h (this plays a role mainly when considering optimal growth).

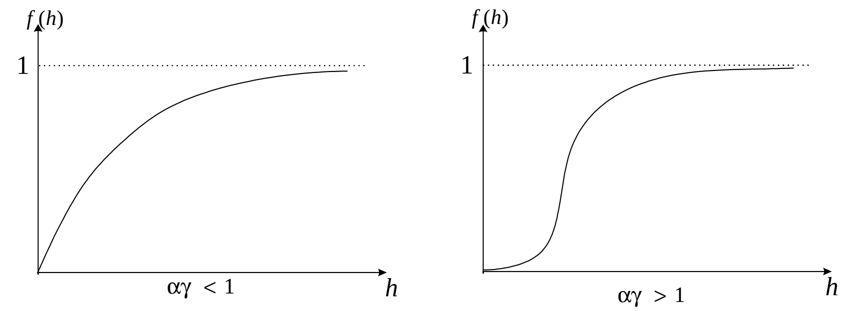


Figure 2: R&D production function

labor.[7] Research lab j at t thus chooses the number of researchers h_j to maximize the excess of this present value over the research cost $w_{At}h_j$:

$$\max_{h_j} \int_t^{t+T(h_j)} [A_j - w_{Y\tau}]e^{-r(\tau-t)}d\tau - w_{At}h_j,$$

where $T(h_j)$ is the length of time the firm plans to be active on the good market and where we have already assumed that interest rates are constant.

There is free entry to research. This will make sure that at equilibrium the revenues generated by one research lab just cover research wages $w_{At}h_{jt}$. Since each research lab turns into exactly on final-good firm, the number Λ_t of research labs at t determines the extent of entry by new (or rejuvenated) firms into the final-good market. At equilibrium all research labs at t will chose the same knowledge-capital A_t, so that Λ_t determines the number of future final-good competitors with common knowledge-capital A_t.

If there are Λ_t research labs at t, each employing h_t workers, then total labor demand of the research sector is $\Lambda_t h_t = L_{At}$. Thus, while Λ_t measures the extent of entry, h_t determines the productivity of entering firms. Together Λ_t and h_t will determine the curvature of future final good technologies and hence the future intensity of competition and of profits. An increase of total R&D activity L_{At} does not automatically enhance productivity growth of the leading final good technology. It may merely increase the extent of entry.

Once more, note that each research lab turns into a single final-good firm only. It cannot for instance licence the same knowledge-capital to several final-good producers at the same time. A firm which considers to enter the market for final-goods or to upgrade its technology has to perform some research on its own (or to let it done by a down-stream research lab), even if competitors are

[7] Alternatively each research lab sells the exclusive rights to use its innovation to a single final good producer. Competition for these rights will ensure that their market-values correspond to the present value of the expected flow of quasi-profits generated by the corresponding final good producer.

developing similar technics. This reflects the rival part of knowledge which takes into account that to some extent ideas are always embodied in human or physical capital. Nevertheless knowledge keeps its partially public good character: When trying to enter the market or upgrade its knowledge a final-good producer (or its downstream research lab) can built on all knowledge which has been introduced by previous innovators (the term $\int_{-\infty}^{t} A_\tau d\tau$ in (2)). It has however to add some proper effort (h_{jt} in (2)) – not necessarily much, if he contents himself with a rather old-fashioned technology and, depending on the shape of $f(\cdot)$, much more if he wants to come up with an up to date or even leading technology.

2.3 Households

The household sector is modeled as in the standard Ramsey-Cass-Koopmans model. The representative household has the dynastic utility function

$$\int_{t=0}^{\infty} e^{-\rho t} u\left(c_t\right) L_t dt \text{ with } u\left(c_t\right) = \ln c_t$$

where $c_t = C_t/L_t$ is the per capita consumption of the household at t and L_t the number of household members.

Total population and labor-supply L_t grows at an exogenous rate $n := \frac{dL_t/dt}{L_t} \geq 0$. Of the L_t workers L_{Yt} are employed in the final good sector at wage w_{Yt}, while $L_{At} = L_t - L_{Yt}$ are employed in the research sector at wage w_{At}. At present I leave open whether there are two distinct types of workers, with fixed exogenous research intensity $z := L_{At}/L_{Yt}$, or whether all workers are identical and freely choose in which sector to work. In the former case (Section 5) total industrial employment and total R&D activity are exogenously predetermined and the two wages w_{Yt} and w_{At} will typically differ at equilibrium. In the latter case (Section 6) the allocation of labor to the two sectors is determined endogenously such as to equalize the two wages.

Households own the existing final good firms and can acquire shares of new final good firms on the equity market from successful research firms. At each instant they receive the short-run profits of final good firms corresponding to their shares. (Households also own research labs. However, as there is free entry to research, shares in research firms will pay no dividend and will have zero price).

Final good producer assets are priced at their fundamental value, i.e. at the present value of the corresponding flow of future profits evaluated at market interest rates. Under these conditions, a necessary condition of utility maximization given the sequence of interest rates r_t is the standard Euler-equation $\dot{c}_t/c_t = r_t - \rho$.

2.4 Steady State Equilibrium

At each instant of time final good firms maximize instantaneous profits, taking factor prices (w_{Yt}, w_{At}, and r_t) and output price (normalized to 1) as given. Each active research lab j chooses its research input h_{jt} such as to maximize the difference between the value of the final good firms' shares they generate and the cost of research. There is an infinite number of potential research firms, which are indifferent between becoming active or not, when profits are zero (Free Entry to Research). Households maximize dynastic utility within their financial means, taking as given initial asset holdings and the correctly foreseen sequence of wages and interests. A perfectly competitive **equilibrium** given an initial distribution of firms over known $A_{\theta(0)}$, A_0, $\{\Lambda_\tau\}_{\tau\in[\theta(0),0]}$ is defined as a sequence of prices (w_{Yt}, w_{At}, and r_t) and of quantities (h_t, L_{At}, L_{Yt}, Y_t, C_t, A_t, Λ_t, T_t) such that under the listed assumptions the markets for labor, final goods, and shares clear at any instant.

A **steady state equilibrium** is an equilibrium at which r_t, h_t, $l_{At} := L_{At}/L_t$, $l_{Yt} := L_{Yt}/L_t$, $\lambda_t = \Lambda_t/L_t$, and T_t are constant and w_{Yt}, w_{At}, $y_t := Y_t/L_t$, $c_t := C_t/L_t$, and A_t grow exponentially at the same constant rate g.

3 Mechanical steady-state relations

This section describes some steady-state relations between the growth rate g of the economy, the active lifetime of a technology vintage T, the per-capita extent of entry λ, and the per capita amount of resources $l_Y := L_{Yt}/L_t$ and $l_A := L_{At}/L_t$ allocated to the two sectors. Nothing is said here about any causality in these relations, as most of the mentioned variables are endogenous (In Section 5 all of these variables except L_{At} and L_{Yt} are endogenous and in Section 6, also $l_Y := L_{Yt}/L_t$, $l_A := L_{At}/L_t$ or $z = L_{At}/L_{Yt}$ will be determined endogenously).

Throughout the paper we will deal with present values $\int_0^T e^{-x\tau}d\tau$ of a unit-output stream over the interval $[0, T]$, where the constant discount rate x will be the productivity growth rate, the population growth rate, the interest rate, or the utility discount rate. In the following I use the notation

$$I(x,T) := \int_0^T e^{-x\tau}d\tau = \begin{cases} \frac{1-e^{-xT}}{x} \text{ if } x \neq 0 \\ T \text{ if } x = 0 \end{cases} \qquad \text{Notation}$$

By definition at steady state the per-capita amounts of resources allocated to final-good production and to research l_Y and $l_A = 1 - l_Y$ as well as the productivity growth rate g are constant. Let l_A and g be given. I first show how the other constant steady state variables can be mechanically derived without considering the equilibrium behavior of the R&D sector.

As has already been remarked, at steady state $\dot{A}_t = f(h)\int_{-\infty}^t \dot{A}_\tau d\tau = f(h)A_t$ and thus $g := \dot{A}_t/A_t = f(h)$. Thus the R&D production function $f(\cdot)$ defines a one to one relation between h

and the productivity growth rate $g \in [0, 1]$, the number of workers needed to generate a given g is $h(g) := f^{-1}(g)$, or

$$h(g) = b\left(\frac{g^\eta}{1 - g^\eta}\right)^\beta$$

with $\eta = 1/\gamma$, $\beta = 1/\alpha$, and $b = (1/a)^\beta$. Of course h is constant if g is.

Now consider employment in the R&D sector. Since h is the number of workers employed by an individual research lab and $\lambda = \Lambda_t/L_t$, the per-capita number of research labs at t, an immediate relation between l_A, the total per-capita employment in R&D, and λ is given by

$$l_A = \lambda \cdot h(g). \tag{4}$$

Thus for our given $l_A = 1 - l_Y$ and g, the constant per-capita extent of entry $\lambda = l_A/h(g)$ is determined as well. In fact the relation $g = f(\frac{l_A}{\lambda})$ defines a simple steady-state **trade-Off between entry and growth:**Next consider employment in the final-good sector. Each active final-output firm employs

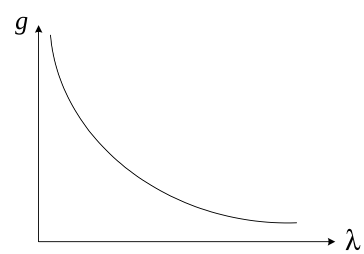


Figure 3: Trade-Off between Entry and Growth: $g = f(\frac{l_A}{\lambda})$

one unit of labor. Thus per-capita employment by firms of vintage $\tau \in [\theta(t), t]$ is $\lambda = \Lambda_\tau/L_\tau$, the per-capita number of firms that have entered at τ. Therefore per-capita employment at t in manufacturing is $l_Y = L_{Yt}/L_t = \int_{\theta(t)}^t \frac{\Lambda_\tau}{L_t} d\tau = \int_{\theta(t)}^t \frac{\Lambda_\tau}{L_\tau}\frac{L_\tau}{L_t} d\tau = \lambda \int_{t-T_t}^t e^{-n(t-\tau)} d\tau$ or

$$l_Y = \lambda \cdot I(n, T). \tag{5}$$

For $n = 0$ steady state per-capita industrial employment simply is the per-capita number of firms' per vintage multiplied by the number of active vintages: $l_Y = \lambda T$. Equation (5) can be rewritten as $T = [-\ln(1 - \frac{l_Y \cdot n}{\lambda})]/n$ if $n \neq 0$ and $T = l_Y/\lambda$ if $n = 0$.

Dividing (4) by (5) yields for the ratio of researchers to industrial labor $z := l_A/l_Y = h(g)/I(n, T)$, or $h(g) = z \cdot I(n, T)$. This relation defines a straightforward stationary state relation between productivity growth g and technology lifetime T which must of course hold at steady-state-REE:[8]

$$g^{\text{Mecha}}(\underset{+}{T}|\underset{+}{z}, \underset{-}{n}, \underset{+}{a}) := f(z \cdot I(n,T)) \tag{6}$$

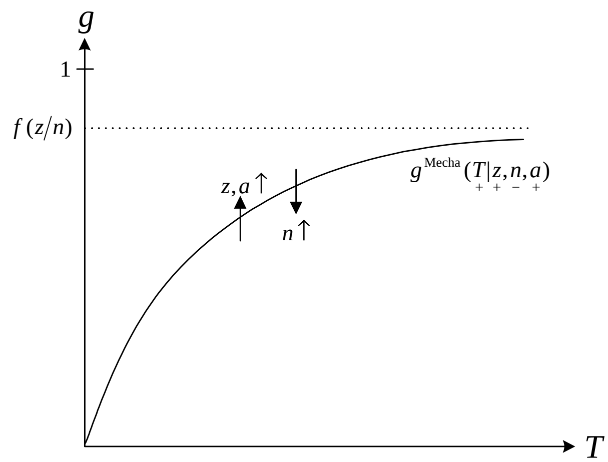


Figure 4: Steady-State Relation between Growth and Vintage Lifetime

Since g, λ, T are constant, production at t can be written as

$$\begin{aligned} Y_t &= \int_{\theta(t)}^{t} \Lambda_\tau A_\tau d\tau = \Lambda_t A_t \int_{\theta(t)}^{t} \frac{\Lambda_\tau}{\Lambda_t}\frac{A_\tau}{A_t} d\tau \\ &= \lambda L_t A_t \int_{\theta(t)}^{t} e^{-(g+n)(t-\tau)} d\tau \\ &= \lambda L_t A_t \frac{1-e^{-(g+n)T}}{g+n}. \end{aligned}$$

Therefore Y_t grows at the rate $g+n$ of the number L_tA_t of labor efficiency units. Consumption and output per labor efficiency unit is

$$\frac{Y_t}{A_t L_t} = \lambda I(g+n,T) \tag{7}$$

Thus, as in the Solow-model, *given* g, T, and λ, the population size L_t of the economy has no impact on the level of per-capita steady-state incomes, while the population growth rate n has a negative effect on these incomes.

Inserting $L_{Yt} = \lambda L_t I(n,T)$ into (7), the equation $\frac{Y_t}{A_t} = \frac{I(g+n,T)}{I(n,T)} L_{Yt}$ describes the long-run steady state relation between industrial employment and income levels.

[8]Equivalently, for $0 < g < f(z/n)$ this can be written as $T(\underset{+}{g}|\underset{-}{z},\underset{+}{n}) = \frac{-\ln[1-nh(g)/z]}{n}$ if $n \neq 0$ and $T(\underset{+}{g}|\underset{-}{z}) = \frac{h(g)}{z}$ if $n = 0$. Once more consider an economy at steady state with constant population ($n = 0$): The constant active lifetime of each knowledge-vintage equals the number of employees per research lab multiplied by the ratio of total employment in R&D to employment in manufacturing.

The short-run relation between L_{Yt} and Y_t exhibits the usual properties of a short-run **neoclassical production function** (see figure 1.1): Short-run aggregate production is a strictly concave and increasing function of labor. Assuming constant **past** λ and g this function is $Y_t(L_Y) = \lambda L_t A_t \frac{1-e^{-(g+n)T(L_{Yt})}}{g+n}$, where $T(L_{Yt})$ is defined by inverting (5). Thus $Y_t(L_Y) = \lambda L_t A_t \frac{1-[1-(nL_Y/\lambda L_t)]^{\frac{g+n}{n}}}{g+n}$ for $L_Y \leq \lambda L/n$ and $Y(L_Y) = \lambda L A_t \frac{1-e^{-gL_Y/\lambda L}}{g}$ if $n = 0$. The marginal product of labor therefore is $\frac{dY_t(L_{Yt})}{dL_Y} = \lambda L_t A_t e^{-(g+n)T} \frac{dT(L_Y)}{dL_Y}$. Inserting $\frac{dT(L_Y)}{dL_Y} = \frac{d\{-\ln[1-(n/\lambda)(L_Y/L_t)]/n\}}{dL_Y} = \frac{1}{\lambda L_t} e^{nT}$ this becomes $\frac{dY_t(L_{Yt})}{dL_Y} = A_t e^{-gT} = A_{\theta(t)}$. At competitive equilibrium final good sector employees are paid at their marginal productivity, thus $w_{Yt} = A_{\theta(t)}$, or

$$\frac{w_{Yt}}{A_t} = e^{-gT}.$$

Thus final good sector wages too grow at the rate of productivity growth. Since y_t grows at the rate g, per capita consumption $c_t = y_t$ also grows at the rate g. Finally, since $g = \dot{c}_t/c_t = r_t - \rho$ is constant, the interest rate $r_t = g + \rho$ is constant too.

Summarizing, fixing the employment ratio $z = l_A/l_Y$ and the productivity growth rate $g < f(z/n)$ also determines constant levels for the variables l_A, l_Y, h, λ, T, y_t/A_t, c_t/A_t, w_{Yt}/A_t and r.

We can now describe the **trade-off between growth and competition** sketched on page 7 of the Introduction in a more precise way (now also allowing $n > 0$): Given l_A and $l_Y = 1 - l_A$ (hence given $z = l_A/l_Y$) the growth rate g can only be raised by raising T well via (the inverse of) Equation 6. Both the increase in g and in T lower the intensity of competition defined by the weighted sum of shares of non-profit incomes from production [9]

$$\begin{aligned} \mathrm{Comp}_t \quad : & = 1 - \frac{1}{\int_{t-\theta(t)}^{t} \Lambda_\tau d\tau} \int_{t-\theta(t)}^{t} \Lambda_\tau \frac{A_\tau - w_{Yt}}{A_\tau} d\tau \\ & = \frac{e^{-gT} I(n-g, T)}{I(n, T)}. \end{aligned}$$

Note that this measure of competition is indeed closely linked to (the inverse of) the mark-up e^{gT} of the price ($= 1$) over the leading firm's marginal cost ($= e^{-gT}$). For $n = 0$ for instance, $\mathrm{Comp}_t = \frac{e^{-gT} I(-g,T)}{T} = \frac{I(g,T)}{T} = I(gT, 1) = \int_0^1 e^{-(gT)\tau} d\tau$.

4 Optimal individual R&D-intensity and free-entry

The two fundamental equilibrium conditions arising from optimal individual research intensity and free entry to research can be derived without further specifying the model. They hold both for exogenous and for endogenous research intensity $l_A = L_{At}/L_t$ and do neither depend on the scale L_t of the economy nor on population growth n. They are valid both for $n = 0$ and for $n > 0$.

[9] A similar negative dependency on g and T results for the simpler measure $1 - \pi_t/Y_t = w_{Yt} L_{Yt}/Y_t = e^{-gT} I(n, T)/I(n + g, T)$.

4.1 Optimal research intensity of the individual innovator

The market value of an innovation is the present value of the expected flow of quasi-profits generated by the corresponding final good producer. Thus the individual research lab j at t chooses h_j to maximize the excess of this present value over the research cost $w_{At}h_j$.

$$\max_{\{h_j:T(h_j)\geq 0\}} \Gamma(h_j) = \max_{h_j} \int_t^{t+T(h_j)} [A_j - w_{Y\tau}]e^{-r(\tau-t)}d\tau - w_{At}h_j,$$

where $T(h_j)$ is the length of time a firm using A_j would be active on the good market and where I have already assumed that interest rates are constant. Note that for sufficiently small h_j, say for $0 < h_j < h_t^{\min}$ the technology A_j achieved by research h_j is less efficient than the least efficient presently active incumbent technology: $A_j < A_{\theta(t)}$, where $\theta(t)$ is the vintage of the oldest active firm at t. An intensity h_j, with $0 < h_j < h_t^{\min}$ will never be chosen, since hiring researchers to come up with already outdated techniques is not profitable (for $h_j > 0$ and $T(h_j) = 0$ profits $\Gamma(h_j) < -w_{At}h_j$ are negative). Perfect competition on the goods markets leads to marginal product real wages $w_{Y\tau} = A_{\theta(\tau)}$, where $\theta(\tau)$ is the vintage of the oldest active firm at τ. Thus the research lab solves

$$\max_{h_j\geq h_t^{\min}} \int_t^{t+T(h_j)} [A_j - A_{\theta(\tau)}]e^{-r(\tau-t)}d\tau - w_{At}h_j.$$

The first order condition (marginal revenue from increasing h equals research wage) is

$$\int_t^{t+T(h_j)} \frac{dA_j}{dh}e^{-r(\tau-t)}d\tau + \frac{dT(h_j)}{dh}[A_j - A_{\theta(t+T(h_j))}]e^{-rT(h_j)} = w_{At}.$$

Appendix 9.1 shows that the first order condition is sufficient if all other steady state equilibrium conditions are satisfied. Since by definition of $\theta(\bullet)$ and $T(h_j)$, $\theta(t+T(h_j)) = t$, the second term is zero such that the condition shortens to

$$\frac{dA_j}{dh} \cdot \int_0^{T(h_j)} e^{-r\tau}d\tau = w_{At}.$$

The intuition for this condition is straightforward: The present value from augmenting future revenues (and profits) of the up-stream final output firm by one unit in each active period (that is $\int_0^{T(h_j)} e^{-r\tau}d\tau$) multiplied by the number of such additional final output units per period realized by one additional unit of research (that is dA_j/dh) must exactly offset the cost of this additional research.

At steady state with constant growth of labor efficiency at rate g, (1) becomes $A_j = f(h_j) \cdot A_t \int_{-\infty}^0 e^{-g\tau}d\tau = f(h_j) \cdot A_t(1/g)$ or, with $g = f(h)$,

$$A_j = \frac{f(h_{jt})}{f(h)} \cdot A_t$$

and

$$\frac{dA_j}{dh} = \frac{f'(h_j)}{f(h)} \cdot A_t.$$

Since at steady state the lifetime T_t is constant too (hence $T_\tau = T = \tau - \theta(\tau)$ for all τ) the marginal revenue condition becomes

$$\frac{f'(h_j)}{f(h)} \cdot \int_0^T e^{-r\tau} d\tau = \frac{w_{At}}{A_t}$$

Thus at (symmetric) steady state equilibrium with $h_j = h_t = h$, $g = f(h)$ for all j and t the marginal revenue condition for h is

$$\varepsilon(g) \cdot I(r, T) = \frac{w_{At}}{A_t} h \tag{8}$$

where as before $I(x, T) := \int_0^T e^{-xt} d\tau = \frac{1-e^{-xT}}{x}$ (with $\frac{1-e^{-0T}}{0} = T$) and where $\varepsilon(g) = \frac{hf'(h)}{f(h)}$ is the elasticity of the R&D production function with respect to h at $h = f^{-1}(g)$.

4.2 Free entry to research and zero profit for research-firms

As before the present value of future quasi profits expected by one innovator at t employing $h_j \geq h_t^{\min}$ workers in research (assuming constant interest rate r) is $\int_t^{t+T(h_j)} [A_j - A_{\theta(\tau)}] e^{-r(\tau - t)} d\tau - w_{At} h_j$ where $T(h_j) \geq 0$ is the length of time the firm plans to be active on the good market.

At equilibrium free entry to research requires these profits to be zero (which can also be interpreted as the condition that marginal revenues from increasing λ equal the cost of research of a single research firm):

$$\int_t^{t+T(h_t)} (A_t - A_{\theta(\tau)}) e^{-r(\tau - t)} d\tau = w_{At} h_t$$

or

$$\int_t^{t+T(h_t)} (1 - \frac{A_{\theta(\tau)}}{A_t}) e^{-r(\tau - t)} d\tau = \frac{w_{At}}{A_t} h_t$$

At steady state with constant lifetime (hence $T(h_t) = T = \tau - \theta(\tau)$ for all τ) and constant growth rate g of labor efficiency A_t the zero profit and again using the notation $I(x, T) := \int_0^T e^{-xt} d\tau = \frac{1-e^{-xT}}{x}$ (with $\frac{1-e^{-0T}}{0} = T$), the 'marginal revenue condition' from increasing λ becomes (see the Appendix for the straightforward calculation)

$$I(r, T) - e^{-gT} I(r - g, T) = \frac{w_{At}}{A_t} h \tag{9}$$

The present value of the final-good output generated by an individual research lab (this is $A_t I(r, T)$) reduced by the present value of future wage bills of the up-stream final good producer ($A_t e^{-gT} I(r - g, T)$) equals the wage bill of the research lab ($w_{At} h_t$).

Why do conditions (9) and (8) not depend directly on the size of the economy (L) nor on its growth rate (n)? As has been explained in the introduction, the relevant features are that neither the technology of the individual final-good producer nor the industrial wage $w_{Yt} = e^{-gT} A_t$ depend on L or n.

4.3 Equal marginal present values

Comparison of the free-entry condition (9) with the optimal-research condition (8) shows that the "marginal present value" from increasing λ must equal that from increasing h:

$$I(r,T) - e^{-gT} I(r-g,T) = I(r,T) \cdot \varepsilon_{fh}(g).$$

Thus, steady state equilibrium must satisfy

$$[1-\varepsilon(g)]\, e^{gT} - \frac{I(r-g,T)}{I(r,T)} = 0 \tag{10}$$

Lemma 1 *At $D=0$, the function $D(g,r,T,\varepsilon) := (1-\varepsilon)e^{gT} - \frac{I(r-g,T)}{I(r,T)}$ is strictly increasing in the first three variables and decreasing in ε.*

Proof. See Appendix 9.2. ■

Intuitively D rises with an increase of T because an increase of T raises the additional present value of increasing λ more than that of increasing h. The first is increased in two ways: Increasing T raises the profitable life-time and therefore the incentive to open a new research-lab and it reduces the future wage-costs because the active firm determining wages will be older when T is larger ($e^{-g\tau}$ reduced for $\tau \in (0,T)$) . The first effect also raises the additional present value of increasing h, however it does so to a weaker degree because the elasticity ε of f is always smaller than one. The second effect (the cost reducing effect) is irrelevant for the marginal present value of increasing h (see the first order condition leading to 8). Thus the (normalized) difference between the two present values is raised.

Similarly an increase of g raises the additional present value of increasing λ more than that of increasing h. In fact, an increase of g *raises* the marginal present value of increasing λ by the mentioned cost-reduction effect ($e^{-g\tau}$ reduced) while it reduces or (leaves unchanged) the marginal present present value of h since $\varepsilon'(g) \le 0$. Thus the (normalized) difference between the two marginal values is raised.

Lemma 2 ***Corollary 3*** *(Equal Marginal Present Values) For every $T>0$ the equilibrium condition $D(g, g+\rho, T, \varepsilon(g)) = 0$ has a unique solution*

$$g^{h\approx\lambda}(\underset{-}{T}|\underset{-}{\rho},\underset{+}{\alpha}). \tag{11}$$

The function is strictly decreasing in T with $\lim_{T\to 0} g^{h\approx\lambda}(T) = 1$ *and* $\lim_{T\to\infty} g^{h\approx\lambda}(T) = g^{\min} := \max\{0, [1-(1/\alpha\gamma)]^{\gamma}\}$.

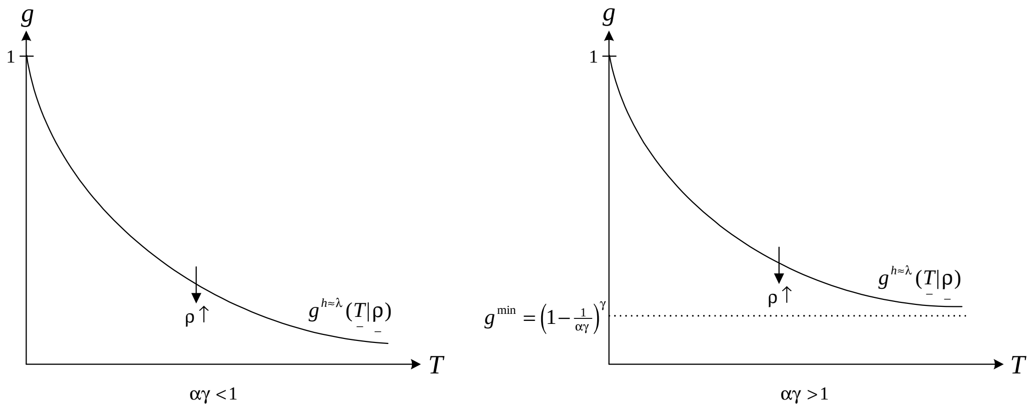


Figure 5: Equalizing Marginal Present Values of h and λ

5 Exogenous global R&D intensity

So far I have not specified how the quantities of resources used in the two sectors (final-good and knowledge-capital) are determined. The present section concentrates on the dimension of endogenous growth which is specific to the perfectly competitive model: The allocation of total research L_{At} among competing research labs, determining the number $\Lambda_t = \lambda L_t$ of competing research labs (and hence the extent of future final good competition) and the research intensity h of each research firm, determining the rate $f(h)$ of growth of new knowledge. To do so I first assume that there are two distinct types of labor. The first is used only in the final good sector, the second is used only in the research sector. The respective total supply shares, $l_Y = L_{Yt}/L_t$ and $l_A = L_{At}/L_t$, are exogenously fixed.

In Section 6 all workers will in principle capable to do both types of tasks, so that the allocation of the total number of workers $L_t = L_{Yt} + L_{At}$ among the two sectors (L_{Yt} and L_{At}) will be endogenous. This adds the standard dimension of the research allocation problem.

In both sections I show that a steady state equilibrium exists and is unique.

5.1 Existence and uniqueness

Collecting the steady state equilibrium conditions from the previous sections provides the following system of 6 equations with the 6 endogenous variables g, h, λ, T, w_{At}/A_t and r (while l_A, $l_Y = 1 - l_A$, and n are exogenously given):

$$g = f(h) \qquad \text{R\&D-function}$$

$$l_Y = \lambda \cdot I(n,T) \qquad \text{manufacturing employment}$$

$$l_A = \lambda \cdot h \qquad \text{R\&D employment}$$

$$\varepsilon(g) \cdot I(r,T) = \frac{w_{At}}{A_t} h \qquad \text{optimal } h$$

$$I(r,T) - e^{-gT} \cdot I(r-g,T) = \frac{w_{At}}{A_t} h \qquad \text{free entry to R\&D}$$

$$g = r - \rho \qquad \text{Euler equation}$$

Section 3 has reduced the first three equations to the falling function $g^{\text{Mecha}}(T)$ (Equation 6). The three last equations have lead to the increasing function $g^{h\approx\lambda}(T)$ in Section 4 (Equation 11). Since these two functions have a unique intersection we have shown:

Proposition 4 ***Existence and Uniqueness.*** *The economy has a unique steady state equilibrium with constant knowledge vintage lifetime T^* and strictly positive productivity and per-capita income growth rate $g^* = f(z \cdot I(n,T^*))$.*

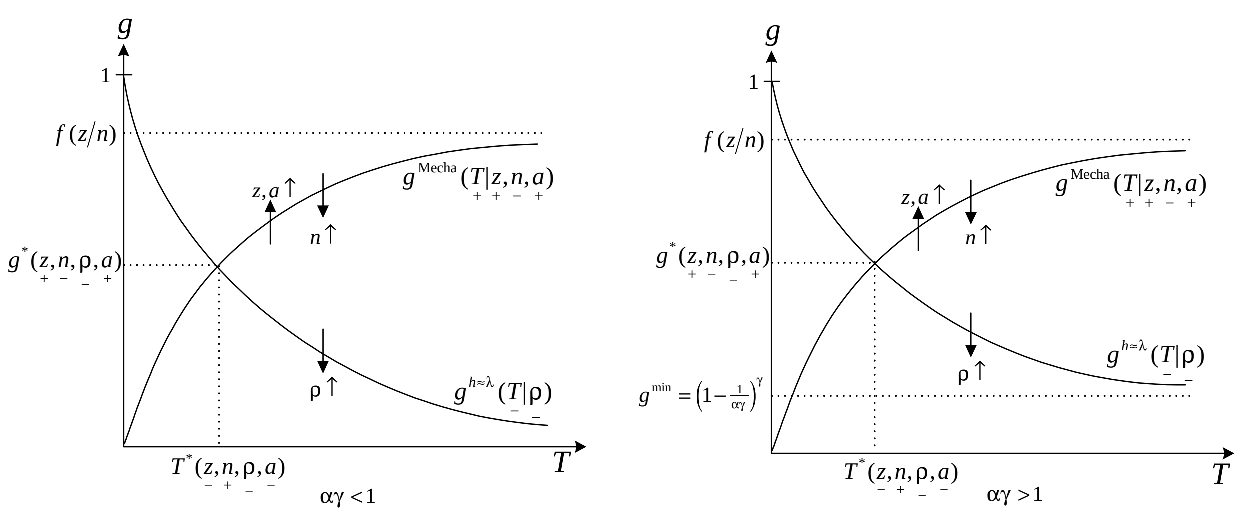


Figure 6: Steady-State REE with Exogenous Global R&D-Intensity

Of course, given g^* and T^* Section 3 determines unique steady state equilibrium values $h(g^*)$, $\lambda^* = l_A/h(g^*)$, $y_t/A_t = c_t/A_t = \lambda^* I(g^*+n,T^*)$, $w_{Yt}/A_t = e^{-g^*T^*}$ and $r = g^* + \rho$. The wages for researchers are given by condition (8): $\frac{w_{At}}{A_t} = \frac{I(r^*,T^*)\varepsilon(g^*)}{h(g^*)}$.

The following comparative static results follow directly from Equation 6 and Equation 11 in Corollary 3:

Proposition 5 *A rise of the of the research intensity $l_A = 1/[1 + (1/z)]$ ...*

- *... **raises** the steady state equilibrium growth rate g^*,*
- *... **reduces** the technology lifetime T^*,*
- *... raises the individual research intensity $h^* = f^{-1}(g^*)$, and*
- *... reduces the per-capita extent of entry $\lambda^* = l_A/h^*$*

A rise of the research efficiency a or a decline of the discount rate ρ ...

- *... **raises** the steady state equilibrium growth rate g^*,*
- *... **reduces** the technology lifetime T^*,*
- *... reduces the individual research intensity $h^* = zI(n, T^*)$, and*
- *... raises the per-capita extent of entry $\lambda^* = l_Y/I(n, T^*)$.*

A decline of the discount rate ρ ...

- *... **raises** the steady state equilibrium growth rate g^*,*
- *... **raises** the technology lifetime T^*,*
- *... raises the individual research intensity $h^* = f^{-1}(g^*)$, and*
- *... reduces the per-capita extent of entry $\lambda^* = l_A/h^*$*

As in the first generation growth models and in contrast to the 'semi-endogenous' growth models, a policy that succeeds in increasing the research intensity l_A has a positive growth effect. This always goes hand in hand with a reduction of technology lifetime.

Furthermore, in contrast to the 'generation 98' growth models, population size not only has no effect on per capita income growth rates. It also has no effect on the *level* of per capita steady state income. In this sense (and in contrast to these models) scale effects are absent in the present model:

Proposition 6 ***Population Size has no Growth-Effect and no Level-Effect.*** *In the absence of population growth a proportional variation of employment in the two sectors raises the extent of entry $\Lambda^* = L\lambda^*$ by the same proportion and has no effect on the growth rate g^* or on the per-labor-efficiency unit income level $(Y_t/A_tL_t)^*$. Similarly, for $n > 0$, a proportional increase of the initial population size (given z) has no effect on λ^*, g^* or $(Y_t/A_tL_t)^*$.*

The two R&D-equilibrium conditions do not depend on the size of the economy. As has been explained the reasons for this are first that the efficient scale size of the individual final-good producer does not depend on the size of the market and second that each individual final-good producer has to perform its own additional research to reach or improve upon the current knowledge frontier. $g^{\mathrm{Mech}}(T|z, n)$ does not depend on the size of the economy because the constancy of Λ_τ/L_t at steady state and the constant population growth rate make the employment equations of both sectors $\frac{L_{At}}{L_t} = \frac{\Lambda_t}{L_t} \cdot h$ and $\frac{L_{Yt}}{L_t} = \int_{t-T}^{t} \frac{\Lambda_t}{L_t} d\tau$ independent of L_t.

However population still has an effect both on per capita income growth rates and levels through its growth rate n:

Proposition 7 *Exogenous total research intensity:* ***Population Growth has negative Growth-Effect****. An increase of population growth n ...*

- ***reduces*** *the steady state equilibrium growth rate* g^**,*
- ***raises*** *the technology lifetime* T^**,*
- *raises the individual research intensity* $h^* = zI(n, T^*)$*, and*
- *reduces the per-capita extent of entry* $\lambda^* = l_Y / I(n, T^*)$

For reasons explained in Section 4 the R&D-equilibrium conditions do not depend on n. $g^{\mathrm{Mech}}(T|z,n)$ depends on n through the manufacturing employment equation $l_Y = \int_{t-T}^{t} \frac{\Lambda_\tau}{L_\tau}\frac{L_\tau}{L_t} d\tau = \lambda \int_{t-T}^{t} e^{-n(t-\tau)} d\tau$ $= \lambda \cdot I(n,T)$: An increase in n reduces the ratio of firms of vintage $\tau < t$ to the number of manufacturing workers at t. Therefore, given g, a larger number of vintages remains active to employ $l_Y = L_{Yt}/L_t$ workers per capita, the active lifetime of knowledge-capital increases. Increasing T for given g, i.e. increasing the active lifetime of knowledge-capital raises the incentives to do research. This allows more entry compatible with zero profits in the research sector: λ increases. Since $l_A = \lambda h$ is constant in the present section, h and hence g fall.

Note that the previous result hinges on the constancy of l_Y and l_A. As we shall see in Section 6, the effect of n on g will be neutralized by an endogenous adoption of $z = l_A/l_Y$, while the effect of n on levels remains.

These comparative statics are consistent with the negative empirical relation between growth and technology lifetime observed by Habakkuk [1962], Williamson [1971], Temin [1966] or Hsieh [2001] if one assumes that observed differences in (g, T) are due to differences of z, a or n:

Corollary 8 ***Negative relation between growth and technology lifetime****. A variation of any of the parameters* z*,* a*, and* n *(given the other parameters) induces a negative relation between* g *and* T*.*

Non-monotonic relation between growth and competition. $\mathrm{Comp}(g, T)$ depends negatively on both g and T. We have seen that g and T are affected in opposite directions by changes in any of the parameters z, a, or n. The net effect depends on which of the two effects dominates. In numerical examples the relation between g and Comp generated by variations of any of the parameters was either positive or non-monotonic.

5.2 Optimal steady state

Is the equilibrium blend of competition and growth optimal? Suppose a well-informed government could choose the per-capita number λ of research firms and their common size h.[10] Is there a steady state allocation of the given resources for research $l_A = \lambda h$ leading to higher overall utility compared to utility from the unique steady state equilibrium?

Final-good output and consumption is $C_t = Y_t = A_t L_Y \frac{1-e^{-\phi(g)}}{\phi(g)}$ at any steady state, with $\phi(g) = T \cdot g = (1/z) \cdot h(g) \cdot g$ and where I assume $n = 0$ for simplicity. The government therefore chooses $g \in [0, 1[$ to maximize

$$
\begin{aligned}
W &= \int_0^\infty e^{-\rho t} \ln C_t dt \\
&= \int_0^\infty e^{-\rho t} \ln \left[A_t L_Y \frac{1 - e^{-\phi(g)}}{\phi(g)} \right] dt \\
&= \int_0^\infty e^{-\rho t} \left[\ln A_0 L_Y + \ln e^{gt} + \ln \frac{1 - e^{-\phi(g)}}{\phi(g)} \right] dt \\
&= \ln A_0 L_Y \int_0^\infty e^{-\rho t} dt + \int_0^\infty e^{-\rho t} g t dt + \ln \frac{1 - e^{-\phi(g)}}{\phi(g)} \int_0^\infty e^{-\rho t} dt \\
&= \ln A_0 L_Y \frac{1}{\rho} + \frac{g}{\rho^2} + \ln \frac{1 - e^{-\phi(g)}}{\phi(g)} \frac{1}{\rho}
\end{aligned}
$$

Thus the government solves

$$
\max_{g \in [0,1[} \widetilde{W} = \frac{g}{\rho} + \ln \frac{1 - e^{-\phi(g)}}{\phi(g)}
$$

Increasing g has a positive *growth effect* on W, measured by the corresponding increase $\frac{\partial \widetilde{W}}{\partial g} = \frac{1}{\rho} > 0$. At the same time increasing g decreases the extent of entry λ, which has a negative *level effect* on W, measured by the corresponding decrease of $\ln \frac{1-e^{-\phi(g)}}{\phi(g)} = \ln Y_t / (A_t L_Y)$. Unsurprisingly, the growth effect on utility is the more important the weaker consumers' impatience ρ.

Proposition 9 *The planner's problem has a strictly positive solution $g^{opt} \in]0, 1[$. Depending on time preference, the planner's solution g^{opt} may be smaller or larger than the equilibrium growth rate g^*.*

Proof. See Appendix 9.4. ■

[10] Note that this government cannot overcome the barriers to knowledge of individual producers by simply publishing the latest knowledge, be it because making the knowledge public is not sufficient for its application, be it because knowledge is always embodied in some form of knowledge carrying rival capital (This is in particular satisfied if we adhere to the embodied knowledge interpretation of Boldine and Levine). Thus private final good producers still have to provide themselves with the knowledge-capital A_j needed to produce final output.

6 Endogenous global R&D intensity

So far the total number of researchers L_{At} or the research intensity $l_A = L_{At}/L_t$ were fixed exogenously. The question was how many research labs were opened with these researchers or, equivalently, how large each single research lab was. In this section total number of researchers L_{At} or the research intensity l_A are determined endogenously. It is now assumed that all workers are equally skilled to perform any task in the economy.

On the one hand this adds as endogenous variable the research intensity l_A (or the research ratio $z = l_A/(1-l_A)$ or $l_Y = 1 - l_A$) and also one additional equilibrium condition: At equilibrium, the wages in both sectors are identical $w_{At}/A_t = w_{Yt}/A_t = e^{-gT}$. The system of steady state equilibrium conditions (with the seven endogenous variables g, h, l_A, λ, T, w_{At}/A_t and r) now is:

$$
\begin{aligned}
g &= f(h) && \text{R\&D-function} \\
1 - l_A &= \lambda \cdot I(n, T) && \text{manufacturing employment} \\
l_A &= \lambda \cdot h && \text{R\&D employment} \\
\frac{w_{At}}{A_t} &= e^{-gT} && \text{(equal wages)} \\
\varepsilon(g) \cdot I(r, T) &= \frac{w_{At}}{A_t} h && \text{optimal } h \\
I(r, T) - e^{-gT} \cdot I(r - g, T) &= \frac{w_{At}}{A_t} h && \text{free entry to R\&D} \\
g &= r - \rho && \text{Euler equation}
\end{aligned}
$$

Inserting the wage equality condition $w_{At}/A_t = e^{-gT}$, the inverse R&D-function $h = h(g)$, and the Euler equation into the two R&D equilibrium conditions (8) and (9) yields two equations in two endogenous variables g and T alone:

$$
\begin{aligned}
I(g + \rho, T) \cdot e^{gT} &= h(g)/\varepsilon(g) && \text{optimal } h \\
I(g + \rho, T) \cdot e^{gT} &= I(\rho, T) + h(g) && \text{free entry to R\&D}
\end{aligned}
$$

or, equalizing the two right hand sides:

$$h = \frac{I(\rho, T)}{(1/\varepsilon) - 1}$$

Therefore

$$g\left(\underset{+}{T}, \underset{-}{\rho}, \underset{+}{a}, \underset{+}{\varepsilon}\right) := f\left(\frac{I(\rho, T)}{1/\varepsilon - 1}\right)$$

which, taking into account the negative dependence of ε on g, leads to the following proposition:

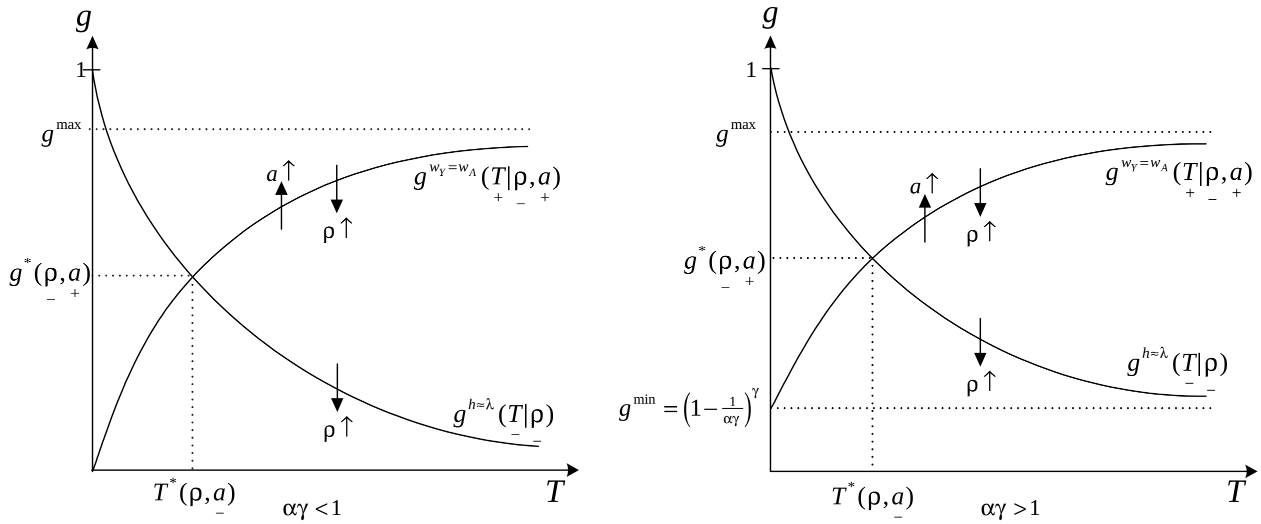


Figure 7: Steady-State REE with Endogenous Global R&D-Intensity

Lemma 10 *Endogenous Global Research Intensity: Equalizing wages defines a positive relation between growth and technology lifetime. For every $T > 0$ the equilibrium condition $w_Y = w_A$ has a unique solution*

$$g\left(\underset{+}{T}, \underset{-}{\rho}, \underset{+}{a}, \underset{+}{\varepsilon}(\underset{-}{g})\right) =: g^{w_Y=w_A}(\underset{+}{T}|\underset{-}{\rho}, \underset{+}{a}). \tag{12}$$

The function is strictly increasing in T with $\lim_{T\to 0} g^{w_Y=w_A}(T) = g^{min} := \max\{0, [1-(1/\alpha\gamma)]^{\gamma}\}$ and $\lim_{T\to\infty} g^{w_Y=w_A}(T) = g^{max} < 1$ defined by $h(g^{max})[1/\varepsilon(g^{max}) - 1] = 1/\rho$.

The positive relation between g and T induced by the global research allocation corresponds to the negative relation between competition and growth in more standard endogenous growth models (with exogenous intensity of competition ususally capture by an constant mark up of prices over marginal costs): As we have seen, an increase of T (given g) increases future profits (reduces the intensity of competition $\text{Comp}(g, T)$) which raises the incentives to enter and to increase h (note that the equality of the two marginal present values has also been used in the derivation of 12), thus raises g.

Together with the negative relation between g and T from Lemma 3 the positive relation of Lemma 10 immediately yields the following

Proposition 11 *The economy has a unique steady state equilibrium with constant knowledge vintage lifetime T^* and strictly positive productivity and per-capita income growth rate $g^* = g^{w_Y=w_A}(T^*)$.*

Of course Equation (6) for $g^{\text{Mecha}}(T) := f(z \cdot I(n, T))$ resulting from the steady-state employment equations $l_Y = \lambda \cdot I(n, T)$ and $l_A = \lambda \cdot h$ remains valid. However, the equation is no longer relevant for the determination of the steady state equilibrium level of g or T! Given (g^*, T^*) Equation (6) determines $z^* = h(g^*)/I(n,T^*)$, $l_A^* = 1/[1+1/z^*]$, $\lambda^* = l_A^*/h(g^*)$. Total research employment is $L_{At}^* = L_t/[1+1/z^*]$, total manufacturing employment is $L_{Yt}^* = L_t/[1+z^*]$, and the research-share in GDP is $\frac{w_{At}L_{At}}{Y_t+w_{At}L_{At}} = \frac{e^{-gT}L_{At}}{\lambda L_t I(g+n,T)+e^{-gT}L_{At}} = \frac{e^{-gT}(l_A/\lambda)}{I(g+n,T)+e^{-gT}(l_A/\lambda)} = \frac{e^{-gT}h(g)}{I(g+n,T)+e^{-gT}h(g)}$.

Proposition 12 *Endogenous total research:* ***Size has no growth and no level effect.*** *An increase of the total labor force L_0 leads to a proportional increase in the extent of entry without affecting per-capita entry, the research intensity l_A^*, per-capita income growth g^*, or the steady-state level of per labor efficiency unit income $(Y_t/A_tL_t)^*$.*

Proposition 13 *Endogenous total research:* ***Negative effect of population growth on per-capita income growth neutralized by positive effect on research intensity.*** *An increase of the population growth rate n ...*

- *... has no influence on g^* and T^*!*
- *... raises the research ration $z^* = \frac{f^{-1}(g^*)}{I(n,T^*)}$ and the global R&D-intensity $l_A^* = \frac{1}{1+1/z^*}$*
- *... raises the per-capita intensity of entry $\lambda^* = l_A^*/h(g^*)$*
- *... reduces per labor-efficiency unit income level $(Y_t/A_tL_t)^*$*
- *... raises the R&D-share in total income $\frac{e^{-g^*T^*}h(g^*)}{I(g^*+n,T^*)+e^{-g^*T^*}h(g)}$!*

The reason behind the absence of any growth effect even of n is that now also the research wage is determined by the marginal productivity of labor in manufacturing, which does not depend on the size of the economy or its growth rate. Furthermore, as before, the two R&D equilibrium conditions do not depend on n and L_0. As has been explained this hinges on the assumptions that allow to provide a sound non-cooperative foundation of perfect-competition in a world of endogenous technical change: Individual firms' technologies exhibit small efficient scales and each final-good producer has to perform his own research (see Section 4).

Remember that in the economy with exogenous global research ratio z, an increase of n reduces g. This is still relevant here, however, as we have just seen this effect is now neutralized by an endogenous adoption of the employment ratio z! Thus: The negative growth effect of n is absorbed by its positive effect on z^*! As a consequence also the R&D-share in total income is increased. In an environment of increasing global resources that are useful for research one my therefore observe a constant per-capita income growth rate despite an increasing R&D-share. This provides a solution to the "Jones-puzzel" which allows to retain the possibility of fully endogenous growth.

Proposition 14 ***Research efficiency has growth effects.*** *A rise of the research efficiency a* ***raises*** *the steady state equilibrium growth rate g^* and* **reduces** *the technology lifetime T^*.*

Thus as in Section 5 a variation of a induces **negative relation between growth and technology lifetime** (This is no longer as clear-cut as before for a variation of ρ, although in numerical examples it remains true for ρ as well). Furthermore the growth effect of a leaves room for growth enhancing policy:

Growth enhancing policy A growth enhancing policy is a subsidy on research wages finances by a lump-sum tax. Suppose the government pays a subsidy of $s < 1$ output units for each output unit a research firm pays to its workers: the wage paid by a research firm then is $w_{At} = (1-s)w_{At}^{\text{worker}}$, where w_{At}^{worker} is the wage received by the worker. Workers must be indifferent between the two sectors, so that $w_{At}^{\text{worker}} = w_{Yt}$ or $w_{At} = (1-s)w_{Yt}$. The relation $w_{Yt} = \frac{dY_t(L_{Yt})}{dL_Y} = A_t e^{-gT}$ is neither affected by the lump-sum tax nor by the research subsidy, so that $w_{At}/A_t = (1-s)e^{-gT}$. With the new interpretation of w_{At} as net wage paid by research firms condition (8) and (9) and thus condition (10) remain unchanged. With $w_{At}/A_t = (1-s)e^{-gT}$conditions (optimal h) and (free entry) must be rewritten to

$$\varepsilon(g) \cdot I(r,T) = e^{-gT}(1-s)h(g) \qquad \text{optimal } h$$

$$I(r,T) - e^{-gT} \cdot I(r-g,T) = e^{-gT}(1-s)h(g) \qquad \text{free entry to R\&D}$$

Already here one can see that reducing $(1-s)$ has the same effect as increasing the R&D-subsidy s has the same effect as increasing the research efficiency a. Since the Euler-equation $g = r - \rho$ too remains unaffected by the lump-sum tax, we have

Theorem 15 ***A subsidy on research wages****, financed by a lump-sum tax,* ***increases the steady state growth rate*** *in the same way as an increase of the research efficiency parameter a.*

Thus despite the absence of scale effects, the present model is a model of endogenous growth not only in the sense that growth occurs due to profit-seeking R&D (as is also the case in 'semi-endogenous' growth models), but also in the sense that the growth rate can be affected by public policy (in contrast to 'semi-endogenous' growth models).

The scale of final-good producers So far the size of an individual final-good firm of vintage τ was normalized to one in the sense, that it employs *one* worker to produce A_τ units of output. To make precise in what sense this was called a 'normalization', I now assume that an individual final-good firm produces φA_τ units of output with κ workers.

In the Appendix 9.5 it is shown that the steady state equilibrium values g^* and T^* do not depend on φ and – more interestingly– that g^* and T^* depend on κ in the same way as on $1/b$: An increase of

κ, i.e. the scale of individual final-good producers has the same effect on the steady state growth rate as a proportional reduction of $b = (1/a)^{1/\alpha}$, i.e. the number of workers needed to generate a given g. In other words, the effect of an increase of κ on the growth rate is neutralized by a proportional increase of b. The reason is that an increase of κ not only reduces a worker's productivity in final-good production but indirectly (that is through its effect on the final-good productivity) also that of a researcher.

The employment ratio now is $z^* = (l_A/l_Y)^* = h(g^*)/\kappa I(n, T^*)$ (since $L_{Yt} = \kappa\lambda L_t I(n, T)$ and $L_{At} = \lambda L_t h(g)$)). The research-share in GDP is $\frac{w_{At}L_{At}}{Y_t + w_{At}L_{At}} = \frac{(\varphi/\kappa)e^{-gT}l_A}{\varphi\lambda I(g+n,T)+(\varphi/\kappa)e^{-gT}l_A} = \frac{e^{-gT}h(g)}{\kappa I(g+n,T)+e^{-gT}h(g)}$ and the extent of entry is $\lambda^* = l_A^*/h(g^*) = 1/[(1 + 1/z^*)h(g^*)] = 1/[h(g^*) + \kappa T^*]$. Thus

Theorem 16 *An increase of the efficiency parameter φ of final-good technologies has no effect on the steady state equilibrium values g^* or T^*. A proportional increase of the scale κ of individual final-good producers has the same positive effect on g^* as a proportional increase of the research efficiency $1/b = a^{1/\alpha}$. An increase of L_0 leave unchanged g^*, T^*, the research-shares in total labor force and in GDP and the extent of entry λ^*. A proportional reduction of κ and b has the same effect except that it induces a proportional increase of extent of entry λ^*.*

The corresponding results hold in the model of Section 5 with exogenous $z = l_A/l_Y$. Here g^* and T^* are determined by $D(r, g, T, \varepsilon) = 0$ and $I(n, T) = (1/z)\,\kappa h(g)$.

7 Scale Effects in previous growth models

As has already been emphasized, the conclusions concerning the effect of population size and growth distinguish the present model from previous endogenous growth models:

Endogenous growth models with growth effect of size. The absence of any buffer between the growth rate and the total amount of research in the first generation of endogenous growth models (Romer [1990], Grossman and Helpman [1991a,b], Aghion and Howitt [1992]) is responsible for the unrealistic and much criticized growth effect of the sheer size of an economy: These models predict that a permanent increase of the supply of resources that can be used for research leads to a proportional permanent increase of per capita growth rate of output. This prediction is at odds with the time series evidence of industrialized countries in the second half of the 20th century in the most developed countries. For instance, the U.S. National Science Foundation (NSF 03-307, December 2002) reports that the U.S. real expenditures in constant 1996 dollars has risen from less than 50 billions in 1953 to more than 250 billions in 2002. Contrary to the prediction of standard endogenous growth models, the growth rate of real US per capita income has remained fairly constant.

Semi-Endogenous Growth models with level effect of size. The strong scale effect of the first generation of endogenous growth models has led some authors, notably Charles Jones in a series of influential articles, to dismiss the assumption that a constant amount of labor devoted to innovating activity can generate a constant growth rate (Jones [1995a,b,2002]], Kortum [1997], Segerstrom [1998]). This assumption is crucial for the possibility of perpetual growth at constant rates in the first generation growth models. In other words, in order to reconcile the observed increase of resources allocated to research with the observed constant growth rates, these articles eliminate the possibility of perpetually balanced growth. Growth is sustainable only in the presence of population growth. A permanent increase of the population growth rate raises the long-run growth rate of per-capita income! A related consequence is that it becomes harder to influence long-run growth rates. Even a permanent subsidy of R&D for instance no longer enhances long-run growth. To emphasize the difficulty to influence the growth rate, Jones labels this class of models 'semi-endogenous'. As has been noted, the present model gives an example of how to accommodate the increasing amount of research resources observed in recent decades within an endogenous growth model that allows for perpetual balanced growth without depending on always growing resources. In contrast to semi-endogenous growth models, in the present model an R&D subsidy (financed by a lump-sum tax) increases the steady state income growth rate.

Endogenous growth models with level effect of size. The effect of size on the steady state *growth rate* has previously been eliminated also in the two-dimensional endogenous growth models of Aghion and Howitt [1998, Chapter 12], Dinopoulos and Thompson [1998], Peretto [1998], Young [1998]. These 'generation 98' growth models add a second growth dimension to the first generation growth models: Resources spent on research can either be used to increase the horizontally differentiated variety of products or to increase the efficiency with which existing varieties are produced. All of these models are based on the horizontally differentiated commodity space à la Spence [1976] and Dixit and Stiglitz [1977]. Jones [1999] has already emphasized that each of these models involves scale effects: An increase in the size of an economy increases its steady state per-capita income levels. The only way to eliminate the effect of size on the growth rate in the Spence-Dixit-Stiglitz based endogenous growth models – apart from requiring a further knife-edge assumption (see Jones [1999] or Li [2002]) – automatically adds a positive level effect of size and a positive growth effect of population growth. These 'side effects' of eliminating the growth effect of size are absent in the present paper.

The Spence-Dixit-Stiglitz version of competition is not only responsible for scale effects. It is also at odds with some of the examples given to motivate the 'generation 98' models or the real phenomenon sometimes intended to be modelled. Young [1998] for instance "seeks to develop a model that incorporates Gilfillan's principle of equivalent innovations". The following quote from Gilfillan

[1935] is already used by Young [1998] to illustrate the concept of equivalent innovations:

> " .... [I]n contraception we find 18 radically different methods indicated in a recent book, without counting minor variations. ... In marine history we recall numerous kinds of sails, all for much the same result ... . [A] configuration of forces [can] call forth a number of independent solutions by different inventors about the same time, some identical and others unlike, even utterly unlike, yet filling the same need".

In other words, "equivalent innovations" lead to products that are perfect substitutes for their users! This corresponds to the version of competition assumed in the present paper. In contrast, in the Spence-Dixit-Stiglitz version of competition, each additional competitor adds a new variety, which lies at a *strictly positive identical distance to all other* varieties. In fact, Gilfillan's [1935] early and ingenious vision of endogenous growth is more closely matched by the competitive endogenous growth framework assumed in the present paper.

8 Scale effects: Inconclusive empirical evidence

What is the empirical evidence concerning the effect of population size and growth on per capita income levels and growth? Kremer [1993] considers 5 'continents' that developed for more than 10,000 years in complete isolation from each other and notes that the initial population sizes show exactly the same ranking as the per capita income growth rates during this period. This suggests that in the very long-run, population size has (had) a positive effect on the growth rate of knowledge and of per capita income. There is rather broad agreement about the absence of such a growth effect in more recent times (see above). The same cannot be said about the effect of population size on the *level* of per capita income and the effect of population growth on per capita income growth. A casual comparison of current population growth rates and income levels in different countries seems to reveal that the effects of population growth rates on per-capita income levels are negative. For example, compare countries like China or India (high population growth rates, low per capita income levels) with much less fertile and richer countries like Japan of the US. Such comparisons point towards a negative level effect of population growth as predicted by the present model. The empirical studies of Barro [1991], Mankiw et.al.[1992], or Backus et.al. [1992] support this casual observation.

However, national differences concerning the legal, social, and political environment not explicitly modeled by the above growth models may be responsible for this negative correlation. Several empirical studies try to control for such national differences (see Jones [2003] for a summary). For instance, Frankel and Romer [1999] and Alacala and Ciccone [2002] control for differences in international trade and find a positive effect of population size on per-capita income levels. In contrast, Sala-I-Martin's

[1997] extensive cross country study shows no systematic effect of size on long-run per capita income. In an econometrically refined version Sala-I-Martin, Doppelhofer, and Miller's [2004] find that out of 67 explanatory variables (including population size and growth rate) "18 are significantly and robustly partially correlated with long-term growth and another three are marginally related." Population sizes and the population growth rate are not significantly related to growth. Even the posterior means of the estimated coefficients *conditional* on inclusion of the population size is almost zero, while the conditional mean for population growth is slightly positive if only 7 or 9 explanatory variables are included but slightly negative when 11 or 16 variables are included! The conclusions from cross-section studies concerning the effect of population size on per-capita income levels thus remain ambiguous.

More importantly, if ideas, commodities, and factors (including scientists) are internationally mobile, it becomes unclear how these cross-country studies can test the predictions of any of the above theoretical models. The fact that large numbers of Chinese or Indian engineers and scientists work in US-research labs for instance indicates that the relevant pool of labor for R&D in the US is not confined to the US population.

Consider the following very stylized scenario, which pushes this idea to the extreme. There are two groups of countries N and S, which are initially (before date 0) completely isolated. Before date 0 the first group of countries, N, has a no population growth 0.5%, a constant research intensity of 1% (ratio of researchers to total labor) and constant per-capita income growth of $g_{\mathrm{N}} = 2\%$. The second group of countries, S, has before date 0 a constant population growth of 2% and constant per capita income. Due to unfavorable institutional conditions no research is undertaken in S before date 0. Date 0 is the day of globalization. From now on the world is integrated. National populations continue to grow at previous rates. Institutional conditions in N remain as good as before, while they may improve in S (this is not essential since researchers too are mobile). The relevant labor market for research firms in S and N now is the global economy.

What are the predictions of the diverse endogenous growth models about the post globalization steady state variables in this example? The first generation of endogenous growth models predicts exploding post globalization income growth rates. The semi-endogenous growth models as well as the 'generation 98' endogenous growth models predict that the new global growth rate g_{G} is *larger* than the past growth rate g_{N} of N. Instead, the competitive endogenous growth model predicts that the new steady state growth rate g_{G} of the global economy remains *unchanged* from the point of view of N ($g_{\mathrm{G}} = g_{\mathrm{N}} = 2\%$). Furthermore, the competitive model predicts that the new research intensity will be *larger* than 1% and that the *world-wide* average per-labor-efficiency unit income would be smaller than the northern pre-globalization level.

Admittedly this scenario has been arranged to bring out favorably the predictions of the present model as compared with recent stylized facts: Not only the *total* amount of resources spent on R&D and the total number of researchers world-wide and in particular in industrialized countries have risen in the past 50 years, but – to a lesser extent – also the *share* of R&D expenditures in total expenditures and the share of researchers in the total labor force. Thus per-capita income growth rates have remained more or less constant, despite an increasing research *share* in GDP and increasing research *intensity*.

Considering current empirical evidence, it seems unwarranted to exclusively bid on models in which continuous per capita income growth depends on continuous population growth or/and on models that are 'inextricably tied to scale effects'. This paper shows that from a theoretical point of view this specialization is not necessary: Endogenous growth without scale effects is possible and does not require population growth.

9 Appendix

9.1 Appendix. Optimal Individual Research Intensity

Necessary first order condition The research lab solves $\max_{h_j \geq h_t^{\min}} \int_t^{t+T(h_j)} [A_j - A_{\theta(\tau)}] e^{-r(\tau-t)} d\tau - w_{At} h_j$. The first derivative is

$$\begin{aligned} \Gamma'(h_j) &= \int_t^{t+T(h_j)} \frac{dA_j}{dh} e^{-r(\tau-t)} d\tau + \frac{dT(h_j)}{dh} [A_j - A_{\theta(t+T(h_j))}] e^{-r(\tau-t)} - w_{At} \\ &= \frac{dA_j}{dh} \int_t^{t+T(h_j)} e^{-r(\tau-t)} d\tau - w_{At}, \end{aligned}$$

since by definition of $\theta(\cdot)$ and $T(h_j)$ the term $A_j - A_{\theta(t+T(h_j))} = 0$ for *any* choice of A_j by j at t: Technology A_j ceases to be active (by definition at $t + T(h_j)$) exactly when the wages w_τ will be determined by A_j.

Sufficiency of the first order condition I show sufficiency of the first order condition already using the fact that at steady state equilibrium all (other) firms are determining the constant steady state variables h, g, and T. The lifespan $T(h_j)$ of firm j (deviating at t) is determined by the condition, that j becomes inactive when its technology A_j matches the least efficient non-deviating incumbents' technology, i.e. by the condition $A_j = A_{t+T(h_j)-T} = e^{g(T(h_j)-T)} A_t$. With $A_j/A_t = f(h_j)/f(h)$ this is $f(h_j) = g e^{g(T(h_j)-T)}$ or

$$T(h_j) = T + \frac{\ln(f(h_j)/g)}{g}.$$

Note that this condition also determines the $h_t^{\min} = f^{-1}(g e^{-gT})$. Furthermore j's lifetime is limited by the upper bound $\lim_{h_j \to \infty} T(h_j) = T + \frac{\ln(1/g)}{g}$. Given that all other firms choose their steady state variables firm j's expected profits are

$$\begin{aligned}
\Gamma(h_j) &= \int_t^{t+T(h_j)} [A_j - A_{\theta(\tau)}]e^{-r(\tau-t)}d\tau - w_{At}h_j \\
&= A_t \int_t^{t+T(h_j)} [\frac{f(h_j)}{g}e^{-r(\tau-t)} - e^{g[(\tau-t)-T]}e^{-r(\tau-t)}]d\tau - w_{At}h_j \\
&= A_t \frac{f(h_j)}{g} \int_0^{T+\frac{\ln(f(h_j)/g)}{g}} e^{-r\tau}d\tau - A_t e^{-gT} \int_0^{T+\frac{\ln(f(h_j)/g)}{g}} e^{-(r-g)\tau}d\tau - w_{At}h_j \\
&= A_t \frac{f(h_j)}{g} \frac{1-e^{-r\left[T+\frac{\ln(f(h_j)/g)}{g}\right]}}{r} - A_t e^{-gT} \frac{1-e^{-(r-g)\left[T+\frac{\ln(f(h_j)/g)}{g}\right]}}{r-g} - w_{At}h_j \\
&= A_t \left[\frac{f(h_j)}{g}\frac{1-e^{-rT}\left(f(h_j)/g\right)^{-r/g}}{r} - e^{-gT}\frac{1-e^{-(r-g)T}\left(f(h_j)/g\right)^{-\frac{r-g}{g}}}{r-g} - \frac{w_{At}}{A_t}h_j\right] \\
&= A_t \left[\frac{1}{rg}f(h_j) + \frac{g^{r/g}}{r(r-g)}e^{-rT}f(h_j)^{-\frac{r-g}{g}} - \frac{w_{At}}{A_t}h_j - \frac{1}{r-g}e^{-gT}\right] \\
&= A_t\left[c_1 f(h_j) + c_2 f(h_j)^{-c_3} - c_4 h_j - c_5\right],
\end{aligned}$$

where $c_1 = 1/rg, c_2 = \left[g^{r/g}/r(r-g)\right]e^{-rT} = \left[g^{r/g}/r\rho\right]e^{-rT}, c_3 = (r-g)/g = \rho/g, c_4 = w_{At}/A_t$ and $c_5 = [1/(r-g)]\,e^{-gT} = [1/\rho]\,e^{-gT}$ are strictly positive numbers, which are exogenously given for the individual research lab j. The research lab j thus chooses $h_j \geq h_t^{\min}$ to maximize $\widetilde{\Gamma}(h_j) := [c_1 f(h_j) + c_2 f(h_j)^{-c_3} - c_4 h_j - c_5]$. First consider the function $\Psi(h_j) := c_1 f(h_j) + c_2 f(h_j)^{-c_3}$ on the complete domain $h_j \geq 0$. The first derivative of Ψ is $\Psi'(h_j) := \left[c_1 - c_2 c_3 f(h_j)^{-c_3-1}\right] f'(h_j)$. The term $\left[c_1 - c_2 c_3 f(h_j)^{-(c_3+1)}\right]$ is strictly increasing from $-\infty$ (when $h_j \to 0$) to $c_1 > 0$ (when $h_j \to \infty$) and is zero for a unique h_j, say h_1. Thus $\Psi'(h_j) < 0$ for $h_j < h_1$ and $\Psi'(h_j) > 0$ for $h_j > h_1$ and with $\Psi'(h_1) = 0$ too, since $f'(h_j) > 0$. Furthermore $\lim_{h_j \to \theta} \Psi'(h_j) = 0$ since $\lim_{h_j \to \theta} f'(h_j) = 0$. The first derivative $\Psi'(h_j)$ has a unique maximum at some $h_j > h_1$, say h_2 with $\Psi''(h_2) = 0$. Since $c_4 > 0$ the

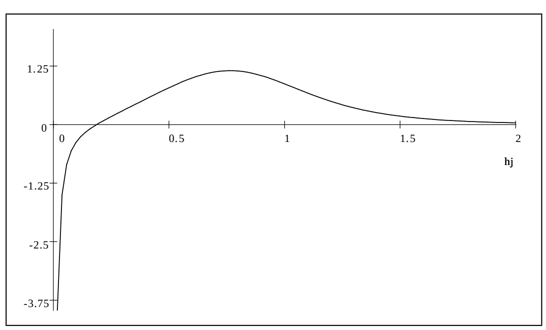


Figure 8: $\Psi'(h_j)$

function $\Psi'(h_j) - c_4$ therefore takes zero value exactly twice if $\Psi'(h_1) > c_4$, once if $\Psi'(h_1) = c_4$ and nowhere if $\Psi'(h_1) < c_4$. We can exclude the latter case because $\widetilde{\Gamma}'(h_j) = \Psi'(h_j) - c_4 = 0$ has at least one solution $h_j = h$ since by construction at steady state all innovators satisfy the first order condition. The case that $\Psi(h_j) - c_4 h_j$ has only one interior extremum is also excluded since $\widetilde{\Gamma}$ would be everywhere strictly decreasing except at h_1 which contradicts $\widetilde{\Gamma}(h_t^{\min}) = -(w_{At}/A_t)h_t^{\min} < 0 = \widetilde{\Gamma}(h_j = h)$ since $T > 0$, thus $h > h_t^{\min}$. Thus $\Psi(h_j) - c_4 h_j$ has two local extrema. The first (which lies between h_1 and h_2) must be a local minimum of $\Psi(h_j) - c_4 h_j$ and hence of $\widetilde{\Gamma}(h_j)$ and the second (which is larger than h_2) must be a local maximum. Because $\lim_{h_j \to \infty} f(h_j)^{-c_3} = \infty$ the same holds for $\Psi(h_j)$ and $\widetilde{\Gamma}(h_j)$. Furthermore $\lim_{h_j \to \infty} \Psi(h_j) = \lim_{h_j \to \infty} \widetilde{\Gamma}(h_j) = -\infty$ because $f(h_j)$ is bounded and $c_4 > 0$. Therefore $\widetilde{\Gamma}(h_j)$ is strictly positive for h_j small, first falls to attain a local minimum, than rises to reach a local maximum and finally falls with increasing h_j.

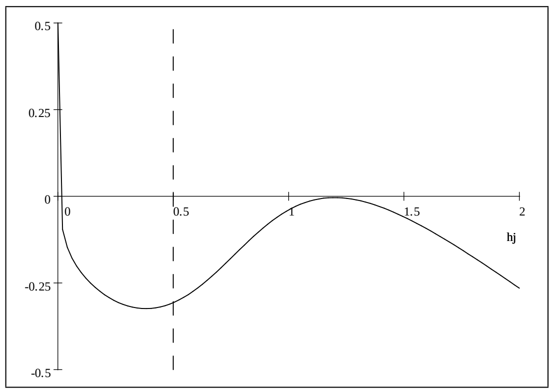


Figure 9: for $h_j \geq h^{\min} > 0$, $\widetilde{\Gamma}(h_j)$ is the individual innovator's profit function at steady state equilibrium

Remember that $\widetilde{\Gamma}(h_j)$ is not relevant for j's maximization for $h_j < h_t^{\min}$. We know that $\widetilde{\Gamma}(h_t^{\min}) = -(w_{At}/A_t)h_t^{\min} < 0$. Therefore at its local minimum $\widetilde{\Gamma}$ must be negative too. Furthermore we know that at steady state equilibrium $\Gamma(h_j) = \Gamma(h) = 0$ so that the first order condition used to determine the steady state must in fact correspond to the local maximum of $\widetilde{\Gamma}(h_j)$ which is a global maximum on the allowed domain $(h_t^{\min}, \infty)$.

9.2 Appendix. Free Entry

The free entree condition is

$$
\begin{aligned}
\int_t^{t+T} (1 - e^{g[\theta(\tau)-t]}) e^{-r(\tau-t)} d\tau &= \frac{w_{At}}{A_t} h \\
\int_t^{t+T} e^{-r(\tau-t)} d\tau - \int_t^{t+T} e^{g[\theta(\tau)-t]-r(\tau-t)} d\tau &= \frac{w_{At}}{A_t} h \\
\int_t^{t+T} e^{-r(\tau-t)} d\tau - \int_t^{t+T} e^{(g-r)(\tau-t)-gT} d\tau &= \frac{w_{At}}{A_t} h, \\
\int_t^{t+T} e^{-r(\tau-t)} d\tau - e^{-gT} \int_t^{t+T} e^{-(r-g)(\tau-t)} d\tau &= \frac{w_{At}}{A_t} h, \\
I(r,T) - e^{-gT} I(r-g,T) &= \frac{w_{At}}{A_t} h.
\end{aligned}
$$

9.3 Appendix.

Proof of Lemma 1

1. Both integrals $I(r-g,T)$ and $I(r,T)$ decrease with the interest rate r. However $I(r-g,T)$ decreases faster than $I(r,T)$, as the 'discount rate' $r-g$ is always smaller than the 'discount rate' r (for $g>0$). Therefore $-I(r-g,T)/I(r,T)$ and hence D are increasing in r, whenever $g>0$.
2. $\partial D(r,g,T,\varepsilon)/\partial g > 0$ at $D=0$ if $\left[(1-\varepsilon)e^{gT}\right] T > \left[\frac{1}{r-g} I(r-g,T) - \frac{T}{r-g} e^{-(r-g)T}\right] \frac{1}{I(r,T)} - \left[\frac{1}{r} I(r,T) - \frac{T}{r} e^{-rT}\right] \frac{I(r-g,T)}{[I(r,T)]^2}$ at $D=0$. Substituting $(1-\varepsilon)e^{gT}$ by $\frac{I(r-g,T)}{I(r,T)}$ and dividing both sides of the inequality by $\frac{I(r-g,T)}{I(r,T)}$ yields $T > \left[\frac{1}{r-g} - \frac{T}{r-g}\frac{e^{-(r-g)T}}{I(r-g,T)}\right] - \left[\frac{1}{r} - \frac{T}{r}\frac{e^{-rT}}{I(r,T)}\right]$, or $T > \left[\frac{1}{r-g} - \frac{Te^{-(r-g)T}}{1-e^{-(r-g)T}}\right] - \left[\frac{1}{r} - \frac{Te^{-rT}}{1-e^{-rT}}\right]$, or $T > \left[\frac{1}{r-g} - \frac{T}{e^{(r-g)T}-1}\right] - \left[\frac{1}{r} - \frac{T}{e^{rT}-1}\right]$. The second term in parentheses is always positive for $r>0$, since $T < \frac{e^{rT}-1}{r} = \int_0^T e^{r\tau} d\tau$. The first term in parentheses is negative if $g>r$, since then $T > \frac{e^{(r-g)T}-1}{r-g} = \int_0^T e^{(r-g)\tau} d\tau$. In this case the desired inequality follows. If $g<r$ I show that already $T > \frac{1}{r-g} - \frac{T}{e^{(r-g)T}-1}$, in which case the inequality is also shown. In fact $T(1+\frac{1}{e^{(r-g)T}-1}) > \frac{1}{r-g}$, or $T(\frac{e^{(r-g)T}}{e^{(r-g)T}-1}) > \frac{1}{r-g}$ or $T > \frac{1-e^{-(r-g)T}}{r-g} = I(r-g,T)$.
3. $\partial D(r,g,T,\varepsilon)/\partial T > 0$ at $D=0$ if $\left[(1-\varepsilon)e^{gT}\right] g > \left[\frac{e^{-(r-g)T}}{I(r,T)} - \frac{e^{-rT}}{I(r,T)}\frac{I(r-g,T)}{I(r,T)}\right]$ at $D=0$. Substituting $(1-\varepsilon)e^{gT}$ by $\frac{I(r-g,T)}{I(r,T)}$ and dividing both sides by $\frac{I(r-g,T)}{I(r,T)}$ yields $g > \left[\frac{e^{-(r-g)T}}{I(r-g,T)} - \frac{e^{-rT}}{I(r,T)}\right]$ or $g > \left[\frac{r-g}{e^{(r-g)T}-1} - \frac{r}{e^{rT}-1}\right]$. Multiplying both sides with $(e^{(r-g)T}-1)(e^{rT}-1)$, this becomes $ge^{(r-g)T}(e^{rT}-1) > (<) re^{(r-g)T}(e^{gT}-1)$ or $\frac{e^{rT}-1}{r} > (<) \frac{e^{gT}-1}{g}$ if $r-g > (<) 0$. This is equivalent to $\int_0^T e^{r\tau} d\tau > (<) \int_0^T e^{g\tau} d\tau$ if $r > (<) g$, which is always satisfied. ■

Proof of Lemma 10

First note that $D<0$ for $\varepsilon(g) = \gamma\alpha(1-g^{1/\gamma}) \geq 1$ $D(g, g+\rho, T, \varepsilon(g)) = [1-\varepsilon(g)]e^{gT} - \frac{I(\rho,T)}{I(\rho+g,T)} < 0$ for all T, so that $D=0$ has no solution for $g < g^{\min} := \max\{0, [1-(1/\alpha\gamma)]^\gamma\}$. For $\lim_{g \underset{>}{\to} g^{\min}} D = -\frac{I(\rho,T)}{I(\rho+g,T)} < -1$. For $\lim_{g\to 1} D = e^T - \frac{I(\rho,T)}{I(1+\rho,T)} > 0$ if $e^T I(1+\rho,T) - I(\rho,T) > 0$ or if $\frac{e^T - e^{-\rho T}}{1+\rho} - \frac{1-e^{-\rho T}}{\rho} > 0$ or if

$\rho e^T - \rho e^{-\rho T} - (1+\rho) + e^{-\rho T} + \rho e^{-\rho T} = \rho(e^T - 1) - (1 - e^{-\rho T}) > 0$ or if $\frac{e^T-1}{1} - \frac{1-e^{-\rho T}}{\rho} = \int_0^T (e^\tau - e^{-\rho\tau}) d\tau > 0$ which is always satisfied since $e^\tau > 1 > e^{-\rho\tau}$ for all $\tau > 0$. Thus $D = 0$ has a solution. ■

9.4 Appendix. Fixed Total Research.

Proof of Theorem 9: $\frac{d\widetilde{W}}{dg} = \frac{1}{\rho} + \frac{\frac{dI(\phi,1)}{dg}\phi'}{I(\phi,1)}$, where $\phi(g) = \frac{1}{z}h(g)g$, $h(g) = b\left(\frac{g^\eta}{1-g^\eta}\right)^\beta$, $\phi' = \frac{d\phi(g)}{dg} = \frac{1}{z}h(g)[1 + \varepsilon_{hg}(g)]$, $\varepsilon_{hg}(g) = 1/\varepsilon(g) = \frac{\eta\beta}{1-g^\eta}$, $I(\phi, 1) = \int_0^1 e^{-\phi\tau} d\tau$.

$\lim_{g\to 0} \varepsilon_{hg}(g) = \eta\beta$, $\lim_{g\to 0} h(g) = 0$, therefore $\lim_{g\to 0} \phi' = 0$. $\lim_{g\to 0} \phi(g) = 0$, $\lim_{g\to 0} I(\phi, 1) = 1$, $\lim_{g\to 0} \frac{dI(\phi,1)}{dg} < 0$. Hence $\lim_{g\to 0} \frac{d\widetilde{W}}{dg} = \frac{1}{\rho} > 0$. Furthermore $\lim_{g\to 0} \widetilde{W} = 0$, $\lim_{g\to 1} \frac{d\widetilde{W}}{dg} = -\infty$ and $\lim_{g\to 1} \widetilde{W} = -\infty$. Thus, $\widetilde{W}$ is positive and increasing for sufficiently small positive g and is strictly negative and decreasing for g sufficiently close to 1. Therefore $\widetilde{W}$ has a maximum $g^{\text{opt}} \in]0, 1[$. Note: $U(g)$ may have two local maxima.

Consider the numerical example $\eta = 1/11$, $\beta = 25$, $b = 1/2$, $z = L_A/L_B = 5\%$. The equilibrium growth rate is $g^* \simeq 2.2209\%$ if the time preference is $\rho = 2\%$ and is $g^* \simeq 2.027\%$, thus slightly smaller, if consumers are much more impatient, with $\rho = 4\%$ With $\rho = 2\%$ the (unique) optimal rate is larger than the equilibrium rate $g^{\text{opt}} \simeq 2.227 > g^*$, while it is smaller for $\rho = 2\%$: $g^{\text{opt}} \simeq 2 < g^*$. While g^{opt} and g^* decline with rising impatience ρ, the optimal rate is declines by more.

Similarly $\eta = 1/6$, $\beta = 38$, $b = 1/2$, $z = L_A/L_B = 5\%$. For $\rho = 1\%$ we have $g^{\text{opt}} \simeq 1.1615 > g^* \simeq 1.1565$ ($T^* \simeq 10.62$ years and $r^* \simeq 2.57\%$). For $\rho = 2\%$ we have $g^{\text{opt}} \simeq 1.1525 < g^* \simeq 1.1563$ ($T^* \simeq 10.04$ years and $r^* \simeq 3.5\%$). For $\rho = 4\%$ we have $g^{\text{opt}} \simeq 1.45 < g^* \simeq 1.156$ ($T^* \simeq 9.799$ years and $r^* \simeq 5.56\%$). ■

9.5 Appendix: Endogenous Research Intensity

Proof of Theorem 16. The present value of the rents accruing to an innovator now are $\int_t^{t+T(h_j)} [\varphi A_j - \kappa w_{Y\tau}] e^{-r(\tau-t)} d\tau - w_{At} h_j$. With $w_{Y\tau} = (\varphi/\kappa) A_{\theta(\tau)}$ this becomes $\int_t^{t+T(h_j)} \varphi[A_j - A_{\theta(\tau)}] e^{-r(\tau-t)} d\tau - w_{At} h_j$. Conditions (8) and (9) become $I(r, T) \cdot \varepsilon(g) = \frac{w_{At}}{A_t}\frac{h}{\varphi}$ and $I(r, T) - e^{-gT} I(r - g, T) = \frac{w_{At}}{A_t}\frac{h}{\varphi}$. Thus, the Condition $D(r, g, T, \varepsilon) := (1 - \varepsilon)e^{gT} - \frac{I(r-g,T)}{I(r,T)} = 0$ remains valid and does not depend on φ or κ. In the model with endogenous research intensity the second equilibrium condition becomes $T(r, g, b/\kappa, \beta, \eta) = \frac{-\ln\{1-(r-g)[h(g)/\kappa][1/\varepsilon(g)-1])\}}{r-g}$, where I have used $w_{At}/A_t = w_{Yt}/A_t = (\varphi/\kappa)e^{-gT}$ (hence $\frac{w_{At}}{A_t}\frac{h(g)}{\varphi} = \frac{h(g)}{\kappa}e^{-gT}$) and as before (8) and (9). Thus the steady state equilibrium values g^* and T^* do not depend on φ. Obviously a variation of φ has the same effect as one of A_0. Furthermore g^* and T^* depend on κ in the same way as on $1/b$. ■

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