No. 12

UNIVERSITY OF COLOGNE

WORKING PAPER SERIES IN ECONOMICS

DEBT NON-NEUTRALITY, POLICY INTERACTIONS, AND MACROECONOMIC STABILITY

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Debt Non-Neutrality, Policy Interactions, and Macroeconomic Stability[1]

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September 17, 2004

Abstract

We study the consequences of non-neutrality of government debt with respect to aggregate demand for short-run macroeconomic stability and for fiscal-monetary policy interactions in an environment where prices are sticky. Assuming either transaction services of government bonds or partial debt repayments, Ricardian equivalence fails because public debt has a negative impact on its total rate of return and thus on private savings. Equilibrium stability then requires real public debt to be stationary, which steers future expectations about prices and output, and rules out self-fulfilling expectations. Under aggressive anti-inflationary monetary policy regimes, macroeconomic fluctuations can then decrease with the share of tax financing. In particular, a balanced budget policy stabilizes the economy under cost-push shocks such that output and inflation variances can be lower than in a corresponding framework where debt is neutral.

JEL classification: E32, E63, E52.
Keywords: Government debt, fiscal and monetary policy rules, stabilization policy, equilibrium uniqueness.

[1] The authors would like to thank Matthew Canzoneri, Jagjit Chadha, Alex Cukiermann, Roger Farmer, Jordi Gali, Berthold Herrendorf, Robert Kollmann, and Massimiliano Marzo for useful suggestions. We further benefitted from comments by participants of the European Economic Association Meeting 2002 in Venice, the Annual Meeting of the German Economic Association 2002 in Innsbruck, the Workshop on "EMU Macroeconomic Institutions and Policies" 2003 in Milan, and the CEPR RTN Conference "Designing a Macroeconomic Policy Framework for Europe" 2004 in Barcelona. An ealier version of the paper circulated under the title "Fiscal-monetary policy interactions and macroeconomic stability".

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1 Introduction

This paper studies the role of public debt and fiscal-monetary policy interactions for macroeconomic stability in models where the stock of outstanding government bonds is non-neutral with respect to aggregate demand. Contemporary public policy debates in Europe show a large and growing concern about macroeconomic implications of government deficits. How seriously the issues related to government indebtedness are taken can be seen from recent discussions surrounding the 'Stability and Growth Pact', which intends to restrict fiscal deficits on EMU member countries in order to ensure the independence of the European Central Bank. At the same time, large parts of the recent academic literature on business cycle theory have concentrated their efforts on the design and analysis of monetary stabilization policies (see Woodford, 2003), while fiscal policies have received comparatively little attention. The majority of this reseach studies short-run stabilization policy in infinite horizon representative agent models with lump-sum taxation, as is true of most New Keynesian sticky price theories (e.g. Clarida et al., 1999). As a consequence, Ricardian equivalence holds in these models, and government debt policy has no bearing on aggregate demand and inflation (as long as the public sector respects its solvency constraint[1]), leaving monetary policy as solely responsible for macroeconomic stabilization.

Most recently, some studies have focused on the assumption that lump-sum taxes are unavailable, with the consequence that government debt matters indirectly for output and inflation determination.[2] In these models, public debt can have an aggregate supply in the short run when it is negatively related to tax distortions. In this paper, we take a different approach and consider cases where the current stock of government bonds *directly* affects the consumption and saving behavior of households, implying that output and inflation are not independent of debt policy in equilibrium. This setup is shown to imply that equilibrium determination and the model's responses to shocks crucially depend on the interaction of monetary and fiscal policies, since the former controls the price and the latter the supply of government bonds, through which both impact on aggregate demand. Under non-neutrality the evolution of public debt introduces history dependence of equilibrium sequences, which steers expectations about future prices and real activity, implying that high shares of tax finance and, in particular, balanced budget policies are – under anti-inflationary monetary policy regimes – recommendable for short-run macroeconomic stabilization.

Specifically, while infinite horizons, lump-sum taxation, and fiscal solvency are assumed throughout, aggregate demand effects of public debt are introduced through either

[1] For deviations from this principle, see e.g. Leeper (1991), Woodford (2001a), or Benhabib et al. (2001), who analyze so-called 'non-Ricardian' fiscal policy regimes. An alternative strategy that gives rise to debt effects is applied in Leith and Wren-Lewis (2000), where policy interactions originate in a Blanchard (1985)-type specification of finite horizons.

[2] See Benigno and Woodford (2003) Kollmann (2004), and Schmitt-Grohe and Uribe (2004) for the analysis of optimal monetary and fiscal policy in models with distortionary taxation and sticky prices.

of two specifications which both imply short-run non-neutrality of public debt. We isolate a mechanism by which the total rate of return on government bonds decreases with the real value of its outstanding stock. The first specification is due to Canzoneri and Diba (2004), who propose to resolve price level indeterminacy under interest rate policy by allowing public debt to provide transaction services. This property is modeled in this paper by assuming transaction costs to be decreasing and convex in both types of government liabilities, money and bonds. The total rate of return on bonds, therefore, consists of interest rate payments and of a real return from lowering transaction costs. As the marginal return from transaction services is decreasing in households' government bond holdings, real public debt is negatively related to its rate of return. Thus, an increase in public debt (ceteris paribus) induces substitution of consumption from the future to the present. This intertemporal substitution mechanism is the specific form in which government debt influences aggregate demand in this model. Alternatively, a second assumption, which establishes an equivalent link between current consumption and real debt, is that the fiscal authority repays only a fraction of debt obligations in each period. The aforementioned intertemporal substitution effect then also emerges when this fraction decreases with the current stock of public debt, which can be interpreted as an ad-hoc specification of a sovereign default probability.[3]

We embed these assumptions in a New Keynesian sticky price model, where the two alternative assumptions mentioned above lead to identical structures.[4] Fiscal and monetary policies are specified in form of simple feedback rules, which is convenient as it allows for a straightforward identification of their effects on macroeconomic stability. The fiscal authority is assumed to levy lump-sum taxes as a percentage of its expenditures, while the central bank sets the nominal interest rate contingent on current inflation.

The following results are derived. First, due to the link between aggregate demand and government debt, the existence of a stable steady state requires the equilibrium sequence of real government bonds to be stationary. This is guaranteed as long as there is a negative feedback from inflation to the real value of the stock of outstanding debt, which is the case when the nominal interest rate policy of the central bank is not too aggressively targeting inflation. Given this, any decline in real public debt tends to reduce aggregate demand (by one of the two mechanisms introduced above) and thus inflation, which in turn causes debt to recover. This also rules out the possibility of local equilibrium multiplicity that is known to characterize many models with nominal interest rate policy: Arbitrary expectations of higher inflation would raise the expectation of future decreases in the real value of government bonds, and thus of a higher rate of return, inducing current demand

[3] Arbitrage freeness then demands the interest rate on government bonds to exceed a risk-free interest rate by a (risk) premium. For a thorough analysis of sovereign default risk in a general equilibrium framework, see Uribe (2002).

[4] Similar relations also arise when households face convex adjustment costs for bond holdings, as assumed in "limited participation" models (see, e.g., Christiano and Eichenbaum, 1992), or in an overlapping generations framework (see Leith and Wren-Lewis, 2000). The latter approach, however, differs from our specification in that public debt is there also non-neutral in the long-run.

and prices to slump, such that inflation expectations cannot be self-fulfilling. Hence, if public debt matters for aggregate demand, monetary policy is relieved from the task of avoiding sunspot equilibria, which is in contrast to the results obtained in corresponding models with neutral debt (see, e.g., Woodford, 2003).

Second, we derive the impulse responses of the model's reactions to interest rate and tax shocks. The model behaves intuitively and in accordance with empirical evidence, in that an unexpected increase in the nominal interest rate and a temporary rise in taxes are contractionary, i.e. cause output and inflation to decline (see Christiano et al., 1999, Mountford and Uhlig, 2002). However, a high degree of debt financing can counteract the effect of an interest rate increase, and a very aggressive monetary policy can overturn the impact of higher taxes. The reason is that in both cases there are large increases in public debt which tend to raise output due to the intertemporal substitution effect.

Third, we analyze the contributions of monetary and fiscal policy to stabilizing fluctuations induced by cost-push shocks, which lead to an immediate rise in inflation, and a decline in output and public debt. A more aggressive interest rate setting reduces (raises) inflation (output) fluctuations, as would also be the case in a corresponding model with neutral debt. The new element here is, however, that a high degree of tax financing always reduces the inflation variance, while it may raise or reduce the output variance depending on the monetary stance. The reason is that a rise in inflation due to a cost-push shock tends to lower the real value of public debt, which reinforces via the intertemporal substitution channel the output contraction. When this channel is more pronounced, the inflation variance declines, while the output variance rises for moderate interest rate policies. This, however, changes under aggressive anti-inflationary monetary policy regimes, where higher shares of tax financing also reduce output fluctuations. We further find that with tight debt control, and in particular under a balanced budget regime, both the variance of output and of inflation can be lower in the present model than in a standard New Keynesian model, where, other things equal, public debt is neutral.

The reason why tight constraints on public debt contribute to macroeconomic stabilization under debt non-neutrality (as indicated by the first and the third result) is that the fluctuations of inflation are limited by the requirement that the equilibrium sequence of real government debt must be stationary. Thus, fiscal policy induces the equilibrium sequences of inflation and output to evolve in a history dependent way. This constitutes an evident analogy to the role of monetary policy in the debt neutral case, where the optimal commitment solution is known to minimize macroeconomic fluctuations by inducing history dependence (see Woodford, 2003).

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 analyzes the uniqueness and stability of the steady state, whereupon section 4 derives the model's behavior under shocks and analyzes output and inflation volatility under cost-push shocks. Section 5 concludes.

2 A sticky price model with non-neutral debt

In this section a model in which the stock of government bonds affects its total rate of return is presented. We separately provide two alternative modelling strategies to introduce this feedback mechanism into an otherwise conventional New Keynesian sticky-price model. First, government bonds are – analogously to money – an argument of a convex transaction cost function. Since the model becomes non-linear in public debt through this assumption, the households' savings/consumption decision is related to their holdings of government bonds. Second, the fiscal authority is assumed to repay only a fraction of its debt obligations, and the size of the fraction is assumed to be decreasing in the total amount of outstanding debt.

2.1 Version A: Transaction services of bonds

Throughout the paper, nominal variables are denoted by upper-case letters, while real variables are denoted by lower-case letters. A bar over a variable denotes a constant steady state value, and a caret operator marks a logarithmic deviation from steady state, $\widehat{x_t} = \log(x_t/\overline{x})$. There is a continuum of households indexed with $j \in [0, 1]$. Households have identical asset endowments and identical preferences. Household j maximizes the expected sum of a discounted stream of instantaneous utilities u :

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{jt}, l_{jt}\right), \tag{1}$$

where E_0 is the expectation operator conditional on the time 0 information set, and $\beta \in (0, 1)$ is the subjective discount factor. The instantaneous utility u is assumed to be increasing in consumption c, decreasing in working time l, strictly concave, twice continuously differentiable, and to satisfy the usual Inada conditions. For analytical simplicity, instantaneous utility u is further restricted to be separable in private consumption c and working time l : $u(c_{jt}, l_{jt}) = v(c_{jt}) - \mu(l_{jt})$.

At the beginning of period t household j is endowed with holdings of money M_{jt-1}, government bonds B_{jt-1}, which are carried over from the previous period, and a portfolio of state contingent claims on other households yielding a (random) payment Z_{jt}. Let $q_{t,t+1}$ denote the period t price of one unit of currency in a particular state of period $t+1$ normalized by the probability of occurrence of that state, conditional on the information available in period t. Then, the price of a random payoff Z_{jt+1} in period $t+1$ is given by $E_t[q_{t,t+1} Z_{jt+1}]$.

Purchases of the consumption good are assumed to be associated with real transaction costs. While it is commonly assumed that only money provides transaction services, here also holdings of government bonds reduce transaction costs. We view this assumption, which is, for example, also applied Bansal and Coleman (1996), Lahiri and Vegh (2003), and, particularly, in Canzoneri and Diba (2004) for a related purpose, as reasonable, since securities serve as collateral for many types of transactions. Our specification of transaction costs h further implies that private debt, which can be freely issued by the households, is different, as it does not provide transaction services. This assumption can for example be rationalized by the asset acquisition policy of many central banks, by which public debt but not private debt is eligible in open market operations (see Lacker, 1997). We assume that the goods market opens before the asset market, such that households rely on the beginning-of-period holdings of government liabilities to reduce transaction costs.[5]

Assumption 1 *The transaction cost function $h(c_{jt}, M_{jt-1}/P_t, B_{jt-1}/P_t)$ satisfies: i) h is non-negative, increasing in c, decreasing in M_{jt-1}/P_t and in B_{jt-1}/P_t, and twice continuously differentiable in all arguments, ii) $h_{cc} \geq 0$, $h_{mm} > 0$, $h_{bb} > 0$, $\lim_{m\to 0} h_m = -\infty$, $\lim_{b\to 0} h_b = -\infty$, and iii) $h_{cm} = h_{cb} \, (= h_{mb}) = 0$.*

Part *iii)* implies that the transaction cost function is separable in all arguments (as in Lahiri and Vegh, 2003). We further assume that transaction costs are private costs that are paid to a particular sector whose only function is to rebate its receipts immediately to the household sector through lump-sum transfers, such that transaction costs do not show up in the aggregate resource constraint. Both assumptions ensure that there is no direct (wealth) effect of money and bond holdings on consumption. Nonetheless, there is an effect of government bond holdings on consumption that operates exclusively through intertemporal substitution, which will be explained below.

In order to introduce supply side disturbances, we assume that households monopolistically supply differentiated labor services. Differentiated labor services l_j are transformed into aggregate labor input l_t, which can be employed for the production of the final good. The transformation is conducted via the aggregator $l_t^{1-1/\vartheta_t} = \int_0^1 l_{jt}^{1-1/\vartheta_t} dj$. The elasticity of substitution between differentiated labor services $\vartheta_t > 1$ varies exogenously over time. Cost minimization then leads to the following demand for differentiated labor services l_{jt},

$$l_{jt} = \left(\frac{w_{jt}}{w_t}\right)^{-\vartheta_t} l_t, \qquad \text{with} \qquad w_t^{1-\vartheta_t} = \int_0^1 w_{jt}^{1-\vartheta_t} dj, \tag{2}$$

where w_{jt} and w_t are the individual and the aggregate real wage rate, respectively. Household j faces a lump-sum tax $P_t\tau_t$ (where P is the aggregate price level), labor income $P_t w_{jt} l_{jt}$ and dividends $D_{j,it}$ from monopolistically competitive firms indexed by $i \in [0, 1]$. After the goods market is closed, the financial market opens where households can either invest in nominal bonds B_{jt} at the price $1/R_t$, in money holdings M_{jt}, or in nominal state contingent claims. Household j's flow budget constraint reads

$$\begin{aligned} &M_{jt} + B_{jt}/R_t + E_t\left[q_{t,t+1} Z_{jt+1}\right] + P_t c_{jt} + P_t h(c_{jt}, M_{jt-1}/P_t, B_{jt-1}/P_t) \\ &\leq P_t w_{jt} l_{jt} + Z_{jt} + B_{jt-1} + M_{jt-1} - P_t\tau_t + \int_0^1 D_{j,it} di. \end{aligned} \tag{3}$$

[5] Note that the partial derivative of h with respect to the real value of beginning-of-period t money (bond) holdings M_{jt-1}/P_t (B_{jt-1}/P_t) is denoted by h_m (h_b).

It maximizes (1) subject to (2), (3), and a borrowing constraint $\lim_{s\to\infty} E_t q_{t,t+s}(M_{jt+s} + B_{jt+s} + Z_{jt+1+s}) \geq 0$, for given initial values $M_{j(-1)} = M_{-1} > 0$, $B_{j(-1)} = B_{-1} > 0$, and $Z_{j0} = Z_0$. The first order conditions for the household's problem are given by

$$\lambda_{jt}(1 + h_c(c_{jt})) = \upsilon'(c_{jt}), \tag{4}$$

$$\xi_t^{-1} w_{jt} \lambda_{jt} = \mu'(l_{jt}), \tag{5}$$

$$E_t \left\{ \frac{\lambda_{jt+1}}{\lambda_{jt}} \left[\left(1 - h_b(b_{jt}\pi_{t+1}^{-1})\right) \frac{R_t}{\pi_{t+1}} \right] \right\} = 1/\beta, \tag{6}$$

$$\beta \left(\lambda_{jt+1}/\lambda_{jt}\right) \pi_{t+1}^{-1} = q_{t,t+1}, \tag{7}$$

and $\beta E_t \left[\lambda_{jt+1}\pi_{t+1}^{-1}\left(1 - h_m(m_{jt}\pi_{t+1}^{-1})\right)\right] = \lambda_{jt}$, where λ_{jt} is the Lagrange multiplier on the budget constraint, $\pi_t = P_t/P_{t-1}$ is the gross inflation rate, $m_{jt} \equiv M_{jt}/P_t$ and $b_{jt} \equiv B_{jt}/P_t$ are real cash and government bond holdings, respectively, and $\xi_t \equiv \vartheta_t/(\vartheta_t - 1)$ denotes the wage mark-up, which is assumed to follow an exogenous stochastic process (see below). Further, the transversality condition $\lim_{s\to\infty} E_t q_{t,t+s}(M_{jt+s} + B_{jt+s} + Z_{jt+1+s}) = 0$ is required to hold.

Equations (4) and (5) are first order conditions for consumption and labor supply. The central model element can be seen in equation (6), which is the first order condition for bond holdings. Here, the growth rate of the shadow price of wealth λ_{jt} is related to the expected total rate of return on government bonds (given in the square brackets), consisting of the real interest rate R_t/π_{t+1}, and the marginal benefit from transaction services $\left(1 - h_b(b_{jt}\pi_{t+1}^{-1})\right)$. By the assumption $h_{bb} > 0$ the latter is decreasing in the stock of real bonds. Thus, a higher stock of bonds reduces its rate of return, which triggers the same intertemporal reallocation of consumption as a real interest rate reduction, since it requires the growth rate $\lambda_{jt+1}/\lambda_{jt}$ to rise. This relation, which will be called the intertemporal substitution effect of government debt henceforth, is the basis of the relevance of debt for aggregate demand.

Equation (7) holds for each state in period $t+1$, and determines the price of one unit of currency for a particular state at time $t+1$ normalized by the conditional probability of occurrence of that state in units of currency in period t. Since all households face the same stochastic discount factor $q_{t,t+1}$, they can completely share consumption risks, i.e., their growth rates of $\lambda_{jt+1}/\lambda_{jt}$ are identical (see 7). Further, assuming identical initial asset endowments, it follows that shadow prices equalize across households $\lambda_{jt} = \lambda_t$. By (4) and (6), consumption and bond holdings are, therefore, also identical between households. We thus omit the indices j in what follows, except for the idiosyncratic working time l_{jt}, which is monopolistically supplied to firms at a wage rate w_{jt}. The interest rate on a risk-free portfolio between t and $t+1$ is given by $R_t^f \equiv 1/E_t q_{t,t+1}$. Arbitrage freeness then implies the risk-free interest rate R_t^f to exceed the interest rate on government bonds R_t, for strictly positive bond holdings (see 6). It should be noted that the central bank will be assumed to set R_t, such that R_t^f can be separately determined in equilibrium. This assumption contains a major departure of the present model from related studies, which regularly assume that the central bank sets the nominal rate of return on a one-period risk-free portfolio, where the stock of assets has no bearing on the rate of return (see Woodford, 2003). Here, in contrast, by setting the nominal rate on an asset that yields an additional non-pecuniary benefit through its ability to facilitate transactions, the central bank's policy interferes with the supply of public debt, since both jointly affect its total rate of return by (6).

The final consumption good is an aggregate of differentiated goods produced by monopolistically competitive firms indexed with $i \in [0,1]$. The CES aggregator of differentiated goods is defined as $y_t^{\frac{\epsilon-1}{\epsilon}} = \int_0^1 y_{it}^{\frac{\epsilon-1}{\epsilon}} di$, with $\epsilon > 1$, where y_t is the number of units of the final good, y_{it} the amount produced by firm i, and ϵ the constant elasticity of substitution between these differentiated goods. Let P_{it} and P_t denote the price of good i set by firm i and the price index for the final good. The demand for each differentiated good is $y_{it} = (P_{it}/P_t)^{-\epsilon} y_t$, with $P_t^{1-\epsilon} = \int_0^1 P_{it}^{1-\epsilon} di$. A firm i produces good y_i employing a technology which is linear in the labor input: $y_{it} = l_{it}$ (note that $l_t = \int_0^1 l_{it} di$). Hence, labor demand satisfies: $mc_{it} = w_t$, where mc denotes real marginal costs. Nominal stickiness is present in form of staggered price setting as developed by Calvo (1983). Each period firms may reset their prices with the probability $1-\phi$ independently of the time elapsed since the last price setting. The fraction $\phi \in [0,1)$ of firms are assumed to adjust their previous period's prices according to the simple rule $P_{it} = \overline{\pi} P_{it-1}$, where $\overline{\pi}$ denotes the average inflation rate. Firms are assumed to maximize their market value, which equals the expected sum of discounted dividends $E_t \sum_{s=0}^{\infty} q_{t,t+s} D_{it+s}$, where $D_{it} \equiv (P_{it} - P_t mc_{it}) y_{it}$ and we assumed that firms also have access to contingent claims. In each period a measure $1-\phi$ of randomly selected firms set new prices $\widetilde{P}_{it}$ as the solution to $\max_{\widetilde{P}_{it}} E_t \sum_{s=0}^{\infty} \phi^s q_{t,t+s} (\overline{\pi}^s \widetilde{P}_{it} y_{it+s} - P_{t+s} mc_{t+s} y_{it+s})$, s.t. $y_{it+s} = (\overline{\pi}^s \widetilde{P}_{it})^{-\epsilon} P_{t+s}^{\epsilon} y_{t+s}$. The first order condition for the optimal price setting of re-optimizing producers is given by

$$\widetilde{P}_{it} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{s=0}^{\infty} \phi^s \left[q_{t,t+s} y_{t+s} P_{t+s}^{\epsilon+1} \overline{\pi}^{-\epsilon s} mc_{t+s} \right]}{E_t \sum_{s=0}^{\infty} \phi^s \left[q_{t,t+s} y_{t+s} P_{t+s}^{\epsilon} \overline{\pi}^{(1-\epsilon)s} \right]}, \tag{8}$$

where we used $mc_{it} = mc_t$. The linear approximation of this first order condition and $P_t^{1-\epsilon} = \phi \left(\overline{\pi} P_{t-1} \right)^{1-\epsilon} + (1-\phi) \widetilde{P}_t^{1-\epsilon}$ at the steady state for a given initial price level $P_{-1} > 0$ is known to lead to $\widehat{\pi}_t = \chi \widehat{mc}_t + \beta E_t \widehat{\pi}_{t+1}$, with $\phi\chi = (1-\phi)(1-\beta\phi)$, while aggregate output is given by $y_t = (P_t^*/P_t)^{\epsilon} l_t$, where $(P_t^*)^{-\epsilon} = \int_0^1 P_{it}^{-\epsilon} di$ and thus $(P_t^*)^{-\epsilon} = \phi \left(\overline{\pi} P_{t-1}^* \right)^{-\epsilon} + (1-\phi) \widetilde{P}_t^{-\epsilon}$ (see Yun, 1996).

The public sector consists of the fiscal authority and the central bank. The fiscal authority issues risk-less one-period bonds B_t at the price $1/R_t$ paying B_t units of currency in period $t+1$, receives a transfer τ_t^c from the central bank, and collects lump-sum taxes τ_t from households,

$$B_{t-1} = B_t / R_t + P_t \tau_t + P_t \tau_t^c. \tag{9}$$

We abstract from any government expenditures on goods, such that the services on outstanding debt are the only flow that needs to be financed, either by issuing new debt or by raising taxes. To facilitate the isolation of debt effects, we assume that the fiscal authority sets taxes according to a simple feedback rule. In particular, we specify the level of taxes as a fraction of debt service costs $\frac{i_t}{1+i_t}B_{t-1}$ net of central bank transfers,

$$P_t\tau_t = \kappa_t \, \frac{i_t B_{t-1}}{1+i_t} - P_t\tau_t^c, \tag{10}$$

where $1 + i_t = R_t$. We further assume that κ_t satisfies $\kappa_t = \kappa \exp(\varepsilon_{\kappa t})$, where $\kappa \in (0, 1]$ and $\varepsilon_{\kappa t}$ is an i.i.d. random variable with mean zero. The stochastic feedback parameter κ_t thus specifies how actively the government seeks to collect funds from the private sector to finance its debt burden. Using (9) and (10), the evolution of government debt can be summarized by

$$B_t = (1 + (1 - \kappa_t)i_t)\, B_{t-1}. \tag{11}$$

Note that $\kappa_t = 1$ is the case of a budget that balances in every instant, such that nominal government bonds are constant over time: $B_{t-1} = B_t$. Thus, κ_t measures the share of government expenditures that are financed through taxation as opposed to debt. It is crucial to note that this specification of tax policy, in particular the property $\kappa > 0$, ensures fiscal solvency, $\lim_{s\to\infty} B_{t+s}\Pi_{v=1}^{s} R_{t+v}^{-1} = 0$. Thus, fiscal policy regimes considered in our analysis are not related to so-called non-Ricardian policy regimes, which are known to affect equilibrium determination under lump-sum taxes (see Benhabib et al., 2001).

The central bank transfers seigniorage to the fiscal authority, $P_t\tau_t^c = M_t - M_{t-1}$, and controls the nominal interest rate R_t on government bonds. To minimize the model's complexity, we assume that the central bank sets R_t in the most simple way contingent on current inflation, subject to a monetary policy shock,

$$R_t = R(\pi_t, \varepsilon_{rt}) = \mathcal{R}\pi_t^{\rho_\pi} \exp(\varepsilon_{rt}), \qquad \rho_\pi > 0,\ R_t \geq 1, \tag{12}$$

where ε_{rt} is assumed to be i.i.d. with mean zero. We further assume the support of all shocks to be small enough such that there exists a constant $\mathcal{R}$ that the central bank chooses to ensure that $R_t \geq 1$ holds for all t. Hence, the nominal interest rate is non-negatively related to the inflation rate through the elasticity ρ_π.

Since all households are identical, in the aggregate their asset holdings entirely consist of government liabilities and the indices for the labor market variables can be omitted. Further, using that transaction costs are private ($y_t = c_t$) and that money is irrelevant for the determination of the remaining variables, a competitive equilibrium of the model can be defined as follows.

Definition 1 *A rational expectations equilibrium is a set of sequences* $\{y_t,\ l_t,\ \pi_t,\ P_t^*,\ P_t,\ \widetilde{P}_t,\ mc_t,\ w_t,\ b_t,\ R_t\}_{t=0}^{\infty}$ *satisfying the firms' first order conditions* $mc_t = w_t$, *(8) with* $\widetilde{P}_{it} = \widetilde{P}_t$, *and* $P_t^{1-\epsilon} = \phi\,(\overline{\pi}P_{t-1})^{1-\epsilon} + (1-\phi)\,\widetilde{P}_t^{1-\epsilon}$, *the households' first order conditions*

$\mu'(l_t)\upsilon'(y_t)^{-1}(1+h_c(y_t)) = \xi_t^{-1}w_t$, *and*

$$\beta E_t\left[\left[\upsilon'(y_{t+1})(1+h_{c,t+1})\right]^{-1}\pi_{t+1}^{-1}\left(1-h_b(b_t\pi_{t+1}^{-1})\right)\right]R_t = \left[\upsilon'(y_t)(1+h_{c,t})\right]^{-1}, \qquad (13)$$

and $\pi_t = P_t/P_{t-1}$, *the aggregate resource constraint* $y_t = (P_t^*/P_t)^{\epsilon}l_t$, *where* $(P_t^*)^{-\epsilon} = \phi\left(\overline{\pi}P_{t-1}^*\right)^{-\epsilon} + (1-\phi)\widetilde{P}_t^{-\epsilon}$, *and the transversality condition, for fiscal and monetary policy satisfying* $b_t = (1+(1-\kappa_t)i_t)b_{t-1}\pi_t^{-1}$ *and (12), and given sequences of* $\{\varepsilon_{\kappa t}\}_{t=0}^{\infty}$, $\{\varepsilon_{rt}\}_{t=0}^{\infty}$, *and* $\{\xi_t\}_{t=0}^{\infty}$ *and initial values* $P_{-1} > 0$, $P_{-1}^* > 0$, *and* $b_{-1} \equiv B_{-1}/P_{-1} > 0$.

As implied by definition 1, the equilibrium sequence of public debt cannot separately be determined from the equilibrium sequences of the other variables, which leads to the failure of Ricardian equivalence. This property is due to the combined assumptions that (i) the quantity of debt affects the household's intertemporal consumption decision by $h_b < 0$, and that (ii) the central bank sets the price of public debt R_t. In contrast, real debt would be neutral and thus indetermined if either of these assumptions failed, in which case the equilibrium would be independent of fiscal policy (given that the latter is solvent).

2.2 Version B: Partial debt repayment

The non-neutrality of public debt can also be derived from a second approach where we abstain from assuming that bonds provide transaction services (i.e. set $h_b = 0$), but which leads to an equivalent structure through a different mechanism. Assume that the fiscal authority only partially repays its debt obligations. Specifically, only a fraction $1-\delta$ with $\delta \in (0,1]$ per unit of public debt is repaid in every period, where δ is an increasing function of the real value of the beginning of period (aggregate) stock of public debt $\delta'(B_{t-1}/P_t) > 0$. This assumption can either be interpreted as a deterministic debt repayment rule of the fiscal authority, which aims at alleviating the households' tax burden in states where public indebtedness is high, or as a probability of sovereign default which rises with the stock of public debt. The latter interpretation, which we view as more appealing, relates to a modelling strategy applied to induce stationarity of open economy models by introducing a domestic default probability increasing with foreign debt (see, e.g., Schmitt-Grohe and Uribe, 2001). It should, however, be noted that a default probability interpretation is not literally compatible with our specification of public policy, which ensures government solvency.[6] The flow budget constraint of household j is then given by

$$\begin{aligned} &M_{jt} + B_{jt}/R_t + E_t\left[q_{t,t+1}Z_{jt+1}\right] + P_tc_{jt} + P_th(c_{jt}, M_{jt-1}/P_t) \\ &\leq P_tw_{jt}l_{jt} + Z_{jt} + (1-\delta(B_{t-1}/P_t))\,B_{jt-1} + M_{jt-1} - P_t\tau_t + \int_0^1 D_{j,it}di. \end{aligned}$$

The first order conditions for government bonds changes to $\lambda_t = R_t\beta E_t(\lambda_{t+1}\frac{1-\delta(b_t/\pi_{t+1})}{\pi_{t+1}})$, where we used that $\lambda_{jt} = \lambda_t$. Further, the public sector budget constraint now reads $P_tg_t + M_{t-1} + (1-\delta)\,B_{t-1} = P_t\tau_t + B_t/R_t + M_t$. The fiscal authority rebates the savings

[6] For an endogenous derivation of a probability of sovereign default, see Uribe (2002).

from partial debt repayment in a lump-sum way, such that taxes satisfy $P_t\tau_t = \kappa_t(i_t\frac{B_{t-1}}{1+i_t}) - (P_t\tau_t^c + \delta B_{t-1})$. As a consequence, public debt again evolves according to (11). A rational expectations equilibrium is then characterized as in definition 1, except for equation (13), which is now replaced by

$$\beta E_t\left[\left[\upsilon'(y_{t+1})\left(1+h_{c,t+1}\right)\right]^{-1}\pi_{t+1}^{-1}\left(1-\delta(b_t/\pi_{t+1})\right)\right]R_t = \left[\upsilon'(y_t)\left(1+h_{c,t}\right)\right]^{-1}. \quad (14)$$

In contrast to the former version, the risk free interest rate R_t^f cannot exceed the nominal interest rate on government bonds R_t. However, the dynamic behavior of the equilibrium sequences in version A and B are qualitatively identical for $b_t > 0$, since $\delta' > 0$ in version B has the same consequences as $h_{bb} > 0$ in version A.

3 Steady state and stability

Before turning to the main task of discussing the implications of debt non-neutrality for macroeconomic stabilization below, we characterize the model's equilibrium given in definition 1 by discussing the conditions that lead to the existence, uniqueness, and local (saddle point) stability of its steady state.

3.1 Steady state

A deterministic steady state ($\varepsilon_{rt} = \varepsilon_{\kappa t} = 0$ and $\xi_t = \overline{\xi}$) of the model is characterized by constant values for output, inflation, and government bonds. The latter requirement is the distinguishing feature here, since under Ricardian equivalence the stock of bonds would not be constrained by the assumptions needed to guarantee the existence of a steady state.

The steady state of model A can be reduced to a set of conditions relating output, inflation, and real debt. Due to the assumption that transaction costs are private and separable, the first order conditions on consumption and labor, and the aggregate resource constraint uniquely determine steady state output by

$$\mu'(\overline{y})\upsilon'(\overline{y})^{-1}(1 + h_c(\overline{y})) = (\vartheta - 1)(\epsilon - 1)/(\vartheta\epsilon), \quad (15)$$

where bars indicate steady state values of endogenous variables.[7] A steady state further requires $b_t = \overline{b}$ and $\pi_t = \overline{\pi}$ (see 11 and 13). The fiscal and monetary policy specification leads to the restriction

$$\overline{\pi} = 1 + (1-\kappa)(\mathcal{R}\overline{\pi}^{\rho_\pi} - 1), \quad (16)$$

on the steady state inflation rate. Whether condition (16) has a unique or multiple solutions for the steady state inflation rate depends on both policy parameters. The equilibrium condition for bond holdings (13) for version A can be used to uniquely determine

[7] In contrast, steady state output is in general not independent of real wealth (debt) in a finite horizon set-up (see, e.g., Leith and Wren-Lewis, 2000), such that real public debt and, thus, inflation affect real activity also in the long run.

the steady state level of government bonds for a given steady state inflation rate,

$$h_b(\overline{b}/\overline{\pi}) = 1 - \overline{\pi}/\left[\beta \mathcal{R} \overline{\pi}^{\rho_\pi}\right]. \tag{17}$$

The steady state inflation rate and, thus, the steady state level of government bonds, is determined by (16). As policy satisfies $\kappa \in (0, 1]$ and $\overline{R} = \mathcal{R}\overline{\pi}^{\rho_\pi} \geq 1$, we know that $G(\overline{\pi}) \equiv (1 + (1-\kappa)(\mathcal{R}\overline{\pi}^{\rho_\pi} - 1)) - \overline{\pi}$ is strictly positive for $\overline{\pi} \to 0$. Hence, $G(\overline{\pi}) = 0$ has a unique solution if $G'(\overline{\pi}) < 0 \Leftrightarrow \rho_\pi < \left[\left(\mathcal{R}\overline{\pi}^{\rho_\pi - 1}\right)(1-\kappa)\right]^{-1}$. Using that (17) and $h_b < 0$ imply $\mathcal{R}\overline{\pi}^{\rho_\pi - 1} < 1/\beta$, a sufficient condition for the existence and uniqueness of a steady state inflation rate for version A is given by

$$\rho_\pi < \beta/(1-\kappa). \tag{18}$$

If (18) is satisfied, the model further exhibits a unique steady state level of government bonds. For version B, the condition for existence and uniqueness of the steady state is slightly different, as the risk-free interest rate cannot exceed the interest rate on government bonds. In particular, the necessary and sufficient condition for the existence and uniqueness of a steady state inflation rate for version B is given by $\rho_\pi < \frac{\beta(1-\overline{\delta})}{1-\kappa}$, which is more restrictive than (18).

The existence of a steady state relies on the two effects of inflation on public debt. On the one hand, the real value of nominal debt decreases with inflation. On the other hand, higher inflation induces the central bank to raise the nominal interest rate such that the fiscal authority might issue new debt to finance additional interest rate payments. If $G'(\overline{\pi}) < 0$, then there exists an inflation rate where both effects exactly offset each other, such that real public debt is constant. If, however, interest rate policy is too aggressive (high ρ_π) for a given fiscal stance κ, or equivalently if the share of tax financed fiscal expenditures is too small (low κ) for a given monetary stance ρ_π, then $G(\overline{\pi})$ can be increasing in inflation. For $\overline{b} > 0$, any rise in inflation, which causes the central bank to increase the nominal interest rate and, thus, raises interest payments on outstanding debt, is followed by an issuance of government bonds that leads to a rise in real public debt. Thus, the only value for real public debt which remains constant would be zero. This, however, is not a feasible equilibrium value, since assumptions on the transaction cost function leads to a strictly positive value of government bond holdings of households.

The properties of the steady state for version A are summarized in the following proposition.

Proposition 1 (Steady state) *Assume that fiscal and monetary policy satisfy (18). Then a steady state of the model described in definition 1 exists and is uniquely determined. It is characterized by i)* $\overline{y} > 0$, $\overline{\pi} \geq 1$, *and* $\overline{b} > 0$*; ii)* $\partial\overline{y}/\partial\kappa = 0, \partial\overline{\pi}/\partial\kappa < 0$, *and* $\partial\overline{b}/\partial\kappa \lesseqgtr 0 \Leftrightarrow \rho_\pi \gtreqless 1 - \Psi$, *where* $\Psi \equiv \frac{h_{bb}}{1-h_b}\frac{\overline{b}}{\overline{\pi}} > 0$*; and iii)* $\partial\overline{y}/\partial\rho_\pi = 0, \partial\overline{\pi}/\partial\rho_\pi > 0$, *and* $\partial\overline{b}/\partial\rho_\pi \gtreqless 0 \Leftrightarrow \rho_\pi \gtreqless 1 - (\Psi + \Upsilon)$, *where* $\Upsilon \equiv \frac{\overline{\pi}(1+\ln\overline{\pi})}{\partial\overline{\pi}/\partial\rho_\pi} > 0$.

Proof. The steady state condition (15) determines $\overline{y}$ independently of the policy parameters, such that $\partial\overline{y}/\partial\kappa = \partial\overline{y}/\partial\rho_\pi = 0$. Condition (16) implies that $\partial\overline{\pi}/\partial\kappa = (\overline{R}-1)/G'(\overline{\pi}) < 0$ and $\partial\overline{\pi}/\partial\rho_\pi = -[(1-\kappa)\overline{R}\ln\overline{\pi}]/G'(\overline{\pi}) > 0$, given that (18) ensures $G'(\overline{\pi}) < 0$. Condition (17) can then be used to derive the impact on $\overline{b}$. As $\partial\overline{b}/\partial\kappa = \left(\partial\overline{b}/\partial\overline{\pi}\right)\left(\partial\overline{\pi}/\partial\kappa\right)$ and $\left(\partial\overline{b}/\partial\overline{\pi}\right) = \overline{b}/\overline{\pi} - \overline{\pi}\left(1-\rho_\pi\right)/\left(\overline{h_{bb}}\beta\overline{R}\right) \gtreqless 0 \Leftrightarrow \frac{\overline{h_{bb}}}{1-\overline{h_b}}\frac{\overline{b}}{\overline{\pi}} \gtreqless (1-\rho_\pi)$, we can conclude that $\partial\overline{b}/\partial\kappa \gtreqless 0 \Leftrightarrow \rho_\pi \lesseqgtr 1-\Psi$. From (17), we obtain $\partial\overline{b}/\partial\rho_\pi = \overline{b}\overline{\pi}^{-1}\left(\partial\overline{\pi}/\partial\rho_\pi\right) + \overline{\pi}\left(\overline{h_{bb}}\beta\overline{R}\right)^{-1}\left[\overline{\pi}\left(1+\ln\overline{\pi}\right) - \left(1-\rho_\pi\right)\left(\partial\overline{\pi}/\partial\rho_\pi\right)\right] \gtreqless 0 \Leftrightarrow \rho_\pi \gtreqless 1-(\Psi+\Upsilon)$. ■

Output and (equivalently) consumption are not affected by monetary or fiscal policy measures in the steady state. Assuming that (18) is satisfied, steady state inflation unambiguously rises with the reactiveness of monetary policy and declines with a permanent rise in the fiscal policy parameter κ governing the proportion of tax financing. The effects on public debt are not unambiguous. A rise in κ leads to a decline in real public debt if and only if the inflation elasticity of the interest rate rule is sufficiently aggressive, $\rho_\pi > 1-\Psi$. The latter has a further (direct) impact on public debt via (17), making real public debt increase with ρ_π for $\rho_\pi > 1-(\Psi+\Upsilon)$. It should be noted that the results summarized in proposition 1 also apply for version B, where $\rho_\pi < \frac{\beta(1-\overline{\delta})}{1-\kappa}$ replaces condition (18) and the composite parameter Ψ is defined as $\Psi \equiv \frac{\overline{\delta'}}{1-\overline{\delta}}\frac{\overline{b}}{\overline{\pi}} > 0$.

Summing up, while the fiscal authority can reduce nominal debt by raising the share of tax financing (see 11), its influence on the real value of outstanding debt crucially relies on the reaction of inflation and thus on the stance of monetary policy. Note that in the balanced budget case $\kappa = 1$, from (16) steady state inflation is $\overline{\pi} = 1$.

3.2 Local dynamics

Next, we turn to the analysis of the local dynamics in the neighborhood of a steady state (assuming $\phi > 0$). The purpose is to find the conditions under which the rational expectations equilibrium path is locally unique and stable, such that the steady state is a saddle point. A crucial feature of the model is the relevance of real debt as a predetermined state variable, which evolves in a history dependent way. Since the evolution of this state variable is affected by the realizations of inflation, the requirement that all equilibrium sequences have to be stable imposes a restriction on current and future inflation – which would not appear in an equilibrium under debt neutrality. As a consequence, the model under debt non-neutrality appears to behave in a fundamentally different way compared to the latter, regardless of the magnitude of the partial effect of debt on output growth, which is measured by the elasticity Ψ. In particular, the history dependence steers future inflation expectations and leads to a condition for stability and uniqueness of local equilibria, i.e., for a saddle path configuration, that is not related to the Taylor-principle that characterizes otherwise comparable models with debt neutrality (see Woodford, 2001b).

Log-linearizing the equilibrium conditions given in definition 1 at the steady state, which is characterized in proposition 1, leads to the following set of approximate equilibrium conditions in $\widehat{y}_t$, $\widehat{\pi}_t$, $\widehat{b}_t$, and $\widehat{R}_t$:

$$\sigma\widehat{y}_t = \sigma E_t\widehat{y}_{t+1} - \widehat{R}_t + (1-\Psi)\, E_t\widehat{\pi}_{t+1} + \Psi\widehat{b}_t, \qquad \Psi > 0, \tag{19}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \omega\widehat{y}_t + \widehat{\varphi}_t, \tag{20}$$

$$\widehat{b}_t = \widehat{b}_{t-1} + \eta\widehat{R}_t - \widehat{\pi}_t - \varepsilon_{\kappa t}, \qquad \eta \in [0,1) \text{ and } \partial\eta/\partial\kappa < 0, \tag{21}$$

$$\widehat{R}_t = \rho_\pi\widehat{\pi}_t + \varepsilon_{rt}, \tag{22}$$

together with an exogenous stochastic process for the cost-push shock assumed to follow $\widehat{\varphi}_t \equiv \chi\widehat{\xi}_t = \rho_c\widehat{\varphi}_{t-1} + \varepsilon_{ct}$, with $\rho_c \in (0,1)$, where ε_{ct} is a white noise innovation, and $\sigma \equiv -\frac{\overline{u_{cc}c}}{\overline{u_c}} + \frac{\overline{h_{cc}c}}{1+\overline{h_c}} > 0$, $\omega \equiv \chi(\sigma + \frac{\overline{u_{ll}l}}{\overline{ul}}) > 0$, $\eta \equiv \frac{(1-\kappa)\overline{R}}{1+(1-\kappa)(\overline{R}-1)}$, given sequences for $\varepsilon_{\kappa t}$, ε_{rt}, and ε_{ct}. Recall that $\Psi \equiv \frac{\overline{h_{bb}}}{1-\overline{h_b}}\frac{\overline{b}}{\overline{\pi}} > 0$ in version A and $\Psi \equiv \frac{\overline{\delta'}}{1-\overline{\delta}}\frac{\overline{b}}{\overline{\pi}} > 0$ in version B. Equation (19) specifies the evolution of real aggregate demand as a function of the nominal interest rate and inflation. If debt were neutral, consumption growth would only depend on the real interest rate; crucially, this is different here as real debt $\widehat{b}_t$ enters the demand equation. Equation (21) is the law of motion of real debt, i.e. the log-linearized flow budget constraint of the composite government sector. Note that the composite parameter $\eta(\kappa)$ is strictly decreasing in κ. Finally, equation (22) gives the log-linearized nominal interest rate feedback rule of the central bank. The following proposition states the qualitative local dynamic properties of the model, for a small weight of debt in the aggregate demand constraint (19), $\Psi \leq 2$.

Proposition 2 (Local dynamics) *Suppose that there exists a steady state and that $\Psi \leq 2$. Then, the model's local approximation (19)-(22) has a unique equilibrium converging to the steady state if and only if $\rho_\pi < 1 + \frac{\kappa}{(1-\kappa)\overline{R}}$.*

Proof. The deterministic version of the model (19)-(22) can be summarized as

$$\begin{pmatrix} \widehat{y}_{t+1} \\ \widehat{\pi}_{t+1} \\ \widehat{b}_t \end{pmatrix} = \begin{pmatrix} -(\sigma\beta)^{-1}\omega(\Psi-1)+1 & \Xi & -\Psi/\sigma \\ -\frac{1}{\beta}\omega & \frac{1}{\beta} & 0 \\ 0 & \eta\rho_\pi - 1 & 1 \end{pmatrix} \begin{pmatrix} \widehat{y}_t \\ \widehat{\pi}_t \\ \widehat{b}_{t-1} \end{pmatrix} = A \begin{pmatrix} \widehat{y}_t \\ \widehat{\pi}_t \\ \widehat{b}_{t-1} \end{pmatrix},$$

where $\Xi \equiv \frac{1}{\sigma}\left(\rho_\pi - \Psi\left(\eta\rho_\pi - 1\right) + \left(\Psi - 1\right)/\beta\right)$. Since there is one predetermined state variable $(\widehat{b}_{t-1})$, while the other two variables can jump, a saddle path configuration obtains if the matrix A has exactly one eigenvalue with modulus smaller than one. The characteristic polynomial of A reads $H(X) = X^3 - (\sigma\beta)^{-1}(\sigma + \omega + 2\sigma\beta - \Psi\omega)X^2 - (\sigma\beta)^{-1}(\Psi\eta\omega\rho_\pi - \omega - \sigma\beta - \omega\rho_\pi - 2\sigma)X - (\sigma\beta)^{-1}(\sigma + \omega\rho_\pi)$. As the determinant of A is strictly larger than one, $\det(A) = -H(0) = (\sigma\beta)^{-1}(\sigma + \omega\rho_\pi) > 1$, A exhibits at least one unstable eigenvalue. Given that $H(1) = (\sigma\beta)^{-1}(1 - \eta\rho_\pi)\Psi\omega$, there is at least one stable (and positive) eigenvalue lying between zero and one if $1 - \eta\rho_\pi > 0$. As $H(-1) = \omega(\sigma\beta)^{-1}[\Psi(1+\eta\rho_\pi) - 2(1+\rho_\pi) - 4\frac{\sigma}{\omega}(1+\beta)]$, we know that $H(-1) < 0$ for $\Psi \leq 2$ and $\eta < 1$, and that the third eigenvalue is unstable. Hence, the model exhibits exactly one stable and positive eigenvalue if and only if $1 - \eta\rho_\pi > 0 \Leftrightarrow \rho_\pi < 1 + \frac{\kappa}{(1-\kappa)\overline{R}}$. ■

The condition for saddle path stability in proposition 2 departs fundamentally from the one known from the corresponding model with debt neutrality (e.g. Clarida et al., 1999), where the interest rate policy would have to be active to ensure sadde path stability. Here, stability requires that the central bank does not raise the nominal interest rate too much in response to inflation, and the precise meaning of what is too much depends on the fiscal parameter κ. Essentially, both policy authorities must ensure that the partial effect of inflation on future debt, $\partial\widehat{b}_t/\partial\widehat{\pi}_t = \eta\rho_\pi - 1$ (from equations 21 and 22) is negative, which in rearranged form gives the condition stated in the proposition.

To see why, recall that debt is positively related to demand. Hence, if debt were temporarily higher than in the steady state, this would tend to raise future inflation. To bring debt back to its steady state, higher inflation must reduce the real value of outstanding bonds. But policy interferes with this stabilization mechanism. If the central bank raises the nominal interest rate in response to higher inflation, this increases the burden of public debt service costs on the fiscal budget, since the government would have to finance additional interest payments on existing debt. If $\kappa < 1$, not all of this additional expenditure is financed through taxation, but a fraction is covered by issuance of new debt. Since these add to existing debt holdings, they trigger the intertemporal substitution effect of real debt: higher real debt reduces the marginal return from transaction services such that consumption is ceteris paribus shifted to the present. Thus, an aggressive interest rate policy (high ρ_π) can induce the economy to evolve on a divergent path, unless $\rho_\pi < 1 + \frac{\kappa}{(1-\kappa)\overline{R}} \Leftrightarrow \partial\widehat{b}_t/\partial\widehat{\pi}_t < 0$.

If this condition is fulfilled, indeterminacy cannot occur in this model, for arbitrary expectations of rising inflation then imply that future debt is reduced, which tends to lower demand and, therefore, prices, such that inflation expectations cannot be self-fulfilling. Some notable implications of this result are summarized in the following corollary.

Corollary 1 *(1) The model exhibits a unique and saddle point stable steady state if monetary and fiscal policy satisfy (18). (2) This is the case if fiscal policy runs a balanced budget in the sense $\kappa = 1 \Rightarrow B_t = B_{t-1}$, or if (3) monetary policy pegs the nominal interest rate at an arbitrary constant $R_t = \overline{R} > 1$.*

Part (1) states that the condition for uniqueness of the steady state, $\rho_\pi < \beta/(1-\kappa)$ from (18), is sufficient for the saddle path stability condition $\rho_\pi < 1 + \frac{\kappa}{(1-\kappa)\overline{R}}$ from proposition 2 to hold. Thus, there is local equilibrium determinacy if the steady state is unique. Parts (2) and (3) of the corollary state that both a balanced budget fiscal rule and a nominal interest rate peg are sufficient for a unique and saddle point stable steady state to prevail. In these cases, the policy interaction vanishes, and stability is ensured by one of the policy rules alone. If the interest rate is pegged, an inflation increase unambiguously reduces the real interest rate, such that debt emissions by the government decline, and the resulting demand slump stabilizes inflation. If the budget is balanced in nominal terms, it can be seen from (21) that $\eta = 0$ and thus the evolution of real government debt is driven by inflation alone. Hence, higher than average inflation automatically reduces the real value of debt, increases (by $h_{bb} < 0$) the return from transaction services, and induces postponement of consumption to the future such that inflation is kept down.

4 Macroeconomic fluctuations and public debt

In this section the impact of debt non-neutrality on the cyclical behavior of output and inflation under cost-push shocks and the role of policy interactions for macroeconomic stabilization are discussed. First, however, we briefly summarize the main effects of fiscal and monetary policy measures.

4.1 Shock responses

We derive the responses of core variables to cost-push shocks, $\widehat{\varphi}_t$, and to fiscal and monetary policy shocks, $\varepsilon_{\kappa t}$ and ε_{rt}. The policy shocks are studied to assess if the model behaves in way compatible with empirical evidence, while cost-push shocks are analyzed to pave the ground for the discussion of policy trade-offs and the performance of differently parameterized rules in the next section. To this end, we derive the state space representation of the log-linearly approximated model (19) to (22) in the endogenous variables $\widehat{b}_t$, $\widehat{\pi}_t$, and $\widehat{y}_t$, given the state variables $\widehat{b}_{t-1}$, ε_{rt}, $\varepsilon_{\kappa t}$, $\widehat{\varphi}_t$, which immediately delivers the impulse responses. Throughout this section it is assumed that condition (18) is satisfied, such that there is a unique and saddle point stable steady state and that the fundamental solution is the unique solution of the model. The model is solved applying the method of undetermined coefficients. Let $\delta_{yb} \equiv \partial\widehat{y}_t/\partial\widehat{b}_{t-1}$, $\delta_{yr} \equiv \partial\widehat{y}_t/\partial\widehat{\varepsilon}_{rt}$, $\delta_{y\kappa} \equiv \partial\widehat{y}_t/\partial\widehat{\varepsilon}_{\kappa t}$, and $\delta_{yc} \equiv \partial\widehat{y}_t/\partial\widehat{\varphi}_t$ be the solution coefficients describing the impact of the state variables on output (analogous definitions apply for the solution coefficients with respect to inflation and bonds). The following proposition summarizes the qualitative properties of the coefficients for small debt elasticities, $\Psi < 1$.

Proposition 3 (Impulse responses) *Suppose that $\Psi < 1$ and (18) are satisfied. Then*

1. $\partial\widehat{b}_t/\partial\varepsilon_{\kappa t} = \delta_{b\kappa} < 0$, $\partial\widehat{\pi}_t/\partial\varepsilon_{\kappa t} = \delta_{\pi\kappa} < 0$, and if $\rho_\pi < \widetilde{\rho_\pi} : \partial\widehat{y}_t/\partial\varepsilon_{\kappa t} = \delta_{y\kappa} < 0$,
2. $\partial\widehat{b}_t/\partial\varepsilon_{rt} = \delta_{br} > 0$, and if $\eta < \widetilde{\eta} : \partial\widehat{\pi}_t/\partial\varepsilon_{rt} = \delta_{\pi r} < 0$ and $\partial\widehat{y}_t/\partial\varepsilon_{rt} = \delta_{yr} < 0$,
3. $\partial\widehat{b}_t/\partial\widehat{\varphi}_t = \delta_{bc} < 0$, $\partial\widehat{\pi}_t/\partial\widehat{\varphi}_t = \delta_{\pi c} > 0$ and $\partial\widehat{y}_t/\partial\widehat{\varphi}_t = \delta_{yc} < 0$,

and $\partial\widehat{b}_t/\partial\widehat{b}_{t-1} = \delta_b \in (0,1)$, $\partial\widehat{\pi}_t/\partial\widehat{b}_{t-1} = \delta_{\pi b} > 0$, *and* $\partial\widehat{y}_t/\partial\widehat{b}_{t-1} = \delta_{yb} > 0$, *where* $\widetilde{\rho_\pi} \equiv (1-\Psi)/\beta > 0$ *and* $\widetilde{\eta} \equiv \left(\Psi + \rho_\pi + (1-\delta_b)\left[\frac{\sigma}{\omega}(1+\beta-\beta\delta_b) + 1 - \Psi\right]\right)^{-1} > 0$.

Proof. See appendix.

Proposition 3 shows that the model generates reasonable results: in response to a temporary rise in taxes ($\varepsilon_{\kappa t} > 0$), public debt and, by the intertemporal substitution effect of debt, also inflation declines (see part 1.). As the central bank reacts to the latter by lowering the nominal interest rate, the output response crucially depends on monetary policy reactiveness. If the central bank is not too aggressive, for which $\rho_\pi < \widetilde{\rho_\pi}$ is sufficient, the expansionary impact of the decline in the nominal interest rate is dominated by the contractionary effect of lower debt, such that output decreases.

A contractionary monetary policy shock, i.e. a positive innovation to the nominal interest rate rule ($\varepsilon_{rt} > 0$), raises the interest rate burden on outstanding bonds, which leads to a future rise in debt. The response of output and inflation is generally ambiguous (see part 2.). The reason is that if a sufficiently large portion of government expenditures is tax financed ($\eta < \widetilde{\eta}$), then inflation and output decline in response to a monetary contraction. Otherwise, with heavy deficit finance the implied large rise in public debt can cause an increase in inflation and output due the positive intertemporal substitution effect of debt on private consumption. Finally, part 3. of the proposition states that a cost-push shock leads to a decline in output and a rise in inflation, while the latter causes a reduction in real public debt.

Summing up, for monetary and fiscal policy feedback rules which do not feature extreme parameter values, in the sense that ρ_π is not too high and κ is not too low, the model's predictions about responses to interest rate and tax shocks qualitatively accord to the evidence, based on vector autoregressions, provided by Christiano et al. (1999) for federal funds rate shocks and by Mountford and Uhlig (2002) for tax cut shocks.

4.2 Output and inflation volatility

Having established that the model works in an intuitive and empirically plausible way, we want to assess the impact of public debt non-neutrality on the cyclical behavior of the model under cost-push shocks. These are chosen as the only driving force of dynamics since it is well known that exogenous cost changes imply a trade-off for monetary policy with respect to the stabilization of inflation and output. The focus here is on the modification that debt non-neutrality and fiscal policy brings about in this respect. Thus, for the rest of the section, the policy innovations are set equal to zero, $\varepsilon_{rt} = \varepsilon_{\kappa t} = 0$, such that the fiscal policy stance is constant $\kappa_t = \kappa$ and the interest rate feedback rule reduces to $\widehat{R}_t = \rho_\pi \widehat{\pi}_t$.

To facilitate comparisons with corresponding models where debt is neutral, we restrict our attention to the case where the effect of public debt on the intertemporal substitution of consumption is small. When debt is non-neutral, the fiscal policy stance is relevant for the cyclical properties of macroeconomic variables. Thus, the interaction between fiscal and monetary policies affects for the variances of output and inflation in our model, whereas only monetary policy is responsible for macroeconomic fluctuations when debt is neutral. The distinguishing feature of debt non-neutrality is that the evolution of public debt has to follow a stationary path, which imposes a restriction on feasible equilibrium sequences of inflation and consumption. This raises the question how macroeconomic fluctuations are altered due to the relevance of public debt, and if changes in the fiscal stance are – corresponding to changes in the monetary stance – associated with a trade-off with regard to the variances of output and inflation.

4.2.1 A flexible price example

Before we turn to this question, which will be answered by using numerical methods, we present a simplified example, for which one can easily examine how the relevance of public debt affects macroeconomic fluctuations. In order to facilitate the derivation of analytical results, we apply the flexible price version of the model that is sufficient to reveal the main principle. Thus, we assume that the probability of firms not receiving a price signal equals zero $\phi = 0$, such that the equilibrium conditions for the log-linearized version of the model are given by (19), (21), (22), and the static condition $\widehat{y}_t = -(\sigma + \overline{u_{ll}}\overline{l}/\overline{u_l})^{-1}\widehat{\xi}_t$. Public debt does, evidently, not affect the equilibrium behavior of output regardless whether debt is neutral or not. Comparing both versions, it turns out that inflation is less volatile under debt non-neutrality when the cost-push shock is not too strongly autocorrelated. The following proposition summarizes the particular condition for this result for the example of an interest rate peg.

Proposition 4 (Variances under $\phi = 0$) *Suppose that prices are flexible and that the central bank pegs the nominal interest rate $\rho_\pi = 0$. Then the inflation variance is smaller under debt non-neutrality if $\rho_c^{-1} > -\Psi + 1 + \sqrt{2}$.*

Proof. Under an interest rate peg and flexible prices, the model with $\Psi > 0$ can be reduced to the following system of equations in inflation and real public debt: $-\gamma\,(1-\rho)\,\widehat{\xi}_t = (1-\Psi)\,E_t\widehat{\pi}_{t+1} + \Psi\widehat{b}_t$ and $\widehat{b}_t = \widehat{b}_{t-1} - \widehat{\pi}_t$, where $\gamma = \sigma(\sigma + \overline{u_{ll}}\overline{l}/\overline{ul})^{-1} > 0$. Applying the method of undetermined coefficients for a generic solution form featuring real public debt as a state variable $\widehat{b}_t = \delta_b\widehat{b}_{t-1} + \delta_{be}\widehat{\xi}_t$ and $\widehat{\pi}_t = \delta_{\pi b}\widehat{b}_{t-1} + \delta_{\pi e}\widehat{\xi}_t$, leads to the following fundamental solution $\delta_b = 0$, $\delta_{\pi b} = 1$, $\delta_{be} = -\delta_{\pi e}$ and $\delta_{\pi e} = \gamma\,(1-\rho_c)\,/(1-\rho_c+\Psi\rho_c)$, and to an inflation variance satisfying $var_\pi = 2\delta_{\pi e}^2 var_\xi$. When debt is neutral ($\Psi = 0$) the solution for inflation reads $\delta_{\pi e} = -\gamma\,(1-\rho_c)\,/\rho_c$, and its variance is $var_\pi = \delta_{\pi e}^2 var_\xi$. Hence, inflation is more volatile in the latter case if $2\left[\gamma\,(1-\rho_c)\,/\,(1-\rho_c+\Psi\rho_c)\right]^2 < \left[(1-\rho_c)\,\rho_c^{-1}\gamma\right]^2 \Leftrightarrow \rho_c^{-1} > -\Psi + 1 + \sqrt{2}$. ■

This demonstrates that for the flexible price case with an interest peg, the relevance of public debt may cause inflation to exhibit a smaller variance. The reason for this is that a stable rational expectations equilibrium path must – under debt non-neutrality – be associated with a stationary sequence of real public debt, which requires a sequence of inflation that induces the latter (state) variable to evolve on a mean reverting path. To see this, consider, first, a cost increasing shock for the case where debt is neutral. This leads to an immediate decline in output, which has to be accompanied by a rise in the real rate of return from government bonds and, thus, by a decline in expected future inflation. Under debt non-neutrality this cannot be an equilibrium outcome, as a decline in inflation would tend to raise real public debt that, on the other hand, would tend to lower the total real marginal return from bond holdings and would therefore even amplify the increase in expected future inflation (see 19). Thus, stationarity requires a positive and mild inflation response, such that real public debt can decline in a way that is consistent with the expected rise in future inflation. Once the cost-push shock has died out, inflation has, by (19), to be lower than in the steady state, inducing the real value of public debt to converge back to its steady state value. Thus, debt non-neutrality induces a history dependence by which inflation depends on the real value of beginning-of-period debt. This is responsible for a reduction of the inflation variance if the autocorrelation of cost-push shocks is sufficiently small, $\rho_c^{-1} > -\Psi + 1 + \sqrt{2}$. Otherwise, the recovery of real debt and, thus, of inflation are temporally extended in a way that causes its variances to be higher than in the case where debt is neutral.

4.2.2 Numerical results

We return to the sticky-price case in this section and investigate the role of government debt for macroeconomic fluctuations by means of calculating variances for empirically plausible parameter values. The impact of different (fiscal and monetary) policy parameters on macroeconomic volatility is assessed by comparing the variances of output and inflation relative to the variance of their source, i.e., the cost-push shock process φ_t. Using the coefficients of the model's solution discussed in proposition 3 (see the proof to proposition 3 for the exact definitions), the variances of the model's endogenous variables output (var_y), inflation (var_π) and government debt (var_b) can easily be expressed in relation to the exogenous variance of the cost-push shock, $var_\varphi = (1-\rho_c^2)^{-1} var_{\varepsilon_c}$, where var_{ε_c} is the variance of the white noise innovation to φ_t. The resulting expressions are $var_b/var_\varphi = (1-\delta_b^2)^{-1}\delta_{bc}^2$, $var_\pi/var_\varphi = \left(\delta_{bc}^2\delta_{\pi b}^2\right) / \left(1-\delta_b^2\right) + \delta_{\pi c}^2$, and $var_y/var_\varphi = \left(\delta_{bc}^2\delta_{yb}^2\right) / \left(1-\delta_b^2\right) + \delta_{yc}^2$.

For convenience, we present the results in graphical form applying a set of deep parameters in accordance with values often found in the literature. In particular, we set preference parameters equal to $\sigma = \vartheta = 2$ and $\beta = 0.99$, the average (quarterly) gross nominal interest rate to $\overline{R} = 1.01$, the autocorrelation of cost-push shocks to $\rho_c = 0.9$, and the fraction of non-optimally price adjusting firms to $\phi = 0.8$, where the latter value accords to estimates in Galí and Gertler (1999). Since the empirical evidence on the interest rate and aggregate demand effects of debt is quantitatively inconclusive (see Engen and Hubbard, 2004), we set the transaction cost elasticity equal to $\Psi = 0.05$ for the benchmark specification. To assess the impact of this value on the variances of inflation and output, a sensitivity analysis with respect to its influence is presented in the last part of this section. These parameter values are then applied to compute the solution coefficients δ_b, δ_{bc}, $\delta_{\pi b}$, $\delta_{\pi c}$, δ_{yb}, and δ_{yc} for varying policy parameters ρ_π and κ.

Figure 1 displays the relative variance of inflation var_π/var_φ, and figure 2 the relative variance of output var_y/var_φ , each for various values of the fiscal feedback parameter κ and the inflation elasticity ρ_π of the nominal interest rate on government bonds. Evidently, the interest rate policy parameter ρ_π involves the usual policy trade-off when the model is driven by cost-push shocks, in that higher values of ρ_π lower the variance of inflation, but increase the variance of output. What is new here is the influence of the fiscal policy parameter κ: a higher value of κ, i.e. a higher share of tax financing, reduces changes in public debt and is generally associated with a lower inflation variance, while it has an

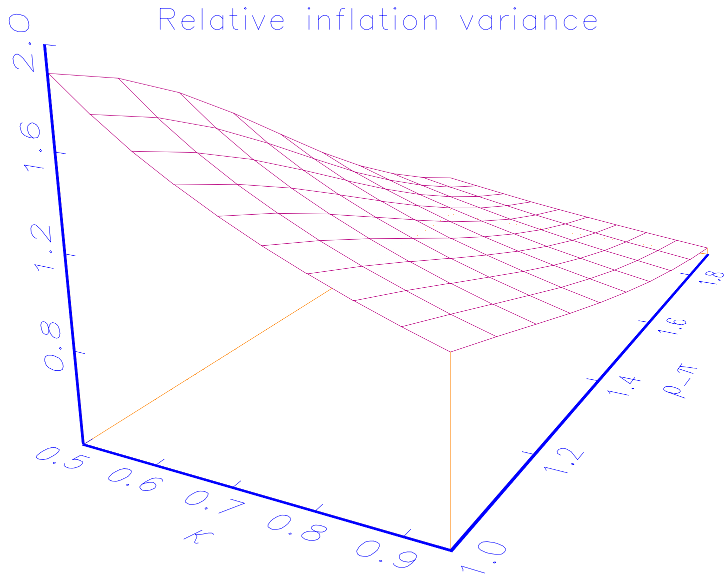

Figure 1: Relative inflation variance.

ambiguous (but generally small) influence on the output variance. The lowest inflation volatility is achieved with a balanced budget policy.

Before turning to explanations, it is useful to compare the performance of different stabilization policies to the case where debt is neutral. Therefore, figure 3 shows the relative output and inflation variances for selected monetary and varying fiscal policy parameters in comparison to the latter case, which is labelled DN (for *d*ebt *n*eutral).[8] Recall that the DN model, which can be summarized by (22), $\sigma\widehat{y}_t = \sigma E_t\widehat{y}_{t+1} - \rho_\pi \widehat{R}_t + E_t\widehat{\pi}_{t+1}$, and $\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \omega\widehat{y}_t + \widehat{\varphi}_t$, accords to the prototype New Keynesian model by Clarida et al. (1999). Given that public debt is irrelevant in this model, the fundamental solution exhibits no endogenous state variable and is characterized by the following coefficients on cost-push shocks: $\widetilde{\delta}_{yc} = -\frac{\rho_\pi - \rho_c}{\omega(\rho_\pi - \rho_c) + (1-\beta\rho_c)(1-\rho_c)\sigma}$ and $\widetilde{\delta}_{\pi c} = \frac{\sigma(1-\rho_c)}{\omega(\rho_\pi - \rho_c) + (1-\beta\rho_c)(1-\rho_c)\sigma}$. The relative output and inflation variances, which are given by $var_y/var_\varphi = \widetilde{\delta}^2_{yc}$ and $var_\pi/var_\varphi = \widetilde{\delta}^2_{\pi c}$, are displayed for $\rho_\pi = 1.5$ by the solid horizontal lines in figure 3.

Figure 3 further displays relative variances of the model with non-neutral debt for three different values of the monetary policy feedback parameter, $\rho_\pi \in \{1.25, 1.5, 1.75\}$, which ensure a commovement of the nominal and the real interest rate (only points where the parameter combination entail a saddle path stability are shown). Not surprisingly, higher ρ_π values reduce the inflation variance and raise the output variance; this effect

[8] In this case, $\Psi = 0$, steady state inflation is determined by $\overline{\pi} = \overline{R}\beta$ and is, thus, independent of fiscal policy.

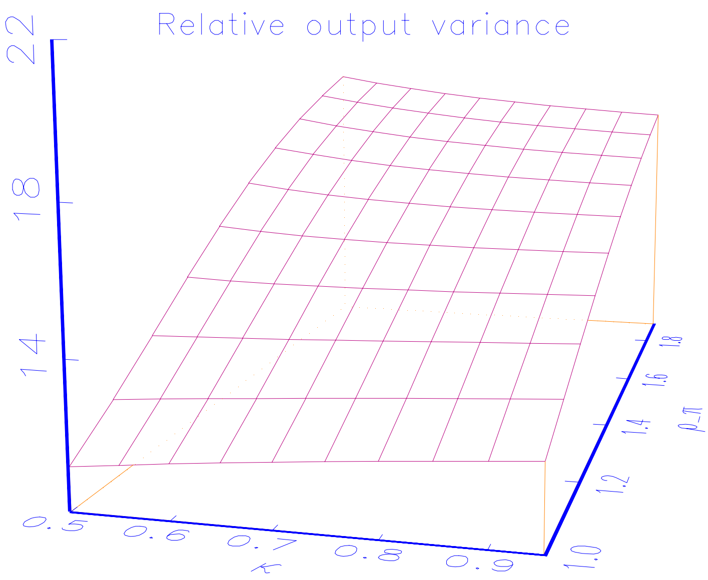

Figure 2: Relative output variance.

is already well known from the DN case. What is new here is seen by comparing the solid lines in figure 3. These show that for a given monetary policy stance, in this case for the example value $\rho_\pi = 1.5$ common to both solid lines, the variances of both output and inflation are lower for $\Psi > 0$ (solid line marked with squares) than in the DN case (solid line without squares) if the share of tax financing κ is sufficiently high. Thus, the relevance of government debt for demand determination appears to stabilize inflation and output fluctuations when tax policy contributes to a relatively smooth evolution of real debt (through a high κ value). The reason is that if a cost-push shock hits the economy, output declines while inflation rises. As has been shown above, the inflation increase reduces the real value of public debt (despite the positive partial effect from a higher real interest rate). The debt reduction exerts a negative intertemporal substitution effect on consumption, which tends to exacerbate the output contraction and to mitigate the rise in inflation caused by cost-push shocks. In equilibrium, the dampening effect on (future) inflation is strong enough to limit the real interest rate increase so much that, in the end, the output variance can be even lower than in the DN case.

This mechanism corresponds to the one outlined for the flexible price case in proposition 4 for an interest rate peg in the presence of price flexibility. As there, the central point here is that if government debt is relevant for the determination of the equilibrium values of inflation and output, the equilibrium response of inflation is constrained by the requirement that real debt must return to its steady state value subsequent to a shock. With an active monetary policy, this implies that during the adjustment process future

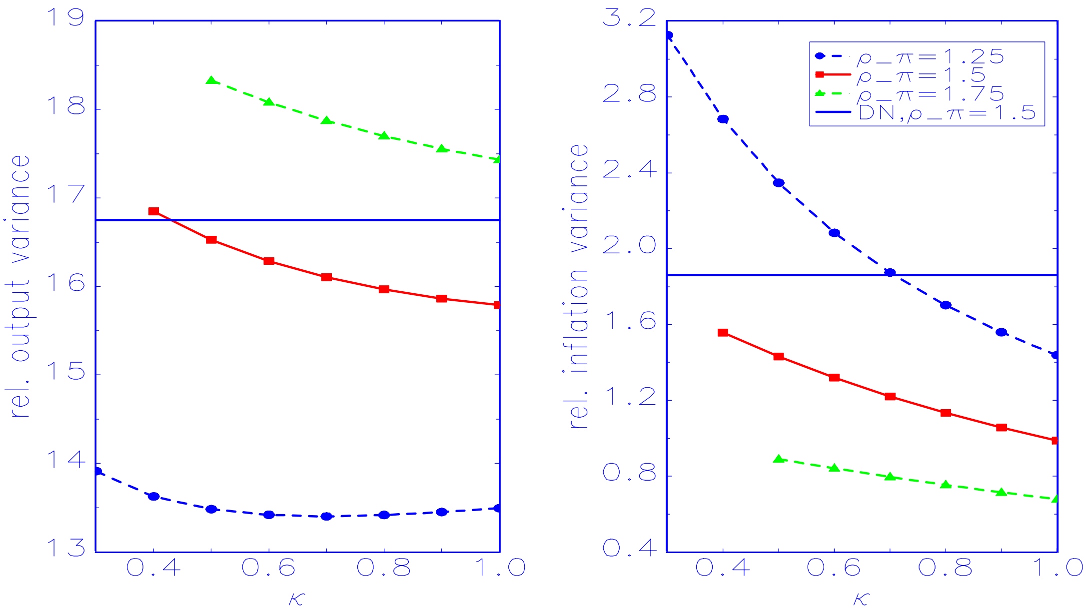

Figure 3: Relative inflation and output variances, comparison to debt neutral (DN) case.

real rates of interest must be lower than in steady state, which – with a large enough value for ρ_π – also reduces the impact of the shock on consumption and thus can mitigate output volatility. Thus, the history dependence of the equilibrium sequences introduced by fiscal policy can smooth fluctuations, which in the sticky-price case discussed here also holds for high shock autocorrelations. This result is related to the well-known principle for optimal monetary policy in the debt neutral case, where a monetary policy must be history dependent to implement the fully optimal allocation under commitment (see Woodford, 2003).

Figure 3 further shows that the inflation variance is always declining in the tax financing share κ, while the output variance is ambiguously linked to κ for a lower inflation elasticity of the interest rate rule, $\rho_\pi = 1.25$. In fact, the inflation variance reaches a minimum in the balanced budget case $\kappa = 1$. The reason is that with a nominally balanced budget the negative influence of inflation on the real value of debt is strongest, and the mechanism described above is maximal. The effects on the output variance are ambiguous, since debt reduction on the one hand reduces output partially, but the resulting inflation decrease makes room for lower real interest rates. Given an aggressive monetary policy (high ρ_π), however, there is no trade-off involved in fiscal policy: both the output and inflation variance decrease in κ and are minimized by a balanced budget policy ($\kappa = 1$).[9]

Finally, figure 4 shows the same numerical experiment for different values of the co-

[9]It should noted that the latter fiscal policy regime further minimizes the average distortion from the nominal rigidity as it implies the aggregate price level to be constant in the steady state, $\overline{\pi} = 1$.

efficient Ψ which parameterizes the aggregate demand effect of public debt, assuming a monetary policy coefficient of $\rho_\pi = 1.5$. As Figure 4 reveals, the attenuation of the inflation variance attributable to the non-neutrality of debt is stronger when the intertemporal substitution effect of debt is quantitatively more pronounced, i.e. with higher Ψ. The effect on the output variance is ambiguous. When the share of tax financing is small, the output variance can even rise with the elasticity as demonstrated for $\kappa = 0.4$. Provided that the government finances a sufficiently large share of its outlays through taxation, the output and inflation variances are lower than in the corresponding debt neutral case.

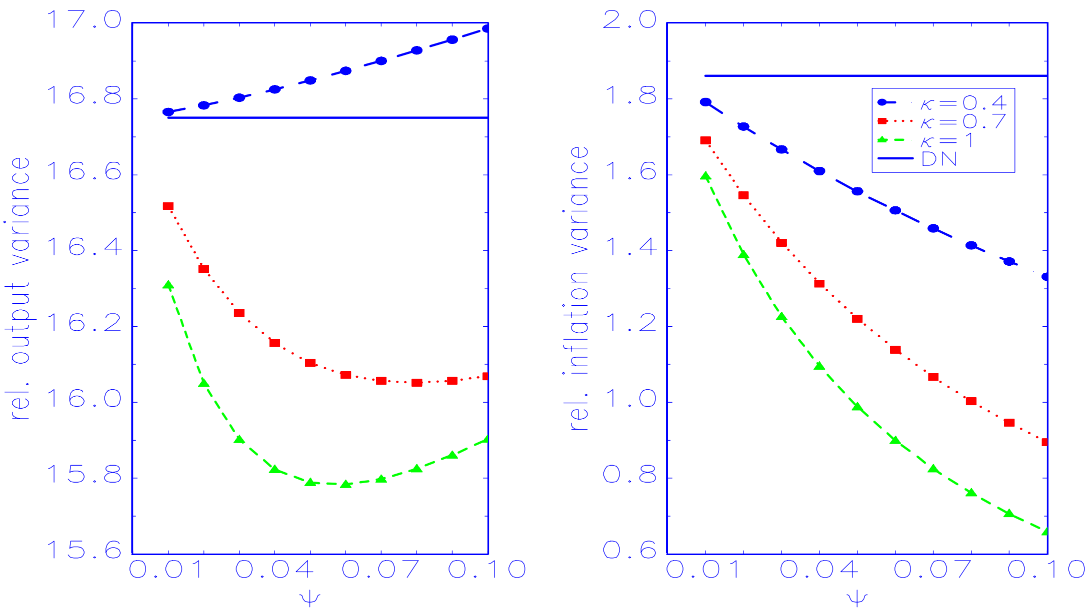

Figure 4: Influence of Ψ on relative output and inflation variances ($\rho_\pi = 1.5$).

5 Conclusion

This paper has explored the consequences of public debt non-neutrality for the short-run dynamics of a sticky-price business cycle model, and examined the interaction of fiscal and monetary policy. Government debt matters for aggregate demand determination through assumptions that imply the (negative) dependence of the rate of return on government bonds on the real value of their outstanding stock. This is the case, e.g., if government bonds yield transactions services, or if higher debt leads to lower repayment ratios. In both cases, a rise in public debt leads to a decline in the total rate of return, exerting an expansionary intertemporal substitution effect on (consumption) demand, thus implying a tendency for rising inflation. There is fiscal-monetary policy interaction, in that the central bank's interest rate reaction to changes in inflation influences the amount of payments on existing debt that the government has to finance. The composition of government finance among taxation and debt issuance in turn feeds back on the equilibrium values of output and inflation.

It is shown that even small effects of public debt on consumption growth can lead to results which substantially depart from those known for comparable business cycle models with neutral debt. Non-neutrality of debt implies that the equilibrium sequence of real bonds must be stationary, which constrains the admissible equilibrium solutions of inflation. This implies that the central bank's reaction to inflation must not be too large if the steady state is to be unique and saddle point stable, or the tax financing share used by the government may not be too low. Otherwise, if the expansionary impact of higher debt were associated with aggressive interest rate increases, the resulting surge in debt service costs would require the government to issue more debt, which might cause equilibrium sequences to become divergent. However, a balanced budget fiscal policy that keeps the nominal stock of bonds constant inevitably leads to equilibrium uniqueness and stability, since higher inflation then reduces the real value of debt irrespective of the monetary policy stance.

Further, while policy effects are generally in accordance with empirical evidence, debt non-neutrality is found to have an impact on the performance of simple policy rules in stabilizing the inflation and output volatility arising from cost-push shocks. The well known trade-off that these impart on monetary policy, which can only lower inflation variance at the cost of augmenting output variance, is existent here as well. Whether fiscal policy also faces a trade-off depends on the monetary policy stance. For an aggressive anti-inflationary monetary policy regime, inflation and output variances are simultaneously minimized for high shares of tax finance and, in particular, for a balanced budget regime. Inflation and output variances can even be lower than in the debt neutrality case, as the relevance of public debt imposes a restriction on admissible equilibrium values, introducing a history dependence in an otherwise forward looking environment. Thus, provided that public policy is conducted under tight debt constraints, macroeconomic fluctuations can be reduced by a fiscal policy induced history dependence, which relates to the requirement that a fully optimal monetary policy has to be history dependent in a corresponding framework where debt is neutral.

Appendix: Proof of proposition 3

To derive qualitative properties of the impulse response of the endogenous variables $X_t = (\widehat{b}_t,\ \widehat{\pi}_t,\ \widehat{y}_t)'$ to policy and cost-push shocks, we apply the fundamental solution of the model which features the state variables $S_t = (\widehat{b}_{t-1},\ \varepsilon_{rt},\ \varepsilon_{\kappa t},\ \widehat{\varphi}_t)'$. In what follows we assume that (18) is satisfied, such that the fundamental solution is the unique solution to (19)-(22). The model is then solved applying the method of undetermined coefficients for the elements of Δ defined by

$$X_t = \begin{pmatrix} \delta_b & \delta_{br} & \delta_{bk} & \delta_{bc} \\ \delta_{\pi b} & \delta_{\pi r} & \delta_{\pi k} & \delta_{\pi c} \\ \delta_{yb} & \delta_{yr} & \delta_{\pi k} & \delta_{\pi c} \end{pmatrix} \cdot S_t = \Delta \cdot S_t.$$

Given that (18) is assumed to be satisfied, we already know from proposition 2 that $\delta_b \in (0, 1)$. Hence, we aim at deriving the solutions for the remaining elements of Δ as functions of δ_b. The two other coefficients describing the structural part of the solution are given by

$$\delta_{\pi b} = \frac{1-\delta_b}{1-\eta\rho_\pi} > 0, \qquad \delta_{yb} = \frac{1-\beta\delta_b}{\omega}\frac{1-\delta_b}{1-\eta\rho_\pi} > 0,$$

which are unambiguously positive as (18) ensures $\eta\rho_\pi < 1$ (see proof of proposition 2). The coefficients governing the impact responses to the fiscal policy shocks ($\varepsilon_{\kappa t}$) are

$$\begin{aligned} \delta_{bk} &= -\frac{\rho_\pi\omega + \sigma}{(1-\eta\rho_\pi)\,\Gamma + \rho_\pi\omega + \sigma} \in (-1, 0), \qquad \delta_{\pi k} = -\frac{\Gamma}{(1-\eta\rho_\pi)\,\Gamma + \rho_\pi\omega + \sigma} < 0, \\ \delta_{yk} &= -\frac{\Psi + \delta_{\pi b}(1-\Psi-\rho_\pi\beta) + \delta_{yb}\sigma}{(1-\eta\rho_\pi)\,\Gamma + \rho_\pi\omega + \sigma}, \end{aligned}$$

where $\Gamma \equiv \omega(\Psi + \delta_{yb}\sigma) + \delta_{\pi b}\left[\omega(1-\Psi) + \beta\sigma\right] > 0$. Inspecting the solution for δ_{yk}, immediately reveals that $\rho_\pi < (1-\Psi)/\beta$ is sufficient for $\delta_{yk} < 0$. The coefficients on the monetary policy shock (ε_{rt}), are given by

$$\begin{aligned} \delta_{br} &= \frac{\omega\,(1-\eta\rho_\pi) + (\rho_\pi\omega + \sigma)\,\eta}{(1-\eta\rho_\pi)\,\Gamma + \rho_\pi\omega + \sigma} > 0, \qquad \delta_{\pi r} = -\frac{\omega - \eta\Gamma}{(1-\eta\rho_\pi)\,\Gamma + \rho_\pi\omega + \sigma}, \\ \delta_{yr} &= -\frac{1}{\Gamma}\frac{(\omega-\eta\Gamma)\,[\delta_{\pi b}(1-\Psi) + \Psi + \delta_{yb}\sigma] + \beta\delta_{\pi b}\,(\Gamma+\sigma)}{(1-\eta\rho_\pi)\,\Gamma + \rho_\pi\omega + \sigma}, \end{aligned}$$

Thus, a low value for η (high κ) satisfying $\eta < \widetilde{\eta}$, where $\widetilde{\eta} \equiv (\Psi + \rho_\pi + (1-\delta_b)\,[\frac{\sigma}{\omega}\,(1+\beta-\beta\delta_b) + 1-\Psi])^{-1} > 0$, is sufficient to ensure $\omega - \eta\Gamma > 0 \Leftrightarrow \delta_{\pi r} < 0$ and also guarantees $\delta_{yr} < 0$. Finally, the coefficients on the cost-push shocks ($\widehat{\varphi}_t$) are given by

$$\begin{aligned} \delta_{bc} &= -\frac{(1-\eta\rho_\pi)(1-\rho_c)\sigma}{\Theta} < 0, \qquad \delta_{\pi c} = \frac{(1-\rho_c)\sigma}{\Theta} > 0, \\ \delta_{yc} &= -\frac{\omega\,(\rho_\pi(1-\eta\Psi) + 1 - (\delta_b+\rho_c)(1-\Psi)) + \sigma(1-\delta_b)(1-\beta\delta_b)}{\omega\Theta} < 0, \end{aligned}$$

where $\Theta \equiv -\omega[(\delta_b+\rho_c)(1-\Psi) + \rho_\pi(\eta\Psi-1) - 1] + \sigma[(1-\beta\rho_c)(2-\delta_b-\rho_c) + \beta(1-\delta_b)^2] > 0$, given that $1 < \rho_\pi < 1/\eta$ and $\Psi < 2$. These properties of the solution coefficients are summarized in the proposition. ■

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