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FALSE CONSENSUS VOTING AND WELFARE REDUCING POLLS

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Abstract

We consider a process of costly majority voting where people anticipate that others have similar preferences. This perceived consensus of opinion is the outcome of a fully rational Bayesian updating process where individuals consider their own tastes as draws from a population. We show that the correlation in preferences lowers expected turnout. The intuition is that votes have a positive externality on those who don't participate, which reduces incentives to participate. We study the effects of the public release of information ("polls") on participation levels. We find that polls raise expected turnout but reduce expected welfare because they stimulate the "wrong" group to participate. As a result, polls frequently predict the wrong outcome. While this lack of prediction power is usually attributed to an imperfect polling technology, we show it may result from the reaction of rational voters to the poll's accurate information.

Keywords: Majority Voting, Correlated Preferences, False Consensus, Pre-election Polls

1. Introduction

Pre-election polls regularly predict the wrong outcome. In the 1993 general elections in Australia, for instance, almost every opinion poll had the Liberal party pegged as the winner but Labor went on to win.[1] In 2002, most polls got it wrong on the mid-term elections in the US where Democrats were supposed to retain control of the Senate but in fact lost widely. In key elections in many countries, opinion polls have shown as much as a 15-point margin of blunder. Indeed, polling's biggest goof ever resulted in one of the largest political upsets in US history. On the morning after the 1948 presidential election, the Chicago Daily Tribune's headline read "DEWEY DEFEATS TRUMAN." The headline, still seared into the world's collective memory more than 50 years later, was based on polls that predicted a victory of New York Governor Thomas E. Dewey over Harry S. Truman, the incumbent president.

Burns Roper, son of pioneering pollster Elmo Roper, and George Gallup Jr., co-chairman of the Gallup organization and son of another polling giant, review the events surrounding the 1948 presidential election.[2] "The polls predicted a Dewey victory of between 5 to 15 percentage points," Roper recalls, "and the labor vote was energized as Democrats worried about Dewey's strength in pre-election polls while Republicans felt their candidate would win 'so they played golf that day'." After the election the polling pioneers re-examined their methods and gradually moved away from quota sampling, which questioned a set number of people from different ethnic and age groups, toward random sampling. But half a century later, Burns Roper and George Gallup Jr. doubt whether their fathers' poll methodologies were seriously flawed. "I don't think the polls were wrong in terms of measuring national sentiment," says Roper, "clearly they were wrong in determining the election. I think the 1948 polls were *more accurate* than the 1948 election."

In this paper we present a simple two-candidate voting model to formalize Roper's intuition.[3] We assume that potential voters differ in their beliefs about the level of support for either candidate and that they use the information conveyed by the poll to refine their expectations. Belief heterogeneity occurs in our model because individuals, unsure about others' preferences, use their own tastes as information in guessing what others like.[4] In other words, individuals

[1] The polls were so unanimous that the Australian press referred to the election as "unlosable" by Labor.

[2] See http://www.hollandsentinel.com/stories/101998/new_pollstrs.html.

[3] See Campbell (1999) for an alternative explanation of the unexpected outcome of the 1948 presidential election. Campbell shows that an election can be won by a minority of "zealous" voters who have either a large stake in the outcome, or very small participation costs, relative to the rest of the electorate. In contrast to the analysis of this paper, polls play no role in Campbell's model.

[4] For example, Ross, Greene, and House (1977) report a study where students were asked whether they would

see their own tastes as draws from a population and *rationally* use this information the same way as they would use any other random sample of size one. Note that this pulls individuals' beliefs about others' tastes towards their own tastes, thus creating a so called "false consensus effect" (Ross, Greene, and House, 1977): people who engage in a given behavior estimate that behavior to be more common than people who engage in alternative behaviors.[5]

The false consensus effect is a robust phenomenon that has been observed for a wide range of preferences and opinions (e.g. Mullen *et al.*, 1985). It seems especially strong when applied to political opinions. Brown (1982) reports the choices of 179 psychology students who had to indicate their preferred candidate in the 1980 US presidential election: Anderson, Carter, or Reagan. In addition, they had to estimate the percentage of students in the class believed to prefer each candidate. Supporters of all three candidates estimated significantly higher support for their own candidate compared to the predictions of the rest of the class.[6] Morwitz and Pluzinsky (1996) report a strong false consensus bias for the 1992 US presidential election and the 1993 New York mayoral election.[7] Finally, a similar picture arises from an empirical study of the 1992 Constitutional Referendum in Canada (Baker *et al.*, 1995), where those planning to vote "Yes" predicted a significantly higher proportion of "Yes" votes than those planning to vote "No," and vice versa. The extent of the consensus effect in the Canadian referendum is noticeable especially because a substantial monetary reward was offered for accurate estimates.

While the occurrence of the false consensus effect in voting processes has been well documented, its impact on voting decisions has not yet been studied. In this paper, we present the first analysis of majority voting when decision-makers rationally anticipate that others have similar tastes. In other words, individuals with opposite preferences will have conflicting views about which alternative is more likely to be favored by the majority.[8]

walk around campus wearing a sign. While those who agreed estimated 62% of the others would, those who refused thought 67% would refuse.

[5]Dawes (1990) first noted that the definition put forth by Ross *et al.* (1977) does not necessarily justify the term "false." As explained in the text, when someone's decision is driven by her taste, which is seen as a draw from a population, then it is perfectly rational to use this decision the same way as any other random sample of size one. Only when the information about one's own taste is overweighed is the perceived consensus false. In this paper we assume rational voters that do not overweight (own) information.

[6]Anderson supporters estimated 30.4% of the class preferred their candidate, while Carter and Reagan supporters estimated 24.4% and 22.0% favored Anderson. Similarly, Carter supporters predicted 38.3% would favor their candidate while Anderson and Reagan supporters estimated 34.1% and 33.6% favored Carter. More dramatic differences were observed for the estimates of support for Reagan: those favoring Reagan predicted this to be 44.5% while Anderson and Carter supporters' estimates were 35.4% and 37.9%.

[7]Clinton supporters' estimates for (Clinton, Bush) support were (94.2%, 5.8%), while Bush supporters' estimates were (55.9%, 44.1%). Similarly, in the 1993 New York mayoral election, Dinkins supporters' estimates for (Dinkins, Giuliani) support were (84.2%, 15.8%) and Giuliani supporters' estimates were (45.8%, 54.2%).

[8]Myatt (2002) considers the case where individuals receive informative signals about support levels, which remain uncertain even in large electorates. In his model, voting is costless and there are more than two

In our model, voting is voluntary: individuals first choose whether or not to participate in the decision-making process, and those who participate then vote for one of the alternatives. We assume that participation is costly so that it may not be socially optimal for everyone to vote. Individuals' preferences reflect idiosyncratic tastes, i.e. our model is one of private values. In this case, voting against one's preferred alternative is strictly dominated by not participating. Hence, the voting decision for those who participate is straightforward: one should vote sincerely for one's preferred alternative.

While the model is *ex ante* symmetric with respect to alternatives and individuals, voters' preferences are correlated. We show that this correlation reduces participation. One reason is that correlation in preferences lowers the benefits of participating since it lowers the chances of being pivotal. To see this, consider first the case when preferences are independent and suppose an individual believes that exactly one of the others will vote. Then the chance of being pivotal is one-half, i.e. the probability that the other voter has opposing preferences. If, however, preferences are perfectly correlated, the chance of being pivotal is zero when one other person votes. In addition, when preferences are independent and both alternatives are equally likely to be favored, votes have no positive externality on those who don't participate, since any vote is equally likely to be in favor or against their preferred alternative. In contrast, with correlated preferences any vote is more likely to be in favor and thus creates a positive externality on those who don't vote. It is this positive externality that further reduces the incentives to participate.[9]

Our main interest concerns the impact of pre-election polls on voting behavior. When people use this public information to update their perceptions of others' preferences, it will correct the views of some and reinforce others. As a result, the public information will reduce the participation incentives of those who are reinforced in their belief that they belong to the majority. And it will stimulate participation of the minority group who realize they overestimated the support for their preferred alternative. We show that the net effect of polls is to raise expected turnout. However, the increase in participation levels due to polls is welfare reducing because the release of public information stimulates the "wrong" group to participate. Our results clearly demonstrate the possible negative effects of pre-election polls and may explain why some countries bar them close to an election date.[10]

candidates. Myatt shows that multi-candidate support can arise in his model, in contrast with Duverger's Law. We thank Tom Palfrey for pointing Myatt's paper out to us.

[9]Note that irrespective of whether preferences are correlated, one person's decision to participate creates a negative externality for others who participate, since it reduces their chances of being pivotal.

[10]For example, in Italy and Luxembourg no polls are allowed in the month before the election. Other countries with shorter bans include, Peru, Poland, Venezuela, and Switzerland. On March 9 of this year, Russia reversed

We mainly focus on the case when the public release of information reduces all (perceived) correlation in preferences, i.e. when large polls are conducted. In the resulting model, everyone knows others' preferences are independent of their own and everyone favors one alternative with probability $p \geq 1/2$ and the other alternative with complementary probability. This type of preference uncertainty is akin to the one studied by Ledyard (1981, 1984) and Palfrey and Rosenthal (1985).[11,12] Our model is closest to that of Börgers (2004) who proves existence of a unique symmetric equilibrium when preferences are independent and both alternatives are equally likely to be favored.[13] We extend Börgers' analysis to the case where one alternative is favored by a majority of the electorate. We show that, for some range of parameter values, there is a unique equilibrium in totally mixed strategies for which total expected turnout is *independent* of p. Moreover, the expected turnouts of both groups are *equal*, implying that members of the "smaller" group participate more frequently to offset the advantage of the "larger" group. The election therefore ends in a tie in expectation, with negative implications for expected welfare. We believe these results are novel to the literature.

The paper is organized as follows. Section 2 explains the model and provides an equilibrium analysis of simple majority voting. In section 3, we compare equilibrium and optimal levels of voting. Section 4 studies the effects of public information release. Section 5 concludes. Proofs of the propositions can be found in the Appendix.

2. Equilibrium Voting

There are $n \geq 2$ individuals labelled $i = 1, \ldots, n$. Individuals can choose to participate in the decision-making process at a cost, $c > 0$, which gives them the opportunity to vote for one of two alternatives, B (blue) or R (red). Individuals who do not wish to participate bear

an earlier relaxation of the ban on pre-election polls. More generally, in a recent survey of 66 countries around the world, the Foundation for Information/ESOMAR finds that 30 countries have an embargo on the publication of poll results on or prior to election day.

[11] In Ledyard's (1984) model, each individual knows her own characteristics, i.e. her cost and "location", the spatial positions of both alternatives, and the distribution of others' costs and locations. Ledyard proves existence of a symmetric equilibrium, but the possibility of multiple symmetric equilibria is not studied. Ledyard also considers the case where the spatial positions of the alternatives are endogenized. He proves that the resulting equilibrium is optimal in the sense that candidates choose identical positions which maximize welfare and no one votes.

[12] Palfrey and Rosenthal (1985) consider two types, or groups, of individuals: within a group, individuals have identical preferences, but preferences are opposite between groups. They derive conditions for a (possibly non-unique) symmetric equilibrium and study its limit properties as the size of the electorate diverges.

[13] This result sharply contrasts with the multiplicity of equilibria in participation games with equal and known group sizes, see Palfrey and Rosenthal (1983).

no costs and do not vote. The outcome of the voting process is determined by simple majority (with a random tie-breaking rule) and applies to all n individuals irrespective of whether they participated. Individual i's utility is 1 if her preferred outcome wins and -1 otherwise. From these utilities the cost of participating is subtracted only if individual i participates.

The model is *ex ante* symmetric both with respect to alternatives and individuals in the following sense. First, "nature" selects one of two possible states, 0 or 1, both of which are equally likely. The realization of this state is not observed by the electorate, but individuals do receive conditionally independent signals about the state. If the state is 0, individuals receive a b signal with probability $p \geq 1/2$ and an r signal with probability $1-p$.[14] Similarly, when the state is 1, individuals receive r signals with probability p and b signals with probability $1-p$. Individuals' preferred alternative simply correspond to the color of the signal they receive. Note that, independent of whether the realized state is 0 or 1, an individual with a preference for one color rationally anticipates that others are more likely to favor that same color. Indeed, a simple application of Bayes' rule shows that

$$P(\text{other prefers blue} \,|\, \text{I prefer blue}) \;=\; p^2 + (1-p)^2, \tag{2.1}$$

and the same equation holds for red. The right side of (2.1) is a convex function of p, which is minimized at $p = 1/2$ where its value is $1/2$. Hence, individuals believe others are more likely to prefer the same color whenever $p > 1/2$, while preferences are independent at $p = 1/2$. In our setup, correlation of preferences arises via a fully rational Bayesian updating process.

We next describe equilibrium behavior. Since our model is one of private values and voting is costly, voting against one's true preference is strictly dominated by not participating. Hence, those who participate vote sincerely. We assume that the decision to participate is independent of one's label and of one's preferred color as the model is *ex ante* symmetric. In other words, we restrict attention to symmetric Bayesian-Nash equilibria.

Let γ denote the probability that an individual participates. This probability will vary with the cost of voting, c, and the size of the electorate, n. First note that with a finite number of people, there is always some benefit to voting (i.e. some chance of being pivotal) even when all others participate. Hence, for low costs $c \leq \underline{c}$, everyone participates with probability 1 (see Proposition 1). In contrast, when $c \geq \overline{c} = 1$, the equilibrium probability of voting is 0. To see

[14]By restricting $p \geq 1/2$, the 0-state makes a blue outcome more likely while the 1-state is more conducive to red outcomes. This link between states and colors plays no role in what follows, and nothing would change if it were reversed. This could be done, for instance, by considering values of p less than $1/2$. To avoid trivial duplication, we henceforth assume $p \geq 1/2$.

this, suppose all individuals different from i decide not to participate. Then i's expected utility of participating is $1 - c$, while the expected utility of not participating is $\frac{1}{2} \cdot (1) + \frac{1}{2} \cdot (-1) = 0$.

Henceforth, we consider the more interesting case $\underline{c} < c < \overline{c}$ so that both "staying out" and participating occur with strictly positive probability. In the terminology of Palfrey and Rosenthal (1983), we will consider only *totally mixed* equilibria in which no individual plays a pure strategy. Of course, individuals are only willing to mix between participating and not participating if they are indifferent, which implies that the expected benefit of participating equals the cost. The benefit of voting is zero unless the vote makes a difference, i.e. when it is pivotal. There are two instances when individual i's vote is pivotal: either i's preferred color is one vote short, in which case i's vote creates a tie, or i's vote breaks a tie. In the former case, i's utility is raised from -1 to 0 and in the latter case it increases from 0 to 1. So the benefit of being pivotal is 1 in both cases.

It remains to determine the probability of being pivotal, which depends on how many others participate. Suppose, without loss of generality, that i's preferred alternative is B and consider the case with an even number, $2k$, of other participants. Then i's vote can only be pivotal when k of the others prefer B and the remaining k prefer R. The probability of this event is

$$P_{piv}(2k) \;=\; \binom{2k}{k}\, P(k \text{ others prefer } B,\, k \text{ others prefer } R \,|\, i \text{ prefers } B),$$

and a straightforward application of Bayes' rule shows that this probability can be written as[15]

$$P_{piv}(2k) \;=\; \binom{2k}{k}\, p^k (1-p)^k. \tag{2.2}$$

Similarly, if the number of other participants is odd, $2k+1$, then i's vote is pivotal only when k others vote for B and $k+1$ others vote for R. The probability of being pivotal in this case can be worked out as[16]

$$P_{piv}(2k+1) \;=\; \binom{2k+2}{k+1}\, p^{k+1} (1-p)^{k+1}. \tag{2.3}$$

[15] An individual with a preference for B will have a subjective probability p for state 0 and $1-p$ for state 1. In state 0, the probability that exactly k out of $2k$ voters favor B is $\binom{2k}{k} p^k(1-p)^k$ and the same is true in state 1. Hence, the probability of being pivotal is $p\binom{2k}{k} p^k(1-p)^k + (1-p)\binom{2k}{k} p^k(1-p)^k$, which yields (2.2).

[16] A B-voter will have a subjective probability p for state 0 and $1-p$ for state 1. The probability that exactly k out of $2k+1$ other voters favor B is $\binom{2k+1}{k} p^k(1-p)^{k+1}$ in state 0, and $\binom{2k+1}{k} p^{k+1}(1-p)^k$ in state 1. Hence, the probability of being pivotal is $2\binom{2k+1}{k} p^{k+1}(1-p)^{k+1} \;=\; \binom{2k+2}{k+1} p^{k+1}(1-p)^{k+1}$.

Some important insights can be gleaned from (2.2) and (2.3). First, note that $p(1-p)$ is a concave function that is maximized at $p = 1/2$. This implies that the probability of being pivotal for a given number of other participants decreases with p for $p \geq 1/2$. Second, comparing (2.2) and (2.3) shows that $P_{piv}(2k-1) = P_{piv}(2k)$ and $P_{piv}(2k+1) < P_{piv}(2k)$ for all $k \geq 1$.[17] Hence, the probability of being pivotal (weakly) decreases in the number of other participants. This insight can be used to prove uniqueness of the symmetric equilibrium (see Börgers, 2004) and to derive its comparative statics properties.[18]

Proposition 1. *In the unique symmetric Bayesian-Nash equilibrium, individuals participate with probability $\gamma^*(n,c,p)$ and those that participate vote sincerely for their preferred alternative.*

(i) If $c \leq \underline{c} = P_{piv}(n-1)$ then $\gamma^*(n,c,p) = 1$, i.e. everyone participates.

(ii) If $c \geq \overline{c} = 1$ then $\gamma^*(n,c,p) = 0$, i.e. no one participates.

(iii) If $\underline{c} < c < \overline{c}$ then $0 < \gamma^*(n,c,p) < 1$ satisfies

$$\sum_{k=0}^{n-1} \binom{n-1}{k} (\gamma^*(n,c,p))^k (1-\gamma^*(n,c,p))^{n-1-k} P_{piv}(k) \;=\; c. \tag{2.4}$$

The equilibrium probability of participating, $\gamma^*(n,c,p)$, is decreasing in the degree of correlation in preferences, p, the cost of participating, c, and the size of the electorate, n.

3. Welfare Analysis

Here we compare the equilibrium level of participation with the socially optimal level that maximizes the sum of players' utilities.[19] Consider first the case of $n = 2$ and $\underline{c} < c < \overline{c}$, where $\underline{c} = 2p(1-p)$ and $\overline{c} = 1$. The equilibrium level of participation that follows from (2.4) is given by

$$\gamma^*(2,c,p) \;=\; \frac{1-c}{p^2 + (1-p)^2}.$$

[17] It is readily verified that $P_{piv}(2k+1) = \frac{k+1/2}{k+1}(4p(1-p))P_{piv}(2k) < P_{piv}(2k)$.

[18] There is a simple connection between our complete-information setup with symmetric costs and models where participation costs are privately known and distributed according to some known distribution function, $F(\cdot)$. Equilibrium behavior in the latter models can be characterized by a threshold cost, c^*: individuals with costs below c^* participate, while those with higher costs stay out. The necessary condition that determines the equilibrium threshold level c^* is simply (2.4) with γ^* replaced by $F(c^*)$, see Palfrey and Rosenthal (1985).

[19] This is formally equivalent to maximizing the *ex ante* expected utility of individuals, since our model is completely symmetric.

To derive the socially optimal level, note that the sum of players' utilities is positive only when individuals have the same preferences, which occurs with probability $p^2 + (1-p)^2$. Both individuals' utilities are 1 when at least one person participates while their utilities are zero if neither participates. Hence, expected welfare is $W(\gamma) = 2(\gamma^2 + 2\gamma(1-\gamma))(p^2 + (1-p)^2) - 2\gamma c$, and maximization of W with respect to γ yields the socially optimal level

$$\gamma^o(2, c, p) \;=\; 1 - \frac{c/2}{p^2 + (1-p)^2}.$$

When $p = \frac{1}{2}$, the equilibrium level of participation exceeds the optimal level and expected welfare is zero: $W(\gamma^*(2, c, \frac{1}{2})) = 0$. In contrast, when preferences are perfectly aligned ($p = 1$), equilibrium participation is less than socially optimal and welfare is strictly positive. The next proposition generalizes to arbitrary electorate sizes.

Proposition 2. *Equilibrium participation is too high (low) when preferences are independent (perfectly correlated). In equilibrium, expected welfare is zero when preferences are independent and strictly positive when preferences are correlated.*

Börgers (2004) first demonstrated that when preferences are independent and both alternatives are equally likely to be favored, voting is excessive (even though it is voluntary). The intuition is that the decision of one person to participate creates a negative externality for other participants since it reduces their chances of being pivotal. This negative externality is not incorporated enough by individuals, which is why they participate more than is socially optimal.

A formal proof of the welfare results can be found in the Appendix, but the intuition is quite simple. Individuals randomize between voting or not, so their expected payoffs of these two options must be equal. Consider a voter who stays out. With independent preferences there is a fifty-fifty chance that the election outcome matches the voter's preference, so her expected utility is zero. The same is true for other voters that stay out and for those that participate, so total welfare is zero. In contrast, when preferences are correlated, a voter who stays out has strictly positive utility because the election outcome is more likely to match her own preference.

4. Polls

Here we study the effect of the public disclosure of information on the decision to participate. Intuitively, when information is released that makes state 0, say, more likely, then those that favor blue are reinforced in their opinion that others also favor blue. As a result, they will tend to participate less. In contrast, those that favor red adjust the likelihood that others favor the same color downwards, resulting in a higher propensity to participate. Below we determine the effects of public information disclosure on expected turnout and welfare.

Let $\mathcal{I}$ be a publicly observable signal, which provides information about the likelihood of the realized state $z = 0, 1$. For instance, when a poll is conducted among a thousand randomly selected people, $\mathcal{I}$ could be the percentage that favors a certain presidential candidate.[20] We define the likelihood-ratio $\alpha \equiv P(\mathcal{I} \,|\, 0)/P(\mathcal{I} \,|\, 1)$ and focus, without loss of generality, on the case $\alpha \geq 1$ so that the 0 state is more likely. Note that when $\alpha = 1$, the signal $\mathcal{I}$ provides no information and the model is identical to that of the previous section. When $\alpha \to \infty$, people know for sure that the realized state is 0 and that another's preferred choice is blue with probability p and red with probability $1 - p$.

Before we present a general analysis consider again the case $n = 2$. Note that the presence of the public signal $\mathcal{I}$ alters (2.1), i.e. the chance that another person favors the same color. We now have

$$P(\text{other prefers blue} \,|\, \text{I prefer blue, public signal } \mathcal{I}) \;=\; \frac{\alpha p^2 + (1-p)^2}{\alpha p + (1-p)}, \tag{4.1}$$

while

$$P(\text{other prefers red} \,|\, \text{I prefer red, public signal } \mathcal{I}) \;=\; \frac{p^2 + \alpha(1-p)^2}{p + \alpha(1-p)}. \tag{4.2}$$

Since $p \geq 1/2$, an increase in the likelihood-ratio, α, raises (lowers) the probability that others favor blue (red) given that I prefer blue (red). Let $P(B \,|\, B, \mathcal{I})$ and $P(R \,|\, R, \mathcal{I})$ denote the probabilities in (4.1) and (4.2) respectively. Someone's vote is pivotal when the other person has opposite preferences (irrespective of whether the other participates) or when the other has the same preferences but doesn't participate. So the probability, γ_B, that an individual who favors blue will participate satisfies $P(B \,|\, B, \mathcal{I})(1 - \gamma_B) + (1 - P(B \,|\, B, \mathcal{I})) = c$, or

$$\gamma_B^*(2, c, p) \;=\; \frac{1 - c}{P(B \,|\, B, \mathcal{I})},$$

and similarly

$$\gamma_R^*(2, c, p) \;=\; \frac{1 - c}{P(R \,|\, R, \mathcal{I})}.$$

[20] We make the implicit assumption that pre-election polls elicit truthful responses. However, responses would also reveal true preferences when people lie with known probability λ as long as $\lambda \neq \frac{1}{2}$. (When $\lambda = \frac{1}{2}$ polls would always predict a tie.)

A more precise information signal that raises the likelihood of the 0 state therefore reduces (raises) participation incentives for those that favor blue (red).

It is interesting to compare the impact of the public information release on equilibrium versus socially optimal levels of participation. It is readily verified that the welfare maximizing levels of participation after the public signal $\mathcal{I}$ is released become

$$\gamma^o_B(2,c,p) \;=\; 1 - \frac{c/2}{P(B\,|\,B,\mathcal{I})},$$

and

$$\gamma^o_R(2,c,p) \;=\; 1 - \frac{c/2}{P(R\,|\,R,\mathcal{I})}.$$

Hence, the public information signal $\mathcal{I}$ affects equilibrium and optimal levels in an *opposite* manner. Information that makes blue more likely reduces the participation-incentives for those that favor blue, but it raises the value of a vote for blue (since more people benefit), which is why the socially optimal level of participation rises. Likewise, such information raises the equilibrium levels of participation for those that favor red, while their votes are welfare reducing since they lower the chance that the majority wins. For the $n = 2$ case, it is straightforward to compute the welfare level that results in equilibrium and show it declines with α.[21] In other words, *polls unambiguously reduce expected welfare.*

The analysis of polls becomes more complicated for general electorate sizes. We can, however, derive results for the interesting case $\alpha = \infty$, i.e. when the release of the public information signal, $\mathcal{I}$, resolves all uncertainty about the state. This would occur, for instance, when a poll is taken among a large number of people, which is commonly the case. In the resulting model, everyone knows that (i) others' preferences are independent of their own and that (ii) each other person favors B with probability $p \geq 1/2$ and R with probability $1-p$.

First, consider again the $n = 2$ case. The equilibrium participation probabilities given above limit to $\gamma^*_B = \frac{1-c}{p}$ and $\gamma^*_R = \frac{1-c}{1-p}$ respectively. Hence, without correlation in preferences, the expected turnout of the two groups is the same: $2p\gamma^*_B = 2(1-p)\gamma^*_R$. Conditional on either state, each alternative therefore has the same chance of winning. The latter property generalizes to electorates of arbitrary size, a novel result that contrasts with earlier work in this field. We say

[21] When those that favor blue/red participate with probability $\gamma^*_B(2,c,p)$ and $\gamma^*_R(2,c,p)$ respectively, then welfare is given by

$$W \;=\; 2(1-c)\frac{(1-2p)^2(p^2+(1-p)^2)}{(\alpha p^2+(1-p)^2)(p^2+\alpha(1-p)^2)},$$

which decreases with α. Notice that welfare is 0, independent of α, when $p = 1/2$ (see also Proposition 2).

an equilibrium is *quasi-symmetric* when individuals with the same preference participate with the same probability.

Proposition 3. *Suppose it is commonly known that each individual favors B with probability $p \geq \frac{1}{2}$ and R with probability $1-p$. In the unique quasi-symmetric Bayesian-Nash equilibrium, the participation rates of those who prefer B and R are given by $\gamma_B^*(n,c,p) = \gamma^*(n,c,\frac{1}{2})/(2p)$ and $\gamma_R^*(n,c,p) = \gamma^*(n,c,\frac{1}{2})/(2(1-p))$ respectively, with $\gamma^*(n,c,\frac{1}{2})$ determined in Proposition 1. Those that participate vote sincerely for their preferred alternative.*

In the absence of any public information, expected turnout is equal to $n\gamma^*(n,c,p)$, while after the poll total expected turnout is $n(p\gamma_B^* + (1-p)\gamma_R^*) = n\gamma^*(n,c,\frac{1}{2})$. Recall from Proposition 1 that $\gamma^*(n,c,p)$ is decreasing in p for $p \geq \frac{1}{2}$, so expected turnout is higher after the poll.

Note, however, that members of the "minority color," red, participate more frequently. Indeed, after the poll one should expect as many red as blue votes, even though only a $(1-p)$ minority of the population favors red. In contrast, in the absence of a poll those that favor blue or red participate with the same probability, and the minority color is only expected to receive a fraction $1-p$ of all votes. In other words, polls raise expected turnout (and, hence, expected costs) but lower the chance that the majority color wins.

Proposition 4. *The public release of information, which eliminates all correlation in preferences, raises expected turnout but lowers expected welfare.*

To grasp the intuition behind the decline in expected welfare, consider the *ex ante* expected utility of voter i. Without loss of generality condition on state 0, so that voter i is more likely to favor B. Others' preferences are stochastically independent, but they also more likely favor B. In equilibrium, voter i plays a mixed strategy in which she randomizes between participating and not participating. Therefore, she must be indifferent between these two strategies and we can condition on the event where she does *not* participate. Without a poll, all voters' probability of participating is independent of their preference, and therefore others are more likely to vote for B than R. Hence, B is more likely to win, and voter i's expected utility, conditional on not participating, is greater than zero. With a poll, each alternative is equally likely to win (Proposition 3) and voter i's expected utility, conditional on not participating, is zero.

5. Conclusions

In this paper we present a simple model where voters have probabilistic preferences over two alternatives. Voters' tastes are correlated because they do not know the exact support for either candidate. Instead, they use their own tastes to guess others' preferences. In such an environment, pre-election polls can play a role as they help refine voters' expectations.

The contributions of our paper are two-fold. First, we analyze simple majority voting with correlated preferences and show that there exist a unique symmetric Bayesian-Nash equilibrum (Proposition 1). This contrasts with earlier work in this area (e.g. Palfrey and Rosenthal, 1985) and extends the results of Börgers (2004) who considers the special case when both alternatives are equally likely to be favored. Like Börgers we find that voting is excessive when preferences are independent. However, correlation in preferences reduces participation incentives because one person's vote creates a positive externality for others. As a result, participation levels are less than socially optimal when the degree of correlation is high enough (Proposition 2).

Second, we show that after a pre-election poll, there exist a unique quasi-symmetric Bayesian-Nash equilibrium (Proposition 3). In this equilibrium, members of the minority participate more frequently and the election is predicted to end in a tie. Moreover, overall turnout is predicted to rise. Polls thus have a negative effect on welfare since they increase participation costs and reduce the chance that the majority wins (Proposition 4).

Our results are derived under the assumption that voters' benefits are correctly measured by the chance of being pivotal. Many political scientists doubt the empirical validity of this rational choice assumption. Despite this critique, the predicted effects of polls make intuitive sense and may explain why many countries ban their use close to an election. According to Burns Roper's, the upset in the 1948 Dewey vs. Truman presidential election was not due to flawed polls, which "measured national sentiment more accurately than the elections." Rather it was caused by the electorate's reaction to the polls accurate information: "Democrats were energized as they worried about Dewey's strength in pre-election polls, while Republicans 'played golf that day' as they were sure their candidate would win."

A. Appendix

Proof of Proposition 1. Uniqueness of the symmetric equilibrium follows since an increase in γ^* raises the expected number of other participants, in the sense of first-order stochastic dominance. Since $P_{piv}(k)$ is decreasing in k, the expected benefit of being pivotal on the left side of (2.4) therefore falls with γ^*. Finally, the left side of (2.4) is equal to $\bar{c} = 1$ at $\gamma^* = 0$ and equal to $\underline{c}$ at $\gamma^* = 1$. Hence, for $\underline{c} < c < \bar{c}$ there is a unique $0 < \gamma^* < 1$ satisfying (2.4). Recall that the probability of being pivotal is decreasing in p for $p \geq 1/2$. Hence, when p increases, probability has to shift towards lower values of k to maintain (2.4) so γ^* has to fall. Similarly, when the participation cost, c, rises, the left side of (2.4) has to increase so γ^* should decrease. Finally, an increase in the size of the electorate, n, raises the number of participants in the sense of first degree stochastic dominance and γ^* has to fall to maintain (2.4). *Q.E.D.*

Proof of Proposition 2. Expected welfare is simply measured by the total utility of the group minus total costs. If γ denotes the probability of participation, then expected welfare can be written as

$$W \;=\; \sum_{k=0}^{n} \binom{n}{k} \gamma^k (1-\gamma)^{n-k}\, W(k) - n\gamma c \tag{A.1}$$

where $W(k)$ is the expected group, or electorate's, benefit when k individuals vote. Optimizing with respect to γ gives the necessary condition for the socially optimal level of participation, γ^o:

$$\sum_{k=0}^{n-1} \binom{n-1}{k} (\gamma^o)^k (1-\gamma^o)^{n-1-k} \Big(W(k+1) - W(k) \Big) \;=\; c. \tag{A.2}$$

We next determine $W(k)$. Suppose l people vote for R and $k-l$ vote for B. The group benefit depends on how many people prefer R and B. Consider those cases in which r individuals prefer R and $n-r$ prefer B, where $r \geq l$ and $n-r \geq k-l$. The group benefit is then $(n-2r)$ when $l < k-l$ so that B is the majority color, and the group benefit is $(2r-n)$ when $l > k-l$ so that R is the majority color.[22] We can combine the different cases by restricting $l = 0, \cdots, \lfloor \frac{k-1}{2} \rfloor$, where $\lfloor x \rfloor$ denotes the integer part of x, and assign the minority group of l voters once to the R group and once to the B group. This way, $W(k)$ can be written as

$$\begin{aligned} W(k) \;&=\; \sum_{l=0}^{\lfloor \frac{k-1}{2} \rfloor} \sum_{r=l}^{n-k+l} \binom{k}{l} \binom{n-k}{r-l} p^{n-r} (1-p)^r (n-2r) \\ &-\; \sum_{l=0}^{\lfloor \frac{k-1}{2} \rfloor} \sum_{r=k-l}^{n-l} \binom{k}{l} \binom{n-k}{r+l-k} p^{n-r} (1-p)^r (n-2r) \end{aligned}$$

To understand the binomials that appear in these expressions, note, for instance, that in the top line we are drawing l people that prefer R from k voters and $r-l$ people that prefer R from $n-k$ non-voters. This can be done in $\binom{k}{l}\binom{n-k}{r-l}$ ways.

[22] Note that the group utility is zero when there are an equal number of B and R voters.

It is useful to redefine the summation variable $r \to r - l$ in the first line, and $r \to r - k + l$ in the second line. Then $W(k)$ becomes

$$\begin{aligned} W(k) &= \sum_{l=0}^{\lfloor\frac{k-1}{2}\rfloor} \sum_{r=0}^{n-k} \binom{k}{l}\binom{n-k}{r} p^{n-r-l}(1-p)^{r+l}(n-2r-2l) \\ &- \sum_{l=0}^{\lfloor\frac{k-1}{2}\rfloor} \sum_{r=0}^{n-k} \binom{k}{l}\binom{n-k}{r} p^{n-r-k+l}(1-p)^{k+r-l}(n-2k-2r+2l) \end{aligned}$$

The sum over r can now be done and the result is given by

$$\begin{aligned} W(k) &= \sum_{l=0}^{\lfloor\frac{k-1}{2}\rfloor} \binom{k}{l}\Big(p^l(1-p)^{k-l} + p^{k-l}(1-p)^l\Big)(k-2l) \\ &\qquad + (n-k)(2p-1) \sum_{l=0}^{\lfloor\frac{k-1}{2}\rfloor} \binom{k}{l}\Big(p^{k-l}(1-p)^l - p^l(1-p)^{k-l}\Big) \end{aligned} \tag{A.3}$$

The top line represents the benefit of the k votes to those that participate, while the bottom line measures the positive externality of these votes for non-participants. To glean some further insight, it is useful to decompose the benefits of a vote to those that participate into a selfish benefit to the voter and a benefit to other voters. For this we need to distinguish whether an even or an odd number of people vote. It is readily verified that for even k (A.3) is equal to

$$W_{even}(k) \;=\; kP_{piv}(k-1) + n(2p-1)^2 \sum_{l=0}^{\lfloor\frac{k}{2}\rfloor-1} \binom{2l}{l} p^l(1-p)^l \tag{A.4}$$

while for odd k (A.3) equals

$$W_{odd}(k) \;=\; kP_{piv}(k-1) + (n-k)(2p-1)^2 P_{piv}(k-1) + n(2p-1)^2 \sum_{l=0}^{\lfloor\frac{k}{2}\rfloor-1} \binom{2l}{l} p^l(1-p)^l \tag{A.5}$$

Note that $W(k) = kP_{piv}(k-1)$ when $p = \frac{1}{2}$ and $W(k) > kP_{piv}(k-1)$ when $p > \frac{1}{2}$.

First, consider the case of no correlation: $p = 1/2$. We have $W(k) = kP_{piv}(k-1)$ for all k and it is readily verified that $W(k+1) - W(k) = P_{piv}(k)$ when k is even and $W(k+1) - W(k) = 0$ when k is odd. Let $W_{piv}(k) \equiv W(k+1) - W(k)$, then optimal and equilibrium levels follow from $\sum_{k=0}^{n-1}\binom{n-1}{k}(\gamma^o)^k(1-\gamma^o)^{n-1-k}W_{piv}(k) = c$ and $\sum_{k=0}^{n-1}\binom{n-1}{k}(\gamma^*)^k(1-\gamma^*)^{n-1-k}P_{piv}(k) = c$ respectively. Note that $W_{piv}(k) = P_{piv}(k)$ when k is even and $W_{piv}(k) < P_{piv}(k)$ when k is odd. Moreover, $W_{piv}(k)$ falls with k for those k for which it is positive. Hence, more mass should be put on low values of k so $\gamma^o < \gamma^*$. Since $W(k) = kP_{piv}(k-1)$ expected welfare in (A.1) becomes

$$\begin{aligned} W &= \sum_{k=0}^{n} \binom{n}{k} (\gamma^*)^k (1-\gamma^*)^{n-k}\, W(k) - n\gamma^* c \\ &= n\gamma^* \Big\{ \sum_{k=0}^{n-1} \binom{n-1}{k} (\gamma^*)^k (1-\gamma^*)^{n-k-1}\, P_{piv}(k) - c \Big\} \;=\; 0, \end{aligned}$$

where the final equality follows from the definition of γ^*. Since $W(k) > kP_{piv}(k-1)$ when $p > \frac{1}{2}$, it follows that $W > 0$ in this case.

Finally, when preferences are perfectly aligned, all individuals favor the same alternative. This situation is equivalent to a "volunteer's dilemma" game, where all individuals receive a utility of 1 when at least one person makes the costly decision to participate. It is straightforward to show that the equilibrium level of participation for the volunteer's dilemma game is given by $\gamma^* = 1 - c^{\frac{1}{n-1}}$, while the socially optimal level is $\gamma^o = 1 - (c/n)^{\frac{1}{n-1}} > \gamma^*$. To see how strong the incentives to free ride are, note that the probability that no one participates is $(1-\gamma^*)^n = c^{\frac{n}{n-1}}$. So even when the size of the electorate diverges to infinity, there is a chance $c > 0$ that the preferred alternative is not chosen. *Q.E.D.*

Proof of Proposition 3. First, take the point of view of someone who prefers B. Suppose k others also prefer B, while $n-k-1$ others favor R and a total of l others vote. If l is odd then a vote for B can be pivotal only when $(l+1)/2$ others vote for R and $(l-1)/2$ others vote for B. Similarly, when l is even, a vote for B is pivotal only when $l/2$ others vote for R and $l/2$ others vote for B. These cases can be combined by noting that when l others vote, a vote for B is pivotal only when $\lfloor\frac{l+1}{2}\rfloor$ vote for R and $\lfloor\frac{l}{2}\rfloor$ vote for B. Since there is a total of k others who favor B we necessarily have $k \geq \lfloor\frac{l}{2}\rfloor$. Likewise, we have $n-k-1 \geq \lfloor\frac{l+1}{2}\rfloor$ or $k \leq n-1-\lfloor\frac{l+1}{2}\rfloor$.

Let γ_B^* and γ_R^* denote the equilibrium probabilities of participating for B and R respectively. Equating the expected probability of being pivotal for a B-voter to the cost of voting yields:

$$\sum_{l=0}^{n-1} \sum_{k=\lfloor\frac{l}{2}\rfloor}^{n-1-\lfloor\frac{l+1}{2}\rfloor} \binom{n-1}{k} p^k (1-p)^{n-k-1} \times$$
$$\binom{k}{\lfloor\frac{l}{2}\rfloor}\binom{n-k-1}{\lfloor\frac{l+1}{2}\rfloor}(\gamma_B^*)^{\lfloor\frac{l}{2}\rfloor}(1-\gamma_B^*)^{k-\lfloor\frac{l}{2}\rfloor}(\gamma_R^*)^{\lfloor\frac{l+1}{2}\rfloor}(1-\gamma_R^*)^{n-k-1-\lfloor\frac{l+1}{2}\rfloor} = c.$$

It is useful to redefine the summation variable $k \to k - \lfloor\frac{l}{2}\rfloor$. After this redefinition, the binomial expressions can be worked out as

$$\frac{(n-1)!}{\lfloor\frac{l}{2}\rfloor!\,\lfloor\frac{l+1}{2}\rfloor!\,k!\,(n-k-l-1)!} = \binom{n-1}{l}\binom{l}{\lfloor\frac{l}{2}\rfloor}\binom{n-l-1}{k},$$

where we used $\lfloor\frac{l}{2}\rfloor + \lfloor\frac{l+1}{2}\rfloor = l$. Combining terms we can rewrite the pivotal equation for a B-voter as

$$\sum_{l=0}^{n-1} \binom{n-1}{l}\binom{l}{\lfloor\frac{l}{2}\rfloor}(p\gamma_B^*)^{\lfloor\frac{l}{2}\rfloor}((1-p)\gamma_R^*)^{\lfloor\frac{l+1}{2}\rfloor} \times$$
$$\sum_{k=0}^{n-1-l} \binom{n-l-1}{k}(p(1-\gamma_B^*))^k((1-p)(1-\gamma_R^*))^{n-l-1-k} = c.$$

The sum over k can now readily be done, yielding

$$(p(1-\gamma_B^*) + (1-p)(1-\gamma_R^*))^{n-1-l} = (1 - p\gamma_b^* - (1-p)\gamma_R^*)^{n-1-l},$$

so that the pivotal equation for a B-voter becomes

$$\sum_{l=0}^{n-1} \binom{n-1}{l} \binom{l}{\lfloor \frac{l}{2} \rfloor} (p\gamma_B^*)^{\lfloor \frac{l}{2} \rfloor} ((1-p)\gamma_R^*)^{\lfloor \frac{l+1}{2} \rfloor} (1 - p\gamma_B^* - (1-p)\gamma_R^*)^{n-1-l} = c. \tag{A.6}$$

Likewise, for an R-voter we have

$$\sum_{l=0}^{n-1} \binom{n-1}{l} \binom{l}{\lfloor \frac{l}{2} \rfloor} (p\gamma_B^*)^{\lfloor \frac{l+1}{2} \rfloor} ((1-p)\gamma_R^*)^{\lfloor \frac{l}{2} \rfloor} (1 - p\gamma_B^* - (1-p)\gamma_R^*)^{n-1-l} = c. \tag{A.7}$$

Notice that if we define $\gamma_B^* = \gamma^*/(2p)$ and $\gamma_R^* = \gamma^*/(2(1-p))$, the pivotal equations for B-voters and R-voters reduce to a single equation:

$$\sum_{l=0}^{n-1} \binom{n-1}{l} (\gamma^*)^l (1-\gamma^*)^{n-1-l} \binom{l}{\lfloor \frac{l}{2} \rfloor} \left(\frac{1}{2}\right)^l = c.$$

The solution is given by $\gamma^*(n, c, \frac{1}{2})$ because, for all l, $P_{piv}(l) = \binom{l}{\lfloor \frac{l}{2} \rfloor} \left(\frac{1}{2}\right)^l$ when $p = \frac{1}{2}$.

We next prove uniqueness of the totally-mixed equilibrium. Define $\gamma_1 = p\gamma_B^*$ and $\gamma_2 = (1-p)\gamma_R^*$ and suppose, in contradiction, there exists an equilibrium for which $\gamma_1 \neq \gamma_2$. Notice that the pivotal equations (A.6) and (A.7) for B and R voters can be written as

$$\sum_{l=0}^{n-1} \binom{n-1}{l} \binom{l}{\lfloor \frac{l}{2} \rfloor} \gamma_1^{\lfloor \frac{l}{2} \rfloor} \gamma_2^{\lfloor \frac{l+1}{2} \rfloor} (1 - \gamma_1 - \gamma_2)^{n-1-l} = c, \tag{A.8}$$

and

$$\sum_{l=0}^{n-1} \binom{n-1}{l} \binom{l}{\lfloor \frac{l}{2} \rfloor} \gamma_1^{\lfloor \frac{l+1}{2} \rfloor} \gamma_2^{\lfloor \frac{l}{2} \rfloor} (1 - \gamma_1 - \gamma_2)^{n-1-l} = c. \tag{A.9}$$

Without loss of generality assume $\gamma_1 < \gamma_2$. Taking the difference of (A.8) and (A.9) yields

$$\sum_{l=0}^{n-1} \binom{n-1}{l} \binom{l}{\lfloor \frac{l}{2} \rfloor} \gamma_1^{\lfloor \frac{l}{2} \rfloor} \gamma_2^{\lfloor \frac{l}{2} \rfloor} (\gamma_2 - \gamma_1)^{\lfloor \frac{l+1}{2} \rfloor - \lfloor \frac{l}{2} \rfloor} (1 - \gamma_1 - \gamma_2)^{n-1-l} = 0,$$

a contradiction since $(\gamma_2 - \gamma_1)^{\lfloor \frac{l+1}{2} \rfloor - \lfloor \frac{l}{2} \rfloor} = 0$ for even l and $(\gamma_2 - \gamma_1)^{\lfloor \frac{l+1}{2} \rfloor - \lfloor \frac{l}{2} \rfloor} > 0$ for odd l. Hence, $\gamma_1 = \gamma_2$. *Q.E.D.*

Proof of Proposition 4. Let $p > \frac{1}{2}$. Before the poll, turnout is given by $n\gamma^*(n, c, p)$, and after the poll, turnout is $n\gamma^*(n, c, \frac{1}{2})$. Recall from Proposition 2 that $\gamma^*(n, c, p)$ is decreasing in p, so turnout is higher after the poll.

To determine expected welfare, we closely follow the setup in the proof of Proposition 2. Suppose l people vote for R and $k-l$ vote for B, and consider those cases in which r individuals prefer R and $n-r$ prefer B, where $r \geq l$ and $n-r \geq k-l$. The group benefit is then $(n-2r)$ when $l < k-l$ so that B is the majority color, and the group benefit is $(2r-n)$ when $l > k-l$ so that R is the majority color. We can combine the different cases by restricting $l = 0, \cdots, \lfloor \frac{k-1}{2} \rfloor$

and assign the minority group of l voters once to the R group and once to the B group. This way, expected welfare, W, can be written as

$$\begin{aligned}
W \;=\; & \sum_{k=0}^{n} \sum_{l=0}^{\lfloor \frac{k-1}{2} \rfloor} \sum_{r=l}^{n-k+l} \frac{n!}{l!(k-l)!(r-l)!(n-k-r+l)!} \times \\
& \qquad (1-\gamma_R)^{r-l}\,(1-\gamma_B)^{n-k-r+l}\,\gamma_R^l\,\gamma_B^{k-l}\,p^{n-r}(1-p)^r(n-2r) \\
- & \sum_{k=0}^{n} \sum_{l=0}^{\lfloor \frac{k-1}{2} \rfloor} \sum_{r=k-l}^{n-l} \frac{n!}{l!(k-l)!(r-k+l)!(n-r-l)!} \times \\
& \qquad (1-\gamma_R)^{r-k+l}\,(1-\gamma_B)^{n-r-l}\,\gamma_R^{k-l}\,\gamma_B^{l}\,p^{n-r}(1-p)^r(n-2r) \\
- & \; n(p\gamma_B + (1-p)\gamma_R)c.
\end{aligned}$$

Again it is useful to redefine the summation variable $r \to r-l$ in the first line, and $r \to r-k+l$ in the second line. Expected welfare can then be written as

$$\begin{aligned}
W \;=\; & \sum_{k=0}^{n} \sum_{l=0}^{\lfloor \frac{k-1}{2} \rfloor} \binom{n}{k}\binom{k}{l}((1-p)\gamma_R)^l(p\gamma_B)^{k-l} \times \\
& \qquad \sum_{r=0}^{n-k} \binom{n-k}{r}((1-p)(1-\gamma_R))^r(p(1-\gamma_B))^{n-k-r}\,(n-2r-2l) \\
- & \sum_{k=0}^{n} \sum_{l=0}^{\lfloor \frac{k-1}{2} \rfloor} \binom{n}{k}\binom{k}{l}(p\gamma_B)^l((1-p)\gamma_R)^{k-l} \times \\
& \qquad \sum_{r=0}^{n-k} \binom{n-k}{r}((1-p)(1-\gamma_R))^r(p(1-\gamma_B))^{n-k-r}\,(n-2k-2r+2l) \\
- & \; n(p\gamma_B + (1-p)\gamma_R)c.
\end{aligned}$$

The sum over r can now be done and the result is

$$W \;=\; \sum_{k=0}^{n}\binom{n}{k}(\gamma^*(n,c,\tfrac{1}{2}))^k(1-\gamma^*(n,c,\tfrac{1}{2}))^{n-k}\left(\frac{1}{2}\right)^{k-1}\sum_{l=0}^{\lfloor \frac{k-1}{2} \rfloor}\binom{k}{l}(k-2l) - nc\gamma^*(n,c,\tfrac{1}{2}),$$

were we used $p\gamma_B^* = (1-p)\gamma_R^* = \frac{1}{2}\gamma^*(n,c,\frac{1}{2})$. It is readily verified that

$$\left(\frac{1}{2}\right)^{k-1}\sum_{l=0}^{\lfloor \frac{k-1}{2} \rfloor}\binom{k}{l}(k-2l) \;=\; kP_{piv}(k-1)$$

when $p = 1/2$. Hence, expected welfare equals

$$W \;=\; n\gamma^*(n,c,\tfrac{1}{2})\Big\{\sum_{k=0}^{n-1}\binom{n-1}{k}(\gamma^*(n,c,\tfrac{1}{2}))^k(1-\gamma^*(n,c,\tfrac{1}{2}))^{n-k-1}\,P_{piv}(k) - c\Big\} = \;0,$$

independent of p. *Q.E.D.*

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