No. 7

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IMPULSE BALANCE EQUILIBRIUM AND FEEDBACK IN FIRST PRICE AUCTIONS

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Impulse Balance Equilibrium and Feedback in First Price Auctions

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April 2004

Abstract

Experimental sealed-bid first-price auctions with private values in which feedback on the losing bids is provided yield lower revenues than auctions where this feedback is not given. The concept of weighted impulse balance equilibrium, which is based on a principle of *ex post* rationality and incorporates a concern for social comparison, captures the data.

JEL classification: C7 ; C9

Keywords: Auctions; Overbidding; Feedback; Experiments; Ex-post rationality; Bounded rationality; Social comparison

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I. Introduction

Feedback in a repeated auction environment can substantially affect outcomes, even when the feedback has no strategic information value. As a consequence, the feedback made available to the bidders is an important design variable. This paper experimentally investigates sealed-bid first-price auctions with private values under different feedback conditions. A novel, behaviorally founded equilibrium concept captures the observed bids' sensitivity to feedback.

Isaac and Walker (1985) compare sealed-bid private-value first-price auctions with four bidders, in which feedback is given only on the price, to other auctions in which bidders are additionally informed about the losing bids after the auction. They find that prices generated with limited feedback are higher than those generated under the full feedback condition. Dufwenberg and Gneezy (2002), independently of our study, investigate sealed-bid first-price auctions with and without feedback on losing bids in a common value environment (the value was identical to all bidders, which was commonly known), and also find the revenue-reducing effect of feedback on losing bids. In this paper, we provide further support of the feedback effect in private value auctions with two competing bidders. Taken together, these studies suggest that the examined feedback conditions elicit a robust behavioral effect.[1] Yet, the effect cannot easily be explained by standard notions of rational bidding, because in all experiments, the strategic links between successive auctions were minimized. In particular, bidders were rematched into new bidder groups from auction to auction, and bidders' values were either identical and commonly known, or for each bidder and in each auction independently drawn.

While Isaac and Walker (1985) do not attempt to theoretically explain their observation, Dufwenberg and Gneezy (2002) argue intuitively that, while the feedback effect is inconsistent with standard theory, it may be in line with a certain form of signaling behavior.[2] In this paper, as an alternative, we propose an equilibrium explanation. However, our approach is very different from standard economic equilibrium models. It is based on the concept of weighted impulse balance that is in turn based on a simple principle of *ex-post* rationality similar to Selten and Stoecker's (1986) learning direction theory (see Selten, 1998, and the references cited therein for evidence). Weighted impulse balance involves one parameter that might, in our auction context,

[1] Cox et al. (1984) and Battalio et al. (1990), however, find no effect of relatively minor variations in their information conditions on bidding behavior (Kagel, 1995, chapter IE; see also Dufwenberg and Gneezy, 2000).
[2] Plott (1982) discusses related signaling hypotheses that have been put forward in very early experimental Industrial Organization work; for more recent related work see also Huck et al. (2000).

be interpreted as a measure of concern for relative standing.[3] It organizes the feedback effect – along with the fact that bids in first-price auctions tend to be higher than at risk neutral Nash equilibrium[4] – quite well, both qualitatively and quantitatively.

II. Experimental game and design

The model underlying the experiment is a symmetric sealed-bid first-price auction with private values. The values v_1 and v_2 of the two bidders 1 and 2 are uniformly and independently distributed over the interval [0, 100]. (Actually, values were decimal numbers with at most two digits after the decimal point.) Both bidders simultaneously and independently make bids b_1 and b_2. The higher bid b_j wins the auction, and the profit of the winner j is $v_j – b_j$. The loser receives nothing. In case of a tie, $b_1 = b_2$, the auction winner is selected randomly. In order to avoid losses bids above the value are not permitted (of course, such bids would be dominated in the game theoretic sense), but otherwise no restrictions are imposed on bids. After each auction, each bidder is informed about whether he won the auction, the price, and his payment for that auction. Under treatment *NF* no additional feedback is given, but under treatment *F* feedback on the opponent's bid is supplied to the winner of the auction. Altogether eight sessions with 12 subjects each were run for 140 rounds with one auction in each round. The subjects of one session belonged to two independent subject groups of six participants each.

For reasons that will become clear later, the 140 rounds were divided into 28 *weeks* with five rounds each. At the beginning of each week a value was drawn randomly for each subject. Then each subject played against each of the five other participants of the same subject pool in a random order, with the restriction that a subject could not be matched with the same opponent twice in a row. The subjects did not know about the division into two independent subject pools. The impression was conveyed that in each week they would play against opponents randomly selected among the other eleven participants (see Appendix for the instructions given to the

[3] As will become clear later, the model's parameter can also be interpreted in terms of a 'utility of winning', a kind of 'loss aversion', or a mix of these and related concerns. Our data do not allow distinguishing between these concerns. However, recent evidence by Bolton and Ockenfels (2000), Fehr and Schmidt (1999), and Morgan et al. (2003) suggests that the social comparison explanation of our parameter is likely to be part of the right interpretation.
[4] Bidding above risk neutral Nash equilibrium (RNNE), a phenomenon called 'overbidding', is well-studied. Cox, Roberson and Smith (1982) proposed the constant relative risk averse model (CRRAM) to account for overbidding. The model assumes that subjects maximize the expected value of a utility of the form $u(x) = x^r$, where u is the utility index, x is the amount of money gained, and r is the risk aversion parameter. Bidders are assumed to play Nash equilibrium strategies on this basis. The model is consistent with overbidding (see Cox, Smith and Walker, 1988), but a number of studies challenged CRRAM's explanation (see, e.g., Harrison, 1989, Kagel and Levin, 1993, Kagel

subjects). The interaction was anonymous and formal via computer terminals, utilizing Fischbacher's (1998) z-Tree software tool. The computers were placed in three-sided cubicles, and neither the other subjects nor the experiment's monitor could watch anyone make their choices. In total, 96 subjects participated in four sessions under treatment *NF* and four sessions under treatment *F*, yielding eight independent observations per treatment.

Each money unit in the experiment was worth 2 German Pfennige (approximately 1 American or 1 European cent). The total payoff of each subject was the sum of his winnings over all 140 auctions plus 10 German Marks show up fee. No session lasted longer than 1.5 hours and average payoffs were 49 German Marks ($23) with a standard deviation of 11 German Marks ($5).

III. Experimental results

III.1 Overbidding and feedback effect

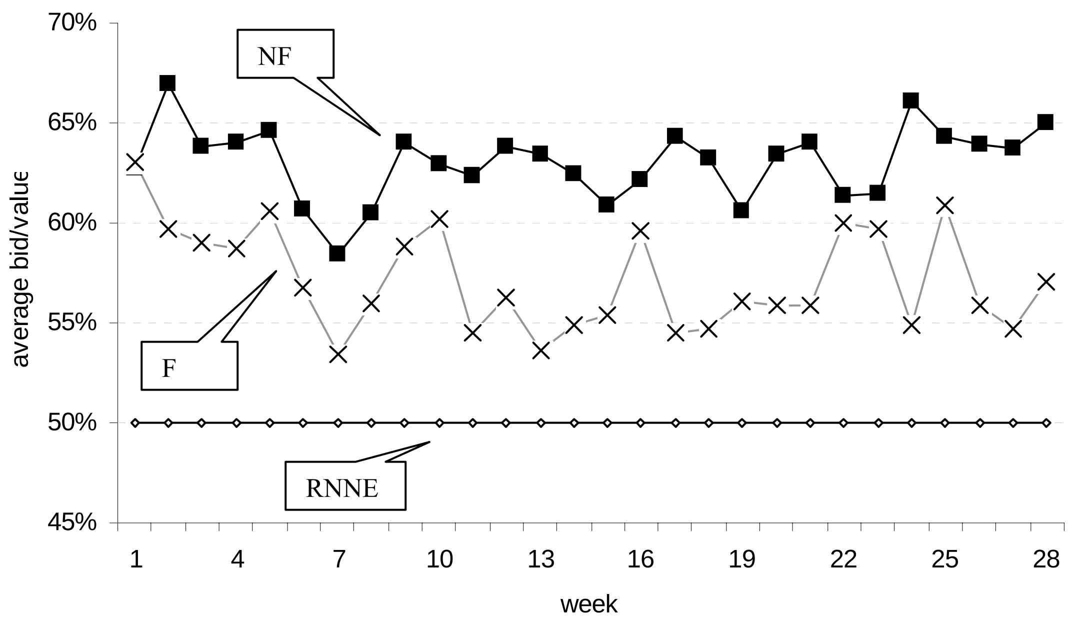

Figure 1. Average relative bids over weeks

Figure 1 shows the actual average relative bids in weeks 1 to 28 separately for the two treatments *F* (with feedback on losing bid) and *NF* (without feedback on losing bid), as well as

and Roth, 1992; Kagel, 1995, surveys the discussions). However, neither risk aversion nor social utility models based on full rationality provide different predictions for each of our treatments.

the average relative bids predicted by the risk neutral Nash equilibrium (RNNE). The relative bid of a subject in a round is his bid as a percentage of his value. The average relative bid in a week is the average of all relative bids of all subjects under the same treatment *F* or *NF* in this week. Therefore, each data point in the graphic corresponds to 240 bids (and 48 values) of eight groups of six subjects for five rounds. In the risk neutral Nash equilibrium, the relative bid in a two-person auction is 50 percent regardless of the feedback condition.[5]

Figure 1 shows that with the exception of the first week the curve for feedback treatment *F* is entirely below the curve for treatment *NF*, and that the line for the risk neutral Nash equilibrium is entirely below the line for treatment *F*. The overall average relative bids are 63.1 percent for treatment *NF* and 57.2 percent for treatment *F*. The one-sided Mann-Whitney *U* test applied to average relative bids of subject groups rejects the null hypothesis of equal average relative bids across treatments *F* and *NF* at the three percent level. A similar comparison of average revenues or, in other words, of average high bids yields significance at the one percent level. A comparison of actual relative bids and the predictions of the risk neutral Nash equilibrium yields significance levels of .01 for treatment *NF* and .04 for treatment *F*. Recall that these results are in line with earlier findings. Bidding above RNNE ('overbidding') is the most common outcome in first-price private value auctions (Kagel, 1995), and a comparison of the feedback conditions in the four-bidder case by Isaac and Walker (1985) yielded an analogous feedback effect.[6]

III.2 Ex-post rationality

Impulse balance theory, analogous to learning direction theory (Selten and Stoecker, 1986), postulates that the adjustment tends to follow a principle of *ex-post* rationality. It applies to repeated decision tasks in which feedback information after each period permits causal inferences about what might have been better last time. In our context, if a higher (lower) bid in the last round might have been better, the bid tends to be increased (decreased) in the current round.[7] In

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[5] We have chosen a figure showing average relative bids rather than other data since RNNE, CRRAM and impulse balance theory all lead to constant predictions for average relative bids.

[6] Isaac and Walker (1985, p. 141) conclude from their experimental findings that "Central to the results of this study is the consistent evidence of the previous two studies [Walker, Smith and Cox 1983, and DeJong, Forsythe and Uecker 1984] which suggest that, in sealed bid (offer) auctions, the more limited the between market publicized information the lower the profits for those making bids to buy. […] If such evidence is found to be robust, this relationship has important implications for the evolution of market institutions in private economies, as well as policy implications for government implemented auctions."

[7] Learning direction theory does not predict that every single change of the decision parameter will obey *ex-post* rationality, but only that observed changes will be more than randomly expected in the indicated direction; see Selten (1998) for a more general version of the theory.

particular, in rounds 2 to 5 of a week in our experiment a bidder can look back on his experience from the previous round *with the same value*, facing one of three possible experience conditions:

- *Lost opportunity*: The own bid was smaller than the price, but the value was higher than the price.
- *Overpayment*: The own bid won.
- *Outpriced value*: The price was higher than the own value.

Fixing values within weeks allows us to derive predictions about bid changes. In particular, *ex-post* rationality suggests that, for a given value, the bid will tend to increase in the lost opportunity condition, while it will tend to decrease in the overpayment condition (because in this case a smaller bid might have been sufficient in order to win the auction). In the outpriced value condition, *ex-post* rationality suggests an unchanged bid as the most frequent response.

	percent	change of bid (percent)		
experience condition	(number)	increase	decrease	unchanged
lost opportunity	22.6 (2415)	57.5	12.1	30.4
overpayment	50.0 (5359)	17.5	47.0	35.5
outpriced value	27.4 (2930)	26.6	22.6	50.8

Table 1. Experience conditions and bid changes

Table 1 shows the percentages of different responses to the three experience conditions in rounds 2 to 5 of each week. Since there is no indication that these bid change patterns differ across treatments, we present aggregate results only. The few cases in which both bidders bid the same amount are not included.[8] The prediction of *ex-post* rationality for these .02 percent of all observations are different for treatments *NF* and *F*. Under treatment *NF* such cases belong to the overpayment condition since as far as the bidder knows he could have won at a lower price.

[8] Strictly speaking, because values were decimal numbers with two digits after the decimal point, not only the tied bid case warrant special treatment but also the case where bidder 1 bids b and bidder 2 bids $b + 0.01$ or $b - 0.01$. However, this never happened in our data set.

Under treatment *F*, on the other hand, the case of equal bid must be regarded as a separate experience condition, which, however, is not explored explicitly in view of its very rare occurrence.

The most frequent responses to the different experience conditions are on the main diagonal. The numbers in Table 1 cannot be taken at face value, however. Suppose that the bid of a subject is determined by a constant bid function plus random error. If the value is constant, a low bid is more likely to yield a lost opportunity than a high bid. At the same time, a low bid is more likely to be followed by a higher bid than a high bid. A similar argument applies to the overpayment condition and a subsequent decrease of the bid. We refer to this as the "regression effect" because it reminds us of the regression to the mean. The regression effect alone will already produce diagonal elements in the first two data rows of Table 1 that are higher than randomly expected. In order to control for the regression effect, we tested the hypothesis that bids change as a result of experience as opposed to the hypothesis that the bids are a constant function of the value plus a random term. Our test rejects the error model and indicates a strong and systematic influence of experience (as predicted by the principle of *ex-post* rationality).[9]

The numbers on the main diagonal of Table 1 suggest that subjects do not respond equally strongly to the experience conditions. In particular, the probability that a lost opportunity is followed by a bid increase is greater, by about 10 percentage points, than the probability that an overpayment is followed by a bid decrease. In fact, we find that for 12 out of 16 independent subject groups the average surplus for the lost opportunity condition is greater than that for the overpayment condition. By the binomial test this is significant at the 2.7 percent level. Evidently, the lost opportunity condition motivates subjects more strongly to change their bid in the expected direction than the overpayment condition does. Many interpretations suggest themselves, including a 'utility of winning' and social comparison processes. In particular, in the case of lost opportunity the forgone profit is connected to an unfavorable relative standing because the competitor won the auction (see, e.g., Bolton and Ockenfels, 2000, Fehr and Schmidt, 1999, and Morgan et al., 2003). For a bidder who wins and therefore experiences the

[9] For every subject and every week we considered all possible permutations with respect to the order of the bids the subject made during that week, keeping the order of bids of the opponent constant. For each permutation of the bids we counted how often we hit the main diagonal of Table 1. The *surplus* of a subject in a week is the difference of the number of cases on the diagonal for the actual order of bids minus the average number in all permutations. Learning direction theory predicts that this surplus tends to be positive, whereas the null hypothesis states that the mean surplus is zero. For each of the 16 independent subject groups the average surplus over all six subjects and all 28

overpayment condition, the forgone profit comes with an advantageous relative standing. Thus, from the perspective of these social comparison models, responses are conceivably stronger for the lost opportunity condition.[10]

IV. Weighted impulse balance theory

IV.1 Basic idea

Impulse balance theory is an attempt to make *quantitative* predictions about the central tendency of the stationary distribution of bids on the basis of the principle of *ex-post* rationality without making use of a full-fledged learning model.[11] Thus, as described earlier, impulse balance theory is applicable to situations in which a subject repeatedly has to make a decision on the same parameter, and in which the feedback permits conclusions about what would have been a better choice of the parameter in the last period. It is postulated that the decision has a tendency to shift into the direction suggested by this counterfactual comparison.

We speak of an *upward impulse* if a higher profit could have been made by a higher value of the parameter but not by a lower one. Analogously, a *downward impulse* is experienced if a higher profit could have been made by a lower value of the parameter but not by a higher one. The strength of the impulse is the amount of forgone profit. The basic idea of impulse balance theory is that the size of the shift of the parameter tends to be proportional to the strength of the impulse. Formally, in our context, an impulse balance equilibrium (henceforth IBE) is a bid function for each player, computed on the basis of balancing the expected upward and downward impulses. Thus, the equilibrium bid functions should be thought of in terms of reactions to the (balanced) impulses; they do not involve strategic reactions to the opponents' strategies.

In the following section we propose a modification of impulse balance theory that permits different weights for downward and upward impulses. We shall define a *weighted* IBE that depends on the parameter λ called the downward impulse weight. This parameter is the number of upward impulse units equivalent to one downward impulse unit. The greater λ, the more the

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weeks is positive. By the binomial test this result is significant on the one-percent level for both treatments separately.

[10] Cason and Friedman (1997) also identified stronger responses to upward impulses in two-sided sealed bid call auctions.

[11] Impulse balance theory has been first applied by Selten, Abbink and Cox (2002). Cason and Friedman (1997 and 1999) also extend the basic ideas of *ex-post* rationality and learning direction theory by relating how much offers change to how much can be gained by changing offers in a two-sided sealed bid call auction, but do not further formalize these ideas. Learning direction theory is related to adaptive learning models such as Crawford (1995) and

average bid is drawn downward. Weighted impulse balance requires that the expected upward impulse be equal to λ times the expected downward impulse. Thus, if responses are stronger for upward impulses (that is, in our context, for the lost opportunity condition), λ will be smaller than one.

When applying weighted impulse balance theory to our data, we must distinguish between treatments *F* and *NF*. Under treatment *F* the subjects see the amounts of upward and downward impulses. A comparison of expected amounts is behaviorally reasonable. Under treatment *NF*, however, the amount of the downward impulse is not visible to the winner since feedback on the losing bid is not available. Therefore, under this treatment the principle of weighted impulse cannot be applied to expected amounts of impulses. Instead of this, we shall apply it to expected numbers of impulses. Under treatment *NF* subjects can still observe whether there is an upward or downward impulse even though amounts are not visible in the case of downward impulses.

As an alternative to our impulse measure in treatment *F* one might also assume that subjects (re)act as if they believe that the expected size of a downward impulse is equal to the expected size of an (observable) upward impulse in the *NF* condition. This would not alter our results.

Assuming rational expectations with respect to the expected impulse sizes, however, is inconsistent with both the nature of our model of *boundedly* rational decision making and our experimental results, as we will demonstrate below. In this respect, it is important to note that the application of IBE involves degrees of freedom. Yet, this also holds for models of perfectly rational decision making. RNNE, for example, assumes risk neutrality, and CRRAM constant relative risk aversion. These assumptions appear not more plausible (or less critical to the model) than our hypotheses about information processing.

IV.2 The model

We look at a symmetric sealed-bid private-value first-price auction with n bidders. (While in our experiments $n = 2$, the generalization in the model allows us to apply the model to Isaac and Walker's experimental setting with $n = 4$.) The values v are uniformly and independently distributed over [0, 1]. Let x be the value of bidder 1 and y be the maximum of the values of bidders 2 to n. Then, the density of y is $f(y) = (n-1)y^{n-2}$. Moreover, let p be the price or, equivalently, the highest bid of all bidders. Our analysis is based on the assumption that each

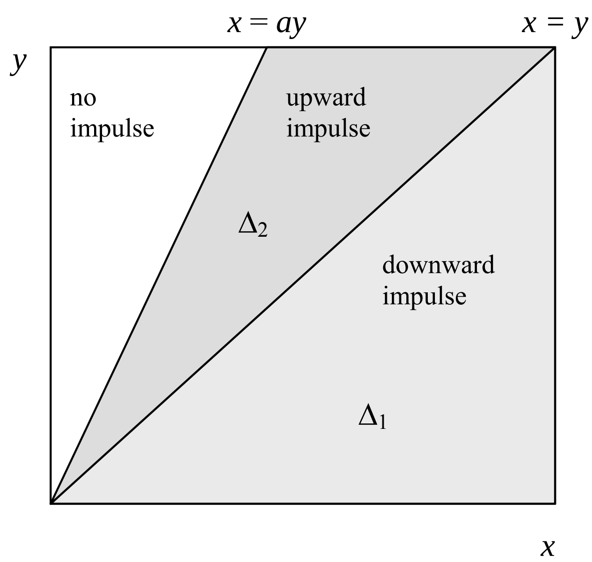
learning models that take into account forgone payoffs such as Camerer and Ho (1999); see chapter 6 in Camerer (2003) for a survey of descriptive learning theories.

bidder has the same homogeneously linear bidding function $b(v) = av$ with $0 \leq a \leq 1$ where $b(v)$ is the bid at the value v. The decision parameter responding to the impulses is the bid function parameter a. That is, bidders pick an entire linear bid function in response to impulses rather than simply choose bids.

A different approach would be to define an IBE in terms of strategies for each value. However, it is highly desirable from the perspective of learning theories that an observation in one very specific situation has some impact on behavior in other situations (virtually all learning theories have this property, including standard rational learning approaches). In addition, from an empirical point of view, there is evidence that bid functions are adjusted holistically in Selten and Buchta (1999).

A non-linear specification would be, while more difficult to solve, preferable. Note, however, that alternative bidding models, RNNE and CRRAM, also lead to bidding functions of this kind; for RNNE we have $a = (n - 1)/n$, and CRRAM yields $a = (n - 1)/(n - 1 + r)$. Moreover, our own data and earlier empirical results suggest that linearity is a good approximation of actual bidding functions (e.g., Cox et al., 1988), even if some deviations like non-monotonicities can be observed (Selten and Buchta, 1999). Finally, as we will show in Section V.III, our linear model fits the data very well. Thus, a richer model could not substantially increase the predictive success, but it would come at considerably higher computational costs.

The diagram in Figure 2 shows the value x of bidder 1 with $0 \leq x \leq 1$ on the abscissa and the maximum value y of the other bidders with $0 \leq y \leq 1$ on the ordinate. We distinguish three regions in this diagram. In region Δ_1, x is greater than y so that bidder 1 wins. In this region bidder 1 observes a downward impulse. In region Δ_2, the maximum value of the others is greater than bidder 1's value, but the highest bid ay of the others is still smaller than x. Here, an upward impulse is observed. The remaining region corresponds to the outpriced value condition in which there is no impulse. The table below the diagram in Figure 2 describes the three regions. The difference between the treatments F and NF is that the amount of a downward impulse is not known.



regions	*characterization*	*amount of impulse* F	NF
downward impulse: Δ_1 (overpayment)	$p = ax > ay$	$ax - ay$	not known
upward impulse: Δ_2 (lost opportunity)	$p = ay < x$	$x - ay$	$x - ay$
no impulse	$p = ay > x$	0	0

Figure 2. Regions and amounts of impulses

IV. 3 Weighted impulse balance equilibrium under treatment F

The expected downward impulse E_- is computed as follows:

$$
\begin{aligned}
E_- &= a\int_{\Delta_1}(x-y)f(x)f(y)dxdy \\
&= a\int_0^1\int_0^x (x-y)(n-1)y^{n-2}dydx \\
&= \frac{a}{n(n+1)}.
\end{aligned}
$$

For the expected upward impulse E_+ we obtain

$$E_+ = \int_{\Delta_2} (x - ay) f(x) f(y) dxdy$$
$$= \int_0^1 \int_{ay}^{y} (x - ay)(n-1) y^{n-2} dxdy$$
$$= \frac{(1-a)^2 (n-1)}{2(n+1)}.$$

Weighted IBE requires

$$E_+ = \lambda E_- .$$

This impulse balance equation is a quadratic equation for the slope *a* of the bid function. We obtain

$$a = 1 + \frac{\lambda}{n(n-1)} - \sqrt{(1 + \frac{\lambda}{n(n-1)})^2 - 1} .$$

The solution for *a* is in the interval (0,1) for all positive values of the downward impulse weight λ. The other root of the quadratic equation is greater than 1 and therefore outside the interval of admissible values for *a*.

IV. 4 Weighted impulse balance equilibrium under treatment NF

Under treatment *NF* the expected number of downward impulses P_- is

$$P_- = \frac{1}{n} .$$

This is due to the fact that all bidders use the same bid function and therefore have the same chance 1/*n* of winning the auction. The expected number of upward impulses P_+ is the area of the region Δ_2:

$$P_+ = \int_{\Delta_2} f(x) f(y) dxdy$$
$$= \int_0^1 \int_{ay}^{y} (n-1) y^{n-2} dxdy$$
$$= \frac{(1-a)(n-1)}{n}.$$

Weighted IBE requires

$$P_+ = \lambda P_- .$$

This is a linear equation for *a*. We obtain

$$a = 1 - \frac{\lambda}{n-1} .$$

a is in the interval (0,1) if $0 < \lambda < n - 1$. In our experiments we have $n = 2$ which means that we must have $\lambda < 1$ if *a* is positive.

V. Explaining the experimental regularities

V.1 Explaining overbidding

In the risk neutral Nash equilibrium, the bidding function is $b(v) = (n - 1)v/n$. Overbidding therefore requires $a > (n-1)/n$. Depending on the value of the downward impulse weight λ the IBE predicts overbidding. Let us turn first to treatment *F* and assume $a > (n-1)/n$. Then we have

$$E_- = \frac{a}{n(n+1)} > \frac{n-1}{n^2(n+1)}, \text{ and}$$

$$E_+ = \frac{(1-a)^2(n-1)}{2(n+1)} < \frac{n-1}{2n^2(n+1)} .$$

This yields

$$\lambda = \frac{E_+}{E_-} < \frac{1}{2} .$$

Thus, under treatment *F* overbidding results in weighted IBE if and only if λ is smaller than 1/2. Under treatment *NF*, on the other hand, $a > (n - 1)/n$ is equivalent to $1 - \lambda/(n - 1) > (n - 1)/n$ and therefore to $\lambda < (n - 1)/n$. This condition is necessary and sufficient for overbidding in IBE under treatment *NF*.

In sum, overbidding requires $\lambda < 1$ regardless of whether we look at treatment *F* or *NF*, and regardless of the number of bidders $n > 1$. This means that upward impulses must be given more weight than downward ones. The root of this asymmetry – and therefore of overbidding – may be the social comparison process described in Section III.2 that explains why forgoing profits by paying 'too much' and winning the auction yields a weaker response than forgoing profits by bidding 'too little' and losing the auction.

V.2 Explaining the feedback effect

We now turn to the observation that under treatment *F* bids tend to be lower than under treatment *NF*. We show that this effect follows from impulse balance theory if the downward impulse weight is in the region connected to overbidding. Let S_- be the conditional expectation of the amount of the downward impulse under the condition that such an impulse occurs. Similarly, let S_+ be the conditional expectation of the amount of an upward impulse if it occurs. We have

$$S_- = E_- / P_- = \frac{a}{n+1}, \text{ and}$$

$$S_+ = E_+ / P_+ = \frac{(1-a)n}{2(n+1)}.$$

Thus, $S_- > S_+ \Leftrightarrow \frac{a}{1-a} > \frac{n}{2}$, which holds if $a > \frac{n}{n+2}$. Since $\frac{n}{n+2} \leq \frac{n-1}{n}$ for all $n > 1$, overbidding implies that the conditional average amount of a downward impulse is larger than the conditional average amount of an upward impulse. This explains the treatment effect. Because conditional downward impulses (S_-) are always larger than conditional upward impulses (S_+), taking into account both amount *and* numbers of impulses yields on average a stronger downward impulse than taking into account only the numbers. In other words, subjects whose behaviors follow the principles of impulse balance and who cannot observe the relative size of downward and upward impulses behave as if they systematically underestimate downward impulses. As a consequence, bids under treatment *NF* are higher than bids under treatment *F*.

V.3 Quantitative estimation and prediction

On the basis of our data we close our discussion of the weighted IBE with a rough estimate of the value of the parameter λ. We use the relationship between the slope *a* and the weight λ and insert the average slope $\overline{a}$ in our data. This is done for each treatment separately. We obtain $\lambda = (1 - \overline{a})/(2\overline{a}) = .32$ for treatment *F*, and $\lambda = 1 - \overline{a} = 0.37$ for treatment *NF*. Both treatments yield slightly different estimates of λ which seem to be near enough to each other, however, in order to justify the assumption that the two values of λ are the same for both treatments. Accordingly, we estimate λ as the average λ = .34 of both estimates.[12]

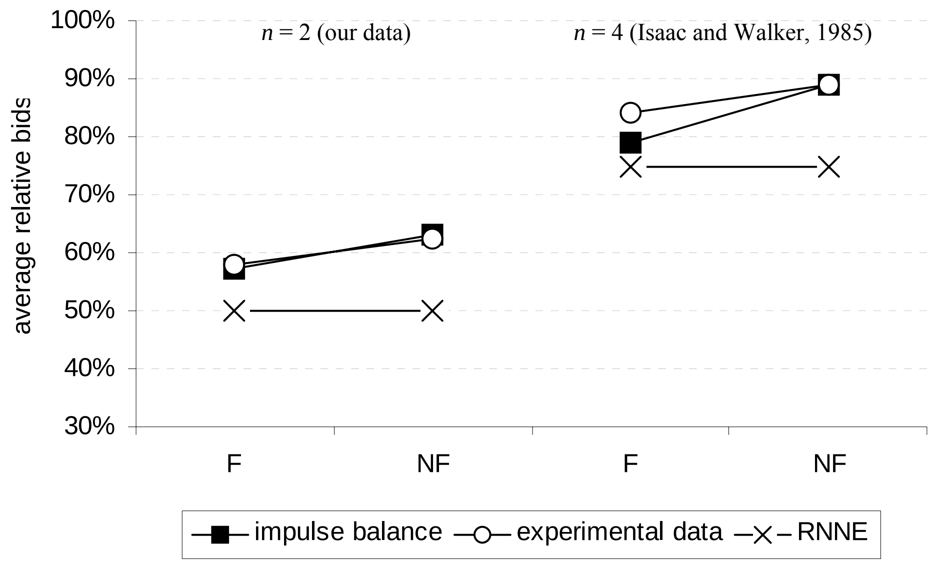


Figure 3. Average relative bids and theoretical predictions

Based on this estimate of λ, Figure 3 shows empirical and theoretical values of average relative bids for both treatments in our experiments and those of Isaac and Walker (1985), who used a comparable experimental design except for the number of bidders (2 in our and 4 in Isaac and Walker's experiment) and that values changed from round to round (whereas in our experiment they changed from week to week). Overbidding and lower slopes for treatment *F* are correctly reproduced for both experiments. Furthermore, we obtain a good quantitative fit of our data. Actual average bids are 57 and 63 percent for treatments *F* and *NF*, respectively, while predicted average bids are 58 and 62 percent, respectively. Using the estimate of the downward impulse weight λ from our data, we also correctly predict the average relative bid under Isaac and Walker's *NF* treatment, and only somewhat underestimate the average relative bid under their *F* treatment. Actual values are 84 and 89 percent for treatments *F* and *NF*, respectively, while predicted values are 79 and 89 percent, respectively.

[12] Whether the parameter should be, theoretically, constant across treatments or not must eventually depend on its exact interpretation. Our interpretation of λ as reflecting a concern for social comparison suggests a constant value. However, there is no reason to assume that λ cannot change with experiment design parameters such as, say, the framing of the setting.

VI. Conclusions

A key lesson drawn from the economic design of auctions is that the details of market rules can substantially affect outcomes (e.g., Klemperer, 2002, Milgrom 2003, Roth and Ockenfels, 2002). One of the details, relevant in repeated game contexts, is what kind of feedback about the market participants' historic behavior should be given.[13]

This paper introduces the concept of weighted impulse balance equilibrium that incorporates a concern for social comparison and avoids the specification of a full-fledged learning model. Weighted impulse balance equilibrium correctly predicts lower average relative bids and thus lower revenues when feedback on losing bids is available on the assumption that the downward impulse weight is in the range where overbidding occurs. The role of feedback cannot be explained by optimization approaches because the additional feedback on the losing bid supplied under treatment *F* is irrelevant for the maximization of profits on the basis of the rules of the game. Note that this feedback is also inconsequential for pure reinforcement theories in which probabilities of bids only depend on experienced past profits (Roth and Erev, 1995). An adequate explanation of bidding behavior seems to require a picture of human learning that is neither completely mechanistic nor hyper-rationalistic. Simple processes of cause and reasoning, as the *ex-post* rationality principle used in learning direction theory, seems to be an adequate approach.

The theory of weighted impulse balance equilibrium shows that quantitative behavioral equilibrium concepts can be devised which may serve as benchmarks in the evaluation of experiments. A particularly simple application would be private-value second-price auctions. There, the Nash equilibrium in weakly dominant strategies (bidding one's value) is also an impulse balance equilibrium because in equilibrium there are no lost opportunities and the winner's payoffs can never be increased by bidding less than one's value.[14] However, the impulse equilibrium concept is in principle applicable to all games in which players repeatedly decide on

[13] For instance, what kind of feedback should be given by internet auction houses? eBay.com recently changed its feedback policy and now provides the complete bidding history of completed auctions, whereas Amazon.com's auction house still only documents the last bid of each bidder (see Ockenfels and Roth, forthcoming), while Germany's largest online auction house, Ricardo.de, does not offer feedback on completed auctions at all. Similar questions arise in competition policies. The Commission of the European Union, for instance, considers the publication of historic market data such as transaction prices as anti-competitive, whereas the Danish Competition Council advocated this kind of data to promote market transparency and thus competitiveness (see Huck et al., 2000, and the references therein).

[14] While Kagel (1995) reports overbidding in experimental second-price auctions, recent work seems to suggest that this tendency disappears for experienced bidders in dynamic second-price auctions (Garrat et al., 2002, Ariely et al., 2004).

one parameter and in which the feedback environment allows conclusions about what would have been the better choice in the last period.

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Appendix: Instructions to the subjects (translation from the German)

Instructions

General information

The purpose of this session is to study how people make decisions. If at any time you have questions, raise your hand and a monitor will assist you. From now until the end of the session, unauthorized communication of any nature with other participants is prohibited.

During the session you will participate in auctions that will give you an opportunity to make money. You will be paid your earnings plus a DM 10 show up fee at the end of the session. Decisions and payments are confidential: No other participant will be told about your actions during the game or the amount of money you make.

Description of the game

Each participant will have the opportunity to submit one bid in each of 140 subsequent auctions. In each auction, there will be exactly two bidders. The higher bid wins. The winner receives a payoff, the loser receives nothing in that auction.

What is the value of the item auctioned off to me?

The precise value of the (fictitious) item differs across bidders and between auctions. However, in each auction, before you submit your bid, you will be told exactly what the commodity is worth to you, i.e., what DM amount we will pay you if you win the auction. Specifically, your value is randomly drawn such that each value between 0.00 and 100.00 money units has the same probability of being drawn. The values of all bidders are independently determined, so that generally each bidder has a different value. (But you won't know the values of the other bidders.) The value does not change for five auctions. After every five auctions, new values are randomly drawn for all bidders.

What will my earnings be?

If you submit the higher bid in an auction, you win. Your payment from this game is then your value minus your bid. (Therefore, bids higher than your value may lead to losses and are therefore not allowed.) In case your bid is smaller than the bid of your opponent, you lose and won't receive any payment for this auction. In case of a tie, a chance move randomly determines the winner. Your earnings, which are paid at the end of the session, are the sum of payments over all 140 auctions. Each money unit earned in the auction is worth 2 Pfennige.

Who is the other bidder?

All pairings are anonymous. Your identity will not be revealed to the persons you are playing with either before, during or after the game. In each auction, your opponent will be selected randomly. However, you won't be paired with the same person twice in a row. Note that by this matching scheme, the values of your (changing) opponents generally also change from round to round.

[TREATMENT *F*] *What feedback is there after each auction?*

After each round you will be informed as to whether you won the auction, the price (i.e., the winning bid), the opponent's bid, and your payment from that auction.

[TREATMENT *NF*] *What feedback is there after each auction?*

After each round you will be informed as to whether you won the auction, the price (i.e., the winning bid), and your payment from that auction.

Good luck!