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CAN MONEY MATTER FOR INTEREST POLICY?

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Can Money Matter for Interest Rate Policy?

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Abstract

In this paper it is shown that money can matter for macroeconomic stability under interest rate policy, if transactions frictions are specified in a consistent way. We develop a sticky price model with a shopping time specification, which induces the marginal utility of consumption to depend on the (predetermined) stock of money held at the beginning of the period. Saddle path stability is then ensured by a passive interest rate policy, whereas activeness is associated with an explosive equilibrium path unless the central bank reacts to changes in beginning-of-period real balances. When the central bank aims at minimizing macroeconomic distortions, real balances enter the interest rate feedback rule under discretionary optimization. If it is alternatively assumed that end-of-period money provides transaction services, money can be neglected for interest rate policy in order to implement the optimal plan. However, the equilibrium under the targeting rule is likely to be indetermined, allowing for endogenous fluctuations, which can be avoided by the central bank implementing the optimal plan with an interest rate feedback rule featuring beginning-of-period real balances.

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1 Introduction

Which role should be assigned to monetary aggregates when the short-run nominal interest rate serves as the central bank's instrument (operating target)? Recent contributions to monetary business cycle analysis have brought out a consensus framework, i.e., the New Keynesian model (see Clarida et al., 1999), in which monetary aggregates hardly play a substantial role for the evolution of core macroeconomic aggregates, such as output and inflation. This has even led to a – now widely applied – strategy to omit money from models, which are, nonetheless, applicable for short-run monetary policy analysis. As a consequence thereof, efficient interest rate policies are completely independent from the evolution of monetary aggregates (see, e.g., Woodford, 2003a). In this paper, we identify conditions within a New Keynesian type framework under which it is desirable that the central bank considers real balances as an indicator for interest rate policy. Thereby, uniqueness and stability of the equilibrium path as well as the minimization of a welfare based loss function serve as the criteria for monetary policy.

The origin of our line of arguments is the presumption that transactions of goods are associated with costs being alleviated by holdings of non-interest bearing money which serves as a medium of exchange. As, for example, demonstrated by McCallum (2001), a consistent application of this concept implies that the equilibrium sequences of output and inflation, on the one hand, and real balances, on the other hand, cannot be determined independently.[3] Yet, the quantitative impact of real balances on the evolution of other macroeconomic aggregates is often found to be very small (see, e.g., Ireland, 2002),[4] suggesting the negligence of money – even if "theoretically incorrect" (McCallum, 2001, page 149) – to be a reasonable approximation. In contrast to this view, it is shown in this paper that a relevant role of money is, in fact, consistent with small real balance effects, i.e., effects of real money holdings on the propensity to consume, as it originates in the treatment of money as a stock variable in dynamic general equilibrium models. We will demonstrate that the predetermined stock of money held by the households at the beginning of the period is not necessarily irrelevant for the equilibrium behavior of the households and as an indicator for a stabilizing interest rate policy, if transactions frictions are not neglected.

To reveal the potential relevance of money for interest rate policy, we introduce a conventional shopping time specification, which can equivalently be written by money entering the utility function in a non-separable way, as, for example, in Den Haan (1990). Households

[3] Similar arguments can be found in Woodford (2003a, chapter 4.3.2).

[4] Other studies indicate that money significantly contributes to the prediction of inflation and consumption, see, e.g., Estrella and Mishkin, (1997), Stock and Watson (1999), Nelson (2003), or Rudebusch and Svensson (2002) for the US. Similar conclusions can be drawn from recent analyses of Euro area data finding that monetary aggregates have a potentially significant role in providing information about current real output (see Coenen et al., 2002) and contains independent predictive content for inflation (see Gerlach and Svensson, 2003).

are willing to carry over money from one period to the other, even when it is dominated in rate of return, as they rely on the stock of money held when they enter the goods market in order to economize on shopping time associated with purchases of consumption goods. As we assume – for our benchmark specification – that the goods market opens before the asset market, beginning-of-period money holdings enter the shopping time function and, thus, the utility function, as in McCallum and Nelson (1999) or Lucas (2000). Given that real balances affect the marginal utility of consumption, aggregate demand and labor supply depend on the predetermined value of beginning-of-period money, implying that the equilibrium sequences of output and inflation are, in general, history dependent, regardless whether monetary policy is conducted in a backward or forward looking way. Concisely, the fundamental solution of any variable, including the nominal interest rate depends on beginning-of-period (lagged) realizations of real balances, serving as a relevant endogenous state variable.[5]

To assess the role of this state variable on the stabilizing properties of interest rate policy, we, firstly, derive conditions for saddle path stability when the central bank applies an interest rate feedback rule solely featuring inflation and beginning-of-period real balances as indicators. Thereby, it turns out that the mere reliance of the households' behavior on this state variable, is responsible for the non-applicability of the so-called Taylor-principle, which applies for the standard New Keynesian model (see Clarida et al., 1999, or Woodford, 2001). In particular, it is shown that a passive interest rate rule ensures equilibrium uniqueness and stability (saddle path stability), while activeness gives rise to explosive equilibrium sequences.[6] If, however, the central bank increases the nominal interest rate with beginning-of-period real balances, activeness does not necessarily destabilize the economy. While a simultaneous rise in the nominal and the real interest rate induces households to decrease the level and the growth rate of real balances, the feedback from real balances alleviates the rise in the nominal interest rate and, thus, avoids macroeconomic aggregates to evolve on divergent paths. By considering money as an indicator, the central bank can, therefore, raise the likelihood for its interest rate setting to be associated with a stable and unique equilibrium path.

The second part of our monetary policy analysis refers to the case, where the central bank aims at minimizing a quadratic loss function, which increases with the variances of output, inflation, and the nominal interest rate, under discretion, as in Woodford (2003b).[7]

[5] Though, the role of money for short-run interest rate policy is examined in several recent contributions, none of them considers the case where real money is a relevant predetermined state variable (see, for example, Dotsey and Hornstein, 2003, Ireland, 2003, Nelson, 2003, or Woodford, 2003a).

[6] Similarly, activeness can lead to explosiveness when physical capital is introduced in a sticky price model (see Dupor, 2001).

[7] As shown by Woodford (2003a) for an isomorphic model, such a loss function can be derived from a second order Taylor expansion of households' welfare.

While this procedure is known to lead to an entirely forward looking monetary policy regime, the fundamental solution of the optimal plan exhibits a history dependence induced by the backward looking behavior of the households. As a consequence, the realizations of the nominal interest rate under discretionary optimization depend on beginning-of-period real balances, implying that the latter naturally enters (together with inflation) an optimal interest rate feedback rule. These interest rate rules are typically characterized by small coefficients on real balances and saddle path stability.[8]

In the last part of the paper, we consider a specification of the model with an alternative timing of markets. In particular, it is assumed that the asset market opens before the goods market, such that the end-of-period stock of money enters the shopping time function, as in Brock (1974).[9] As a consequence, the households' behavior is independent of beginning-of-period real balances and the fundamental solution of the model does not exhibit an endogenous state variable, provided that monetary policy is non-backward looking. In this case, the Taylor-principle applies, i.e., an interest rate rule featuring inflation as the only indicator ensures equilibrium uniqueness if it is active. A simple interest rate rule is further sufficient to implement the optimal plan under discretion. If the transactions friction is neglected, such a rule can lead to equilibrium uniqueness ruling out endogenous fluctuations (see Clarida et al., 1999). If, however, the central bank cares for the transactions friction, this type of policy rule fails to implement the optimal plan under discretion in a unique way. Moreover, the central bank's aim to minimize the distortion induced by transactions frictions, which is measured by the variance of the nominal interest rate, can even cause the equilibrium under the targeting rule to be indetermined.[10] In this case, the central bank can restore uniqueness and, thus, rule out endogenous fluctuations by applying a backward looking feedback rule, by which the nominal interest rate responds to changes in beginning-of-period real balances. Thus, money can – even for entirely forward looking households – play a useful role as an indicator for interest rate policy, if those frictions that are responsible for a positive demand for money are not neglected.

The remainder of the paper is structured as follows. The model is developed in section 2. In section 3, we assess the local dynamic implications of interest rate policy for the linearized version of the benchmark model and derive interest rate feedback rules under discretionary optimization of a quadratic loss function. In section 4, we conduct corresponding analyses for the alternative timing of markets. Section 5 concludes.

[8] The policy rule coefficients can even take negative values, which is also found by Schmitt-Grohé and Uribe (2004) for optimized interest rate rules in a sticky price model with transactions frictions and distortionary taxation.

[9] See Carlstrom and Fuerst (2001) for a critical discussion of the assumption that the end-of-period stock of money provides transaction services. For their analysis of determinacy conditions on interest rate rules, they disregard the relevance of predetermined money.

[10] In contrast, the equilibrium of a standard New Keynesian model under the targeting rule, where transcations frictions are absent, can be shown to be uniquely determined (see Jensen, 2002).

2 A sticky price model with transactions frictions

In this section we present a model where money provides transactions services, modelled by a shopping time specification. The model can then equivalently be written as a money-in-the-utility function model where the marginal utility of consumption depends on real money balances, as, for example, in Den Haan (1990). The equilibrium sequences of all endogenous variables can, therefore, not be separately determined from the path of real money balances. Throughout the paper nominal (real) variables are denoted by upper-case (lower-case) letters. For steady state values, the time index is omitted.

The **timing of events** can be summarized as follows. At the beginning of period t households are endowed with holdings of two assets, risk-free government bonds B_{t-1} and non-interest bearing money M_{t-1}, carried over from the previous period. Then an aggregate (cost push) shock realizes and goods are produced with labor supplied by the households. After goods are produced, the market for the final consumption good opens where households purchase goods and monopolistically competitive firms set their prices in a staggered way. The purchases of goods are assumed to be associated with transaction costs, i.e., shopping time. The stock of money held before the goods market closes, is assumed to reduce the required shopping time, such that households are willing to accumulate a positive amount of money. The central bank, which aims at stabilizing the economy, adjusts the current nominal interest rate in response to changes in macroeconomic aggregates, for example, in the current aggregate inflation rate.[11] After the goods market is closed, households enter the asset market with residual holdings of money.[12] There, they receive the payoff on their assets and they adjust their holdings of bonds and money, which are carried over to the next period. This timing of events implies that the stock of money held at the beginning of the period provides transaction services, i.e., enters the shopping time function, as in Den Haan (1990) or Lucas (2000).[13] Portfolio adjustments in the asset market determine the end-of-period stock of money M_t, which is held – even when money is dominated in rate of return by bonds – to reduce transactions costs in the subsequent period.

There is a continuum of **households** indexed with $j \in [0,1]$. Their utility is assumed to rise with consumption of the final good c and of leisure x. The objective of household j is given by: $\max E_0 \sum_{t=0}^{\infty} \beta^t \upsilon(c_{jt}, x_{jt})$, where $\beta \in (0,1)$ denotes the discount factor and E_0 the expectation operator conditional on the information in period 0. The instantaneous utility

[11] We abstain from specifying interest rate policy in a forward-looking way, which is shown by Bernanke and Woodford (1997) and Carlstrom and Fuerst (2001) to be a potential source for macroeconomic instabilities induced by endogenous fluctuations.

[12] In section 4, the timing of markets is reversed.

[13] A corresponding specification, where the beginning-of-period stock of money enters the utility function can, for example, be found in Woodford (1990) or McCallum and Nelson (1999).

function υ exhibits constant intertemporal elasticities of substitution:

$$\upsilon(c_{jt}, x_{jt}) = (1-\sigma)^{-1} c_{jt}^{1-\sigma} + (1-\vartheta)^{-1} x_{jt}^{1-\vartheta}, \quad \sigma \geq 1 \text{ and } \vartheta \geq 0. \tag{1}$$

Purchases of consumption goods require transaction services. These services are produced with money holdings and with shopping time, which can also be rationalized by a Baumol-Tobin inventory theoretic approach to money demand.[14] Total time endowment, which consists of leisure, working time, and shopping time, is normalized to equal one: $x_t = 1 - l_t - s_t$, where s denotes shopping time and l working time. The shopping time function satisfies the following standard properties (see, e.g., Ljungqvist and Sargent, 2000):

$$s_{jt} = H\left(c_{jt}, A_{jt}/P_t\right), \tag{2}$$

where $H : R_+^2 \to [0,1]$, $H_c > 0, H_{cc} > 0$, $H_a < 0$, $H_{aa} > 0$ and $H_{ca} \leq 0$. While the shopping time is increasing in consumption, it is decreasing in real money balances A_t/P_t, where A_t denotes the relevant stock of money holdings that reduces transaction costs and P_t the aggregate price level in period t. Replacing leisure in (1) by $x_t = 1 - l_t - H_t$, we can rewrite the utility function as $u(c_{jt}, l_{jt}, a_{jt}) = \upsilon(c_{jt}, 1 - l_{jt} - H(c_{jt}, a_{jt}))$, where $a_t \equiv A_t/P_t$.

Given that the goods market opens before the asset market, the stock of nominal balances held at the beginning of the period t reduces the shopping time associated with consumption expenditures in period $t : A_t = M_{t-1} \Rightarrow a_t = m_{t-1}/\pi_t$, where $m_{t-1} \equiv M_{t-1}/P_{t-1}$ denotes beginning-of-period real balances. In section 4 of this paper, we alternatively assume that the goods market opens after the asset market, implying that the end-of-period stock of money enters the shopping time function ($A_t = M_t$). To simplify the derivation of analytical results, it is further assumed that utility is linear in leisure, $\vartheta = 0$, which is also assumed for determinacy analyses under interest rate policy in Dupor (2001) and Carlstrom and Fuerst (2003). This assumption further facilitates comparisons with studies on optimal monetary policy (see Clarida et al., 1999, or Woodford, 2003a), where utility is commonly assumed to be separable in working time ($u_{cl} = u_{al} = 0$). The households' objective can therefore be written as:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(c_{jt}, l_{jt}, m_{jt-1}/\pi_t\right)\right], \tag{3}$$

$$\text{where } u\left(c_{jt}, l_{jt}, \frac{m_{jt-1}}{\pi_t}\right) = \left[\frac{c_{jt}^{1-\sigma}}{1-\sigma} - H\left(c_{jt}, \frac{m_{jt-1}}{\pi_t}\right)\right] + \left(1 - l_{jt}\right),$$

and $\pi_t \equiv P_t/P_{t-1}$ denotes the gross rate of inflation. The remaining properties of $u(c_t, l_t, a_{jt}) = u\left(c_{jt}, l_{jt}, m_{jt-1}/\pi_t\right)$ can be summarized as follows: $u_c = \upsilon_c - H_c$, $u_a = -H_a > 0$, $u_{cc} =$

[14] Precisely, an equivalence between these two specifications requires a unit income elasticity of money demand, which will be assumed in the following sections for demonstrative purposes.

$\upsilon_{cc} - H_{cc} < 0$, $u_{aa} = -H_{aa} < 0$, and $u_{ca} = -H_{ca} \geq 0$. In this paper, we are predominantly interested in the role of money for interest rate policy for the case, where the shopping time function and, thus, the utility function are non-separable between consumption and real balances. As an parametric example, we will apply Lucas's (2000) shopping time specification in the subsequent sections of the paper, which exhibits a unit consumption/income elasticity of money demand, implying $H_{ca} < 0$. In this case, there is a real balance effect, causing marginal utility of consumption to increase with real balances ($u_{ca} > 0$), which can also be rationalized by considering real resource costs of transactions (see Feenstra, 1996, or McCallum, 2001). Specifying real money balances and consumption as complements can, thus, be viewed as "the most plausible assumption" (Woodford, 2003a, page 112),[15] while "theoretical considerations suggest" a separable utility function to be actually "misspecified" (McCallum, 2001, page 157).

We assume that households monopolistically supply differentiated labor services as in Clarida et al. (2002). The differentiated labor services l_j are transformed into a composite labor input l, which can be employed for the production of the final good. The transformation is conducted via the aggregator: $l_t^{1-1/\eta_t} = \int_0^1 l_{jt}^{1-1/\eta_t} dj$, with $\eta_t > 1$. The elasticity of substitution between differentiated labor services η_t is allowed to vary exogenously over time,[16] leading to changes in the labor market conditions which affect the costs of goods producing firms. Cost minimization with respect to differentiated labor services then leads to following demand for l_j:

$$l_{jt} = \left(\frac{w_{jt}}{w_t}\right)^{-\eta_t} l_t, \qquad \text{with} \qquad w_t^{1-\eta_t} = \int_0^1 w_{jt}^{1-\eta_t} dj. \tag{4}$$

As households are identical, the indexation of households' variables is subsequently omitted except for their idiosyncratic working time l_j and the real wage rate w_j. The households own final goods producing firms and, thus, receive their profits ω_t. They have access to risk free one period bonds B, which serve as a store of value dominating money in rate of return by the nominal interest rate $R \equiv 1 + i > 1$. Households further receive wage payments and a government transfer τ. The budget constraint of household j is given by

$$P_t c_t + B_t + M_t \leq R_t B_{t-1} + M_{t-1} + P_t w_{jt} l_{jt} + P_t \tau_t + P_t \omega_t. \tag{5}$$

Maximizing (3) subject to the budget constraint (5), labor demand (4) and a no-Ponzi-game condition, $\lim_{i\to\infty} E_t[(B_{t+i} + M_{t+i})\, \Pi_{v=1}^{i} R_{t+v}{}^{-1}] \geq 0$, for given initial values B_{-1} and M_{-1} leads to the following first order conditions for consumption, money, leisure, and bonds,

[15] Woodford (2003a) further argues that "*if utility is obtained from holding money, this must be because of money balances facilitate transactions, and it is hardly sensible that the benefits of such balances should be independent of the real volume of transactions that a household actually undertakes.*" (op. cit., p. 300).

[16] A decline in η_t leads, for example, to an exogenous increase in labor market competitiveness.

given that $a_t = m_{t-1}/\pi_t$:

$$\lambda_t = c_t^{-\sigma} - H_c\left(c_t, m_{t-1}/\pi_t\right), \tag{6}$$

$$E_t \frac{\lambda_{t+1}(R_{t+1}-1)}{\pi_{t+1}} = E_t \frac{-H_a\left(c_{t+1}, m_t/\pi_{t+1}\right)}{\pi_{t+1}}, \tag{7}$$

$$\mu_t = w_{jt}\lambda_t, \tag{8}$$

$$\lambda_t = \beta E_t \frac{\lambda_{t+1}R_{t+1}}{\pi_{t+1}}, \tag{9}$$

where $\mu_t \equiv \frac{\eta_t}{\eta_t - 1}$ denotes the variable markup over the perfectly competitive real wage governed by a stationary stochastic process and λ the shadow price of wealth. Furthermore, the budget constraint (5) holds with equality and the transversality condition must be satisfied, $\lim_{i\to\infty} E_t\left[\lambda_{t+i}\beta^{t+i}\left(b_{t+i} + m_{t+i}\right)\right] = 0$, where $b_t \equiv B_t/P_t$. It should be noted that consumption depends on beginning-of-period real balances, according to (6). In contrast, if the end-of-period stock of money is – as in the section 4 – assumed to enter the shopping time function ($A_t = M_t$), consumption will not depend on a predetermined value of real balances, since the first order condition that corresponds to (6) then features m_t instead of m_{t-1}/π_t.

The final consumption good is produced by competitive **firms**, which aggregate differentiated goods produced by monopolistically competitive firms indexed with $i \in [0,1]$. The aggregation technology is given by

$$y_t^{1-1/\epsilon} = \int_0^1 y_{it}^{1-1/\epsilon} di, \quad \text{with} \quad \epsilon > 1, \tag{10}$$

where y is the number of units of the final good, y_i the amount produced by firm i, and ϵ the constant elasticity of substitution between these differentiated goods. Let P_i and P denote the price of good i set by firm i and the price index for the final good. The demand for each differentiated good is derived by minimizing the total costs of y : $y_{it} = \left(P_{it}/P_t\right)^{-\epsilon} y_t$, with $P_t^{1-\epsilon} = \int_0^1 P_{it}^{1-\epsilon} di$. Firm i produces good y_i with a technology which is linear in the composite labor input: $y_{it} = l_{it}$. We introduce a nominal stickiness in form of staggered price setting as developed by Calvo (1983). Each period firms may reset their prices with the probability $1-\phi$ independent of the time elapsed since the last price setting. The fraction ϕ of firms are assumed to adjust their previous period's prices according to $P_{it} = \pi P_{it-1}$, where π denotes the average inflation rate. The linear approximation to the corresponding aggregate supply constraint at the steady state, is given by

$$\widehat{\pi}_t = \chi \widehat{mc}_t + \beta E_t \widehat{\pi}_{t+1}, \qquad \text{with } \chi \equiv (1-\phi)(1-\beta\phi)\phi^{-1} > 0, \tag{11}$$

where $\widehat{x}$ denotes the percent deviation from the steady state value x of a generic variable x_t, $\widehat{x} = \log(x_t) - \log(\overline{x})$, and mc real marginal costs of differentiated goods producing firms.

The demand for aggregate labor input in a symmetric equilibrium relates real marginal costs to the real wage rate:

$$mc_t = w_t. \tag{12}$$

The **public sector** consists of a monetary and a fiscal authority. The latter is assumed to issue one-period bonds, earning the net interest $(R_t - 1)B_{t-1}$, while the former issues money. The consolidated flow budget constraint of the public sector is given by $B_t + M_t = R_t B_{t-1} + M_{t-1} + P_t \tau_t$. Public policy is assumed to ensure government solvency: $\lim_{i\to\infty}(B_{t+i} + M_{t+i})E_{t+i}\Pi_{v=1}^{i}\left(R_{t+v}\right)^{-1} = 0$. The monetary authority is assumed to set a sequence of nominal interest rates $\{R_t\}_{t=0}^{\infty}$. Thereby, we restrict the sequence of interest rates to satisfy $R_t > 1$ $\forall t$ and to be consistent with the steady state condition $R = \pi/\beta$.

The **rational expectations equilibrium** of the model is a set of sequences $\{\pi_t$, w_t, m_t, λ_t, R_t, c_t, l_t, $mc_t\}_{t=0}^{\infty}$ satisfying (i) the household's first order conditions (6)-(7) and (8)-(9), (ii) optimal price setting approximated by (11) and the aggregate labor demand (12); (iii), the aggregate resource constraint, $l_t = c_t$, a sequence for $\{\mu_t\}_{t=0}^{\infty}$ and for the monetary policy instrument $\{R_t\}_{t=0}^{\infty}$, and the transversality condition for given initial values M_{-1} and P_{-1}.

3 Interest rate policy in the benchmark model

In this section, we aim at disclosing the role of money as an indicator for monetary policy and for local determinacy and stabilization of macroeconomic aggregates under interest rate feedback rules. The analysis is conducted for the log-linear approximation of the model at the steady state. The first part of this section presents the rational expectations equilibrium of the linearized version of the model and introduces a parametric example for the shopping time function, which is taken from Lucas (2000). The subsequent part examines the local dynamic properties of the model under simple interest rate feedback rules, featuring inflation and real money balances as indicators. In the last part of this section we examine interest rate policy under discretionary optimization of a quadratic loss function and describe implied feedback rules for the nominal interest rate.

3.1 The linearized model

The benchmark model where beginning-of-period real balances enter the shopping time function is log-linearized at the steady state, which is characterized by the following conditions: $\pi = R\beta$, $u_l(c) = u_c(c, m/\pi)\left(\epsilon - 1\right)/(\mu\epsilon)$, and $u_c(c, m/\pi)\left(R - 1\right) = u_a(c, m/\pi)$, and a consistent interest rate policy. The rational expectations equilibrium of the linearized and reduced model can then be defined as follows.

Definition 1 *A rational expectations equilibrium of the log-linear approximation to the* *model with $A_t = M_t$ at the steady state is a set of sequences $\{\widehat{c}_t, \widehat{m}_t, \widehat{\pi}_t\}_{t=0}^{\infty}$ satisfying*

$$\widehat{\pi}_t = \chi\sigma_c\widehat{c}_t - \chi\varepsilon_{ca}\widehat{m}_{t-1} + \chi\varepsilon_{ca}\widehat{\pi}_t + E_t\beta\widehat{\pi}_{t+1} + \chi\widehat{\mu}_t, \quad (13)$$

$$\sigma_c\widehat{c}_t - \varepsilon_{ca}\widehat{m}_{t-1} + \varepsilon_{ca}\widehat{\pi}_t = \sigma_c E_t\widehat{c}_{t+1} - \varepsilon_{ca}\widehat{m}_t + (\varepsilon_{ca}+1)\,E_t\widehat{\pi}_{t+1} - E_t\widehat{R}_{t+1}, \quad (14)$$

$$(\varepsilon_{ca}+\sigma_a)\,\widehat{m}_t = -zE_t\widehat{R}_{t+1} + (\sigma_c+\phi_{ac})\,E_t\widehat{c}_{t+1} + (\varepsilon_{ca}+\sigma_a)\,E_t\widehat{\pi}_{t+1}, \quad (15)$$

where $z \equiv \frac{R}{R-1}$, $\varepsilon_{ca} \equiv \frac{\overline{au}_{ca}}{\overline{u}_c}$, $\sigma_c \equiv -\frac{\overline{cu}_{cc}}{\overline{u}_c}$, $\sigma_a \equiv -\frac{\overline{au}_{aa}}{\overline{u}_a}$, and $\phi_{ac} \equiv \frac{\overline{cu}_{ac}}{\overline{u}_a}$, and the transversality condition, given a sequence of $\{\widehat{\mu}_t\}_{t=0}^{\infty}$ satisfying $\widehat{\mu}_t = \rho\widehat{\mu}_{t-1} + \varepsilon_{\mu t}$, where $\varepsilon_{\mu t}$ is i.i.d. with $E_{t-1}\varepsilon_{\mu t} = 0$, a policy $\{\widehat{R}_t\}_{t=0}^{\infty}$ with $R_t > 1\forall t$, and an initial value $m_{-1} = M_{-1}/P_{-t} > 0$.

The equilibrium conditions (13)-(15) reveal that beginning-of-period real balances, $\widehat{m}_{t-1}$, are for $H_{ca} < 0 \Rightarrow \varepsilon_{ca} > 0$ non-negligible for the determination of inflation, consumption, and end-of-period real balances, $\widehat{m}_t$. The predetermined value for real balances imposes, by (6), a restriction on current consumption and, therefore, on the remaining variables. It should further be noted that the coefficients on real balances and on consumption in the aggregate supply constraint (13) exhibit opposite signs.[17] Consumption tends to raise real marginal costs and, thus, current inflation, whereas real balances tend to reduce real marginal costs. This effect stems from the fact that consumption and real balances are Edgeworth complements, implying that the marginal utility of consumption rises with real money holdings. In this case, households, who aim at equalizing the ratio of the marginal utilities of consumption and of leisure to the ratio of their prices, are willing to reduce leisure for a given real wage. Equivalently, real marginal costs of firms decline in this case, as the real wage demanded by households declines for a given labor supply. A higher value for beginning-of-period real balances is, thus, associated with lower current inflation, whereas a rise in nominal money growth is consistent with higher current inflation given that real balances in (13) are predetermined.

Throughout the remainder of the paper, we aim at deriving characteristics of stabilizing feedback rules for the nominal interest rate in an environment where transactions frictions, which are responsible for a positive demand for money, are non-negligible. The specification of the model given in definition 1 is, however, not always suited for the presentation of the results. Therefor and to provide numerical examples, we introduce the following parametric form for the shopping time function H, which is taken from Lucas (2000):

$$s_t = H\left(\frac{P_t c_t}{A_t}\right) = \gamma\frac{c_t}{a_t}, \quad (16)$$

$$\Rightarrow \varepsilon_{ca} = (R-1)a/c,\ \ \sigma_c = \sigma\left(\varepsilon_{ca}+1\right),\ \ \sigma_a = 2,\ \text{and}\ \phi_{ac} = 1.$$

[17] Similarly, the growth rate of consumption increases and the growth rate of real balances decreases with the real interest rate (see 14), as the shadow price of wealth declines (rises), by (6), with consumption (real balances).

The shopping time specification (16) is associated with a unit elasticity of money demand with respect to consumption/income and an interest elasticity of money demand equal of 0.5, which is also found in empirical studies (see Lucas, 2000). It further corresponds to the Baumol-Tobin inventory-theoretic approach to money demand.[18] According to the latter, which encompasses Clower's (1967) cash-in-advance constraint as a limiting case, the parameter γ can be interpreted as the time costs per trip to the bank, while the number of these trips equals c_t/a_t. Thus, for $A_t = M_{t-1}$ the marginal utility of consumption rises with real money balances held at the beginning of the period, by $\frac{\partial u_c}{\partial m_{t-1}} = \gamma m_{t-1}^{-2}\pi_t > 0$. The specification (16) further implies that the steady state elasticity of the marginal utility of consumption with respect to real balances ε_{ca} is strictly positive and relatively small for reasonable values for the net interest rate $(R-1)$.[19]

3.2 Real balances and local stability

Before we turn to the analysis of the local dynamics for the benchmark model, we briefly consider an alternative version of the model, which corresponds to specifications that can often be found in recent contributions to the literature on monetary (interest rate) policy analysis. Suppose that the shopping time function is separable, $H_{ca} = 0$, such that $\varepsilon_{ca} = \phi_{ac} = 0$. Then the model in definition 1 reduces to a standard New Keynesian model, containing the so-called New Keynesian Phillips curve, $\widehat{\pi}_t = \chi\sigma\widehat{c}_t + \beta E_t\widehat{\pi}_{t+1} + \chi\mu_t$, and a forward looking aggregate demand constraint $\sigma\widehat{c}_t = \sigma\widehat{c}_{t+1} - E_t\widehat{R}_{t+1} + E_t\widehat{\pi}_{t+1}$. Applying a simple Taylor-type interest rate rule, $\widehat{R}_t = \rho_\pi\widehat{\pi}_t$, the model is known to exhibit a unique rational expectations equilibrium path converging to the steady state if and only if $\rho_\pi \in (1, \widetilde{\rho_\pi})$,[20] where $\widetilde{\rho_\pi} \equiv 1 + 2(1+\beta)\chi^{-1}$ takes extremely large values for reasonable price rigidities. Turning to the general version of the model presented in definition 1, we want to examine if activeness is still necessary for a saddle path configuration if consumption and, thus, inflation depends on the predetermined value of real balances, $H_{ca} > 0 \Leftrightarrow u_{ca} > 0$. To reveal the role of the latter for local stability, we allow the interest rate to be set contingent on realizations of the state variable, $\widehat{m}_{t-1}$:

$$\widehat{R}_t = \rho_\pi\widehat{\pi}_t + \rho_m\widehat{m}_{t-1}, \quad \rho_\pi, \rho_m \geq 0. \tag{17}$$

The rule (17) is sufficiently general for the purpose of this paper, as it provides a generic form for an interest rate feedback rule under discretionary optimization, which will be derived in the following subsection. Applying (17) and eliminating consumption, the model given in definition 1 can be reduced to the following 2×2 system in inflation and real

[18] A recent application of the inventory-theoretic approach can be found in Alvarez et al. (2003) for the analysis of the short-run behavior of money, velocity, and prices.

[19] Thus, the intertemporal substitution elasticity of consumption σ_c approximately equals σ (see 1).

[20] The proof for an isomorphic model can be found in Carlstrom and Fuerst (2001, section 5) or Woodford (2003a, proposition 4.5).

balances:

$$\phi_2\widehat{\pi}_t = \phi_1\widehat{m}_{t-1} + \beta E_t\widehat{\pi}_{t+1} + \chi\widehat{\mu}_t, \tag{18}$$

$$(\phi_3 - \rho_m)\,\widehat{m}_t - \phi_5 E_t\widehat{\pi}_{t+1} = \phi_3\widehat{m}_{t-1} - \phi_4\widehat{\pi}_t, \tag{19}$$

where $\phi_1 \equiv \chi(\sigma_c\frac{\varepsilon_{ca}+\sigma_a}{\sigma_c+\phi_{ac}} - \varepsilon_{ca}) + \chi\frac{\sigma_c}{\sigma_c+\phi_{ac}}z\rho_m$, $\phi_2 \equiv 1 + \phi_1 - \chi\frac{\sigma_c}{\sigma_c+\phi_{ac}}z(\rho_m + \rho_\pi)$, $\phi_3 \equiv \phi_1/\chi$, $\phi_4 \equiv \phi_3 - \frac{\sigma_c}{\sigma_c+\phi_{ac}}z(\rho_m + \rho_\pi)$, and $\phi_5 \equiv \phi_4 - (1 - \rho_\pi)$. As revealed by the two conditions (18) and (19), the model exhibits exactly one predetermined variable, $\widehat{m}_{t-1}$, implying that a stable and unique equilibrium path (saddle path stability) requires one stable and one unstable eigenvalue. Given that the model can be shown to exhibit at least one unstable eigenvalue, equilibrium indeterminacy cannot occur. Hence, interest rate policy faces the problem of avoiding the equilibrium path to become divergent (explosive) rather than to rule out self-fulfilling expectations. The conditions for saddle path stability are presented in the following proposition for a shopping time function satisfying (16).[21]

Proposition 1 *Suppose that the central bank sets the nominal interest rate according to (17) and that $\pi \in [1,\, 2\beta)$. Then the equilibrium path of the model satisfying (16) is saddle path stable if and only if*

$$\begin{array}{ll} i.) & \rho_\pi < 1 + \rho_m\Delta_1, \quad \textit{or} \\ ii.) & \rho_\pi > \widetilde{\rho_\pi} + \rho_m\Delta_2, \end{array}$$

where $\Delta_1 \equiv \frac{\chi((z-2)\sigma_c+\varepsilon_{ac})-(1-\beta)(1+\sigma_c)}{\chi(2\sigma_c-\varepsilon_{ac})}$ and $\Delta_2 \equiv \frac{(1+\beta)((2z-1)\sigma_c-1)+\chi((z-2)\sigma_c+\varepsilon_{ac})}{\chi(2\sigma_c-\varepsilon_{ac})} > \Delta_1$, and is for i. non-oscillatory and for ii. oscillatory. Otherwise, $\rho_\pi \in (1 + \rho_m\Delta_1,\, \widetilde{\rho_\pi} + \rho_m\Delta_2)$, the equilibrium path is explosive.

Proof. See appendix 6.1.

The result presented in proposition 1 reveals that passiveness, $\rho_\pi < 1$, ensures saddle path stability when the central bank does not react to changes in beginning-of-period real balances, $\rho_m = 0$. Hence for a simple interest rate policy satisfying $\widehat{R}_t = \rho_\pi\widehat{\pi}_t$ the Taylor-principle does not apply, as activeness, $\rho_\pi > 1$, which is necessary for a standard New Keynesian model ($H_{ca} = 0$), destabilizes the economy in our benchmark model with $H_{ca} < 0$. This finding, which relates to Dupor's (2001) result for a sticky price model with physical capital, reveals that the existence of a relevant state variable is able to revert the stability condition under interest rate policy. Correspondingly, the threshold $\widetilde{\rho_\pi}$ serves for the separable case as an upper bound for an inflation elasticity that ensures determinacy, whereas it provides a lower bound for the inflation elasticity in the benchmark case if $\rho_m = 0$ (see part *ii.* of proposition 1). In what follows, we focus on the case where the equilibrium path is non-oscillatory, which refers to part *i.* of proposition 1.

[21] Note that the proof refers to the case where disinflation and extremely high steady state inflation rates are disregarded, $\pi \in [1, 2\beta)$, for convenience.

To get an intuition for the result, consider first the standard case, where money enters the utility function in a separable way ($H_{ca} = 0$). In this case, activeness is necessary for determinacy. When inflation is expected to rise, interest rate policy has to raise the nominal and the real interest rate to rule out expectations to become self-fulfilling. Otherwise, when the nominal interest rate is raised by less than one for one with inflation, the real interest rate declines, inducing households to save less and to raise current consumption, while the negative consumption growth rate implies a subsequent return to the steady state. The rise in aggregate demand raises the required labor input and, thus, real marginal costs, such that inflation indeed rises and the initial change in expectations is self-fulfilling.

Now suppose that the end-of-period stock of money enters a shopping time function with $H_{ca} < 0$. In this case the aggregate demand constraint relates the growth rate of the predetermined variable, $\widehat{m}_{t-1} - \widehat{m}_t$, to the real interest rate (see 14). In contrast to the growth rate of consumption, the growth rate of real balances is negatively related to the latter, as the shadow price of wealth increases (decreases), by (6), with real balances (consumption). Further, a higher nominal interest rate reduces the level of real balances held by the households (see 15). Hence, an active interest rate policy, which leads – for higher inflation – to a rise in the nominal and the real interest rate, causes a decline in the level and in the growth rate of real balances. The associated decline in the marginal utility of consumption then induces households to substitute consumption in favor of leisure, such that labor becomes more costly. According to (13), the decline in real balances tends firms to raise their prices, such that the initial rise in inflation is reinforced. This, however, induces a further rise in the interest rates implying that the equilibrium path does not converge back to the steady state.

In order to avoid the economy to evolve on an explosive path, the central bank should either apply a passive interest rate policy, or it should raise the nominal interest rate with beginning-of-period real balances, when $\Delta_1 > 0$ (see part *i.* in proposition 1). In the former case, a rise in inflation leads to a rise in the nominal interest rate, which is – ceteris paribus – associated with a smaller level of real balances. The real interest rate, however, declines such that real balances grow, by (14), implying a convergence back to the steady state. Similarly, a central bank can stabilize the economy even for $\rho_\pi > 1$ by reacting positively to real balances, $\rho_m > 0$. In this case, the decline in the level and in the growth rate of real balances, calls the central bank to raise the nominal interest rate by, de facto, less than one for one with inflation. Concisely, if the central bank accounts for real balances when it sets its instrument, then the adjustment of the interest rates is less pronounced and avoids the history dependent evolution of real balances to become explosive.

For a positive feedback from beginning-of-period real balances to be actually stabilizing, the composite parameter Δ_1 has to be strictly positive. This condition is, however, hardly restrictive and is guaranteed if prices are not extremely rigid $\chi > \widetilde{\chi}$, where

$\widetilde{\chi} \equiv (1-\beta)\frac{1+\sigma_c}{(z-2)\sigma_c+\varepsilon_{ac}}$. Given that $\partial\widetilde{\chi}/\partial\varepsilon_{ca} < 0$, the lower bound $\widetilde{\chi}$ cannot exceed $\widetilde{\chi}_{\varepsilon_{ca}=0} = (1-\beta)\frac{1+\sigma}{(z-2)\sigma}$. Hence, the condition for χ presented in the following corollary is sufficient to ensure that $\Delta_1 > 0$ and that $\rho_m > 0$ raises the likelihood for (moderate) inflation elasticities (see part *i.* in proposition 1) to be associated with saddle path stability.

Corollary 1 *If $\chi > (1-\beta)\frac{1+\sigma}{(z-2)\sigma}$, then there exists for each interest rate rule with $\rho_\pi > 1$ one $\widetilde{\rho_m} > 0$ such that any interest rate rule with $\rho_m > \widetilde{\rho_m}$ ensures the equilibrium path of the model with (16) to be saddle path stable.*

Values for the fraction ϕ of non-optimizing price setters that are consistent with the estimates in Clarida and Gertler (1999), are clearly sufficient to satisfy the condition given in corollary 1 for reasonable values for the remaining parameters. For example, the parameter values $\beta = 0.9926$, $\varepsilon_{ca} = 0.01682$, $\sigma_c = 2$, $\pi = 1$, and $\phi = 0.8$, which will be introduced in detail in the subsequent section, imply $\chi = 0.0515$, that is apparently larger than $\widetilde{\chi} = 8.34 \times 10^{-5}$.[22] The coefficient Δ_1 in part *i.* of proposition 1 then equals 66.75, implying that an interest rate rule with $\rho_\pi = 1.5$ (2) is associated with a saddle stable equilibrium path if the coefficient on money satisfies $\rho_m > 0.0075$ (0.015). Hence, even very small responses of the interest rate to changes in real balances help to avoid instabilities, which arise for high inflation elasticities.

3.3 Interest rate policy under discretionary optimization

In this section we extend the analysis of interest rate policy to the case where the central bank sets the nominal interest rate in accordance with its aim to stabilize the economy. To allow for the derivation of analytical results and to facilitate comparisons to related studies, we apply a quadratic loss function as the objective of the central bank. As shown by Woodford (2003a), a loss function featuring variances of inflation and output can be obtained from a second-order Taylor expansion of the households' welfare function at a frictionless steady state, given that utility rises with consumption and leisure, as in (1), and that prices are not completely flexible, as implied by (11). However, for a loss function to exclusively depend on the quadratic deviations of inflation and output from their steady state values, there should be no further friction besides the one brought about by the nominal rigidity. Nonetheless, such a loss function can still be viewed as a suitable approximation for cases where additional frictions are sufficiently small.

In our framework there, in fact, exists an additional friction that stems from transactions of consumption goods, i.e., the shopping time assumption, which is obviously not negligible if $H_{ca} < 0$. As shown by Woodford (2003a, proposition 6.8) for an isomorphic money-in-the-utility function specification, the loss function then additionally features the variance

[22] For the upper bound $\widetilde{\chi}$ to become binding, the fraction ϕ has, in fact, to be larger than 0.992.

of the nominal interest rate as an argument:[23]

$$-E_0\frac{1}{2}\sum_{t=0}^{\infty}\beta^t L_t, \quad \text{where } L_t = \hat{\pi}_t^2 + \alpha\widehat{c}_t^2 + \varphi\widehat{R}_t^2, \quad \alpha > 0, \varphi \geq 0, \tag{20}$$

The loss function (20) thus implies that optimal monetary policy should not only minimize the variances of inflation and consumption (output), but also the variance of the nominal interest rate. This is due to the presence of the transactions friction, which implies that changes in the interest rate, i.e., the opportunity costs of money, cause a distortion on optimal asset holdings. Hence, in order to minimize this distortion, that adds to the nominal rigidity brought about by the staggered price setting, the central bank should aim at minimizing fluctuations in the nominal interest rate. As the model features two (non-negligible) distortions, monetary policy faces the trade-off between adjusting the nominal interest rate in a way that minimizes the distortion due to the price rigidity and to choose a smooth path for its instrument to account for the transactions friction induced by the shopping time assumption. The optimal adjustment of the nominal interest rate, thus, crucially relies on the weights in the loss function.

In order to reveal the role of money for a stabilizing interest rate policy, we aim at deriving a feedback rule for the nominal interest rate that implements the optimal plan, i.e., the equilibrium under the targeting rule. Thereby, we presume that the central bank seeks to maximize (20) on a day-to-day basis, i.e., under discretion, as for example in Woodford (2003b). We abstain from analyzing optimal monetary policy under commitment, as this induces a history dependence of monetary policy and, thus, of the optimal plan (see, e.g., Clarida et al., 1999), which might be a potential source for the equilibrium sequences to depend on the lagged realizations of endogenous variables, e.g., real balances. Instead, we focus on a discretionary optimization which is known to lead to a forward looking (Markov) equilibrium in the standard New Keynesian model. In contrast, the interest rate sequence under discretionary optimization is not independent of the history of real balances in the model presented in definition 1. The central bank considers – even under discretion – that consumption and, thus, inflation depend on the beginning-of-period stock of money when it maximizes (20). Hence, an optimal monetary policy under discretion does not ignore its impact on the endogenous state in the subsequent period. Concisely, the central bank's optimality condition for the trade-off between its targets, i.e., the so-called "targeting rule", is forward looking, while the solution for its instrument that implements the optimal plan under discretion evolves in a history dependent way. This result, which holds for any shopping time function satisfying (2) with $H_{ca} < 0$, is derived in appendix 6.2 and is

[23] The nominal interest rate enters the loss function, as real balances are eliminated in the second-oder Talyor expansion of households utility, $u(c_t, l_t, a_t)$, with the money demand function (15), which can be rewritten as $\widehat{a}_t = \frac{\phi_{ac}+\sigma_c}{\varepsilon_{ca}+\sigma_a}\widehat{c}_t - \frac{1/(1-\beta)}{\varepsilon_{ca}+\sigma_a}\widehat{R}_t$.

summarized in the following proposition.

Proposition 2 *Consider that the central bank maximizes (20) s.t. (13)-(15). If* $H_{ca} < 0$*, then the equilibrium path under discretionary optimization is characterized by a sequence for nominal interest rates that is, in general, not independent of the beginning-of-the-period real balances,* $\partial \widehat{R}_t / \partial \widehat{m}_{t-1} \neq 0 \; \forall t$.

To provide examples for interest rate feedback rules under discretionary optimization, we apply the shopping time function (16) and numerical values for the structural parameters. The elasticities σ_a and ϕ_{ac} of the utility function are already determined by the shopping time specification, $\sigma_a = 2$ and $\phi_{ac} = 1$ (see 16). For the intertemporal elasticity of substitution of consumption we apply the same value as for the intertemporal elasticity of substitution of money: $\sigma_c = 2$. The fraction of firms that adjust their prices in an optimal way is set equal to 0.2, which implies $\phi = 0.8$ in accordance with the estimates in Galí and Gertler (1999). The elasticity of substitution between the differentiated goods ϵ is set equal to 6 and the steady state inflation rate is set equal to 1 (see Woodford, 2003a).[24] The discount factor β and the steady state velocity, $v = y/m$, are set to match an average real interest rate of $1.03^{-0.25}$ and the velocity of $M2$ reported for the US in Christiano et al. (2003): $\beta = 0.9926$ and $v = 0.44$. These values, together with the steady state condition $R = \pi/\beta$, imply the steady state elasticity of the marginal utility of consumption with respect to real balances ε_{ca} to equal 1.68×10^{-2}. Further, we assume that the stochastic process for the cost push shock satisfies $\rho = 0.9$ (see, e.g., Clarida et al. 1999).

The last part of the calibration regards the weights in the loss function, α and φ. Following proposition 6.8 in Woodford (2003a), the weights, α and φ, can be identified with

$$\alpha^* = \chi \frac{\epsilon - 1}{\epsilon} \left(\sigma_c - \varepsilon_{ca} \frac{\sigma_c + \phi_{ac}}{\varepsilon_{ca} + \sigma_a} \right) \text{ and } \varphi^* = \chi \frac{\epsilon - 1}{\epsilon} \frac{1}{1 - \beta} \frac{1/v}{\varepsilon_{ca} + \sigma_a}. \tag{21}$$

According to (21), the weight on the variance of the nominal interest rate, $\varphi^* = 6.53$, is more than six times higher than the weight on the inflation variance and 77 times higher than the weight on the consumption variance, $\alpha^* = 0.085$. This relatively large value for φ^* is, however, not due to the real balances effect summarized by the elasticity ε_{ca} that exhibits a reasonably small value ($\varepsilon_{ca} = 0.0168$), sufficing the upper bound (0.02) suggested by Woodford (2003a). It, actually, stems from the model's property that the demand for money, which is applied to eliminate money from the utility function, depends on the *net* nominal interest rate. Hence, the respective coefficient on the gross nominal interest rate has to be premultiplied by $z = (1 - \beta)^{-1}$. It should further be noted that the weight φ^* is strictly larger than zero, even if there are no real balance effects, $H_{ca} = 0$, as the existence

[24] A constant steady state price level is required to minimize the steady state distortion brought about by the price rigidity. Similarly, it is assumed that the fiscal authority eliminates the average distortions due to monopolistic competition in the labor and in the goods market by a transfer system, which is not explicitly modelled (see, e.g., Clarida et al., 2002).

of transactions frictions does not rely on the non-separability of the utility function (see Woodford, 2003a). To reveal the impact of the weight φ on interest rate policy, we apply some artificial values (0, 0.5, 1) and φ^*, holding the remaining parameter values, including $\alpha = \alpha^*$, constant. The feedback rules for the interest rate are derived by eliminating the shock in the fundamental (minimum state variable) solution for the nominal interest rate under discretionary optimization, by applying the fundamental solution for the inflation rate (see appendix 6.2). The coefficients of the interest rate feedback rule, which then takes the form $\widehat{R}_t = \rho_\pi \widehat{\pi}_t + \rho_m \widehat{m}_{t-1}$, are presented in table 1.[25]

Table 1 Policy rule coefficients under discretionary optimization

	$\varphi = 0$	$\varphi = 0.5$	$\varphi = 1$	$\varphi = \varphi^*$
Inflation elasticity ρ_π	0.068	0.12	0.73	-0.014
Money elasticity ρ_m	0.00081	0.0015	0.0093	-0.00016

Note: The weight α is set equal to 0.085 and φ^* equals 6.53.

The feedback rules for $\varphi = 0$, 0.5, and 1 in table 1 satisfy condition *i.* in proposition 1 and are, thus, associated with a stable and unique equilibrium path. The one in the last column, also leads to a saddle path stable equilibrium, even though the policy rule coefficients ρ_π and ρ_m are negative.[26] The values reported in table 1 further reveal some remarkable properties of interest rate policy under discretionary optimization. As summarized in proposition 2, the money elasticity of the nominal interest rate ρ_m is not equal to zero. In the case where the weight in the loss function on the interest rate variance equals zero ($\varphi = 0$), both policy rule elasticities take very small values. Hence, even if we assume that there is no policy trade-off between the stabilization of inflation (and output) and the nominal interest rate, the response of the latter to changes in inflation is far from values, for example, suggested by the Taylor-principle. The reason for this result is that a rise in the nominal interest rate, which would reduce consumption and inflation for $u_{cm} = 0$, induces a decline in future real balances that raises households' demand for leisure and lowers their labor supply. Hence, the rise in the nominal interest rate tends to increase future real marginal costs and, by (13), future inflation, such that forward looking price setters are – ceteris paribus – willing to raise their prices.[27] If the nominal interest rate is lowered in response to the decline in real balances ($\rho_m > 0$), the latter effect on future prices is mitigated.

Raising the weight of the interest rate variance φ, further discloses that the policy rule

[25] Applying Woodford's (2003b) value for the coefficient φ on the interest rate variance in the loss function ($\varphi = 0.236$) leads to values for the policy rule elasticities equal to $\rho_\pi = 0.087$ and $\rho_m = 0.001$.

[26] It should be noted that the stable eigenvalue is positive for all feedback rules given in table 1, indicating that the equilibrium paths are non-oscillatory.

[27] This effect can easily be seen from the aggregate supply constraint (13), when real balances are eliminated with the money demand condition (15): $\widehat{\pi}_t = \chi \frac{\sigma_c \sigma_a - \varepsilon_{ca} \phi_{ac}}{\varepsilon_{ca} + \sigma_a} \widehat{c}_t + \chi \frac{\varepsilon_{ca} z}{\varepsilon_{ca} + \sigma_a} \widehat{R}_t + E_t \beta \widehat{\pi}_{t+1} + \chi \widehat{\mu}_t$.

elasticities change in a non-monotonic way. For moderate values for φ, these elasticities take higher values than for $\varphi = 0$. In these cases the central bank fights the cost push induced rise in inflation by a more pronounced rise in the nominal interest rate, and responds to the future decline in real marginal costs more aggressively, reducing the deviation of real balances from its steady state value. Raising the weight to the value φ^*, the responses to changes in inflation and in real balances again decrease and even take (very) small negative values,[28] as the central bank is now severely aware of minimizing the transactions distortion. In this case, a cost push shock, which raises the current inflation rate, induces the central bank to lower the nominal interest rate, which leads – ceteris paribus – to an increase in future real balances. The implied cost alleviating effect on future inflation counteracts the impact of the (autocorrelated) cost push shocks, such that strong future adjustments of the nominal interest rate are avoided.

4 An alternative timing of markets

In this section we consider an alternative version of the model which differs from the benchmark specification with regard to the timing of markets. In particular, we assume that the asset market opens before the goods market. Accordingly, households can adjust their holdings of money and bonds before they enter the goods market. Hence, the end-of-period stock of money enters the shopping time function, $A_t = M_t \Rightarrow H(c_t, M_t/P_t)$, as for example, in Brock (1974) or in Ljungqvist and Sargent (2000). It should be noted that this specification entails an inconsistency if the shopping time assumption is interpreted as a generalization of a cash-in-advance approach. Assuming the end-of-period stock of money to enter the shopping time function, rather accords to a 'cash-when-I'm-done' concept (see Carlstrom and Fuerst, 2001) than to the latter approach to money demand, given that M_t is the amount of money that is held by households when they *leave* the goods market. This assumption is, on the other hand, able to ensure that a positive amount of money is held even in a finite horizon framework (see, e.g., Buiter, 2002).

In accordance with the assumption that the asset market opens at the beginning of the period, government bonds are non-state contingent such that their payoff in period t now equals $R_{t-1}B_{t-1}$. This assumption can often be found in related contributions (see, e.g., Carlstrom and Fuerst, 2001) and ensures that the loss function (20) is an approximation of households' welfare (see Woodford, 2003a).[29] The first order conditions (6), (7), and (9) then change to $\lambda_t = c_t^{-\sigma} - H_c\left(c_t, m_t\right)$, $(R_t - 1)E_t\frac{\lambda_{t+1}}{\pi_{t+1}}\beta = -E_t H_a\left(c_{t+1}, m_{t+1}\right)$, and $\lambda_t = \beta R_t E_t \frac{\lambda_{t+1}}{\pi_{t+1}}$. Correspondingly, the steady state is characterized by $u_l = u_c\left(\epsilon - 1\right)/(\mu\epsilon)$,

[28] Negative coefficients on inflation are also found by Schmidt-Grohé and Uribe (2004) for optimized simple interest rate rules in a sticky price model with non-negligible transactions frictions and distortionary taxation.

[29] Otherwise, the future (instead of the current) interest rate would enter the loss function.

$\pi = R\beta$, and $u_c(R-1) = Ru_a$, and a rational expectations equilibrium of the linearized model is a set of sequences $\{\widehat{c}_t, \widehat{m}_t, \widehat{\pi}_t\}_{t=0}^{\infty}$ satisfying:

$$\widehat{\pi}_t = \chi\sigma_c\widehat{c}_t - \chi\varepsilon_{ca}\widehat{m}_t + \beta E_t\widehat{\pi}_{t+1} + \chi\mu_t, \tag{22}$$

$$\sigma_c\widehat{c}_t - \varepsilon_{ca}\widehat{m}_t = \sigma_c E_t\widehat{c}_{t+1} - \varepsilon_{ca}E_t\widehat{m}_{t+1} - \widehat{R}_t + E_t\widehat{\pi}_{t+1}, \tag{23}$$

$$(\varepsilon_{ca} + \sigma_a)\,\widehat{m}_t = (\phi_{ac} + \sigma_c)\,\widehat{c}_t + (1-z)\,\widehat{R}_t, \tag{24}$$

and the transversality condition, given a policy $\{\widehat{R}_t\}$ and a stationary exogenous process $\{\widehat{\mu}_t\}$. Apparently, beginning-of-period real balances do not enter the set of equilibrium conditions (22)-(24), indicating that the private sector behavior does not depend on a predetermined variable. Hence, for a simple (Taylor-type) interest rate rule satisfying $\widehat{R}_t = \rho_\pi\widehat{\pi}_t$, the model is entirely forward looking, implying that the equilibrium path is non-divergent and is unique if all eigenvalues are unstable. Activeness is then necessary and sufficient for local determinacy, as in the standard New Keynesian model (see Clarida et al., 1999, or Woodford, 2001). This result, which holds for any shopping time function satisfying (2), is summarized in the following proposition.

Proposition 3 *Suppose that interest rate policy satisfies $\widehat{R}_t = \rho_\pi\widehat{\pi}_t$. Then the equilibrium path of the model (22)-(24) is saddle path stable if and only if $\rho_\pi > 1$. Otherwise, the equilibrium path is indetermined.*

Proof. See appendix 6.3.

The predetermined stock of money is irrelevant in the model (22)-(24) and real balances adjust freely in a way consistent with a stable and unique equilibrium path. As a consequence, the intuition for the result summarized in proposition 3 accords to the line of arguments for the version satisfying $H_{ca} = 0$ (see page 12). As the private sector behavior lacks any history dependence, the equilibrium sequences are forward looking unless monetary policy is backward looking. Hence, when the central bank is non-inertial, as under discretionary optimization, lagged realizations of endogenous variables and, thus, beginning-of-period real balances are irrelevant for nominal interest rate setting. To be more precise, this is guaranteed if the fundamental solution of the equilibrium under discretionary optimization is the unique solution, which requires the equilibrium under the optimal plan to exhibit exclusively unstable eigenvalues. The latter property is, however, not always satisfied and applies if and only if the weight φ in the loss function is sufficiently small such that $\varphi < \overline{\varphi}$, where $\overline{\varphi} \equiv \chi + \frac{1-\beta}{\omega\chi} - \frac{\varepsilon_{ca}}{\omega}\frac{\beta/(1-\beta)}{\varepsilon_{ca}+\sigma_a}$ and $\omega \equiv \frac{(\sigma_c\sigma_a - \varepsilon_{ca}\phi_{ac})^2}{\alpha(\varepsilon_{ca}+\sigma_a)(\varepsilon_{ca}z+\sigma_a)}$. If the weight φ exceeds the threshold $\overline{\varphi}$, there also exist stable non-fundamental solutions featuring an extraneous state variable. This result is summarized in the following proposition for the case $H_{ca} \leq 0$ with $\sigma_c\sigma_a > \varepsilon_{ca}\phi_{ac}$, which is apparently ensured by $H_{ca} = 0$, but also for the specification (16).

Proposition 4 *Consider that the central bank maximizes (20) s.t. (22)-(24) and that* $\sigma_c\sigma_a > \varepsilon_{ca}\phi_{ac}$*. Then the equilibrium path under discretionary optimization is uniquely determined if and only if* $\varphi < \overline{\varphi}$*. In this case, the nominal interest rate is independent of the beginning-of-period real balances,* $\partial\widehat{R}_t/\partial\widehat{m}_{t-1} = 0$*. Otherwise* ($\varphi > \overline{\varphi}$)*, the fundamental solution is not the unique solution and there exist non-fundamental solutions consistent with the optimal plan. These solutions exhibit exactly one stable eigenvalue.*

Proof. See appendix 6.4.

It should be noted that the results presented in proposition 4 do not rely on the presence of real balance effects. For $\varphi < \overline{\varphi}$, the optimal plan under discretion exhibits only unstable eigenvalues and, thus, necessary implies the economy to evolve in a non-history dependent way. Hence, the state space representation of the unique solution for the nominal interest rate takes the form $\widehat{R}_t = R(\widehat{\mu}_t)$. Thus, the mere existence of non-negligible transactions frictions does not (necessarily) call for interest rate policy to respond to changes in real balances. A feedback rule, which relates the central bank's instrument to an endogenous variable, can obviously be written in terms of end-of-period real balances $\widehat{R}_t = R(\widehat{m}_t)$ or, equivalently, in terms of inflation $\widehat{R}_t = R(\widehat{\pi}_t)$. For monetary policy under discretionary optimization there is, thus, no need to consider money as an indicator for interest rate setting. Concisely, for our alternative specification money is irrelevant for monetary policy if $\varphi < \overline{\varphi}$.

Table 2 presents four numerical examples for interest rate feedback rules of the type $R(\widehat{\pi}_t)$, where the shopping time function satisfies (16) and the parameter values are taken from the previous section. The derivation of the feedback rules can be found in appendix 6.5. It should be noted that the welfare based value for the weight of the interest rate variance in the loss function now reads $\varphi_a^* = \chi\frac{\epsilon-1}{\epsilon}\beta\frac{1}{1-\beta}\frac{1/v}{\varepsilon_{ca}+\sigma_a} = \beta\varphi^*$, and is, thus, slightly smaller than in the case of the benchmark model.[30] Remarkably, the interest rate feedback rule is passive even if the central bank neglects the transactions friction, $\varphi = 0$. This result is due the real balance effect that induces a rise in the nominal interest rate to exert an upward pressure on firms' marginal costs, as it lowers households' willingness to hold money and, thus, to supply labor. Raising the value for the weight φ, leads to a monotonic decline in the inflation elasticity, as the central bank becomes increasingly unwilling to adjust the nominal interest rate, i.e., the opportunity costs of holding money instead of bonds. Thus, the four entirely forward looking interest rate feedback rules in table 2 fail to satisfy the condition for determinacy presented in proposition 3. These instrument rules are, on the one hand, consistent with the equilibrium under discretionary optimization. On the other hand, they cannot rule out non-fundamental solutions, which allow for changes in

[30] The difference stems from the property that money demand now satisfies $\widehat{a}_t = \frac{\phi_{ac}+\sigma_c}{\varepsilon_{ca}+\sigma_a}\widehat{c}_t - \beta\frac{1/(1-\beta)}{\varepsilon_{ca}+\sigma_a}\widehat{R}_t$, whereas the interest elasticity in the benchmark model reads $\partial\widehat{a}_t/\partial\widehat{R}_t = -\frac{1/(1-\beta)}{\varepsilon_{ca}+\sigma_a}$ (see 15).

expectations to induce the economy to deviate from the steady state on a stable equilibrium path and can, thus, be self-fulfilling, giving rise to endogenous fluctuations. This property (Equilibrium Indeterminacy) is summarized by the first entries in the last row of table 2.

Table 2 Optimal policy rules under discretion for the alternative model

	$\varphi = 0$	$\varphi = 0.5$	$\varphi = 1$	$\varphi = \varphi_a^*$	$\varphi = \varphi_a^*$
Inflation elasticity ρ_π	0.91	0.46	0.31	0.067	0.0211
Money elasticity ρ_m	—	—	—	—	0.0205
EI under the instrument/targeting rule	yes/–	yes/yes	yes/yes	yes/yes	–/yes

Note: The weight α is set equal to 0.085 and φ_a^* now equals 6.48. "EI" denotes equilibrium indeterminacy.

When the transactions friction is sufficiently large, such that $\varphi > \overline{\varphi}$, there is a potential role for beginning-of-period real balances in the conduct of interest rate policy. The respective condition can easily be satisfied, given that the threshold $\overline{\varphi}$ takes extremely small values, and does not require the existence of real balance effects. The parametrization presented in the previous section, for example, leads to $\overline{\varphi} = 0.0066$. If $\varphi > \overline{\varphi}$, then the equilibrium is, in fact, indetermined under the *targeting* rule, which is summarized by the second entries in the last row of table 2. The reason for this property, which contrasts findings for the standard New Keynesian model,[31] is that the optimal plan requires the central bank to fight changes in inflation only in a moderate way, to avoid exacerbating the distortions due to the transactions friction. This strategy gives rise to multiple solutions for the equilibrium under the targeting rule, i.e., it allows for non-fundamental solutions, which are consistent with the optimal plan under discretion (see proposition 4). These solutions, which are characterized by an arbitrary endogenous variable serving as an extraneous state variable, are feasible as there exists one stable eigenvalue if $\varphi > \overline{\varphi}$. If one assigns the stable eigenvalue to a non-predetermined variable, for example, $\widehat{\pi}_t$, non-fundamental shocks are known to be able to affect macroeconomic aggregates.[32] Hence, when the latter type of solution is not explicitly ruled out, for example, by applying McCallum's (1999) minimum state variable criterion, the economy under the targeting rule is prone to endogenous fluctuations. In contrast, assigning the stable eigenvalue to a lagged endogenous variable, which is predetermined by definition, the equilibrium sequences can evolve in a stable and history dependent way, ruling out endogenous fluctuations.

Suppose that the central bank does not only maximize (20) s.t. (22)-(24) in an discretionary way, but also aims at avoiding endogenous fluctuations. If its objective is characterized by $\varphi > \overline{\varphi}$, implying that a non-fundamental equilibrium path is consistent with

[31] Jensen (2002) shows that the equilibrium under the targeting rule for discretionary optimization is uniquely determined in the standard New Keynesian model (without transactions frictions).

[32] See, for example, Bernanke and Woodford (1997) for an application of this principle.

the optimal plan under discretion, it can easily avoid endogenous fluctuations, by applying a backward looking interest rate rule. An example for such a rule is (17), featuring beginning-of-period real balances and inflation as indicators. Hence, by setting the nominal interest rate in a history dependent way, the central bank can induce the economy to evolve according to a non-fundamental solution, which is, nonetheless, consistent with its optimal plan under discretion. The virtue of this strategy is that interest rate policy can ensure uniqueness of the equilibrium path, as beginning-of-period real balances serve as a predetermined endogenous state variable. The last column of table 2 presents a numerical example for such a rule, for the case where $\varphi = \varphi_a^*$. The derivation can be found in appendix 6.6. The conditions for the policy rule parameters ρ_π and ρ_m, which ensure saddle path stability of the model (22)-(24), are derived in appendix 6.7. They are satisfied for the values of the non-fundamental rule presented in the last column in table 2.

Corollary 2 *Consider that the central bank maximizes (20) with $\varphi > \overline{\varphi}$ s.t. (22)-(24). Then it can implement a unique equilibrium path, which is history dependent and consistent with the optimal plan under discretion, if it applies an interest rate rule satisfying $\widehat{R}_t = \rho_\pi \widehat{\pi}_t + \rho_m \widehat{m}_{t-1}$.*

Hence, even if households are entirely forward looking, there is a useful role for beginning-of-period real balances in the conduct of stabilizing interest rate policies, if the central bank does not ignore the transactions friction under discretionary optimization. It should be noted that this strategy can, in principle, also be applied for lagged realizations of other variables, serving as an indicator for interest rate policy, and also for alternative monetary policy instruments.

5 Conclusion

In this paper we have demonstrated that money can matter for interest rate policy, if transactions of goods are not completely frictionless. As households rely on money holdings to alleviate transactions costs that are measured in form of shopping time, the equilibrium sequences of consumption and inflation are not independent of real balances. If one assumes that the goods market opens before the asset market, then the stock of money held at the beginning of the period provides transaction services. As beginning-of-period real balances are predetermined, they serve – regardless of interest rate policy – as a relevant state variable, implying that a stabilizing interest rate policy should account for changes therein. In particular, it is shown that the likelihood for interest rate policy to be associated with a stable and unique equilibrium path rises, if interest rates are set contingent on beginning-of-period real balances. The latter are further shown to enter interest rate feedback rules under discretionary optimization of a welfare based central bank loss function. Concisely, if money constrains economic activities due to transaction frictions, then the central bank should – according to conventional stability and efficiency criteria – condition interest rates on the real value of monetary aggregates.

If the timing of markets is reversed and the stock of money held at the end of the period enters the shopping time function, money is not necessarily relevant for interest rate policy. In this case the Taylor-principle applies and the optimal plan under discretion can be implemented by a forward looking interest rate rule. If, however, an optimizing central bank accounts for the transactions friction, then the equilibrium under the optimal plan is likely to be indetermined. In this case, the central bank can induce equilibrium uniqueness and, thus, rule out the possibility of self-fulfilling expectations, if it sets the interest rate contingent on beginning-of-period real balances, leading to history dependent equilibrium sequences consistent with the optimal plan. Hence, even if a predetermined stock of money does not constrain current transactions, a central bank may nevertheless wish to use monetary aggregates as an indicator for interest rate policy in order to avoid endogenous fluctuations. In any case, it is the mere existence of a transactions friction rather than its size that can cause money to matter for interest rate policy.

6 Appendix

6.1 Proof of proposition 1

In order to derive the conditions for saddle path stability, the deterministic version of the model (18)-(19) is rewritten as

$$\begin{pmatrix} \widehat{m}_t \\ \widehat{\pi}_{t+1} \end{pmatrix} = \begin{pmatrix} -\frac{\phi_3}{-\phi_3+\rho_m} + \phi_1 \frac{\phi_5}{-\beta\phi_3+\beta\rho_m} & \frac{\phi_4}{-\phi_3+\rho_m} - \phi_2 \frac{\phi_5}{-\beta\phi_3+\beta\rho_m} \\ -\frac{1}{\beta}\phi_1 & \frac{1}{\beta}\phi_2 \end{pmatrix} \begin{pmatrix} \widehat{m}_{t-1} \\ \widehat{\pi}_t \end{pmatrix} = \mathbf{A} \begin{pmatrix} \widehat{m}_{t-1} \\ \widehat{\pi}_t \end{pmatrix},$$

where the characteristic polynomial of $\mathbf{A}$ is given by:

$$F(X) = X^2 + \frac{\phi_1\phi_5 - \phi_2\phi_3 - \beta\phi_3 + \phi_2\rho_m}{(\phi_3 - \rho_m)\,\beta} X + \frac{\phi_2\phi_3 - \phi_1\phi_4}{(\phi_3 - \rho_m)\,\beta}.$$

Applying the definitions of the composite parameters ϕ_i for $i \in \{1, 2, 3, 4, 5\}$ and $\sigma_a = 2$ and $\phi_{ac} = 1$ (see 16), the value of $F(X)$ at $X = 0$ reads:

$$F(0) = \det(\mathbf{A}) = (\Theta\beta)^{-1} \left(z\sigma_c\rho_m + 2\sigma_c - \varepsilon_{ac}\right), \text{ where } \Theta \equiv \rho_m \left((z-1)\,\sigma_c - 1\right) + 2\sigma_c - \varepsilon_{ac}.$$

Hence, $\Theta > 0$ and $F(0) > 0$ is ensured for $z > 2 \Leftrightarrow \pi < 2\beta$, provided that $\sigma_c = \sigma(1+\varepsilon_{ac})$ and $\sigma \geq 1$. Moreover, the determinant of $\mathbf{A}$ is then strictly larger than one, $\det(\mathbf{A}) = F(0) > 1$, indicating that there is at least one unstable root and that indeterminacy (two stable roots) cannot occur. The value of $F(X)$ at $X = 1$ is further given by

$$F(1) = (\Theta\beta)^{-1} \left[(\rho_\pi - 1)\,\chi\,(2\sigma_c - \varepsilon_{ac}) - \rho_m \left(\chi \left((z-2)\,\sigma_c + \varepsilon_{ac}\right) - (1-\beta)\,(1+\sigma_c)\right)\right],$$

implying that $F(1) < 0$ and that is a stable positive eigenvalue, if and only if

$$\rho_\pi < 1 + \rho_m\Delta_1, \text{ where } \Delta_1 \equiv \frac{\chi\left((z-2)\,\sigma_c + \varepsilon_{ac}\right) - (1-\beta)\,(1+\sigma_c)}{\chi\,(2\sigma_c - \varepsilon_{ac})}.$$

To disclose the conditions for the existence of a negative stable root, we further examine the value of $F(X)$ at $X = 1$, which is given by

$$\begin{aligned} F(-1) = (\Theta\beta)^{-1} \; &\{[2\,(1+\beta) - \chi\,(\rho_\pi - 1)]\,(2\sigma_c - \varepsilon_{ac}) \\ &+ \rho_m \left[(1+\beta)\left((2z-1)\,\sigma_c - 1\right) + \chi\left((z-2)\,\sigma_c + \varepsilon_{ac}\right)\right]\}. \end{aligned}$$

implying that $F(-1) < 0$ and that there is a stable negative eigenvalue if and only if

$$\rho_\pi > 1 + \frac{2\,(1+\beta)}{\chi} + \rho_m\Delta_2, \text{ where } \Delta_2 \equiv \frac{(1+\beta)\left((2z-1)\,\sigma_c - 1\right) + \chi\left((z-2)\,\sigma_c + \varepsilon_{ac}\right)}{\chi\,(2\sigma_c - \varepsilon_{ac})} > 0.$$

Given that $\Delta_1 < \Delta_2$, we can conclude that the equilibrium is saddle path stable if and only if $\rho_\pi < 1 + \rho_m\Delta_1$ or $\rho_\pi > \widetilde{\rho_\pi} + \rho_m\Delta_2$, where $\widetilde{\rho_\pi} \equiv 1 + \frac{2(1+\beta)}{\chi}$, and is non-oscillatory if $\rho_\pi < 1 + \rho_m\Delta_1$ and oscillatory if $\rho_\pi > \widetilde{\rho_\pi} + \rho_m\Delta_2$. Otherwise, $\rho_\pi \in (1 + \rho_m\Delta_1, \widetilde{\rho_\pi} + \rho_m\Delta_2)$, there exist two unstable roots indicating an explosive equilibrium path. ■

6.2 Interest rate feedback rules under discretionary optimization

The policy problem for the benchmark model (see definition 1) can be summarized as

$$\max_{\widehat{\pi}_t,\widehat{c}_t,\widehat{m}_t,\widehat{R}_t} -E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2}\left(\hat{\pi}_t^2 + \alpha \widehat{c}_t^2 + \varphi \widehat{R}_t^2\right) \right.$$
$$+ \phi_{1t}\left[\left(1-\chi\varepsilon_{ca}\right)\widehat{\pi}_t - \chi\sigma_c\widehat{c}_t - \beta E_t\widehat{\pi}_{t+1} + \chi\varepsilon_{ca}\widehat{m}_{t-1} - \chi\widehat{\mu}_t\right]$$
$$+ \phi_{2t}\left[\sigma_c\widehat{c}_t - \varepsilon_{ca}\widehat{m}_{t-1} + \varepsilon_{ca}\widehat{\pi}_t - \sigma_c E_t\widehat{c}_{t+1} + \varepsilon_{ca}\widehat{m}_t - \left(\varepsilon_{ca}+1\right)E_t\widehat{\pi}_{t+1} + E_t\widehat{R}_{t+1}\right]$$
$$\left. + \phi_{3t}\left[\left(\varepsilon_{ca}+\sigma_a\right)\widehat{m}_{t-1} + z\widehat{R}_t - \left(\sigma_c+\phi_{ac}\right)\widehat{c}_t - \left(\varepsilon_{ca}+\sigma_a\right)\widehat{\pi}_t\right]\right\}.$$

The first order conditions under discretionary optimization are given by

$$0 = \hat{\pi}_t + \left(1-\chi\varepsilon_{ca}\right)\phi_{1t} + \varepsilon_{ca}\phi_{2t} - \left(\varepsilon_{ca}+\sigma_a\right)\phi_{3t}, \tag{25}$$
$$0 = \alpha\widehat{c}_t - \chi\sigma_c\phi_{1t} + \sigma_c\phi_{2t} - \left(\sigma_c+\phi_{ac}\right)\phi_{3t}, \tag{26}$$
$$0 = \varphi\widehat{R}_t + z\phi_{3t}, \tag{27}$$
$$0 = \beta\chi\varepsilon_{ca}E_t\phi_{1t+1} - \beta\varepsilon_{ca}E_t\phi_{2t+1} + \varepsilon_{ca}\phi_{2t} + \beta\left(\varepsilon_{ca}+\sigma_a\right)E_t\phi_{3t+1}. \tag{28}$$

Eliminating the multiplier ϕ_{3t} with (27) in (25) and in (26) leads to the following conditions for the multiplier ϕ_{2t} and ϕ_{1t}

$$\phi_{2t} \equiv \psi_1\widehat{\pi}_t + \psi_2\widehat{c}_t + \psi_3\widehat{R}_t, \tag{29}$$
$$\phi_{1t} \equiv \psi_4\widehat{\pi}_t + \psi_5\widehat{c}_t + \psi_6\widehat{R}_t, \tag{30}$$

where $\psi_1 \equiv -\left(\varepsilon_{ac} + \chi^{-1}\left(1-\chi\varepsilon_{ac}\right)\right)^{-1}$, $\psi_2 \equiv \alpha\left(\chi\sigma_c\right)^{-1}\left(1-\chi\varepsilon_{ac}\right)\psi_1$,

$$\psi_3 \equiv \left[z^{-1}\varphi\left(\sigma_a+\varepsilon_{ac}\right) + \left(z\chi\sigma_c\right)^{-1}\varphi\left(\sigma_c+\phi_{ac}\right)\left(1-\chi\varepsilon_{ac}\right)\right]\psi_1,$$
$$\psi_4 \equiv \chi^{-1}\psi_1,\ \psi_5 = \alpha\left(\chi\sigma_c\right)^{-1} + \chi^{-1}\psi_2,\ \ \psi_6 = \left(z\chi\sigma_c\right)^{-1}\varphi\left(\sigma_c+\phi_{ac}\right) + \chi^{-1}\psi_3.$$

With (27), (29) and (30), condition (28) can then be written as a targeting rule

$$0 = \beta\varepsilon_{ca}\left(\chi\psi_4-\psi_1\right)E_t\widehat{\pi}_{t+1} + \beta\varepsilon_{ca}\left(\chi\psi_5-\psi_2\right)E_t\widehat{c}_{t+1}, \tag{31}$$
$$+ \beta\left(\varepsilon_{ca}\chi\psi_6 - \varepsilon_{ca}\psi_3 - \left(\varepsilon_{ca}+\sigma_a\right)z^{-1}\varphi\right)E_t\widehat{R}_{t+1} + \varepsilon_{ca}\psi_1\widehat{\pi}_t + \varepsilon_{ca}\psi_2\widehat{c}_t + \varepsilon_{ca}\psi_3\widehat{R}_t.$$

Using the money demand condition (15), consumption is then eliminated in (31), leading to the following condition of the optimal plan under discretion

$$0 = \delta_1 E_t\widehat{\pi}_{t+1} + \delta_2\widehat{m}_t + \delta_3 E_t\widehat{R}_{t+1} - \delta_4\widehat{\pi}_t - \delta_5\widehat{m}_{t-1} - \delta_6\widehat{R}_t, \tag{32}$$

where $\delta_1 \equiv -\beta\varepsilon_{ca}\left(\chi\psi_5-\psi_2\right)\dfrac{\varepsilon_{ca}+\sigma_a}{\sigma_c+\phi_{ac}}$, $\delta_2 \equiv -\delta_1$,

$$\delta_3 \equiv \beta\left(\varepsilon_{ca}\left(\chi\psi_5-\psi_2\right)z\left(\sigma_c+\phi_{ac}\right)^{-1} + \varepsilon_{ca}\chi\psi_6 - \varepsilon_{ca}\psi_3 - \left(\varepsilon_{ca}+\sigma_a\right)z^{-1}\varphi\right)$$
$$\delta_4 \equiv -\varepsilon_{ca}\left(\psi_1 - \psi_2\frac{\varepsilon_{ca}+\sigma_a}{\sigma_c+\phi_{ac}}\right),\ \ \delta_5 \equiv -\varepsilon_{ca}\psi_2\frac{\varepsilon_{ca}+\sigma_a}{\sigma_c+\phi_{ac}},\ \delta_6 \equiv -\left(\varepsilon_{ca}\psi_2\frac{z}{\sigma_c+\phi_{ac}} + \varepsilon_{ca}\psi_3\right).$$

Applying (13) and (14), where consumption is eliminated by (15), the equilibrium conditions under discretionary optimization can, therefore, be summarized by

$$\delta_1 E_t\widehat{\pi}_{t+1} + \delta_2\widehat{m}_t + \delta_3 E_t\widehat{R}_{t+1} = \delta_4\widehat{\pi}_t + \delta_5\widehat{m}_{t-1} + \delta_6\widehat{R}_t, \quad (33)$$

$$\beta E_t\widehat{\pi}_{t+1} = (1-\chi\delta_7)\,\widehat{\pi}_t + \chi\delta_7\widehat{m}_{t-1} - \chi\delta_9\widehat{R}_t - \chi\mu_t, \quad (34)$$

$$(\delta_7+1)\,E_t\widehat{\pi}_{t+1} - \delta_7\widehat{m}_t + (\delta_9-1)\,E_t\widehat{R}_{t+1} = \delta_7\widehat{\pi}_t - \delta_7\widehat{m}_{t-1} + \delta_9\widehat{R}_t, \quad (35)$$

where $\delta_7 \equiv \varepsilon_{ca} - \sigma_c\dfrac{\varepsilon_{ca}+\sigma_a}{\sigma_c+\phi_{ac}}$ and $\delta_9 \equiv \sigma_c\dfrac{z}{\sigma_c+\phi_{ac}}$.

Apparently, the system (33)-(35) exhibits a backward looking element as the current realizations of consumption and inflation are not independent of $\widehat{m}_{t-1}$. Hence, the fundamental (minimum state) solution of the optimal allocation features the eigenvalue, δ_m, and reads

$$\widehat{m}_t = \delta_m\widehat{m}_{t-1} + \delta_{m\mu}\mu_t, \quad (36)$$

$$\widehat{\pi}_t = \delta_{\pi m}\widehat{m}_{t-1} + \delta_{\pi\mu}\mu_t, \quad (37)$$

$$\widehat{R}_t = \delta_{Rm}\widehat{m}_{t-1} + \delta_{R\mu}\mu_t, \quad (38)$$

where μ_t satisfies $E_t\mu_{t+1} = \rho\mu_t$. We proceed by applying the solution (36)-(38) for (33)-(35), to identify the structural composition of the undetermined coefficients. This leads to the following conditions:

$$0 = \delta_{m\mu}\left(\delta_2 + \delta_1\delta_{\pi m} + \delta_3\delta_{Rm}\right) + \delta_{\pi\mu}\left(\delta_1\rho - \delta_4\right) + \delta_{R\mu}\left(\delta_3\rho - \delta_6\right), \quad (39)$$

$$0 = \chi + \chi\delta_9\delta_{R\mu} + \beta\delta_{\pi m}\delta_{m\mu} + \delta_{\pi\mu}\left(\beta\rho + \chi\delta_7 - 1\right), \quad (40)$$

$$0 = \delta_{\pi\mu}\left(\rho\left(\delta_7+1\right) - \delta_7\right) + \delta_{R\mu}\left(\rho\left(\delta_9-1\right) - \delta_9\right) + \delta_{m\mu}\left(\delta_{\pi m}\left(\delta_7+1\right) - \delta_7 + \delta_{Rm}\left(\delta_9-1\right)\right), \quad (41)$$

$$0 = \delta_2\delta_m - \delta_5 + \delta_{\pi m}\left(\delta_1\delta_m - \delta_4\right) + \delta_{Rm}\left(\delta_3\delta_m - \delta_6\right), \quad (42)$$

$$0 = \chi\delta_9\delta_{Rm} - \chi\delta_7 + \delta_{\pi m}\left(\beta\delta_m - 1 + \chi\delta_7\right), \quad (43)$$

$$0 = \delta_7\left(1-\delta_m\right) + \delta_{\pi m}\left(\delta_m\left(\delta_7+1\right) - \delta_7\right) + \delta_{Rm}\left(\delta_m\left(\delta_9-1\right) - \delta_9\right), \quad (44)$$

The three conditions (42)-(44), are sufficient to characterize the deterministic part of the solution, δ_m, δ_{Rm}, and $\delta_{\pi m}$. Precisely, with (42)-(44) we can express δ_{Rm} and $\delta_{\pi m}$ as functions of the eigenvalue δ_m :

$$\delta_{\pi m} = -\frac{\delta_2\delta_m - \delta_5 + \frac{\delta_7}{\delta_9}\left(\delta_3\delta_m - \delta_6\right)}{\delta_1\delta_m - \delta_4 + \frac{1}{\chi\delta_9}\left(\delta_3\delta_m - \delta_6\right)\left(1 - \beta\delta_m - \chi\delta_7\right)}, \quad (45)$$

$$\delta_{Rm} = \frac{\delta_5 - \delta_2\delta_m - \chi\delta_4\delta_7 - \chi\delta_5\delta_7 - \beta\delta_5\delta_m + \chi\delta_1\delta_7\delta_m + \chi\delta_2\delta_7\delta_m + \beta\delta_2\delta_m^2}{\delta_3\delta_m - \delta_6 - \chi\delta_4\delta_9 + \chi\delta_6\delta_7 + \beta\delta_6\delta_m + \chi\delta_1\delta_9\delta_m - \chi\delta_3\delta_7\delta_m - \beta\delta_3\delta_m^2}. \quad (46)$$

Note that inserting (45) and (46) in (44) delivers a cubic equation in δ_m, the characteristic equation, which is only numerically examined in this paper.

Turning to the remaining coefficients on the exogenous state, $\delta_{m\mu}$, $\delta_{\pi\mu}$, and $\delta_{R\mu}$, we have to apply the conditions (39)-(41). At first, the coefficient $\delta_{m\mu}$, which is not of further interest for our analysis, is eliminated by (39) in (40) and in (41). The coefficients $\delta_{\pi\mu}$ and $\delta_{R\mu}$ can then be written as functions of δ_m, δ_{Rm}, and $\delta_{\pi m}$:

$$\delta_{R\mu} = -\chi\Omega_1^{-1}, \tag{47}$$

$$\delta_{\pi\mu} = \Omega_2\chi\Omega_1^{-1}, \tag{48}$$

$$\text{where } \Omega_1 \equiv \left(\chi\delta_9 - \beta\delta_{\pi m}\zeta_1^{-1}\left(\rho\delta_3 - \delta_6\right)\right) - \Omega_2\left(\beta\rho + \chi\delta_7 - \beta\delta_{\pi m}\zeta_1^{-1}\left(\rho\delta_1 - \delta_4\right) - 1\right),$$
$$\Omega_2 \equiv \left(\rho\left(\delta_9 - 1\right) - \delta_9 - \zeta_1^{-1}\left(\rho\delta_3 - \delta_6\right)\zeta_2\right)\left(\rho\left(\delta_7 + 1\right) - \delta_7 - \zeta_1^{-1}\left(\rho\delta_1 - \delta_4\right)\zeta_2\right)^{-1},$$
$$\zeta_1 \equiv \delta_2 + \delta_1\delta_{\pi m} + \delta_3\delta_{Rm},\ \zeta_2 \equiv \delta_{\pi m}\left(\delta_7 + 1\right) - \delta_7 + \delta_{Rm}\left(\delta_9 - 1\right).$$

Thus, the fundamental solution for the interest rate $\widehat{R}_t = \delta_{Rm}\widehat{m}_{t-1} + \delta_{R\mu}\mu_t$ is determined by (46) and (47), and the solution for inflation $\widehat{\pi}_t = \delta_{\pi m}\widehat{m}_{t-1} + \delta_{\pi\mu}\mu_t$ by (45) and (48). We can then derive an interest rate feedback rule of the form $\widehat{R}_t = \rho_\pi\widehat{\pi}_t + \rho_m\widehat{m}_{t-1}$ by eliminating the exogenous state with $\mu_t = \frac{1}{\delta_{\pi\mu}}\widehat{\pi}_t - \frac{\delta_{\pi m}}{\delta_{\pi\mu}}\widehat{m}_{t-1}$ in the fundamental solution for the interest rate. The feedback rule is, thus, given by

$$\widehat{R}_t = \rho_\pi\widehat{\pi}_t + \rho_m\widehat{m}_{t-1}, \tag{49}$$
$$\text{where } \rho_\pi = \delta_{R\mu}/\delta_{\pi\mu} \text{ and } \rho_m = \delta_{Rm} - \delta_{R\mu}\delta_{\pi m}/\delta_{\pi\mu}.$$

To obtain numerical examples for the feedback rule (49), we set the coefficient δ_m equal to a stable eigenvalue of the matrix $\mathbf{A}$ (see 33-35):

$$\mathbf{A} = \begin{pmatrix} \delta_1 & \delta_2 & \delta_3 \\ \beta & 0 & 0 \\ \delta_7 + 1 & -\delta_7 & \delta_9 - 1 \end{pmatrix}^{-1} \begin{pmatrix} \delta_4 & \delta_5 & \delta_6 \\ 1 - \chi\delta_7 & \chi\delta_7 & -\chi\delta_9 \\ \delta_7 & -\delta_7 & \delta_9 \end{pmatrix},$$

where a saddle path configuration, i.e., stability and uniqueness of the fundamental solution (36)-(38), requires $\mathbf{A}$ to exhibit exactly one stable eigenvalue. Then, one can derive values for δ_{Rm} and $\delta_{\pi m}$ by (45) and (46), and for $\delta_{\pi\mu}$ and $\delta_{R\mu}$ by (48) and (47). With these coefficients, one obtains the weights for the feedback rule given in (49).

6.3 Proof of proposition 3

Eliminating inflation with the money demand condition (24) and the nominal interest rate with $\widehat{R}_t = \rho_\pi\widehat{\pi}_t$, the model (22)-(24) can be reduced to the following 2×2 system in real balances and consumption:

$$\beta E_t\widehat{\pi}_{t+1} = \left(1 - \chi\varpi_b\rho_\pi\right)\widehat{\pi}_t - \chi\left(\sigma_c - \varpi_a\right)\widehat{c}_t - \chi\widehat{\mu}_t, \tag{50}$$

$$\left(1 + \varpi_b\rho_\pi\right)E_t\widehat{\pi}_{t+1} + \left(\sigma_c - \varpi_a\right)E_t\widehat{c}_{t+1} = \left(1 + \varpi_b\right)\rho_\pi\widehat{\pi}_t + \left(\sigma_c - \varpi_a\right)\widehat{c}_t, \tag{51}$$

where $\varpi_a \equiv \varepsilon_{ca}\frac{\phi_{ac}+\sigma_c}{\varepsilon_{ca}+\sigma_a}$ and $\varpi_b \equiv \varepsilon_{ca}\frac{z-1}{\varepsilon_{ca}+\sigma_a}$. The model (50)-(51) exhibits no predetermined variable, implying that saddle path stability requires two unstable eigenvalues. To derive the respective conditions, the deterministic version of the model (50)-(51) is rewritten as

$$\begin{pmatrix}\widehat{\pi}_{t+1}\\ \widehat{c}_{t+1}\end{pmatrix} = \begin{pmatrix} -\frac{-1+\chi\varpi_b\rho_\pi}{\beta} & -\chi\frac{\sigma_c-\varpi_a}{\beta} \\ \frac{-1+\chi\varpi_b\rho_\pi-\varpi_b\rho_\pi+\varpi_b^2\rho_\pi^2\chi+\beta\rho_\pi+\beta\rho_\pi\varpi_b}{\beta(\sigma_c-\varpi_a)} & \frac{\chi+\chi\varpi_b\rho_\pi+\beta}{\beta}\end{pmatrix}\begin{pmatrix}\widehat{\pi}_t\\ \widehat{c}_t\end{pmatrix} = \mathbf{A}\begin{pmatrix}\widehat{\pi}_t\\ \widehat{c}_t\end{pmatrix}.$$

The characteristic polynomial of $\mathbf{A}$ can be simplified to $F(X) = X^2 - \beta^{-1}(\beta+1+\chi)X + \beta^{-1}(1+\chi\rho_\pi)$. The determinant of $\mathbf{A}$ reads $\det(\mathbf{A}) = \beta^{-1}(1+\chi\rho_\pi)$ and is, thus, strictly larger than one for $\rho_\pi \geq 0$. The trace, $\mathrm{trace}(\mathbf{A}) = \beta^{-1}(\beta+1+\chi)$, is strictly positive, and

$$\begin{aligned}\det(\mathbf{A}) - \mathrm{trace}(\mathbf{A}) &= -\beta^{-1}(-\chi\rho_\pi+\chi+\beta),\\ \det(\mathbf{A}) + \mathrm{trace}(\mathbf{A}) &= \beta^{-1}(2+\chi\rho_\pi+\chi+\beta) > 0.\end{aligned}$$

Hence, $\det(\mathbf{A}) - \mathrm{trace}(\mathbf{A}) > -1 \Leftrightarrow 1 < \rho_\pi$ and $\det(\mathbf{A}) + \mathrm{trace}(\mathbf{A}) > -1$, revealing that activeness is necessary and sufficient for saddle path stability. ■

6.4 Proof of proposition 4

To establish the claims made in the proposition, we first characterize the optimal plan under discretionary optimization for the model (22)-(24). The policy problem reads

$$\begin{aligned}\max_{\widehat{\pi}_t,\widehat{c}_t,\widehat{m}_t,\widehat{R}_t} -E_0\sum_{t=0}^{\infty}\beta^t\Big\{\frac{1}{2}\left(\hat{\pi}_t^2+\alpha\widehat{c}_t^2+\varphi\widehat{R}_t^2\right) + \phi_{1t}\left[\widehat{\pi}_t-\chi\sigma_c\widehat{c}_t+\chi\varepsilon_{ca}\widehat{m}_t-\beta E_t\widehat{\pi}_{t+1}-\chi\widehat{\mu}_t\right]\\ +\phi_{2t}\left[\sigma_c\widehat{c}_t-\varepsilon_a\widehat{m}_t-\sigma_c\widehat{c}_{t+1}+\varepsilon_{ca}E_t\widehat{m}_{t+1}+\widehat{R}_t-E_t\widehat{\pi}_{t+1}\right]\\ +\phi_{3t}\left[(\varepsilon_{ca}+\sigma_a)\widehat{m}_t-(\phi_{ac}+\sigma_c)\widehat{c}_t+(z-1)\widehat{R}_t\right]\Big\}.\end{aligned}$$

The first order conditions under discretionary optimization are, therefore, given by

$$\widehat{\pi}_t + \phi_{1t} = 0, \tag{52}$$

$$\alpha\widehat{c}_t - \chi\sigma_c\phi_{1t} + \sigma_c\phi_{2t} - (\phi_{ac}+\sigma_c)\phi_{3t} = 0, \tag{53}$$

$$\varphi\widehat{R}_t + \phi_{2t} + \phi_{3t}(z-1) = 0, \tag{54}$$

$$\chi\varepsilon_{ca}\phi_{1t} - \varepsilon_{ca}\phi_{2t} + (\varepsilon_{ca}+\sigma_a)\phi_{3t} = 0, \tag{55}$$

Eliminating the multiplier ϕ_{1t} with (52) in (53) and isolating the multiplier ϕ_{2t} and multiplier ϕ_{3t}, leads to

$$\phi_{3t} = \frac{\alpha}{\phi_{ac}+\sigma_c z}\widehat{c}_t + \frac{\chi\sigma_c}{\phi_{ac}+\sigma_c z}\hat{\pi}_t - \frac{\sigma_c\varphi}{\phi_{ac}+\sigma_c z}\widehat{R}_t, \tag{56}$$

$$\phi_{2t} = -\frac{z-1}{\phi_{ac}+\sigma_c z}\alpha\widehat{c}_t - \frac{z-1}{\phi_{ac}+\sigma_c z}\chi\sigma_c\hat{\pi}_t - \frac{\sigma_c+\phi_{ac}}{\phi_{ac}+\sigma_c z}\varphi\widehat{R}_t. \tag{57}$$

Applying (56) and (57), for ϕ_{3t} and ϕ_{2t} in (55) delivers the targeting rule

$$0 = \alpha \widehat{c}_t + \chi\kappa\hat{\pi}_t - \varphi\kappa\widehat{R}_t, \quad \text{where } \kappa \equiv \frac{\sigma_c\sigma_a - \varepsilon_{ca}\phi_{ac}}{\varepsilon_{ca}z + \sigma_a}. \tag{58}$$

Hence, for $\varepsilon_{ca} = \phi_{ac} = \varphi = 0$, the targeting rule equals to one derived in Clarida et al. (1999). Eliminating money with $\widehat{m}_t = \frac{\phi_{ac}+\sigma_c}{\varepsilon_{ca}+\sigma_a}\widehat{c}_t - \frac{z-1}{\varepsilon_{ca}+\sigma_a}\widehat{R}_t$ and consumption with $\widehat{c}_t = \frac{\kappa\varphi}{\alpha}\widehat{R}_t - \frac{\chi\kappa}{\alpha}\hat{\pi}_t$, the equilibrium sequences for the nominal interest rate and inflation under discretionary optimization satisfy

$$\beta E_t\hat{\pi}_{t+1} = (1+\chi\gamma_1)\,\hat{\pi}_t - \gamma_2\chi\widehat{R}_t - \chi\mu_t, \tag{59}$$

$$(\gamma_1 - 1)\,E_t\hat{\pi}_{t+1} - \gamma_2 E_t\widehat{R}_{t+1} = \gamma_1\hat{\pi}_t - (\gamma_2+1)\,\widehat{R}_t, \tag{60}$$

$$\text{where } \gamma_1 \equiv \frac{\chi\kappa}{\alpha}\frac{\varepsilon_{ca}z+\sigma_a}{\varepsilon_{ca}+\sigma_a} \text{ and } \gamma_2 \equiv \frac{\varphi\kappa}{\alpha}\frac{\varepsilon_{ca}z+\sigma_a}{\varepsilon_{ca}+\sigma_a} + \varepsilon_{ca}\frac{z-1}{\varepsilon_{ca}+\sigma_a}.$$

Suppose in what follows that $\sigma_c\sigma_a > \varepsilon_{ca}\phi_{ac}$, which is for example satisfied for (16), implying $\sigma_c\sigma_a - \varepsilon_{ca}\phi_{ac} = (2\sigma-1)\,\varepsilon_{ca} + 2\sigma > 0$. Then the composite parameters κ, γ_1, and γ_2 are strictly positive. The deterministic version of the equilibrium conditions under discretionary optimization (59) and (60) reads

$$\begin{pmatrix} \widehat{\pi}_{t+1} \\ \widehat{R}_{t+1} \end{pmatrix} = \begin{pmatrix} \frac{1}{\beta}(1+\chi\gamma_1) & -\frac{1}{\beta}\gamma_2\chi \\ \frac{\gamma_1+\chi\gamma_1^2-1-\chi\gamma_1-\gamma_1\beta}{\beta\gamma_2} & -\frac{\gamma_2\chi\gamma_1-\gamma_2\chi-\beta\gamma_2-\beta}{\beta\gamma_2} \end{pmatrix} \begin{pmatrix} \widehat{\pi}_t \\ \widehat{R}_t \end{pmatrix} = \mathbf{A} \begin{pmatrix} \widehat{\pi}_t \\ \widehat{R}_t \end{pmatrix},$$

where the characteristic polynomial of $\mathbf{A}$ is given by

$$F(X) = X^2 - (\beta\gamma_2)^{-1}(\gamma_2 + \gamma_2\chi + \beta\gamma_2 + \beta)\,X + (\beta\gamma_2)^{-1}(\gamma_2 + 1 + \chi\gamma_1)\,. \tag{61}$$

The determinant of $\mathbf{A}$ is strictly larger than one, $F(0) = \det(\mathbf{A}) = (\beta\gamma_2)^{-1}(\gamma_2 + 1 + \chi\gamma_1) > 1$, indicating that there exists at least one unstable eigenvalue. The trace of $\mathbf{A}$ is given by $\text{trace}(\mathbf{A}) = (\beta\gamma_2)^{-1}(\gamma_2 + \gamma_2\chi + \beta\gamma_2 + \beta)$. To disclose the conditions for equilibrium uniqueness, we further examine

$$\det(\mathbf{A}) + \text{trace}(\mathbf{A}) = \gamma_2^{-1}\beta^{-1}(\beta + 2\gamma_2 + \beta\gamma_2 + \chi\gamma_1 + \chi\gamma_2 + 1) > 0,$$
$$\det(\mathbf{A}) - \text{trace}(\mathbf{A}) = \gamma_2^{-1}\beta^{-1}(\chi\gamma_1 - \beta\gamma_2 - \beta - \chi\gamma_2 + 1)\,,$$

ensuring that $\det(\mathbf{A})+\text{trace}(\mathbf{A}) > -1$. Hence, the equilibrium under discretionary optimization is uniquely determined if and only if $\gamma_1 > \gamma_2 - (1-\beta)/\chi \Leftrightarrow \det(\mathbf{A}) - \text{trace}(\mathbf{A}) > -1$. This condition can be written as a restriction on the loss function weight φ :

$$\varphi < \overline{\varphi}\,, \text{ where } \overline{\varphi} \equiv \chi + \frac{1-\beta}{\omega\chi} - \frac{\varepsilon_{ca}}{\omega}\frac{\beta/(1-\beta)}{\varepsilon_{ca}+\sigma_a}, \text{ and } \omega \equiv \frac{(\sigma_c\sigma_a - \varepsilon_{ca}\phi_{ac})^2}{\alpha(\varepsilon_{ca}+\sigma_a)(\varepsilon_{ca}z+\sigma_a)}, \tag{62}$$

where we used that $z - 1 = \beta/(1-\beta)$ and $\pi = 1$. Hence, if (62) is satisfied, both roots are unstable and the fundamental solution, which exhibits no endogenous state variable, is the unique solution. The equilibrium path under discretionary optimization is then forward-looking and satisfies $\partial\widehat{R}_t/\partial\widehat{m}_{t-1}=0$. If $\varphi>\overline{\varphi}$, the system (59) and (60) exhibits one unstable and one stable eigenvalue and, therefore, allows for a stable non-fundamental solution, featuring exactly one endogenous state variable. This completes the proof of the proposition. ■

6.5 Interest rate feedback rules for the alternative model

Rather than to reinterpret the targeting rule (58) as a policy rule, $\widehat{R}_t=\frac{\chi\kappa}{\kappa\varphi}\widehat{\pi}_t+\frac{\alpha}{\kappa\varphi}\widehat{c}_t$, we aim at deriving an interest rate feedback rule of the form $\widehat{R}_t=\rho_\pi\widehat{\pi}_t$. For this, we apply the fundamental solution for the optimal plan under discretion. The latter is characterized by (58) and (22)-(24), and is entirely forward looking. The fundamental (minimum state) solution thus reads

$$\widehat{\pi}_t=\delta_{\pi\mu}\mu_t,\quad\text{and}\quad\widehat{R}_t=\delta_{R\mu}\mu_t,\tag{63}$$

where $E_t\mu_{t+1}=\rho\mu_t$. Inserting the fundamental solution (63) in (59) and (60) leads to the following conditions for the coefficients $\delta_{\pi\mu}$ and $\delta_{R\mu}:\delta_{\pi\mu}\left(\beta\rho-(1+\chi\gamma_1)\right)+\gamma_2\chi\delta_{R\mu}+\chi=0$ and $\left((\gamma_1-1)\rho-\gamma_1\right)\delta_{\pi\mu}+\left((\gamma_2+1)-\gamma_2\rho\right)\delta_{R\mu}=0$. Thus, these coefficients satisfy

$$\delta_{R\mu}=-\chi\left[(\gamma_1-1)\rho-\gamma_1\right]\zeta^{-1}\quad\text{and}\quad\delta_{\pi\mu}=\chi\left[(\gamma_2+1)-\gamma_2\rho\right]\zeta^{-1},$$

where $\zeta\equiv-\gamma_2\rho(1+\chi)+(1+\gamma_2)(1-\beta\rho)+\chi\gamma_1+\gamma_2\rho^2\beta$. The interest rate feedback rule given by $\widehat{R}_t=(\delta_{R\mu}/\delta_{\pi\mu})\,\widehat{\pi}_t$, therefore satisfies

$$\widehat{R}_t=\rho_\pi\widehat{\pi}_t,\quad\text{where }\rho_\pi=\frac{\rho+\gamma_1(1-\rho)}{1+\gamma_2(1-\rho)}.\tag{64}$$

Hence, if $\gamma_1>1+\gamma_2$ the optimal rule under discretion is active, $\rho_\pi>1$, and thus associated with equilibrium uniqueness (see proposition 3). By using the definitions of γ_1 and γ_2, one can rewrite the condition for activeness as $(\chi-\varphi)\,\kappa/\alpha>1$. For $\sigma_c\sigma_a>\varepsilon_{ca}\phi_{ac}\Rightarrow\kappa>0$, which is for example ensured by (16), activeness of the feedback rule (64), therefore, requires the weight on the interest rate variance to be sufficiently small for a given degree of price rigidity.

6.6 Implementing the optimal plan with a backward-looking rule

The optimal allocation for model with the alternative timing of markets, which is given by (59) and (60), exhibits one stable eigenvalue if $\varphi>\overline{\varphi}$ (see proof of proposition 4). Then the fundamental solution is not unique and an alternative solution with a lagged endogenous variable as an extraneous state is also stable/feasible. In this case, the optimal plan can alternatively be implemented by a backward looking interest rate feedback rule of the type (17). To identify this rule, the allocation under discretionary optimization is rewritten by replacing inflation with

$$\hat{\pi}_t = -\kappa_2 \widehat{m}_t + \kappa_3 \widehat{R}_t, \text{ where } \kappa_2 \equiv \left(\frac{\phi_{ac} + \sigma_c}{\varepsilon_{ca} + \sigma_a} \frac{\kappa\chi}{\alpha} \right)^{-1} \text{ and } \kappa_3 \equiv \frac{\varphi}{\chi} - \frac{z-1}{\phi_{ac} + \sigma_c} \frac{\alpha}{\kappa\chi}, \quad (65)$$

which stems from the money demand condition, $\widehat{m}_t = \frac{\phi_{ac}+\sigma_c}{\varepsilon_{ca}+\sigma_a} \widehat{c}_t - \frac{z-1}{\varepsilon_{ca}+\sigma_a} \widehat{R}_t$, and the targeting rule, $\widehat{c}_t = \frac{\kappa\varphi}{\alpha} \widehat{R}_t - \frac{\kappa\chi}{\alpha} \hat{\pi}_t$, in condition (59) and (60), such that the equilibrium conditions under the optimal plan can be summarized by

$$-\kappa_2 \beta E_t \widehat{m}_{t+1} + \kappa_3 \beta E_t \widehat{R}_{t+1} = -\left(1 + \chi\gamma_1\right) \kappa_2 \widehat{m}_t + \left(\kappa_3 + \chi\kappa_4\right) \widehat{R}_t - \chi\mu_t, \quad (66)$$

$$-\left(\gamma_1 - 1\right) \kappa_2 E_t \widehat{m}_{t+1} + \left(\kappa_4 - \kappa_3\right) E_t \widehat{R}_{t+1} = -\kappa_5 \widehat{m}_t + \left(\kappa_4 - 1\right) \widehat{R}_t, \quad (67)$$

$$\text{where } \kappa_4 \equiv \gamma_1 \kappa_3 - \gamma_2 \text{ and } \kappa_5 \equiv \frac{\sigma_a + \varepsilon_{ac} z}{\sigma_c + \phi_{ac}} \kappa.$$

The eigenvalues of (66)-(67) can again be derived from the characteristic equation $F(X) = 0$, where $F(X)$ is given in (61). Now suppose that $\varphi > \overline{\varphi}$. Then there exists exactly one stable eigenvalue (see proposition 4), implying that the following non-fundamental solution

$$\widehat{m}_t = \delta_m \widehat{m}_{t-1} + \delta_{m\mu} \mu_t, \quad (68)$$

$$\widehat{R}_t = \delta_{Rm} \widehat{m}_{t-1} + \delta_{R\mu} \mu_t, \quad (69)$$

and $\widehat{\pi}_t = \delta_{\pi m} \widehat{m}_{t-1} + \delta_{\pi\mu} \mu_t$, is feasible. Applying the generic solution form (68)-(69) for (66) and (67) leads to the following conditions for the undetermined coefficients

$$0 = \kappa_5 \delta_m - \kappa_2 \delta_m^2 \left(\gamma_1 - 1\right) + \delta_{Rm} \left(\delta_m \left(\kappa_4 - \kappa_3\right) - \kappa_4 + 1\right), \quad (70)$$

$$0 = \delta_{R\mu} \left(\kappa_4 - \rho \left(\kappa_4 - \kappa_3\right) - 1\right) + \delta_{m\mu} \left(\kappa_2 \left(\rho + \delta_m\right) \left(\gamma_1 - 1\right) - \delta_m \left(\kappa_4 - \kappa_3\right) - \kappa_5\right), \quad (71)$$

$$0 = \kappa_2 \delta_m \left(\chi\gamma_1 + 1\right) - \beta\kappa_2 \delta_m^2 + \delta_{Rm} \left(\beta\kappa_3 \delta_m - \chi\kappa_4 - \kappa_3\right), \quad (72)$$

$$0 = \chi + \delta_{R\mu} \left(\beta\rho\kappa_3 - \chi\kappa_4 - \kappa_3\right) + \delta_{m\mu} \left(\beta\kappa_3 \delta_{Rm} - \beta\kappa_2 \left(\rho + \delta_m\right) + \kappa_2 \left(\chi\gamma_1 + 1\right)\right). \quad (73)$$

Condition (70) delivers δ_{Rm} as a function of δ_m. Eliminating δ_{Rm} in (72) leads to a cubic equation in δ_m. Two of the roots are identical with roots of (61), while the last root equals zero, which refers to the fundamental solution (63). Using (70) and combining (71) and (73), the remaining coefficients can be written as functions of a non-zero eigenvalue δ_m :

$$\delta_{Rm} = \left[-\kappa_5 \delta_m + \kappa_2 \delta_m^2 \left(\gamma_1 - 1\right)\right] / \left(\delta_m \left(\kappa_4 - \kappa_3\right) - \kappa_4 + 1\right),$$

$$\delta_{m\mu} = \chi F^{-1} \left(\kappa_4 - \rho \left(\kappa_4 - \kappa_3\right) - 1\right) \varrho^{-1}, \qquad \text{and} \qquad \delta_{R\mu} = -\chi F^{-1},$$

$$\text{where } \varrho \equiv \kappa_2 \left(\rho + \delta_m\right) \left(\gamma_1 - 1\right) - \delta_m \left(\kappa_4 - \kappa_3\right) - \kappa_5,$$

$$F \equiv \beta\rho\kappa_3 - \chi\kappa_4 - \kappa_3 - \left(\beta\kappa_3 \delta_{Rm} - \beta\kappa_2 \left(\rho + \delta_m\right) + \kappa_2 \left(\chi\gamma_1 + 1\right)\right) \left(\kappa_4 - \rho \left(\kappa_4 - \kappa_3\right) - 1\right) \varrho^{-1}.$$

To solve for the feedback rule we further apply the general solution for real balances (68) for (65), leading to the following expression: $\mu_t = -\left(\kappa_2 \delta_{m\mu}\right)^{-1} \hat{\pi}_t - \left(\delta_m / \delta_{m\mu}\right) \widehat{m}_{t-1} +$

$\kappa_3 (\kappa_2 \delta_{m\mu})^{-1} \widehat{R}_t$. Eliminating the exogenous state variable with the latter in the solution $\widehat{R}_t = \delta_{Rm} \widehat{m}_{t-1} + \delta_{R\mu} \mu_t$, leads to the following feedback rule for the nominal interest rate

$$\widehat{R}_t = \rho_\pi \hat{\pi}_t + \rho_m \widehat{m}_{t-1}, \text{ where } \rho_\pi = \frac{\delta_{R\mu}}{\delta_{R\mu}\kappa_3 - \kappa_2 \delta_{m\mu}} \text{ and } \rho_m = \kappa_2 \frac{\delta_{R\mu}\delta_m - \delta_{Rm}\delta_{m\mu}}{\delta_{R\mu}\kappa_3 - \kappa_2 \delta_{m\mu}},$$

which implements a history dependent equilibrium path which is consistent with the optimal plan under discretion. If $\varphi > \overline{\varphi}$, the equilibrium exhibits a saddle path configuration as the number of stable eigenvalues equals the number of predetermined variables.

6.7 Saddle path stability for a backward looking policy rule

In order to demonstrate the determinacy properties of a backward looking policy rule in the model with forward looking households presented in section 4 of the paper, the following proposition summarizes sufficient conditions for saddle path stability under interest rate rules with a non-zero coefficient on beginning-of-period real balances.

Proposition 5 *Suppose that interest rate policy satisfies $\widehat{R}_t = \rho_\pi \widehat{\pi}_t + \rho_m \widehat{m}_{t-1}$. Then the equilibrium path of the alternative model (22)-(24) with (16) is saddle path stable if*

$$\begin{aligned} &i.) \quad \rho_m > 0 \quad \textit{and} \quad \rho_\pi < \min\{1 + \rho_m \Delta_3,\, \rho_m \Delta_4 - \widetilde{\rho_\pi}\}, \quad \textit{or} \\ &ii.) \quad \rho_m > \Delta_5 \quad \textit{and} \quad \rho_\pi < 1 + \rho_m \Delta_3, \end{aligned}$$

where $\Delta_3 \equiv \frac{(z-1)\chi\sigma_c - (1-\beta)(1+\sigma_c)}{(2\sigma_c - \varepsilon_{ca})\chi}$, $\Delta_4 \equiv \frac{(((2z-1)\sigma_c+1)(1+\beta)+(z-1)\chi\sigma_c)}{(2\sigma_c - \varepsilon_{ca})\chi}$, *and* $\Delta_5 \equiv \frac{(2\sigma_c - \varepsilon_{ca})(\chi+1+\beta)}{\sigma_c(z+z\beta-\beta)+1}$.

Proof. Applying a backward looking interest rate rule $\widehat{R}_t = \rho_\pi \widehat{\pi}_t + \rho_m \widehat{m}_{t-1}$ for the model (22)-(24), and eliminating m_{t+1} by $\widehat{m}_{t+1} = \frac{\phi_{ac}+\sigma_c}{\varepsilon_{ca}+\sigma_a} \widehat{c}_{t+1} - \frac{(z-1)\rho_\pi}{\varepsilon_{ca}+\sigma_a} \widehat{\pi}_{t+1} - \frac{(z-1)\rho_m}{\varepsilon_{ca}+\sigma_a} \widehat{m}_t$ leads to a 3×3 system. Its deterministic version reads

$$\begin{aligned} -\chi\varepsilon_{ca}\widehat{m}_t + \beta\widehat{\pi}_{t+1} &= \widehat{\pi}_t - \chi\sigma_c\widehat{c}_t, \\ \rho_m \widehat{m}_{t-1} + \rho_\pi \widehat{\pi}_t + \sigma_c \widehat{c}_t &= (\varepsilon_{ca} + \vartheta(z-1)\rho_m)\widehat{m}_t + (1 + \vartheta(z-1)\rho_\pi)\widehat{\pi}_{t+1} \\ &\quad + (\sigma_c - \vartheta(1+\sigma_c))\widehat{c}_{t+1}, \\ (\varepsilon_{ca} + \sigma_a)\widehat{m}_t &= (1-z)\rho_m \widehat{m}_{t-1} + (1-z)\rho_\pi \widehat{\pi}_t + (\phi_{ac} + \sigma_c)\widehat{c}_t, \end{aligned}$$

where $\vartheta = \varepsilon_{ca}/(\varepsilon_{ca}+2)$. Using that the shopping-time specification (16) implies $\phi_{ac} = 1$ and $\sigma_a = 2$, the eigenvalues of the model are the roots of the characteristic polynomial of $\mathbf{A}$, which is characterized by $\left(\widehat{m}_t \, \widehat{\pi}_{t+1} \, \widehat{c}_{t+1}\right)' = \mathbf{A} \left(\widehat{m}_{t-1} \, \widehat{\pi}_t \, \widehat{c}_t\right)'$ and

$$\mathbf{A} = \begin{pmatrix} -\chi\varepsilon_{ca} & \beta & 0 \\ \varepsilon_{ca} + \vartheta(z-1)\rho_m & 1 + \vartheta(z-1)\rho_\pi & \sigma_c - \vartheta(1+\sigma_c) \\ \varepsilon_{ca}/\vartheta & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 0 & 1 & -\chi\sigma_c \\ \rho_m & \rho_\pi & \sigma_c \\ (1-z)\rho_m & (1-z)\rho_\pi & 1+\sigma_c \end{pmatrix},$$

The characteristic polynomial of A is given by

$$F(X) = X^3 + \frac{-\varepsilon_{ca}\Phi(\chi+1+\beta)+(z-1)\beta\vartheta\rho_m\sigma_c}{\beta\varepsilon_{ca}\Phi}X^2 + \frac{\varepsilon_{ca}\Phi+\Phi\chi\rho_\pi\varepsilon_{ca}+((1-z\chi+\chi-z-\beta z)\sigma_c-\beta)\vartheta\rho_m}{\beta\varepsilon_{ca}\Phi}X + \frac{\vartheta(1+z\sigma_c)}{\beta\varepsilon_{ca}\Phi}\rho_m,$$

where $\Phi = \sigma_c - \vartheta(1+\sigma_c) = \frac{2\sigma_c-\varepsilon_{ca}}{\varepsilon_{ca}+2} > 0$. Hence, $F(0) = -\det(\mathbf{A}) = \frac{\vartheta(1+z\sigma_c)}{\beta\varepsilon_{ca}\Phi}\rho_m$, implying that there is at least one negative eigenvalue if $\rho_m > 0$. For $\rho_m = 0 \Rightarrow \det(\mathbf{A}) = 0$, one stable eigenvalue equals zero and the result in proposition 3 applies. To establish the existence of one positive and stable eigenvalue, $F(X)$ at $X = 1$, given by

$$F(1) = \frac{1}{\beta\varepsilon_{ca}\Phi}\left[\chi\varepsilon_{ca}\Phi(\rho_\pi-1)+\vartheta((1-\beta)(1+\sigma_c)-(z-1)\chi\sigma_c)\rho_m\right],$$

has to be negative, $F(1) < 0$, which is ensured by

$$\rho_\pi < 1+\rho_m\Delta_3, \text{ where } \Delta_3 \equiv ((2\sigma_c-\varepsilon_{ca})\chi)^{-1}((z-1)\chi\sigma_c-(1-\beta)(1+\sigma_c)). \tag{74}$$

To rule out the existence of another stable eigenvalue, $F(X)$ at $X = -1$, given by

$$F(-1) = \frac{-\Phi 2(1+\beta)\varepsilon_{ca}-\Phi(1+\rho_\pi)\chi\varepsilon_{ca}+\rho_m\vartheta([(2z-1)\sigma_c+1](1+\beta)+(z-1)\chi\sigma_c)}{\beta\varepsilon_{ca}\Phi},$$

has to be positive, $F(-1) > 0$, which requires

$$\rho_\pi < \rho_m\Delta_4 - \widetilde{\rho_\pi}, \text{ where } \Delta_4 \equiv \frac{((2z-1)\sigma_c+1)(1+\beta)+(z-1)\chi\sigma_c}{(2\sigma_c-\varepsilon_{ca})\chi}. \tag{75}$$

Hence, the model is saddle path stable if $\rho_m > 0$ and $\rho_\pi < \min\{1+\rho_m\Delta_3, \rho_m\Delta_4-\widetilde{\rho_\pi}\}$. Instead of applying condition (75), it is sufficient for saddle path stability to ensure that ρ_m is sufficiently large such that

$$\rho_m > \Delta_5, \text{ where } \Delta_5 \equiv (2\sigma_c-\varepsilon_{ca})(\chi+1+\beta)(\sigma_c(z+z\beta-\beta)+1)^{-1} > 0,$$

and that (74) is satisfied, as $\rho_m > 0$ and (75) are then guaranteed. This completes the proof of the proposition. ■

It should be noted that the conditions *i.* and *ii.* presented in proposition 5 ensure the single stable eigenvalue to lie between zero and one, such that the equilibrium path is non-oscillatory. Applying the parameter values presented in section 3.3, leads to the following numerical values for the composite parameter $\Delta_3 = 67.24$, $\Delta_4 = 5310$, $\widetilde{\rho_\pi} = 78.41$, and $\Delta_5 = 0.015$. Hence, even small values for both coefficients in the backward looking policy rule, ρ_π and ρ_m, are sufficient to ensure saddle path stability for the model with the alternative timing of markets. This includes the parameter values in the last column in table 2 for $\varphi = \varphi_a^*$.

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