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ON THE RELEVANCE OF OPEN MARKET OPERATIONS

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On the Relevance of Open Market Operations[1]

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Abstract

This paper reexamines the role of open market operations for short-run effects of monetary policy in a New Keynesian framework. The central bank supplies money in exchange for securities that are discounted with the short-run nominal interest rate, while money demand is induced by a liquidity constraint. We allow for a legal restriction by which only government bonds are eligible. Their supply is bounded by fiscal policy that is assumed to be Ricardian. If public debt is dominated in rate of return by private debt, open market operations matter, and an endogenous liquidity premium and a liquidity effect arise. Nominal interest rate setting (including a peg) is then associated with price level and equilibrium uniqueness, regardless whether prices are flexible or set in a staggered way. Thus, the legal restriction overcomes indeterminacies due to an unbounded money supply, as implied by the real bills doctrine. Moreover, it facilitates constant money growth and interest rate policy to be equivalent.

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1 Introduction

Central banks in most industrial countries conduct monetary policy mainly via open market operations, where money is supplied in exchange for risk free securities discounted with a short-run nominal interest rate. Hence, the costs of cash acquisition depend on the current discount rate and the availability of collateral. Monetary theory, however, has not reached a consensus on the effects of open market operations and even claims that they are irrelevant, as for example shown by Wallace (1981) and Sargent and Smith (1987), or Eggerston and Woodford (2003).[3] In accordance with the latter view, the majority of recent contributions to the monetary policy literature abstracts from an explicit specification of open market operations and assumes that money is injected via lump-sum transfers. In this paper open market operations are (re)introduced in a standard monetary business cycle framework and it is shown that the relevance of open market operations depends on whether the set of eligible securities is restricted or not. In particular, when only government bonds are accepted in open market operations, the liquidity puzzle can be resolved and an endogenous liquidity premium on non-eligible securities can be generated. Further, a binding legal restriction avoids price level indeterminacy and equilibrium multiplicity that arise for interest rate policies accompanied by an unbounded money supply and leads to an equivalence between simple money supply and interest rate rules.

The analysis is conducted in an infinite horizon model with identical households, which demand money due to a liquidity constraint. Money is supplied in form of outright sales/purchases and repurchase agreements, where money and interest bearing securities are exchanged. The amount of money supplied in open market operations equals the discounted value of eligible securities. Households can decide on whether to carry over money from one period to the other or to repurchase the securities. The former corresponds to the conventional specification of money, where it is treated as a store of value, while money that is held under repurchase agreements serves as a pure medium of exchange. Households are indifferent between the two types of money holdings, which allows to simplify the analysis by focussing on the case where money is exclusively held under repurchase agreements.[4] Households' financial wealth comprises claims on other households and government bonds carried over from the previous period. We explicitly take into account that real world central banks are typically character-

[3] Dupor (2001) examines the role of open market operations, which are specifed as 'holding fiscal policy constant in the face of a government asset exchange' (see Sargent and Smith, 1987). For this case, he shows that open market operations are not irrelevant, since fiscal policy is non-Ricardian.

[4] This specification of money, which can be interpreted as inside money, relates to the one in Drèze and Polemarchakis (2000) and Dubey and Geanakopolos (2003) and avoids Hahn's (1965) paradox though money is held over a finite horizon.

ized by restrictions on their asset acquisition policy (see, e.g., Kopcke, 2002). In particular, eligible securities are usually constrained to a set of assets with high credit quality in order to avoid any credit risk in the central bank's portfolio and 'opportunities for political pressure to influence the allocation of credit' (see Meyer, 2001). The US Federal Reserve, for example, exclusively accepts securities issued by the Treasury, federal agencies, as well as acceptances and bank bills, which meet high credit quality standards (see Meulendyke, 1998). Recent asset acquisition policy of the US Federal Reserve can even be summarized as 'Treasuries-only' (see Broaddus and Goodfriend, 2001). In the model, a legal restriction is imposed, which constrains money supply in that only government bonds can be used in open market operations. The crucial assumption is that households internalize not only the goods market (cash-in-advance) constraint, but also this money market constraint when they decide on their optimal plan. Then there exists a rational expectations equilibrium where private debt yields a higher interest than public debt and the money market restriction is binding, such that the outstanding stock of government bonds relates to the amount of money supplied in open market operations and exhibits a liquidity value.[5]

In order to facilitate comparisons with the New Keynesian theory, which serves as the predominant framework in the recent literature on monetary policy analysis, the model further allows for prices to be set by monopolistically competitive (retail) firms in a staggered way. When there is no legal restriction on open market operations, the reduced set of linearized equilibrium conditions is isomorphic to the standard New Keynesian model applied in Clarida et al. (1999). In case there is a binding legal restriction on eligible securities, the model exhibits substantial differences. In particular, a monetary injection then reduces the nominal discount rate regardless whether prices are flexible or sticky. Hence, it generates the liquidity effect, for example, reported by Hamilton (1997) or Christiano et al. (1999), which can hardly be reproduced by conventional sticky price models, where the nominal interest rate tends to increase with money supply due to higher expected inflation (see Christiano et al., 1997). While it is known that this so-called 'liquidity puzzle' can – at least temporarily – be solved by allowing for segmentations and information asymmetries in asset markets (see Lucas, 1990, Fuerst, 1992, and Alvarez et al., 2002), the emergence of the liquidity effect in this paper relies on the availability of eligible securities. Due to the assumption that the issuance of public debt is constrained to ensure government solvency,[6] a rise in the supply of money must necessarily be accompanied by a decline in its relative price, i.e., the nominal

[5] Liquidity or transaction services of government bonds are assumed in a more direct way in Bansal and Coleman (1996), Canzoneri et al. (2000), or Lahiri and Vegh (2003).

[6] In other words, the sequence of tax receipts and, thus, public liabilities are restricted to induce a Ricardian fiscal policy regime.

discount (repo) rate. When prices are set in a staggered way, the model further predicts real activity to increase and the spread between the interest rates on private and public debt to decrease with a monetary expansion, if households are risk-averse. Hence, the spread can be interpreted as a liquidity premium on non-eligible securities, contributing to the solution for Weil's (1989) 'risk-free rate puzzle' in the spirit of Bansal and Coleman's (1996) explanation.

When the central bank is assumed to control the nominal discount (repo) rate, which equals the interest rate on government bonds, the analysis discloses that well-known determinacy properties of conventional models relate to the irrelevance of open market operations therein. In particular, an interest rate peg accompanied by an unrestricted money supply leaves (for flexible prices) the price level and (for sticky prices) the rational expectations equilibrium path indetermined, in accordance with the results of Kerr and King (1996) and Benhabib et al. (2001). Hence, these findings affirm that a monetary policy regime with 'no effective limit to the quantity of money', which can be interpreted as a central bank applying the 'real bills doctrine' (see Friedman and Schwartz, 1963), is prone to non-uniqueness of prices and equilibria, as shown by Sargent and Wallace (1982), McCallum (1986), or Smith (1988). On the other hand, if there is a binding legal restriction on the supply of money, nominal interest rate policy is always associated with an uniquely determined price level and an unique rational expectations equilibrium.[7] In particular, equilibrium uniqueness does not require the fulfillment of the so-called Taylor-principle, as it would in the case where open market operations are irrelevant (see Woodford, 2001), implying that the central bank can already stabilize the economy by setting the nominal interest rate rather than being compelled to control the real interest rate. However, when the central bank sets the nominal interest rate contingent on changes in inflation, it should refrain from adjusting the interest rate in an extreme way when debt interest payments are not completely tax financed. Otherwise, it would strongly burden public debt obligations, which – by interfering with interest rate policy – might give rise to a divergent equilibrium path.[8] Macroeconomic stability then requires monetary policy to account for the evolution of public debt.

The stability analysis discloses another implication of the money market constraint, which is concerned with the structural relations between interest rates, money supply, and inflation. Regarding the relation between money supply and the rate of inflation, 'one would expect sensible policy behavior to involve a negative value' (see McCallum, 1999). According to

[7] Given that tax policy is assumed to ensure government solvency, these findings do not relate to determinacy results in Woodford (1994) or Benhabib et al. (2001) for the case where fiscal policy is specified in a non-Ricardian way.

[8] The destabilizing effect of aggressive interest rate policy via 'debt-interest spirals' is also found by Leith and Wren-Lewis (2000) in a framework where public debt is non-neutral due to overlapping generations.

this view, a central bank that aims at stabilizing inflation should implement a sequence of money growth rates, which are negatively related to the sequence of inflation rates, as for example found by Ireland (2003) for the US Federal Reserve policy, whereas a positive relation – indicating an accommodating money supply – does not seem to be consistent with this aim. Within our theoretical framework, it is always possible to identify interest rate rules that are associated with sequences of non-accommodating money growth rates. However, without a legal restriction on money supply an interest rate policy that induces equilibrium (multiplicity) determinacy is accompanied by a sequence of money growth rates, which are (decreasing) increasing with inflation. In contrast, if the legal restriction is binding a simple equivalence principle applies, predicting that a central bank can use non-accommodating money supply or interest rate rules interchangeably without affecting determinacy or altering the equilibrium sequences.[9] Hence, a binding legal restriction on eligible securities in open market operations allows a central bank to implement its money market rate target via a limited supply of reserves and to switch between operating targets leaving the behavior of macroeconomic aggregates unchanged.

The remainder is organized as follows. Section 2 develops the model. In section 3 we present results for the flexible and sticky prices. Section 4 concludes and discusses some further implications.

2 The model

Identical and infinitely lived household-firm units are endowed with government bonds, money, and claims on other households carried over from the previous period. They produce a wholesale good employing labor from all households. Aggregate uncertainty is due to monetary policy shocks, which are realized at the beginning of the period. Then goods are produced and asset markets open, where households can trade without restrictions. Money demand is induced by assuming that purchases of consumption goods are restricted by a liquidity constraint. The central bank supplies money exclusively via open market operations. There, the supplied amount of money equals the discounted value of interest bearing assets, which are deposited at the central bank.[10] Then the goods market opens. After goods have been traded, households can repurchase the securities from the central bank. The remaining amount of money is carried over to the next period. To allow for a nominal rigidity, monopolistically competitive retail firms are introduced that differentiate the wholesale goods and

[9] An analysis of a switch between operating targets in the context of a liquidity trap can be found in Benhabib et al. (2002).

[10] Equivalently, it can be assumed that financial intermediaries or traders engage in open market operations on the behalf of the households.

set their prices in a staggered way. As a consequence, the log-linear approximation of the model nests the standard New Keynesian model presented in Clarida et al. (1999).

Households Lower (upper) case letters denote real (nominal) variables. There is an infinite number of time periods t ($t = 0, 1, 2, ...$). Let $s^t = (s_0, ..., s_t)$ denote the history of events up to date t and $g(s^t|s^{t-1})$ denote probability of state s_t and, thus, of the history s^t conditional on the history s^{t-1} at date $t-1$. The initial state, s^0, is given so that $g(s^0) = 1$. There is a continuum of perfectly competitive household-firm units distributed uniformly over $[0, 1]$. In each period t a household $j \in [0, 1]$ consumes a composite good $c(j, s^t)$ and supplies working time $l(j, s^t) = \int_0^1 l^k(j, s^t)dk$ to household-firm units, where $l^k(j, s^t)$ denotes the working time of household j supplied to household k. It produces a wholesale good $x(j, s^t)$ with the technology

$$x(j, s^t) = \int_0^1 l^j(k, s^t)dk, \tag{1}$$

and sells the wholesale good to retail firms charging a price $P^w(s^t)$ per unit. Household j is assumed to maximize the expected value of the discounted stream of utility stemming from consumption and leisure, which is given by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t g(s^t)\; u(c(j, s^t), l(j, s^t)), \quad \beta \in (0, 1), \tag{2}$$

where β denotes the subjective discount factor. The instantaneous utility function u is assumed to be strictly increasing in consumption c, strictly decreasing in working time l, strictly concave, twice continuously differentiable with respect to both arguments, satisfies the usual Inada conditions, and is additively separable.

We separate the household problem into a intratemporal and an intertemporal part. In the *intratemporal* part they make their optimal decisions on production and on the composition of consumption. Profit maximizing leads to the following demand for labor $l^j(k, s^t)$:

$$P(s^t)w(s^t) = P^w(s^t), \tag{3}$$

where $P(s^t)$ denotes the aggregate price level and $w(s^t)$ the real wage rate. Let $c(j, s^t)$ be consumption of a composite good which is defined as a CES aggregate of differentiated goods $y^j(i, s^t)$, which are bought from retailers indexed with $i \in [0, 1] : c(j, s^t)^{\frac{\epsilon-1}{\epsilon}} = \int_0^1 y^j(i, s^t)^{\frac{\epsilon-1}{\epsilon}} di$, where $\epsilon > 1$ is the constant elasticity of substitution between any two retail goods. Let $P(i, s^t)$ denote the price of the retail good $y(i, s^t)$ and the price of the composite good $P(s^t)$ be given by $P(s^t)^{1-\epsilon} = \int_0^1 P(i, s^t)^{1-\epsilon} di$. Minimizing costs for purchasing a unit of the composite good leads to the following optimal demand for the retail good $y^j(i,s^t)$:

$$y^j(i,s^t) = \left(\frac{P(i,s^t)}{P(s^t)}\right)^{-\epsilon} c(j,s^t). \tag{4}$$

The *intertemporal* part unfolds as follows. In what follows the index j is – except for the supply side variables – disregarded, for convenience, as households are identical. At the beginning of period t households are endowed with financial wealth $A(s^{t-1})$, which comprises government bonds holdings $B(s^{t-1})$, claims on other households $D(s^{t-1})$, and money holdings $M^H(s^{t-1}) : A(s^{t-1}) = B(s^{t-1}) + D(s^{t-1}) + M^H(s^{t-1})$. Both interest bearing assets are assumed to be nominally state contingent leading to a payoff in period t equal to $R^d(s^t)D(s^{t-1})$ and $R(s^t)B(s^{t-1})$. This assumption is introduced, on the one hand, to deliver the conventional specification of the consumption Euler equation. On the other, it will be responsible for households to be indifferent between carrying over money from one period to the other and holding money temporarily under repurchase agreements.[11]

Before agents trade in assets or goods, the aggregate shocks arrive, goods are produced, and wages are credited on checkable accounts at financial intermediaries. Then households enter the assets market, where they can trade with other households and the treasury in an unrestricted way. After the asset market is closed, households can participate in open market operations, where they can exchange interest bearing assets $B^c(s^t)$ for money additions $I(s^t)$. The amount $I(s^t)$ supplied by the central bank equals the discounted value $B^c(s^t)/R(s^t)$:

$$I(s^t) = \frac{B^c(s^t)}{R(s^t)}. \tag{5}$$

Hence, the exchange (repo) rate in open market operations equals the gross nominal interest rate on government bonds. The exchange restriction (5) is assumed to hold for two types of open market operations, namely outright sales and purchases as well as repurchase agreements. The fraction of money traded via repurchase agreements, which is denoted by $M^R(s^t)$, is only held until the end of the period, when the repurchase agreements are settled. Hence, $M^R(s^t)$ is a flow variable and can be interpreted as inside money, as it is the counterpart of securities temporarily deposited at the central bank. Money injections thus satisfy $I(s^t) = M^R(s^t) + M^H(s^t) - M^H(s^{t-1})$.

After households have traded with the central bank, they enter the goods market. Here, they rely on the total amount of money $M(s^t) \equiv M^H(s^t) + M^R(s^t)$, i.e., money held under outright sales/purchases $M^H(s^t)$ and held under repurchase agreements $M^R(s^t)$, and on checkable non-interest bearing accounts at a financial intermediary as means of payment.

[11] This feature will particularly be helpful to derive the model's properties in an analytical way.

These accounts consist of the individual labor income $P(s^t)w(s^t)l(j,s^t)$ net the wage outlays for the own firm $P(s^t)w(s^t)\int_0^1 l^j(k,s^t)dk$. Hence, purchases of goods are subject to the following liquidity constraint:

$$P(s^t)c(s^t) \leq M(s^t) + \left[P(s^t)w(s^t)l(j,s^t) - P(s^t)w(s^t)\int_0^1 l^j(k,s^t)dk\right]. \tag{6}$$

The modification of the Clower (1967) constraint, i.e., the term in the square brackets, is primarily introduced to avoid the cash-credit good distortion between consumption and leisure.[12] Applying a standard cash-in-advance constraint causes the nominal interest rates to distort the optimal consumption-leisure decision of households. While the main results in this paper are not affected by this distortion, it would exacerbate the intended comparison with conventional sticky price models, given that the nominal interest rate would then enter the aggregate supply constraint, i.e., the New Keynesian Phillips curve. The avoidance of this distortion is in fact responsible for the model to nest the standard New Keynesian model, as applied in Clarida et al. (1999).

Households receive cash by selling the wholesale good $x(j,s^t)$ to retail firms and retail firms' profits $P(s^t)\int_0^1 \omega(i,s^t)di$, and have to pay a lump sum tax $P(s^t)\tau(s^t)$. After the goods market is closed, inside money $M^R(s^t)$ is used by the households to repurchase securities from the central bank. Household j's budget constraint is given by

$$\begin{aligned} & D(s^t) + B(s^t) + M^H(s^t) + \left(R(s^t) - 1\right)\left(M^R(s^t) + M^H(s^t) - M^H(s^{t-1})\right) \\ \leq\, & R(s^t)B(s^{t-1}) + R^d(s^t)D(s^{t-1}) + M^H(s^{t-1}) - P(s^t)c(s^t) - P(s^t)\tau(s^t) \\ & + P(s^t)w(s^t)l(j,s^t) - P(s^t)w(s^t)\int_0^1 l^j(k,s^t)dk + P^w(s^t)x(j,s^t) + P(s^t)\int_0^1 \omega(i,s^t)di. \end{aligned} \tag{7}$$

The main novel feature of the model is that the market for money is assumed to be constrained. Considering that asset acquisition of many real world central banks (see Kopcke, 2002), including the US Federal Reserve, is restricted to a set of high credit quality securities, a legal restriction on open market operations is imposed by which only government bonds are accepted by the central bank as collateral for money:

$$B^c(s^t) \leq B(s^t). \tag{8}$$

Such a restriction on the asset acquisition of a central bank is commonly justified by the aim to avoid credit risk in its portfolio and effects on credit allocation (see Meyer, 2001). It actually imposes an upper bound on the supply of money, given by the discounted value

[12] This specification closely follows Jeanne (1998).

of total government bonds held by private sector. In the case where the central bank sets the interest rate, the legal restriction can be viewed as the main difference between a money supply regime, as for example applied by the US Federal Reserve in the recent past, and the so-called 'real bills doctrine', which was applied by the US Federal Reserve during the Great Depression and is characterized by an effectively unlimited quantity of money (see Friedman and Schwartz, 1963). According to this doctrine, the nominal interest rate is held at its target,[13] while money is supplied in exchange for short-term commercial *bills* that are intended to finance *real* transactions and can, thus, rise in a potentially unbounded way.

It is further assumed that households are aware of the fact that their access to cash is restricted by their holdings of government bonds. This restriction would be irrelevant when they can issue private debt with an interest rate not higher than the interest rate on government bonds. However, as the monetary authority (directly or indirectly) controls the latter, a positive spread $R^d(s^t) > R(s^t)$ cannot generally be ruled out, so that the households internalize the constraint (8), which can rewritten as

$$M^R(s^t) + M^H(s^t) - M^H(s^{t-1}) \leq B(s^t)/R(s^t), \tag{9}$$

when they derive their optimal decisions. Maximizing (2) subject to the constraints for goods market (6), the asset market (7), the money market (9), a non-negativity constraint on money held under repurchase agreements, $M^R(s^t) \geq 0$, and a no-Ponzi-game condition $\lim_{i\to\infty} \sum_{s^{t+i}} g(s^{t+i})A(s^{t+i}) \prod_{v=1}^{i} R^d(s^{t+v})^{-1} \geq 0$, for a given initial value of total nominal wealth $A(s^0) > 0$ leads to the following first order conditions for consumption, leisure, holdings of private and public debt, and of money, $M^R(s^t)$ and $M^H(s^t)$:

$$u_c(s^t) = \lambda(s^t) + \psi(s^t), \tag{10}$$
$$u_l(s^t) = -u_c(s^t)w(s^t), \tag{11}$$
$$\lambda(s^t) = \beta \sum\nolimits_{s^{t+1}|s^t} g(s^{t+1}|s^t) \left[R^d(s^{t+1})\lambda(s^{t+1})/\pi(s^{t+1})\right], \tag{12}$$
$$\eta(s^t) = \beta \sum\nolimits_{s^{t+1}|s^t} g(s^{t+1}|s^t) \left[\lambda(s^{t+1}) \left(R^d(s^{t+1}) - R(s^{t+1})\right) /\pi(s^{t+1})\right], \tag{13}$$
$$\xi(s^t) = \left(R(s^t) - 1\right) \lambda(s^t) + R(s^t)\eta(s^t) - \psi(s^t), \tag{14}$$
$$\begin{aligned}\psi(s^t) = {} & R(s^t)\lambda(s^t) - \beta \sum\nolimits_{s^{t+1}|s^t} g(s^{t+1}|s^t) \left[R(s^{t+1})\lambda(s^{t+1})/\pi(s^{t+1})\right] \\ & +R(s^{t+1})\eta(s^{t+1}) - \beta \sum\nolimits_{s^{t+1}|s^t} g(s^{t+1}|s^t) \left[R(s^{t+1})\eta(s^{t+1})/\pi(s^{t+1})\right],\end{aligned} \tag{15}$$

where $\pi(s^t) = P(s^t)/P(s^{t-1})$ denotes the rate of inflation, λ the shadow price of wealth, ψ the

[13] See McCallum (1986) for the relation between an interest rate peg and the real bills doctrine, and for further references.

Lagrange multiplier on the goods market constraint (6), and η the Lagrange multiplier on the money market constraint (9). The equations (14) and (15) give the first order conditions for M^R and M^H, respectively, and the multiplier ξ measures if money is held under repurchase agreements. The household's optimum is further characterized by the constraints (6), (7), and (9),

$$\xi(s^t) \geq 0,\ \xi(s^t)M^R(s^t) = 0, \tag{16}$$

$$\eta(s^t) \geq 0,\ \eta(s^t)\left[b(s^t) - R(s^t)m(s^t)\right] = 0, \tag{17}$$

$$\psi(s^t) \geq 0,\ \psi(s^t)\left[m(s^t) + w(s^t)l(j,s^t) - w(s^t)\int_0^1 l^j(k,s^t)dk - c(s^t)\right] = 0, \tag{18}$$

where $b(s^t) \equiv B(s^t)/P(s^t)$ and $m(s^t) \equiv M(s^t)/P(s^t)$, and the no-Ponzi game condition holding with equality, which provides the transversality condition.

Retailer There is a monopolistically competitive retail sector with a continuum of retail firms indexed with $i \in [0,1]$. Each retail firm, owned by the households, buys a quantity $x^i(j,s^t)$ of the wholesale good produced by household j at price $P^w(s^t)$. To minimize distortions induced by liquidity constraints, it is assumed that households buy coupons for the differentiated consumption goods providing retail firms with cash, which they use to the purchase the wholesale good. We assume that a retailer is able to differentiate the wholesale good without further costs. The differentiated retail good $y(i,s^t) = \int_0^1 x^i(j,s^t)dj$ is then sold at a price $P(i,s^t)$. We assume that retailers set their prices according to Calvo's (1983) staggered price setting model. The retailer changes its price when it receives a signal, which arrives in a given period with probability $(1-\phi)$, where $\phi \in [0,1)$. A retailer who does not receive a signal adjusts its price by the steady state aggregate inflation rate $\overline{\pi}$, such that $P(i,s^t) = \overline{\pi}P(i,s^{t-1})$. A retailer who receives a price change signal in period t chooses a price $\widetilde{P}(i,s^t)$ to maximize the expected sum of future discounted profit streams given by

$$\sum_{v=0}^{\infty}\sum\nolimits_{s^{t+v}|s^t} (\beta\phi)^v\, q(s^{t+v},s^t)\widetilde{\omega}(i,s^{t+v},s^t), \tag{19}$$

where $q(s^{t+1},s^t) \equiv \frac{\lambda(s^{t+1})/P(s^{t+1})}{\lambda(s^t)/P(s^t)} g(s^{t+1}|s^t)$ denotes the stochastic discount factor and $\widetilde{\omega}(i,s^{t+v},s^t)$ real profits in period $t+v$ for own prices not being adjusted after period t : $P(s^t)\widetilde{\omega}(i,s^{t+v},s^t) = \widetilde{P}(i,s^t)y(i,s^{t+v}) - P^w(s^{t+v})\int_0^1 x^i(j,s^{t+v})dj$. Maximizing (19) subject to the demand function (4), taking the price $P^w(s^t)$ of the wholesale good, the aggregate final goods price index $P(s^t)$ and the initial price level $P(s^0)$ as given, yields the following first-order condition for $\widetilde{P}(i,s^t)$

$$\widetilde{P}(i,s^t) = \frac{\epsilon}{\epsilon-1}\frac{\sum_{v=0}^{\infty}\sum_{s^{t+v}|s^t}(\beta\phi)^v\, q(s^{t+v},s^t)x(s^{t+v})P(s^{t+v})^{\epsilon}\overline{\pi}^{-\epsilon v}P^w(s^{t+v})}{\sum_{v=0}^{\infty}\sum_{s^{t+v}|s^t}(\beta\phi)^v\, q(s^{t+v},s^t)x(s^{t+v})P(s^{t+v})^{\epsilon}\overline{\pi}^{(1-\epsilon)v}}, \tag{20}$$

where $x(s^{t+v}) \equiv \int_0^1 x^i(j, s^{t+v})$. Using the simple pricing rule for the remaining fraction ϕ of the firms ($P(i, s^t) = \overline{\pi} P(i, s^{t-1})$), the price index for the final good P_t evolves recursively over time. In a symmetric equilibrium the price level satisfies $P(s^t)^{1-\epsilon} = \phi \left(\overline{\pi} P(s^{t-1})\right)^{1-\epsilon} + (1-\phi) \widetilde{P}(s^t)^{1-\epsilon}$, which can be rewritten as:

$$1 = \phi \left(\overline{\pi} \pi(s^t)^{-1}\right)^{1-\epsilon} + (1-\phi) [\widetilde{P}(s^t)/P(s^t)]^{1-\epsilon}. \tag{21}$$

Public sector The public sector consists of a fiscal and a monetary authority. The monetary authority supplies money in open market operations in exchange for government bonds and transfers the seigniorage to the fiscal authority. The budget constraint of the central bank is given by

$$M^H(s^t) + (R(s^t) - 1) \left[M^R(s^t) + M^H(s^t) - M^H(s^{t-1})\right] = M^H(s^{t-1}) + P(s^t)\tau^c(s^t),$$

where τ^c denotes transfers to the fiscal authority. We consider two monetary policy regimes, which differ with regard to the choice of the operating target being controlled according to simple rules. The first regime is characterized by the central bank controlling the supply of money $\mu(s^t) \equiv M(s^t)/M(s^{t-1})$. In the second regime, which is analyzed in the last part of the paper, the central bank applies the nominal discount (repo) rate $R(s^t)$ as its operating target.

The fiscal authority issues risk free one period bonds earning a gross nominal interest rate $R(s^t)$, collects lump-sum taxes τ from the households, and receives the transfer τ^c from the monetary authority:

$$R(s^t)B(s^{t-1}) = B(s^t) + P(s^t)\tau^c(s^t) + P(s^t)\tau(s^t). \tag{22}$$

Hence, interest rate payments on public debt are the only source of expenditures for the fiscal authority. Fiscal policy is assumed to satisfy the following simple rule which relates interest rate payments on outstanding debt to tax receipts and, for simplicity, to transfers from the central bank:[14]

$$P(s^t)\tau(s^t) = \vartheta(R(s^t) - 1)B(s^{t-1}) - P(s^t)\tau^c(s^t), \quad \vartheta \in (0, 1]. \tag{23}$$

The fiscal policy parameter ϑ governs the portion of government expenditures covered by tax receipts. It thus serves as a measure for fiscal responsiveness: A high value of ϑ indicates fiscal austerity and $\vartheta = 1$ a balanced budget regime. Using the fiscal policy rule (23) to eliminate taxes in the budget constraint (22) leads to the following rule for the supply of

[14] A similar fiscal policy rule, that further allows for $\vartheta = 0$, can be found in Benhabib et al. (2001).

public debt

$$B(s^t) = \left[(1-\vartheta)(R(s^t)-1)+1\right] B(s^{t-1}). \tag{24}$$

Hence, a higher value for the fiscal policy parameter ϑ reduces the growth rate of government bonds. In the subsequent analysis we will focus, for convenience, on the case where money does not serve as a store of value for the households, $M^H(s^t) = 0$. As $\vartheta > 0$ is assumed (see 23), it follows immediately from (24) that solvency of the public sector is guaranteed in this case, as $\lim_{i\to\infty} \sum_{s^{t+i}} g(s^{t+i})[B(s^{t+i}) + M(s^{t+i})] \prod_{v=1}^{i} R(s^{t+v})^{-1} = 0$ is always satisfied. In other words, public policy is Ricardian. It should be noted that this specification of fiscal policy contrasts the one, for example, applied in Dupor (2001), where open market operations are defined as government asset exchanges associated with a constant tax policy (see also Sargent and Smith, 1987, or Schreft and Smith, 1998), implying that public debt can rise with the nominal interest rate policy, i.e., that public policy is non-Ricardian.

Equilibrium Given that households are identical, in equilibrium $D(s^t) = 0$, $l(j, s^t) = l(s^t)$, $x(j, s^t) = x(s^t)$, and $l^j(k, s^t) = l(s^t)$, and as retail firms behave symmetrically: $\widetilde{P}(i, s^t) = \widetilde{P}(s^t)$, $\omega(i, s^t) = \omega(s^t)$, and $y(i, s^t) = y(s^t)$. Market clearing further implies $y(s^t) = x(s^t)$, $y(s^t) = c(s^t)$, and $a(s^t) = b(s^t)$, where $a(s^t) \equiv A(s^t)/P(s^t)$.

Definition 1 *A rational expectations equilibrium of the model is a set of sequences* $\{\lambda(s^t)$, $\psi(s^t)$, $\eta(s^t)$, $\xi(s^t)$, $c(s^t)$, $l(s^t)$, $y(s^t)$, $P(s^t)$, $P^w(s^t)$, $\widetilde{P}(s^t)$, $\pi(s^t)$, $w(s^t)$, $x(s^t)$, $m^H(s^t)$, $m^R(s^t)$, $b(s^t)$, $R(s^t)$, $\mu(s^t)\}_{t=0}^{\infty}$ *satisfying the aggregate version of the production function (1), the labor demand condition (3), the households' first order conditions (10)–(18) combined with (6) and (9), the conditions (20), (21), and* $\pi(s^t) = P(s^t)/P(s^{t-1})$ *for the evolution of aggregate prices, the retail goods production,* $x(s^t) = y(s^t)$*, the aggregate resource constraint,* $y(s^t) = c(s^t)$*, the fiscal policy rule (24), and the transversality condition, for a given monetary policy rule for* $\mu(s^t)$ *or* $R(s^t)$ *and initial values* $P(s_0) > 0$ *(if* $\phi > 0$*) and* $A(s_0) > 0$*.*

In what follows we restrict our attention to the cases where the goods market constraint is binding, $c(s^t) = m(s^t)$. For this, the nominal interest rate on government bonds $R(s^t)$ will be restricted to be larger than one such that $\psi(s^t) > 0$.

3 Results

In this section the role of open market operations for short-run macroeconomic effects of monetary policy is examined. We start by establishing households' indifference between accumulating money or holding money (intratemporally) under repurchase agreements. Using this property, the remainder of the paper focuses, for analytical convenience, on the case where money is exclusively held under repurchase agreements. The first results are then derived for flexible prices. The last part of this section examines the effects of open market operations under rigid prices. To lighten the notion, the reference to the state is suppressed in what follows and E_t denotes the expectation operator conditional on the information in period t.

3.1 Money supply via open market operations

In order to adjust holdings of money, which serves in the aggregate as the single means of payment, households have to engage in open market operations. There, the central bank supplies an amount of money equal to the discounted value of interest bearing securities, which are deposited at the central bank. At the end of the period, after the goods market is closed, households can either repurchase these securities from the central bank or they can carry over money to the next period, such that the securities are held by the central bank. The foregone interest by holding money instead of debt exactly equals the additional cost of money acquisition under repurchase agreements. This property is responsible for households to be indifferent between both types of money holdings, M^H and M^R. This result is summarized in the following proposition.

Proposition 1 (Money holdings) *Households are indifferent between carrying over money from one period to the other and holding money intratemporally under repurchase agreements.*

Proof. In order to establish the claim in the proposition it has to be shown that the multiplier ξ_t on the non-negativity constraint $M_t^R \geq 0$ is equal to zero. Eliminating the multiplier ψ_t on the cash constraint (6) in the first order conditions for money (14) and (15), gives $\xi_t = -\lambda_t + \beta E_t \left[R_{t+1}\lambda_{t+1}/\pi_{t+1}\right] + \beta E_t \left[R_{t+1}\eta_{t+1}/\pi_{t+1}\right]$. Further applying the first order condition for private debt (12) and government bonds (13) proofs that $\xi_t = 0$. ■

The indifference between the two types of money holdings, measured by the multiplier ξ_t on the non-negativity constraint $M_t^R \geq 0$, critically hinges on the assumption that government bonds are nominally state contingent. If, on the other hand, it is assumed that their payoff in period t equals $R_{t-1}B_{t-1}$, implying that they are not nominal (though, still real) state contingent, the multiplier on money holdings under repurchase agreements is in general not equal to zero. Given that the assumed payoff structure induces households to be indifferent between accumulating money or holding money temporarily, $\xi_t = 0$, the following assumption, which substantially simplifies the analysis as money becomes a flow variable, will be applied throughout the remainder of the paper.

Assumption 1 *Money is exclusively held under repurchase agreements, $M_t^H = 0$ $\forall t > 0$, and the initial value of money held by the households is equal to zero, $M_0^H = 0$, such that $I_t = M_t = M_t^R$ $\forall t$.*

It will be shown in what follows that the model features two fundamentally different versions depending on the relevance of open market operations, i.e., on whether the money market constraint (9) enters the set of equilibrium conditions as an equality or an inequality. When open market operations are not legally restricted by (8), such that public and private debt are eligible, open market operations are obviously irrelevant as money can also be acquired in exchange for securities, which can be issued by the households. Even if open market operations are legally restricted by (8), they are irrelevant as long as households' government bonds holdings are sufficiently large such that $B_t \geq B_t^c$ always holds. Given the timing of events in the model, households can afford the latter, if government bonds earn the same interest as private debt ($R_t = R_t^d$). In this case, households can borrow to invest costlessly in government bonds to any amount. In contrast, when the interest rate on government bonds is smaller than the interest rate on private debt, this strategy becomes costly and households are willing to minimize holdings of government bonds. Due to the existence of the money market constraint, which reads $M_t^R (= M_t) \geq B_t/R_t$ under assumption 1, a positive spread $R_t^d > R_t$ can arise in equilibrium, which is associated with a liquidity value of government bonds, indicated by a positive multiplier $\eta_t > 0$. In this case, the money market constraint (9) is binding, $B_t = B_t^c$, indicating that households are only willing to hold government bonds equal to the desired amount of money times the current repo rate, $B_t = R_t M_t$. This result is summarized in the following proposition.

Proposition 2 (Legal restriction) *The money market constraint is binding, $M_t = B_t/R_t$, if the interest rate spread between private and public debt is expected to be positive $E_t[R_{t+1}^d - R_{t+1}] > 0$.*

Proof. Given that $\lambda_t > 0$ is ensured by (10) and (18), the first order condition (13) implies that the multiplier η_t is strictly positive if $E_t[R_{t+1}^d - R_{t+1}] > 0$. Then the complementary slackness condition (17) demands the open market constraint to hold with equality. ■

Whether the open market constraint is binding or not has substantial consequences for the determination and the evolution of government bonds, interest rates, money, and consumption. Suppose that the cash constraint (6) is binding and that the expected spread between the interest rate on private debt and the interest rate on government bonds is positive. According to the result in proposition 2 the open market constraint then demands that money and, thus, consumption is linked to real government bonds by $c_t R_t = B_t/P_t$ and real wealth can be determined, as $A_t = B_t$.[15] If, however, the interest rate spread equals zero, $R_t = R_t^d$, the

[15] It should be noted that an interest rate policy is, in this case, equivalent to a policy regime where the central bank controls the ratio of money to government bonds, as applied in Schreft and Smith's (1998) long-run analysis of open market operations.

money market constraint is not binding and the amount of securities traded in open market operations B_t^c is not directly linked to public debt. This case corresponds to the conventional specification of monetary business cycle models, where there is no legal restriction, such that open market operations and real financial wealth are irrelevant.

3.2 Money, interest rates, and prices

In this subsection the role of open market operations for the relation of money supply and interest rates is examined under flexible prices. Here, we are primarily interested in the ability of the model to generate a liquidity effect, i.e., a decline in the money market rate in response to a monetary injection. While the liquidity effect is commonly found in empirical contributions (see Eichenbaum, 1992, or Hamilton, 1997), it can hardly be reproduced in monetary business cycle models, without referring to segmentations or information asymmetries in asset markets (see Lucas, 1990, Fuerst, 1992, or Alvarez et al., 2002). In any case, the success of these strategies to resolve the so-called liquidity puzzle depends on parameter restrictions that decide on the ability of particular effects, brought about by the asset market frictions, to dominate the expected inflation effect of a monetary injection, that tends to raise the nominal interest rate. On the contrary, it is shown in this section that an unanticipated increase in money supply is unambiguously associated with a liquidity effect, when the money market constraint is binding.

Consider the case where prices are flexible, i.e., the probability of a retailer receiving a price signal is equal to one ($\phi = 0$), and that the central bank exogenously controls the supply of money via open market operations. The growth rate, $\mu_t = m_t\pi_t/m_{t-1}$, is assumed to satisfy

$$\mu_t = \mu^{1-\rho}\mu_{t-1}^{\rho}\exp(\varepsilon_t), \quad \text{where } \rho \in [0, 1), \tag{25}$$

where the innovations ε_t have an expected value equal to zero and are serially uncorrelated. It should be noted that the money supply rule is specified in terms of the growth rate to facilitate comparisons to related studies. Given that money is actually a flow variable in our model, it might be more intuitive to specify a money supply rule in levels, which would further simplify the analysis, leaving the main results unchanged.

For the case where prices are flexible, the solution for most of the variables can immediately be derived. The real wage rate is constant and equals the inverse of the retailers' markup, $w_t = P_t^w/P_t = \frac{\epsilon-1}{\epsilon}$, which immediately implies together with $c_t = y_t = l_t$ that consumption is uniquely pinned down by (11) and, thus, constant.[16] Further, suppose that

[16] This property is actually a virtue of avoiding the cash-credit good distortion between consumption and leisure by applying the modified cash constraint (6).

the nominal interest rate on government bonds exceeds one, $R_t > 1$, implying that the cash constraint is binding, $m_t = c_t$, and that the rate of inflation equals the growth rate of money, $\pi_t = \mu_t$. Then the response of the nominal interest rate(s) on a money supply shock, $\varepsilon_t > 0$, critically hinges on whether the open market constraint is binding or not.

When, the interest rate spread is expected to be equal to zero, $R_t = R_t^d$, the money market constraint is irrelevant, $\eta_t = 0$. Combining the first order conditions for money (14) and (15), which then reads $\psi_t = (R_t - 1)\lambda_t$, for consumption (10) and for bonds (12), gives the consumption Euler equation

$$u_{ct} = \beta R_t^d E_t \left[u_{ct+1}/\pi_{t+1} \right]. \tag{26}$$

Given that consumption is constant, the nominal interest rates satisfy $R_t^{(d)} = E_t \mu_{t+1}/\beta$. Hence, for serially correlated money growth rates, $\rho > 0$, an expansionary money supply shock leads to a rise in the money market rate, due to the so-called expected inflation effect (see Christiano et al., 1997). If, on the other hand, the money market constraint is binding, an inverse relation between money supply and the money market rate arises. For a liquidity effect to occur, it is, however, crucial that fiscal policy is assumed to be Ricardian, i.e., to ensure government solvency by satisfying $\vartheta > 0$.

Suppose that the spread $R_t^d - R_t$ is positive, implying that the money market constraint is binding, $M_t = B_t/R_t$. Then the stock of government bonds outstanding relates to the supply of money and, for a binding goods market constraint, $c_t = m_t$, to consumption expenditures, $b_t = c_t R_t$. Applying the supply rule for government bonds (24), which reads in real terms $\pi_t b_t = \left[(1-\vartheta)R_t + \vartheta\right] b_{t-1}$, and using that consumption is constant under flexible prices, leads to the following relation between money supply and the nominal discount rate $R:$

$$\frac{\vartheta}{R_t} + (1-\vartheta) = \mu_t R_{t-1}^{-1}. \tag{27}$$

Specifying the money supply rule in form of $\mu_{t+1} = \mu_{t+1}(\mu_t, \varepsilon_{t+1})$, as in Alvarez et al. (2002) where money is also not accumulated, would instead lead to a non-backward looking relation, which might be more intuitive as the nominal interest rate is a jump variable. Nevertheless, equation (27) reveals that a rise in the money growth rate μ_t is associated with a decline in the nominal interest rate R_t, provided that we assumed the fiscal authority to satisfy $\vartheta > 0$. If, however, $\vartheta = 0$ would be assumed, which implies that fiscal policy is non-Ricardian, then a money injection would leave the current interest rate unchanged. A non-zero feedback from debt to taxes, $\vartheta > 0$, thus serves as a bound for the supply of eligible securities and is therefore responsible for the price of money, i.e., the nominal discount rate, to decline when money supply rises.

In order to derive the solution for the nominal interest rate, we apply the log-linear approximation to the model at the steady state. A variable with a bar denotes the particular steady state value. The steady state is characterized by constant values for c, π, a, m, R^d, and R given by: $u_c(\overline{c})/[-u_l(\overline{c})] = \epsilon/(\epsilon-1)$, $\overline{m} = \overline{c}$, $\overline{\pi} = \mu$, and $\overline{R^d} = \mu/\beta$, regardless whether the money market constraint is binding or not. Hence, the steady state of the model is always consistent with the 'monetary facts' of McCandless and Weber (1995).[17] If the money market constraint is binding, $\overline{\eta} > 0$, the steady state satisfies

$$\overline{R} = (\mu - \vartheta)/(1-\vartheta) \text{ and } \overline{a} = \overline{c}\overline{R}. \tag{28}$$

Otherwise ($\overline{\eta} = 0$), the repo rate satisfies $\overline{R} = \overline{R}^d$. The existence of a steady state with a binding money market constraint, $\overline{R} < \overline{R}^d \Rightarrow \overline{\eta} > 0$, requires the central bank to choose a small average money growth rate μ for its rule (25) and the fiscal authority to finance a minimum amount of debt obligations with taxes (see 28). The steady state conditions for binding constraints in the money and the goods market ($\overline{R} > 1 \Rightarrow \overline{\psi} > 0$), which immediately follow from the (28) and $\overline{R}^d = \mu/\beta$, are presented in the following proposition.

Proposition 3 (Steady state) *Suppose that the fiscal policy is sufficiently responsive such that $\vartheta \geq 1-\beta$. Then there exists a steady state with binding constraints in the money and the goods market if the central bank chooses an average money growth rate $\mu \in (\beta, \widetilde{\mu})$, with $\widetilde{\mu} \equiv \vartheta/[1-(1-\vartheta)/\beta] \geq 1$.*

It should be noted that the fiscal policy constraint, $\vartheta \geq 1-\beta$, which is imposed to guarantee that the upper bound $\widetilde{\mu}$ is non-negative, is hardly restrictive. The upper (lower) bound on the average money growth rate ensures the money (goods) market constraint to be binding in the steady state. Suppose that public policy satisfies the conditions in proposition 3 and that the support of ε is sufficiently small such that the money market constraint always binds, $\eta_t > 0$. Then, by log-linearizing (27) at the steady state the fundamental solution of the model can be shown to be the unique solution according to the criterion of Blanchard and Kahn (1980). Given that the nominal interest rate is not a predetermined variable, this requires the difference equation (27) to exhibit an unstable eigenvalue. The fundamental solution for the log-linearized model with a binding money market and cash constraint then reveals that an unambiguous liquidity effect arises. The solution for the repo rate for a non-deflationary steady state, $\mu \geq 1$, is presented in the following proposition, where $\widehat{x}_t$ denotes the percentage deviation of a generic variable x from its steady state value $\overline{x} : \widehat{x}_t \equiv (x_t - \overline{x})/\overline{x}$.

[17] For example, money is always neutral in the long-run.

Proposition 4 (Liquidity effect) *The fundamental solution of the log-linear approximation to the model at the steady state with $\overline{\eta} > 0$, $\overline{R} > 1$ and $\mu \geq 1$ is the unique solution and generates a liquidity effect by:*

$$\widehat{R}_t = -(\mu/\vartheta)\widehat{\mu}_t. \tag{29}$$

Proof. Log-linearizing (27) at the steady state with $\overline{\eta} > 0$ and $\overline{\psi} > 0$, leads to $\widehat{R}_t = (\mu/\vartheta)\widehat{R}_{t-1} - (\mu/\vartheta)\widehat{\mu}_t$, implying that the eigenvalue is unstable, given that $\mu \geq \vartheta$ by assumption. As the nominal interest rate is not a predetermined variable, μ_t is the single state variable. Hence, the fundamental solution $\widehat{R}_t = -(\mu/\vartheta)\widehat{\mu}_t$ is the unique solution and predicts an unambiguous liquidity effect. ■

Hence, the model is able to generate a liquidity effect if the money market constraint is binding, whereas the so-called 'liquidity puzzle' arises when open market operations are irrelevant. It should, however, be noted that the consumption Euler equation predicts that the nominal interest rate on private debt rises with the expected inflation rate regardless whether the money market constraint is binding or not. In the latter case both interest rates are identical, whereas the repo rate behaves inversely in the former case. It will be shown in the following subsection that the nominal interest rate R^d will also decrease with money supply when prices are not completely flexible.

The fundamental solution for the nominal interest rate given in (29) further implies that there exists a simple relation between the applied money supply rule (25) and an exogenous interest rate policy: An interest rate peg is equivalent to a constant money growth policy. This property leads to the last result in this subsection, which concerns the determination of the price level. As it is well known, interest rate policy can easily lead to price level indeterminacy, if it does not react to the state of the economy. In particular, an interest rate peg is commonly associated with price level indeterminacy if fiscal policy is assumed to be Ricardian (see, e.g., Benhabib et al., 2001), as in this model. A constant money growth rule, however, differs with regards to its determinacy implications, as shown by Carlstrom and Fuerst (2001, 2003), and provides a nominal anchor facilitating the determination of the price level. In this model, the existence of a nominal anchor rather depends on wether the money market constraint is binding or not, than on the type of monetary policy rule. Given that, an interest rate peg is equivalent to a constant money growth rule for a binding money market constraint, price level determinacy is always ensured. This result is summarized in the following proposition.

Proposition 5 (Price level determinacy) *Suppose that the cash constraint is binding and that the central bank pegs the nominal discount rate $R_t = R$. Then the price level is (in)determined if the money market constraint is (not) binding.*

Proof. Financial wealth is predetermined and satisfies $A_0 > 0$ and $A_t = B_t$ in equilibrium. Hence, it evolves, by (24), according to $A_t = \alpha^t A_0$, where $\alpha \equiv (1-\vartheta)R+\vartheta > 0$. When $\eta_t = 0$, an interest rate peg fixes the inflation rate by $\pi = R\beta$ and the growth rate of real financial wealth is given by $a_t/a_{t-1} = \alpha/(R\beta)$. Its current period real value and, thus, the price level can, however, not be determined. For $\eta_t > 0 \Rightarrow m_t = a_t/R$ and $m_t = c_t$, real wealth equals $a_t = a = Rc$, such that the price level is uniquely determined by: $P_t = A_t/a_t = \alpha^t A_0/(cR)$. ■

The reason why the price level can be determined when the money market constraint binds relies on the property that government bonds provide liquidity services through open market operations, which allows to determine the current real value of financial wealth which is predetermined in nominal terms.[18] Hence, the legal restriction on money supply resolves the problem of price level indeterminacy, which occurs when the nominal interest rate is pegged and money supply is unbounded. The latter case can also be interpreted as a monetary policy regime following the real bills doctrine (see McCallum, 1986). Hence, our finding corresponds to the results in Sargent and Wallace (1982) and Smith (1988), showing that a monetary policy under the real bills doctrine leads to indeterminacies of prices and equilibria in overlapping generations models, whereas legal restrictions on money and credit markets are able to restore determinacy. As, however, stressed by Sargent and Wallace (1982), a Laissez-Faire regime can be welfare enhancing, indicating that 'given that such restrictions, when they are binding, misallocate resources, it follows that there can be a trade-off between achieving price level stability and achieving efficient resource allocation through credit markets' (see Wallace, 1983). In our model with flexible prices, consumption and leisure are constant and identical for both cases, implying that an equilibrium with a legal restriction regime is not Pareto-dominated. This property, which does not necessary apply to the case where prices are rigid, will, however, not further be examined in this paper.

3.3 Monetary policy under staggered price setting

In order to extend the previous analysis for the case where prices are flexible, we assume in this section that prices are set in a staggered way, $\phi > 0$, in accordance with numerous contributions to the recent monetary policy literature (see e.g. Clarida et al. 1999, or Woodford, 2003). On the one hand, the responses to a monetary policy shock should be more realistic than for flexible prices. One the other hand, we expect the properties regarding the price level determination (see proposition 5) to carry over to the determinacy of real variables

[18] This mechanism relates to the result in Canzoneri et al. (2000), where price level indeterminacy is resolved by assuming that government bonds directly enter a cash-in-advance constraint.

and, thus, of the rational expectations equilibrium if prices are not completely flexible. The model then additionally features an aggregate supply constraint stemming from the partial price adjustments of retailers. Log-linearizing (20) and (21), the evolution of the inflation rate can be summarized by the following constraint, i.e., the so-called New Keynesian Phillips curve:[19] $\widehat{\pi}_t = \chi \widehat{mc}_t + \beta E_t \widehat{\pi}_{t+1}$, where $\chi \equiv (1-\phi)(1-\beta\phi)\phi^{-1} > 0$ and $mc_t = P_t^w/P_t (= w_t)$ denotes the retailers' real marginal costs. The equilibrium of the log-linear approximation to the model at a steady state with $\overline{R} > 1$, $\sigma \equiv -\overline{u}_c/(\overline{u}_{cc}\overline{c}) > 0$, and $\upsilon \equiv \overline{u}_l/(\overline{u}_{ll}\overline{l}) > 0$, for the monetary policy rule (29) with $\mu \geq 0$, which can for $\eta_t > 0$ either be interpreted as an exogenous rule for the nominal interest rate or the money growth rate, is defined as follows.

Definition 2 *A rational expectations equilibrium of the log-linear approximation to the sticky price model at the steady state with σ, $\upsilon > 0$, $\overline{R} > 1$, and $\mu \geq 0$ is a set of sequences $\{\widehat{c}_t$, $\widehat{m}_t$, $\widehat{\pi}_t$, $\widehat{R}_t$, $\widehat{a}_t\}_{t=0}^{\infty}$ satisfying (29), $\widehat{m}_t = \widehat{c}_t$,*

$$\widehat{c}_t = \begin{cases} \widehat{a}_t - \widehat{R}_t & \text{if} \quad \eta_t > 0 \\ E_t \widehat{c}_{t+1} - (\widehat{R}_t - E_t \widehat{\pi}_{t+1})/\sigma & \text{if} \quad \eta_t = 0 \end{cases}, \tag{30}$$

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \gamma_1 \widehat{c}_t, \tag{31}$$

$$\widehat{a}_t = \widehat{a}_{t-1} + \gamma_2 \widehat{R}_t - \widehat{\pi}_t \qquad \text{if} \quad \eta_t > 0, \tag{32}$$

where $\gamma_1 \equiv \chi(\sigma + \upsilon) > 0$, $\gamma_2 \equiv (\mu - \vartheta)/\mu \in [0, 1)$, and the transversality condition for a given initial value $a_0 = A_0/P_0 > 0$.

The equilibrium conditions listed in Definition 2 reveal that real financial wealth, i.e., the real value of government bonds outstanding, only affects consumption and inflation in the case where open market operations matter ($\eta_t > 0$). Otherwise ($\eta_t = 0$), the equilibrium sequences of consumption, inflation, real balances, and the nominal interest rate are completely unaffected by real wealth, since they can already be determined by (29)-(31). The model, given by (30)-(31) and a Taylor-rule instead of (29), is in fact isomorphic to the New Keynesian models, as for example applied in Clarida et al. (1999) or in Woodford (2003). The public financing decision, which is represented by the feedback parameter ϑ governing the ratio of tax to debt financing, is therefore irrelevant implying that Ricardian equivalence applies. As the model with $\eta_t = 0$ is characterized by a real bond (wealth) indeterminacy (see Canzoneri and Diba, 2000), condition (32) is then irrelevant for the equilibrium.

In what follows the effects to a monetary policy shock ε_t, where $\widehat{\mu}_t = \rho \widehat{\mu}_{t-1} + \varepsilon_t$, are examined for the version with a binding money market constraint, $\eta_t > 0$, given in Definition 2. In this case, $\widehat{\mu}_t > 0$ measures a money injection associated with an immediate decline in

[19] See, for example, Yun (1996) for the derivation of this constraint.

the nominal discount rate, given that (29) provides the fundamental solution for the nominal discount rate regardless whether prices are sticky or flexible.[20] This version of the model exhibits exactly one relevant predetermined variable, namely, real financial wealth $a_{t-1} = A_{t-1}/P_{t-1}$, such that the state space is given by $s_t = (\widehat{a}_{t-1}, \widehat{\mu}_t)'$. Hence, the fundamental solution for $\widehat{a}_t$, $\widehat{\pi}_t$, $\widehat{c}_t (= \widehat{m}_t)$, $\widehat{R}_t$, and $\widehat{R}_t^d$ takes the form

$$X_t = \mathbf{D} s_t, \tag{33}$$

where $X_t = (\widehat{a}_t, \widehat{\pi}_t, \widehat{c}_t, \widehat{R}_t, \widehat{R}_t^d)'$ and $\mathbf{D}$ is a 5×2 matrix with elements δ_{Xs}. The characteristic polynomial of the model with $\widehat{\eta}_t > 0$ reveals that there exists exactly one stable eigenvalue, given by $\delta_a \equiv \delta_{aa}$, indicating saddle path stability. Hence, the unique determination of the price level for the case where prices are flexible (see proposition 5) carries over to the uniqueness of the rational expectations equilibrium path converging to the steady state for the case where prices are sticky.[21] As a consequence, the fundamental (minimum state) solution (33) is the unique solution of the model. The following proposition summarizes this result and presents sign restrictions for the coefficients in $\mathbf{D}$ (see 33).

Proposition 6 (Fundamental solution) *The fundamental solution of the model given in Definition 2 with $\eta_t > 0$ is the unique solution and is characterized by (i) $\partial\widehat{c}_t/\partial\widehat{\mu}_t > 0$; (ii) $\partial\widehat{\pi}_t/\partial\widehat{\mu}_t > 0 \Leftarrow \vartheta > \underline{\vartheta}$, where $\underline{\vartheta} \equiv 1 - \beta\gamma_1/(1 - \delta_a + \gamma_1) < 1$; (iii) $\partial\widehat{a}_t/\partial\widehat{a}_{t-1} = \delta_a \in (0,1)$ and $\partial\widehat{a}_t/\partial\widehat{\mu}_t = \delta_{a\mu} < 0$; (iv) $\partial\widehat{R}_t/\partial\widehat{\mu}_t = \delta_{R\mu} < 0$; and (v) $\partial\widehat{R}_t^d/\partial\widehat{\mu}_t = -(\sigma - 1)(1 - \delta_a)\delta_{a\mu} + \sigma\delta_{R\mu}$.*

Proof. See Appendix A.

According to the properties of the fundamental solution presented in proposition 6, the model's predictions about monetary policy effects on real activity and prices qualitatively accord to evidence from vector autoregressions, as presented in Christiano et al. (1999). To be more precise, part (i) of proposition 6 predicts that consumption (output) and real balances decline in response to a monetary contraction, $\widehat{\mu}_t < 0$, whereas part (ii) reveals that the price reaction is not unambiguous. For inflation to decline in response to a monetary contraction, the degree of fiscal responsiveness should be sufficiently large, $\vartheta > \underline{\vartheta}$. For example, the parameter values $\beta = 0.99$, $\phi = 0.75$, and $\sigma = \upsilon = 3$ lead to $\underline{\vartheta} \simeq 0.5$. Otherwise, the associated rise in the nominal interest rate on government bonds (see part (iv)), can cause the treasury to increase nominal debt. A stationary sequence of public debt would then require the inflation rate to rise in the future to deflate public debt. As retailers set their

[20] This result is due to the property that one can solve for the nominal discount rate without referring to the aggregate supply constraint (31) if the money market constraint is binding.

[21] This property is discussed in detail when contingent interest rate rules are considered (see proposition 7).

prices in a forward looking way, inflation would then also rise in the impact period. Hence, a small value for the feedback of debt on taxes, $\vartheta < \underline{\vartheta}$, provides an alternative explanation for an inverse price response to a monetary policy shock, which is commonly found in vector autoregressions, and known as the 'price-puzzle' (see Sims, 1991).

The model further predicts that real wealth rises in response to a monetary contraction (see part (iii) of proposition 6), which is mainly caused by the rise in the repo rate due to the existence of the liquidity effect (see part (iv)). Regarding the return on private debt, part (v) of proposition 6 also discloses that $\widehat{R}_t^d$ rises in response to a monetary contraction if the inverse of the elasticity of intertemporal substitution $1/\sigma$ is sufficiently large. Moreover, the solutions for $\widehat{R}_t$ and $\widehat{R}_t^d$ reveal that the spread, $\widehat{R}_t^d - \widehat{R}_t$, rises if $(\sigma - 1)(1 - \delta_a)\delta_{a\mu} > 0$, which is ensured for risk-averse households, $\sigma > 1$. Given that only government bonds can be exchanged for money in open market operations, this spread can be interpreted as a liquidity premium and behaves in an intuitive way: A decline in money supply raises the willingness of risk-averse households to liquidate their securities, such that the liquidity value of government bonds and, thus, the premium on private debt rises.

Corollary 1 *A binding money market constraint is associated with an endogenous liquidity premium if households are risk-avers.*

For the last part of this section, the central bank is assumed to endogenously adjust the nominal discount rate. In particular, interest rate setting is considered to depend on the realizations of the current inflation rate

$$\widehat{R}_t = \rho_\pi \widehat{\pi}_t, \quad \text{where } \rho_\pi \geq 0, \tag{34}$$

which is commonly assumed in recent studies on the determinacy properties of interest rate policy (see Benhabib et al., 2001, 2002, Carlstrom and Fuerst, 2001, or Dupor, 2001), and can be viewed as a simplified version of the rule proposed by Taylor (1993). As prices are rigid, stabilization of inflation rates is in fact a welfare enhancing policy strategy (see Woodford, 2003), which implies that the inflation elasticity ρ_π should be positive, if fiscal policy is sufficiently responsive, $\vartheta > \underline{\vartheta}$. Otherwise, a rise in the nominal interest rate intended to stabilize inflation can cause the opposite, as shown in part (ii) of proposition 6. Hence, a binding money market constraint gives rise to an interaction of fiscal and monetary policy such that an optimal policy analysis might to be more challenging in this environment.[22] In this paper, however, we do not aim to assess the implications of the legal restriction for

[22] For example, the average distortion brought about by price stickiness implies an optimal long-run gross inflation rate equal to one (see Woodford, 2003). This might, however, be incompatible with a binding money market constraint, depending on the prevailing fiscal policy regime (see proposition 3).

households' welfare and continue with the analysis of the local dynamic properties. While an interest rate peg was shown to ensure to saddle path stability (see proposition 6), the same property is not guaranteed for the case where the nominal interest rate is set highly reactive to changes in inflation. In particular, the upper bound for an inflation elasticity, which ensures saddle path stability, depends on the fiscal responsiveness, measured by the feedback parameter ϑ of the tax rule (23). The determinacy properties are summarized in the following proposition.

Proposition 7 (Real determinacy) *Suppose that the central bank sets the nominal discount rate according to (34). Then the rational expectations equilibrium path of the model in Definition 2 with a binding money market constraint is (i) uniquely determined, and (ii) stable if and only if* $\rho_\pi < \overline{\rho_\pi}$*, where* $\overline{\rho_\pi} \equiv 1 + \vartheta[(1-\vartheta)R]^{-1} > 1$.

Proof. See Appendix B.

According to part (i) of proposition 7, the model with a binding money market constraint is in any case associated with a unique rational expectation equilibrium, including a peg $\rho_\pi = 0$, which has already been established in proposition 6. Hence, in contrast to the case where the money market constraint is not binding, the Taylor-principle ($\rho_\pi > 1$) is neither necessary nor sufficient for real determinacy. In the latter case, $\rho_\pi > 1$ avoids a non-fundamentally induced rise in expected inflation to induce a decline in the real interest rate that would lead to a rise in current consumption and, thus, in inflation, which would cause the initial expectation to become self-fulfilling (see, e.g., Woodford, 2001). When the money market constraint is binding, there is another mechanism which rules out sunspot equilibria. A rise in inflation leads in this case to a decrease in real financial wealth by (32) and, as condition (30) implies consumption to rise with real wealth, to a decline in aggregate demand given that the nominal interest rate is non-decreasing in inflation (see 34). Hence, the aggregate demand response tends to lower current inflation by the aggregate supply constraint (31), such that inflation expectations cannot be self-fulfilling.

To get an intuition for the result in part (ii) in proposition 7, consider that the central bank chooses a high inflation elasticity ρ_π and that inflation rises due to a fundamental shock. If tax policy is highly reactive to the evolution of public debt (high ϑ), then the real value of public debt will be reduced by higher prices. If, on the other hand, the fiscal policy regime finances only a small fraction of its debt obligations by taxes (low ϑ), then the associated rise in the nominal interest rate R_t can lead to a rise in real public debt. This, however, corresponds to a rise in the real value of eligible securities held by the households. Thus, households raise their consumption expenditures, since the increase in public debt eases the money market constraint. As the rise in aggregate demand further feeds inflation by (31) the initial inflationary impulse is enhanced. Hence, a highly aggressive interest rate policy might lead to explosive paths when the fiscal feedback is too small.[23] The upper bound $\overline{\rho_\pi}$ given in proposition 7 further reveals that it is sufficient for saddle path stability if the fiscal authority runs a balanced budget policy ($\vartheta = 1 \Rightarrow \overline{\rho_\pi} = \infty$) or if the central bank sets the nominal interest rate in a passive way ($\rho_\pi < 1$).[24]

The stability analysis reveals another remarkable property, which regards the relation of interest rates and money supply. As already shown in proposition 4, there is a simple equivalence principle between interest rates and money supply when the money market constraint is binding. It predicts, that an interest rate rule (34) is associated with a money growth rate which is negatively related to current inflation. Hence, this monetary policy regime can equivalently be described by a money supply rule with a negative inflation elasticity rule, $\partial\widehat{\mu}_t/\partial\widehat{\pi}_t = -\rho_\pi\vartheta/\mu$, which for example relates to the money growth rule in McCallum (1999). Accordingly, a central bank can implement a sequence of interest rates satisfying (34), by supplying money in non-accommodating way, $\partial\widehat{\mu}_t/\partial\widehat{\pi}_t \leq 0$. While this operational procedure, for example, accords to the conduct of monetary policy of the US Federal Reserve (see Meulendyke, 1998), it cannot be reproduced in the model where the money market constraint is not binding. In this case ($\eta_t = 0$), the relation between interest rates and money supply is based on the consumption Euler equation (26). In particular, its linearized version in (30) together with the cash constraint, leads to the following relation between money growth rates and inflation:[25]

$$\eta_t = 0 \Rightarrow \partial\widehat{\mu}_t/\partial\widehat{\pi}_t = (\rho_\pi - 1)\,\sigma^{-1} + 1. \tag{35}$$

According to (35), an active interest rate setting, $\rho_\pi > 1$, is associated with accommodating money growth rates $\partial\widehat{\mu}_t/\partial\widehat{\pi}_t > 0$. On the contrary, an interest rate rule which is accompanied by a non-accommodating money supply, violates the so-called Taylor principle and – as shown by Woodford (2001) in an isomorphic model – allows for multiple rational expectations equilibria. This result is summarized in the following proposition.

Proposition 8 (Equivalence) *A sequence of nominal discount rates satisfying (34) can only be associated with a sequence of non-accommodating money growth rates* $\partial\widehat{\mu}_t/\partial\widehat{\pi}_t \leq 0$ *on an unique rational expectations equilibrium path if the money market constraint is binding.*

In general, one should expect the money supply of a central bank, which aims at stabilizing the economy, to exhibit a non-positive feedback from inflation (see, e.g., McCallum, 1999). Such a money supply regime should – for example according to Ireland's (2003) estimates for

[23] See Leith and Wren-Lewis (2000) for a similar result for a sticky price overlapping generations model.
[24] Note that a non-Ricardian regime ($\vartheta = 0$) would also require passiveness ($\rho_\pi < 1$) to escape explosiveness.
[25] It should be noted that a money-in-the-utility function specification leads to a similar relation.

the US – be associated with money market rates that are negatively related to the inflation rate. For an unrestricted money supply ($\eta_t = 0$), a non-accommodating money supply is, however, associated with nominal interest rates that violate the Taylor-principle, which allows for sunspot equilibria, and that are even negatively related to inflation if households are risk-averse, $\sigma > 1$. Otherwise, an active interest rate policy ($\rho_\pi > 1$), which ensures a saddle path configuration in this case, implies the supply of nominal balances to be accommodating and even real balances to grow with inflation $\partial\widehat{\mu}_t/\partial\widehat{\pi}_t > 1$ (see 35). Hence, a central bank that is unwilling to allow for self-fulfilling expectations and for an accommodating money supply should impose a legal restriction on eligible securities, if it aims at implementing targets for the money market rate by supplying reserves via open market operations.

4 Conclusion

Are open market operations really irrelevant for macroeconomic dynamics, as usually presumed in recent business cycle theory? In this paper it is shown that, when money is the counterpart of discounted securities deposited at the central bank, the relevance of open market operations depends on whether the set of eligible securities is constrained or not. Following the practice of many central banks, a legal restriction on open market operations is introduced, by which only government bonds are accepted as collateral. When this money market constraint is binding, an otherwise standard New Keynesian model exhibits an equilibrium where non-eligible securities are associated with an endogenous liquidity premium. Given that fiscal policy is assumed to be Ricardian such that the supply of eligible securities is not unbounded, money is inversely related to the nominal discount interest rate and a cash injection is associated with a liquidity effect.

Besides its potential to resolve the liquidity and the risk-free-rate puzzle, a binding legal restriction on eligible securities facilitates the unique determination of the price level and the rational expectation equilibrium for interest rate policies, which are associated with indeterminacies when money supply is unrestricted. The reason for this property is that households' financial wealth (public debt) is relevant, as it serves as a collateral for money, and provides a nominal anchor that allows to pin down the price level even for an interest rate peg. The determinacy findings can, therefore, be viewed as a rationale for central banks to impose a restriction on eligible securities rather than to follow an asset acquisition policy in the way recommended by the real bills doctrine, which implies money supply to be effectively unlimited. On the other hand, a policy regime with a binding restriction is likely to be dominated in terms of welfare by a Laissez-Faire regime, as it potentially leads to a misallocation of resources. Hence, the introduction of the money market restriction can be associated with a trade-off between determinacy and optimality. A sensible analysis concerning this trade-off, which is beyond the scope of this paper, might however require a more realistic environment where price rigidities are not the only macroeconomic distortion.

The analysis further reveals a major difference between both regimes that matters for the implementation of interest rate targets. In particular, an interest rate rule, which is aimed to stabilize the economy, implies a non-accommodating money supply only if the legal restriction is binding, while it is associated with money growth rates that rise with inflation if money supply is unrestricted. Moreover, a central bank that aims at implementing its money market rate target via the supply of reserves through open market operations relies in the latter case on its ability to assess the amount of money demanded at the particular *real* interest rate, that actually links the operating target with aggregate demand. This, however, demands the nominal interest rate to be jointly targeted with the inflation rate, as implied by a Taylor-type (1993) interest rate rule. If the central bank, on the other hand, imposes a legal restriction, a direct relation between money supply and the nominal interest rate arises, which is only (quantitatively) affected by the outstanding stock of eligible securities. Hence, the less certain a central bank is about the relation between aggregate demand and the nominal interest rate, the more it should aim to be in control of the assets in open market operations.

Appendix A: Proof of proposition 6

In order to examine the eigenvalues of the model with $\eta_t > 0$ given in Definition 2, it is reduced to a 2×2 system in real wealth, which is a predetermined variable, and inflation:

$$\mathbf{M}_0 \begin{pmatrix} \widehat{a}_t \\ E_t \widehat{\pi}_{t+1} \end{pmatrix} = \mathbf{M}_1 \begin{pmatrix} \widehat{a}_{t-1} \\ \widehat{\pi}_t \end{pmatrix} + \mathbf{M}_e \widehat{\mu}_t \tag{36}$$

$$\text{where} \quad \mathbf{M}_0 = \begin{pmatrix} \gamma_1 & \beta \\ 1 & 0 \end{pmatrix}, \quad \mathbf{M}_1 = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}, \quad \mathbf{M}_e = \begin{pmatrix} -\gamma_1 \pi/\vartheta \\ 1 - \pi/\vartheta \end{pmatrix}.$$

The characteristic polynomial of $\mathbf{M}_0^{-1}\mathbf{M}_1$ is $f(\Lambda) = \Lambda^2 - \frac{\beta+\gamma_1+1}{\beta}\Lambda + \frac{1}{\beta}$. Given that $f(0)$ is equal to $1/\beta$ and, therefore, strictly positive and $f(1)$ is negative $f(1) = -\gamma_1/\beta < 0$, the model exhibits one eigenvalue lying between zero and one, $\Lambda_1 \in (0,1)$ and one unstable eigenvalue, $\Lambda_2 > 1$.

As there is only a single stable eigenvalue, the fundamental solution is the unique solution of the model. Using the general form in (33) to replace the endogenous variables in the equilibrium equations (30)-(32), leads to the following conditions for the undetermined coefficients δ_a, $\delta_{\pi a}$, $\delta_{a\mu}$, and $\delta_{\pi\mu}$ in $\mathbf{D}$:

$$\begin{aligned} &\gamma_1\delta_a + \beta\delta_{\pi a}\delta_a - \delta_{\pi a} = 0, \quad \delta_a - 1 + \delta_{\pi a} = 0, \\ &\gamma_1\delta_{a\mu} + \beta\delta_{\pi a}\delta_{a\mu} + \gamma_1(\mu/\vartheta) - \delta_{\pi\mu} = 0, \quad \delta_{a\mu} + \delta_{\pi\mu} + (\mu - \vartheta)/\vartheta = 0. \end{aligned} \tag{37}$$

where δ_a is the single stable eigenvalue of the model, $\delta_a = \Lambda_1$. Rearranging the conditions in (37), gives the following impact multiplier on inflation and real wealth

$$\delta_{\pi\mu} = [\vartheta\gamma_1 + (\vartheta - \mu)(1 - \delta_a)\beta]/\Gamma, \quad \delta_{a\mu} = -[\gamma_1\mu + (1 - \beta\rho)(\mu - \vartheta)]/\Gamma < 0,$$

where $\Gamma \equiv \vartheta[\gamma_1 + \beta(1 - \delta_a) + (1 - \beta\rho)] > 0$ and $\delta_{\pi a} = 1 - \delta_a$ with $\delta_{\pi a} \in (0, 1)$. The impact multiplier on inflation, $\delta_{\pi\mu}$, is strictly positive if $\vartheta\gamma_1 + (\vartheta - \mu)(1 - \delta_a)\beta > 0$. Using that μ is assumed to be strictly smaller than $\overline{\mu} \equiv \vartheta/[1 - (1 - \vartheta)/\beta]$, it follows that $\delta_{\pi\mu}$ is strictly positive if $\vartheta > 1 - \beta\gamma_1/(1 - \delta_a + \gamma_1)$. The coefficient $\delta_{a\varepsilon}$ is further used together with the solution for $\widehat{c}_t$, $\widehat{c}_t = \delta_a\widehat{a}_{t-1} + (\delta_{a\mu} + \mu/\vartheta)\widehat{\mu}_t$, to derive the impact multiplier on consumption and real balances $\delta_{c\mu}$, which reads

$$\delta_{c\mu} = [\vartheta(1 - \beta\rho) + \mu\beta(1 - \delta_a)]/\Gamma > 0.$$

With these solutions and $\widehat{R}_t = \delta_{R\mu}\widehat{\mu}_t = -(\mu/\vartheta)\widehat{\mu}_t$, one can determine the response of the interest rate on private debt $\widehat{R}^d_t$, by using with the consumption Euler equation, $\widehat{c}_t = E_t\widehat{c}_{t+1} - (\widehat{R}^d_t - E_t\widehat{\pi}_{t+1})/\sigma$. Replacing consumption with the structural relation $\widehat{c}_t = \widehat{a}_t - \widehat{R}_t$ and applying the fundamental solution gives $\widehat{R}^d_t = -[(\sigma - 1)(1 - \delta_a)\delta_{a\mu} + \sigma\mu/\vartheta]\widehat{\mu}_t + \delta_a[\sigma(\delta_a - 1) + \delta_{\pi a}]\widehat{a}_{t-1}$, such that $\partial\widehat{R}^d_t/\partial\widehat{\mu}_t = -[(\sigma - 1)(1 - \delta_a)\delta_{a\mu} + \sigma\mu/\vartheta]$. Hence, the interest rate on private debt declines with $\widehat{\mu}_t$ if and only if $(\sigma - 1)(1 - \delta_a)\delta_{a\mu} + \sigma\mu/\vartheta > 0$, which completes the proof of proposition 6. ■

Appendix B: Proof of proposition 7

When the nominal discount rate is set according to (34), the matrices of the 2×2 model in (36) are unchanged except for the second column of $\mathbf{M}_1$. Its elements are now given by $M_1^{(1,2)} = 1 + \gamma_1\rho_\pi$ and $M_1^{(2,2)} = \gamma_2\rho_\pi - 1$. The characteristic polynomial therefore changes to

$$f(\Lambda) = \Lambda^2 - [(\gamma_1\rho_\pi + 1) - \gamma_1(\gamma_2\rho_\pi - 1) + \beta]\beta^{-1}\Lambda + (\gamma_1\rho_\pi + 1)\beta^{-1}.$$

Apparently, $f(\Lambda)$ is strictly positive at $X = 0$, $f(0) = (1 + \gamma_1\rho_\pi)/\beta > 0$. At $\Lambda = 1$, its sign depends on ρ_π : $f(1) = \gamma_1(\gamma_2\rho_\pi - 1)/\beta$. If $\rho_\pi < 1/\gamma_2 = \overline{\pi}/(\overline{\pi} - \vartheta)$, the model exhibits one stable and one unstable eigenvalue, indicating a saddle path configuration. If $\rho_\pi \geq 1/\gamma_2$, there are either two stable or two unstable eigenvalues. To discriminate between the two cases, the slope at $\Lambda = 1$ is considered: $f'(1) = \beta^{-1}\{\gamma_1[(\gamma_2 - 1)\rho_\pi - 1] - (1-\beta)\}$, revealing that $f'(1) < 0$, given that $\gamma_2 \in [0, 1)$. Thus, both eigenvalues exhibit a real part larger than one. Therefore, equilibrium indeterminacy cannot occur, while, using $\overline{\pi} = (1-\vartheta)R + \vartheta$, saddle path stability prevails if and only if $\rho_\pi < [((1-\vartheta)R + \vartheta]/[(1-\vartheta)R]$. ■

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