Paying for Performance in Hospitals*

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Abstract

A frequent form of pay-for-performance programs increase reimbursement for all services by a certain percentage of the baseline price. We examine how such a “bonus-for-quality” reimbursement scheme affects the wage contract given to physicians by the hospital management. To this end, we determine the bonus inducing hospitals to incentivize their physicians to meet the quality standard. Additionally, we show that the health care payer has to complement the bonus with a (sometimes negative) block grant. We conclude the paper relating the role of the block grant to recent experiences in the American health care market.

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1 Introduction

The provision of quality represents a major concern in the health care sector. Many countries have introduced incentive programs rewarding a better performance (often referred to as Pay(img)-for-Performance, P4P). For example, the U.S. Medicare Programme provides higher transfers to hospitals that perform well according to measurable quality indicators, such as rates of cervical cancer screening and hemoglobin testing for diabetic patients. In the UK, general practitioners who perform well on certain quality indicators, such as the measurement of blood pressure and cholesterol in patients with ischemic heart disease, receive substantial financial rewards. These can amount to about 20% of a general-practitioner’s budget (Doran et al., 2006). Similarly, other health care payers have started to include “rewards for quality” deals, see e.g. AIS (2003) and Leapfrog (2007).

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A frequent form of these P4P-programs reward hospitals (and physicians’ practice groups) financially if a certain quality standard is met (Rosenthal et al., 2004). Often these programs increase the reimbursement for all services by a certain percentage of the baseline price the hospital would be reimbursed otherwise. For example, PROMINA Health system, an Atlanta-area federation of eight hospitals and nearly 4000 employed and outside physicians, has contracted sizable quality reimbursement incentives with about 1500 physicians in affiliated practices (AIS, 2003). If a practice meets a certain level of compliance with quality standards (e.g. that a given percentage of pneumonia patients must receive an antibiotic within four hours of being admitted) they receive reimbursement for all services to CIGNA/PROMINA patients that is set a 5% higher multiple of Medicare reimbursement than the baseline multiple such as specified in PROMINA’s contract with CIGNA HealthCare of Georgia. Other examples can be found in Rosenthal et al (2004).

Providing P4P-programs to hospitals does not automatically bring forward higher quality. This is because hospitals consist of a hierarchy of decision-makers. These decision-makers typically act as separate decision makers within hospitals. For example, hospital managers decide on non-medical resources and on contracts offered to health care employees. These employees again decide on how many patients to treat and on the quality of treatment (e.g. should pneumonia patients be given antibiotic, which blood tests to take etc.). Often the goals of the decision makers differ. Managers might aim to maximize the expected financial surplus of the hospital, while the health personnel might get some direct utility from quality provision. In this paper we address both for-profit and not-for-profit hospital objectives. We stress that the effect of a P4P-program depends on the wage contracts given within hospitals when a multi-tier hierarchy of principal-agent interactions are present. This in turn implies that a rational hospital payer (the sponsor) will choose the parameters of the reimbursement contract taking the consequences for the internal wage contracts into account. Simply speaking, if the sponsor wants to improve on quality, she has to design the hospital’s reimbursement contract in such a way that the hospital manager offers contracts that reward health personnel for exerting more effort on quality.

How should the sponsor design the reimbursement contract so that incentives for higher quality are provided within hospitals? This is the issue we address in this paper. Our model is a three-stage game with three players. At the first stage, a risk neutral sponsor (the health care payer) decides on the hospital’s reimbursement scheme. Instead of assuming that the sponsor adopts an optimal contract approach, see e.g. Baron (1989) and Bolton and Dewatripont (2005), we focus on a type of reimbursement contract widely observed between sponsors and hospitals, namely that the sponsor pays the hospital according to the number of patients treated, but the compensation per treatment depends on whether a certain quality standard is met or not.¹ Accordingly, the sponsor sets the bonus on reimbursement associated with providing the quality standard. In

¹In section 5 we discuss why the sponsor might prefer such a reimbursement contract.
addition to this performance-based payment, the sponsor provides the hospital with a block grant. At the second stage, a risk neutral hospital manager offers the physician a wage contract with bonuses for quality and treatments, respectively. Finally, at the third stage, a risk averse physician chooses his levels of effort on treatments and on quality, respectively. In addition to these active players, fully insured patients seek treatment at the hospital. Patients’ utility is increasing in the quality provided.

We begin with characterizing how the sponsor’s reimbursement scheme, characterized by the bonus, the quality standard, and the block grant, affects the optimal wage contract within the hospital. It turns out that the sponsor must set the bonus sufficiently high to implement the quality standard. Failing this optimal bonus by just a narrow margin, will induce the hospital manager to provide too low quality incentives to the physician. The intuition is that the hospital manager has to offer the physician a bonus contract to ensure that the physician exerts effort on the quality task. The bonus contract exposes the physician to risk, which the hospital management has to compensate for. If the sponsor did not compensate the hospital for this extra risk cost, its management would decide not to implement the quality standard. Empirically, this implies that even when we observe that health care payers include rewards for quality in hospitals’ remuneration contracts, hospitals may choose not to implement the desired quality standard. Furthermore, the block grant should sometimes be used to extract the extra surplus (profit) the P4P-program enables the hospital to generate (in these cases the block grant is negative). Whenever the sponsor abstains from extracting the hospital profit, the introduction of a P4P-program will lead to higher costs for the sponsor and increased profitability of health care providers.

There exists a large literature on how hospital financing affects hospitals’ incentives to provide quality. Commonly this literature assumes that patients’ demand reflects their perceptions of the quality of services offered and that quality is non-contractable. If patients are free to choose providers, patients’ demand can be a powerful mechanism for maintaining standards of services (see e.g. Rogerson, 1994; Ma, 1994; and Chalkley and Malcomson, 1998a). When patient demand does not reflect quality, the optimal contract offered by the payer depends on whether the provider is entirely self-interested or benevolent, having a genuine concern for patient welfare (Chalkley and Malcomson, 1998b). More recently, the literature on hospital financing has adopted the viewpoint that quality can be contracted upon. For instance, Eggleston (2005) examines a model with two quality dimensions, one being contractable and the other not. Eggleston shows that, if one dimension of quality is contractible, whereas the other is not, the introduction of a P4P-program may increase service on the verifiable quality dimension, but decrease service on the non-verifiable one. She argues that incentives for non-verifiable quality can be restored by reducing P4P

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on verifiable quality. Kaarbøe and Siciliani (2008) expand Eggleston’s model and solve for the optimal incentive scheme for contractible quality. None of these papers however investigate the internal organization of hospitals. To our knowledge, only two papers account for the interaction between different decision-makers in a hospital-financing framework, namely Custer et al (1990) and Boadway et al (2004). Both papers study how the optimal payment scheme within a hierarchical model is affected by the interaction between the government and the hospital manager and between the manager and the physician. The former paper assumes full information at all levels and analyzes how the manager’s and physician’s incentive to behave cooperatively or not is affected by the way hospitals are financed. The latter paper considers asymmetric information about patients’ severity and hospital types. Contracts are designed to elicit information about patients’ severity and hospital types in an efficient way, that is, contracts minimize the physician’s and the hospital’s information rent. The reimbursement contracts studied in these papers do not contain P4P-elements.

The paper is organized as follows. In Section 2 we present the model, which we analyze in Section 3. Section 4 extends the model to allow for a hospital with direct quality concerns. In Section 5, we discuss results and assumptions of our analysis. Section 6 concludes.

2 The Model

There are three active players, the sponsor, the (hospital) manager, and the physician. The sponsor provides reimbursement to the hospital, the manager offers an incentive contract to the physician, and the physician provides effort on two tasks: the provision of patient treatment \(z\) and quality \(q\) (both with associated verifiable signals). In addition to these players, fully insured patients, whose utility is increasing in the quality provided, seek treatment at the hospital. We assume that demand for hospital services is so high that the hospital is not demand constrained.

The game has three stages. At the first stage, the risk neutral sponsor decides on the hospital’s reimbursement scheme, \(R(\cdot)\), and on the quality standard \(Q > 0\), given her fixed (exogenous) budget \(B > 0\) available to buy hospital services.

We assume that the sponsor maximizes expected net consumer surplus. Hence, her objective is to minimize the cost of implementing the quality standard. Furthermore, the quality standard is set as the highest one the sponsor can afford given her fixed budget \(B > 0\), and given the hospital breaks even.

The sponsor pays the hospital a performance-based payment according to the number of patients treated, \(z > 0\), but the price the hospital gets per treatment depends on whether realized quality \(q\) meets the quality standard, \(Q > 0\), or not.

\[
R = P(q)z + H, \quad \text{where } P(q) = \begin{cases} 
    P, & \text{if } q \geq Q \\
    1, & \text{otherwise.}
\end{cases}
\]

Accordingly, the sponsor decides on the bonus \(P\) that is paid upon providing the
quality standard. Notice that since the sponsor maximizes net consumer surplus, she will never punish the hospital for providing quality (hence $P \leq 0$). In addition to this performance-based payment, the sponsor provides the hospital a block grant $H \leq 0$ (to ensure that the hospital break even).

At the second stage, the risk neutral manager decides on the physician’s payment, $w$. Her net benefit is given by $R - w$. She only cares for the hospital’s financial surplus, either because it is a for-profit hospital or because the surplus can be spent on perks for staff or on improving facilities.\(^4\) The manager only observes the treatment and quality signals, and uses these signals in the payment scheme she offers the physician. The manager offers the physician a linear wage contract,

$$w = A + \alpha z + \beta q,$$

where $\alpha \geq 0$ and $\beta \geq 0$ represent the incentives on the treatment task $z$ and the quality task $q$, respectively.\(^5\)

At the third stage, the risk averse physician chooses effort exerted on the production of treatment $z$ and on quality $q$, labelled $a$ and $b$, respectively. These choices are private information to the physician. Costs of effort (in monetary units) are denoted $c(a, b)$, where $c(\cdot, \cdot)$ is strictly convex and effort costs are independent across tasks. In order to obtain explicit solutions, let $c(a, b) = \frac{1}{2}a^2 + \frac{1}{2}b^2$.

The physician’s choice of effort produces patient treatment

$$z = fa + \zeta$$

and the quality level

$$q = b + \chi.$$

The noise terms $\zeta$ and $\chi$ represent the effects of uncontrollable events. We assume that $\zeta \sim \mathcal{N}(0, \sigma_\zeta^2)$, $\chi \sim \mathcal{N}(0, \sigma_\chi^2)$, with strictly positive variance of quality noise, $\sigma_\chi^2 > 0$. The parameter $f > 0$ represents the physician’s productivity of treatment effort; the productivity parameter associated with quality production is normalized to one. All noise terms are independent of each other. All parties observe $(z, q)$.

The physician’s utility function is exponential, $u = -\exp\{-r(w - c(a, b))\}$, where the coefficient $r > 0$ measures the physician’s degree of constant absolute risk aversion. With linear compensation, exponential utility, and normally distributed random variables, maximizing expected utility is equivalent to maximizing the physician’s certainty equivalent,

$$CE = \mathbb{E}[w] - c(a, b) - \frac{r}{2} \text{var}[w],$$

where $\mathbb{E}$ is the expectation operator. Without loss of generality we assume that the physician’s outside option is normalized to zero.

\(^4\)In section 4 we show that the model also covers the case where the management cares about the patients (i.e., the quality of treatment provided) or, more specifically, where the utility of the hospital management depends upon a weighted sum of provider profits and quality.

\(^5\)The focus on linear contracts can be justified by appeal to a richer dynamic model in which linear payments are optimal (Holmstrom and Milgrom, 1987).
3 The Analysis

3.1 The Wage Contract

To characterize the optimal linear wage contract inside the hospital, we solve the model by backward induction, starting with the physician’s effort decisions at stage three.

The physician’s problem is given by

\[
\max_{a \geq 0, b \geq 0} \left\{ \mathbb{E}[w] - c(a, b) - \frac{r}{2} \text{var}(w) \right\},
\]

where \( \mathbb{E}[w] = A + fa + b \) represents the physician’s expected wage. Notice that \( \text{var}[w] = \alpha^2 \sigma_z^2 + \beta^2 \sigma_q^2 \). Hence, the first-order conditions for an interior solution read

\[
\alpha f = c_a(a, b) = a, \\
\beta = c_b(a, b) = b,
\]

where \( c_i(a, b) \) denotes the partial derivative of the cost function w.r.t. \( i = a, b \). The second-order conditions are satisfied since the cost function is strictly convex.

The manager can always ensure the physician’s participation by adjusting the fixed wage component such that the physician’s participation constraint holds. For each type of effort (treatment vs. quality), the relative size of productivity and risk determines whether bonus pay has to be complemented with some fixed wage payment or not.

At stage two, the manager maximizes the difference between expected revenue and wage costs subject to the physician’s participation constraint \( CE \geq 0 \). Since this holds with equality, the problem becomes

\[
\max_{A, \alpha \geq 0, \beta \geq 0} \left\{ \mathbb{E}[zP(q)] + H - c(a, b) - \frac{r}{2} \text{var}(w) \right\}
\]

s.t. \( \alpha f = c_a(a, b) \)
\( \beta = c_b(a, b) \).

The first-order conditions of the manager’s optimization problem read:

\[
f^2 (1 + (P - 1) (1 - G (Q - \beta))) = \alpha K_z \quad \text{and} \quad f^2 \alpha (P - 1) g (Q - \beta) = \beta K_q,
\]

where \( G(\cdot) \) denotes the Normal distribution function associated with quality noise, \( g(\cdot) := G'(\cdot) \) is the corresponding Normal density function, and \( \alpha K_z = \alpha (f^2 + r \sigma_z^2) \) and \( \beta K_q = \beta (1 + r \sigma_q^2) \) represent the marginal cost of effort and risk on treatments and quality, respectively. Notice that, since the first-order

\[\text{Expected revenue is } \mathbb{E}[zP(q)] = \mathbb{E}[z] \cdot \mathbb{E}[P(q)] = fa [P (1 - G(Q - b)) + G(Q - b)] = f^2 \alpha (1 + (P - 1) (1 - G (Q - \beta))).\]
conditions (2) represent a non-linear system in $\alpha$ and $\beta$, it is not possible to derive an explicit solution that applies to all values of $Q$ and $P$. In the appendix we show that a solution to (2) exists for all values of $Q \geq 0$ and $P \geq 1$. Furthermore, the second-order conditions for a maximum requires the value of production to be sufficiently large relative to the cost of risk and effort.\footnote{Technically, the assumption reads $f^4 > \pi \sigma_q^2 K_q K_z / 2$.}

The optimal contract characterized by (2) represents the solution provided the manager’s participation constraint hold. Notice that the sponsor can always ensure the manager’s participation by setting the block grant $H$ sufficiently high.

3.2 The Optimal Reimbursement Contract

In this section we derive the optimal reimbursement contract. First we consider the performance-based payment. Second, we determine the size of the block grant.

3.2.1 The Performance-Based Payment

At stage one the sponsor aims at implementing a specific quality standard $Q$ in expected terms,

$$Q = \mathbb{E}[q] = b = \beta \geq 0,$$

where the second equality follows from the production technology and the third from the physician’s first-order condition (1).

In order to induce the manager to offer $\beta = b = Q$, the sponsor must set the bonus $P$ such that the first-order conditions of the manager (2) are satisfied. Inserting $\beta = Q, G(Q - \beta) = 1/2$, and $g(Q - \beta) = 1/ (\sqrt{2\pi}\sigma_q)$ and solving for $\alpha$ and $\beta$, we obtain

$$\alpha = \frac{1 + P}{2} \frac{f^2}{K_z},$$

$$\beta = \frac{(P^2 - 1) f^4}{2K_z K_q \sqrt{2\pi}\sigma_q}.$$

The optimal treatment incentive $\alpha$ is similar to the one obtained in the standard principal agent model with linear contracts (see e.g. Gibbons and Murphy, 1992; Kaarboe and Olsen, 2005). The optimal intensity of the treatment-incentive is reached when the net marginal benefit of increasing $\alpha$ equals the marginal transaction cost of $\alpha$. To see this, note that the expected net marginal benefit of extra effort is $f((P + 1)/2 - c_a(a))$. Furthermore it follows from the physician’s first-order conditions that the rate at which extra effort is supplied for each extra unit of intensity is $f$ (because of $c_{aa}(a) = 1$). Since the physician will choose treatment effort such that $a = \alpha f$, the net marginal benefit is $f^2((P + 1)/2 - \alpha)$. The transaction cost associated with $\alpha$ is the risk premium...
with marginal cost \( \alpha r \sigma^2 z \). Thus, \( f^2((P + 1) / 2 - \alpha) = \alpha r \sigma^2 z \) which is equivalent to \( \alpha = (P + 1) f^2/(2K_z) \).

To obtain the \( Q \)-implementing bonus, we solve \( \beta = Q \) and (5) for \( P \).

**Proposition 1** To implement the quality standard \( Q \) in expectation, the sponsor offers a bonus

\[
P^* = \frac{1}{f^2} \sqrt{f^4 + 2\sqrt{2\pi} \sigma q K_z K_z Q}
\]

(6) to be paid whenever realized quality exceeds \( Q \), i.e. \( q > Q \). We have \( P^* > 1 \) for strictly positive quality standards, \( Q > 0 \).

According to Proposition 1 the sponsor must set the bonus sufficiently high to implement the quality standard. That is, she must compensate the hospital for more than the pure effort cost of providing the quality standard. The reason for this is twofold. First, the manager has to offer the physician a bonus contract to ensure that he exerts effort on the quality task. The bonus contract exposes the physician to risk, which has to be compensated. Second, the increased bonus increases the marginal value of treatments, and hence the optimal incentive on this task. Higher incentives increase the variability of the physician’s wage.

If the sponsor did not compensate the hospital for these two types of extra risk cost, its manager would head for a lower level of expected quality. Examining the out-of-equilibrium behavior, we find that the exact level of expected quality depends on whether the sponsor also adjusts the block grant. If the block grant is not adjusted, zero expected quality is provided. Otherwise expected quality is lower than intended, but still positive. Hence failing to set the right reimbursement incentive by just a narrow margin results in too low quality incentives (\( \beta < \beta^* \)). Empirically, this implies that even if one observes that the sponsor includes rewards for quality in hospitals’ remuneration contracts, their managers may choose not to implement the desired quality standard.

Straightforward calculations establish the following proposition.

**Proposition 2** The bonus \( P^* \) implementing the quality standard \( Q \geq 0 \) in expectation is

(i) strictly increasing in \( Q \geq 0 \),

(ii) increasing in the noisiness of the treatment signal \( \sigma^2 z \), and

(iii) increasing in the noisiness of the quality signal \( \sigma^2 q \).

Monotonicity is strict in case (iii) for \( Q > 0 \), and in case (ii) if \( r > 0 \) and \( Q > 0 \).

**Proof.** Appendix. 

Part (i) says the higher the quality standard, the higher the bonus has to be. This raises risk and effort costs, both of which the sponsor has to compensate.

Part (ii) holds because the incentive to treat patients, \( \alpha \), is decreasing in the noisiness of the treatment signal for any bonus \( P \geq 0 \). Marginal revenue on the quality task would be reduced. But since risk and effort cost on the quality
task remain unaffected, the bonus $P$ must be raised in order to ensure that the hospital management still chooses to implement the quality standard.

Part (iii) is due to the fact that the manager must compensate the physician for the increased wage uncertainty associated with the quality task.

Finally, there is a difference between noise on the treatment signal and noise on the quality signal. While the bonus strictly increases in treatment noise only for strictly positive degrees of risk aversion, $r > 0$; it strictly increases in quality noise even in the absence of risk aversion, i.e. for all $r \geq 0$ (given $Q > 0$). The reason is the following: treatment noise only takes effect via the physician’s risk aversion; quality noise enters both through the physician’s risk aversion and through uncertainty of the bonus; a small increase in quality noise implies that expected marginal revenue decreases for both tasks.  

3.2.2 The Block Grant

We now calculate the block grant required to let the hospital break even. Recall the expected surplus between hospital management and physician:

$$\Pi = E[z] \cdot E[P(q)] + H - c(a^*, b^*) - \frac{r}{2} \text{var}(w^*).$$

Since the sponsor only needs to ensure that the hospital breaks even, the block grant $H$ is set such that $\Pi = 0$. Inserting equilibrium effort, equilibrium incentives and the $Q$-implementing bonus $P^*$, we obtain:

**Proposition 3** The sponsor offers the hospital a block grant

$$H(Q) = -\frac{f^4}{4K_z} - \frac{f^2}{4K_z} \sqrt{f^4 + 2\pi \sigma_q K_q K_z Q} - \frac{2 \pi \sigma_q}{4} K_q Q + \frac{K_q Q^2}{2}.$$  

The block grant has the following properties:

(i) $H(0) = -\frac{f^4}{4K_z} < 0$, (ii) $H'(0) < 0$, (iii) there exists $Q^* > 0$ such that $H'(Q) < 0$ for $Q < Q^*$ and $H'(Q) \geq 0$ for $Q \geq Q^*$, and (iv) $H''(Q) > 0$.

The quality level that minimizes the block grant is

$$Q^* = -\frac{f^4 + 2\pi \sigma_q^2 K_q K_z + f^2 \sqrt{f^4 + 4\pi \sigma_q^2 K_q K_z}}{4\sqrt{2\pi K_q K_z}} > 0.$$  

**Proof.** Appendix.

At $Q = 0$ the block grant is strictly negative. Although no quality is incentivized, an expected rent is created between hospital and physician. Since the sponsor only needs to ensure that the hospital breaks even, the block grant is used to extract the surplus.

The block grant strictly decreases in the quality standard up to some threshold $Q^*$, i.e. for $0 \leq Q < Q^*$, and increases for quality levels above this threshold.

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8To see this, evaluate the left hand side of the manager’s first order conditions (2) at $\beta = Q$ and differentiate them with respect to quality noise $\sigma_q^2$. 

$Q > Q^*$. For large quality levels, the sponsor has to complement the bonus with a positive block grant to ensure the hospital participates.

The following figure illustrates the function $H(Q)$ when $f = 2, r = 0.1, \sigma_q^2 = \sigma_z^2 = 1$.

[Figure 1 about here]

From the figure we see that the block grant is negative for $0 \le Q \lesssim 3.13$ and positive for $Q \gtrsim 3.13$. The block grant has its minimum value when $Q^* \simeq 1.03$.

3.3 Comparative Statics on Incentives

After calculating the optimal reimbursement scheme, we now present comparative statics results on the optimal incentives. Resubstituting $P^*$ into equations (4), and (5), we obtain:

**Proposition 4** (a) The physician’s incentive to treat patients reads

$$\alpha^* = \frac{f^2 + \sqrt{f^4 + 2\sigma_q K_q K_z}}{2K_z}.$$

It

(i) strictly increases in the quality standard $Q \ge 0$,

(ii) decreases in the noisiness of the treatment signal $\sigma^2_z$ (with strict monotonicity for $r > 0$), and

(iii) increases in the noisiness of the quality signal $\sigma^2_q$ (with strict monotonicity for $Q > 0$ and $r \ge 0$).

(b) The physician’s incentive to provide quality is given by $\beta^* = Q$.

**Proof.** Appendix.

The optimal treatment incentive $\alpha^*$ directly inherits most monotonicity properties from the $Q$-implementing bonus $P^*$. Only how the optimal treatment incentive varies with increased noise of the treatment signal remains ambiguous. There are two effects to be taken into account. First, given the sponsor keeps the bonus constant, the optimal incentive is decreasing in $\sigma^2_z$. This is the traditional effect of increased uncertainty on optimal incentives (see, e.g. Gibbons and Murphy, 1992). Second, the sponsor’s response to increased noisiness of the treatment signal is to increase the bonus (as shown in Proposition 2), which makes treatment of patients more valuable for the hospital. The proof shows that the direct effect of increased uncertainty on the incentive dominates the indirect price effect so, overall, the optimal incentive to treat patients is decreasing in the noisiness of the treatment signal.

The physician’s incentive to provide quality equals the quality standard. This directly follows from the production technology ($E[q] = b$) and the physician’s first order conditions ($\beta = c_q(a, b) = b$).
4 Hospitals with direct quality concerns

To cover the case of non-profit hospitals, we extend the model outlined in Section 2 to cover the case where the hospital manager’s objective depends on both hospital profit and the (monetarized) quality of treatment. For the sake of comparison, we leave the physician’s objective unchanged, i.e. the physician has no direct quality concern. In Section 5, we discuss how the model can be adapted to address the case of quality-oriented physicians.

Following Ellis and McGuire (1986) and Ellis (1998), let the manager’s objective function be

$$E[(1 - \xi) zP(q) + \xi q - w],$$

where $$\xi \in (0, 1)$$ represents the weight associated with quality and $$1 - \xi$$ the weight on hospital profit. The constant $$v > 0$$ transforms the units of quality into units of the reimbursement currency. In the following, we focus on the bonus of the reimbursement contract. The block grant can be calculated similarly to the above.

4.1 The Performance-Based Bonus

Correspondingly, the manager’s optimization problem is given as

$$\max_{\alpha, \alpha \geq 0, \beta \geq 0} \left\{ E[(1 - \xi) zP(q) + \xi q - c(a, b) - \frac{r}{2} \text{var}(w)] \right\}$$

s.t. \( \alpha f = c_A(a, b) \) and \( \beta = c_B(a, b) \).

Rewriting the manager’s expected benefit,

$$E[(1 - \xi) zP(q) + \xi q]$$

$$= (1 - \xi) \left( f^2 \alpha (1 + (P - 1)(1 - G(Q - \beta))) + \xi v \beta \right),$$

the hospital’s first order conditions change into

$$(1 - \xi) f^2 (1 + (P - 1)(1 - G(Q - \beta))) = \alpha K_z \quad \text{and} \quad (1 - \xi) f^2 \alpha (P - 1) g(Q - \beta) + \xi v = \beta K_q.$$  \hspace{1cm} \text{(10)}

As the sponsor aims at implementing the quality standard $$Q$$ in expectation, $$E[q] = Q$$, he must set the bonus $$P$$ such that equations (10) are satisfied. Solving these for $$\alpha$$ and $$\beta$$, we obtain

$$\alpha = \frac{(1 - \xi) f^2 1 + P}{K_z} \quad \text{and} \quad \beta = (1 - \xi) \frac{f^2 (P^2 - 1)}{2\sqrt{2\pi}\sigma_K K_z} + \xi v \frac{1}{K_q}.$$  \hspace{1cm} \text{(11)}

Finally, we solve $$\beta = Q$$ for $$P$$ to determine the $$Q$$-implementing bonus.
Proposition 5 To implement the quality standard $Q$ in expectation, the sponsor offers a bonus $P^*_\xi$ for $Q \geq \frac{\xi v}{K_q}$. We have $P^*_\xi > 1$ for $Q > \frac{\xi v}{K_q}$. Moreover, the bonus increases with the hospital’s quality concern $\xi$ if and only if 

$$QK_q > \frac{(\xi + 1) v}{2}.$$ 

Proof. Appendix.

When the manager has direct quality concerns, $\xi > 0$, the sponsor gets the quality level $Q = \frac{\xi v}{K_q}$ for free (which corresponds to a bonus of $P = 1$). In this case the direct quality concern of the manager has a similar effect as a lower outside option. To implement a specific quality level, the sponsor has to pay the less (to ensure the hospital’s participation), the stronger the hospital is directly concerned about quality.

Any level of quality higher than $Q = \frac{\xi v}{K_q}$ requires a bonus larger than one, i.e. $P^*_\xi > 1$. In this case, an increase in the manager’s direct quality concern, $\xi$, has two effects on the manager’s benefit (given that the quality standard remains unchanged). First, it increases the (marginal) direct benefit from quality. Second, it decreases the (marginal) benefit of income. For any given quality standard, the change in the bonus has to balance these two effects. If the first effect dominates, the bonus is reduced (and raised otherwise). The former happens for low levels of the quality standard, $Q \in (\frac{\xi v}{K_q}, \frac{\xi + 1 v}{2(K_q)})$, the latter for higher levels, $Q > \frac{(\xi + 1) v}{2(K_q)}$.

We end this section by showing that the optimal treatment incentive, $\alpha^*$, decreases with the manager’s direct quality concern $\xi$. Differentiating the equilibrium incentive for treatments with respect to $\xi$, we obtain:

Proposition 6 The equilibrium incentive for treatments, $\alpha$, decreases with the direct quality concern of the manager:

$$\frac{\partial \alpha}{\partial \xi} = \frac{f^2}{2K_z} \left( (1 - \xi) \frac{\partial P^*_\xi}{\partial \xi} - (1 + P^*_\xi) \right)$$

$$= \frac{(1 - \xi) f^4 (P^*_\xi + 1) + \sqrt{2\pi v} \sigma_q K_z}{2f^2K_z(1 - \xi) P^*_\xi} < 0.$$ 

Proof. Appendix.

An increase in the manager’s direct quality concern has two effects on the optimal treatment incentive: First, it reduces the (marginal) benefit of the hospital’s expected revenue, given that the bonus remains constant. In this case, the manager will reduce the physician’s incentive to treat patients. This is
a direct effect. Second, the (marginal) benefit of the hospital’s expected revenue depends on the bonus, which might both decrease or increase (see Proposition 5). This is an indirect effect. If the bonus decreases both effects are negative. If the bonus increases, the direct effect dominates. In any case, the number of patients treated decreases with the manager’s direct quality concern $\xi$.

5 Discussion

So far we have assumed that the sponsor maximizes expected net consumer surplus, which is equivalent to minimizing the cost of implementing the quality standard. One implication of this objective is that the sponsor always adjusts the block grant so that the hospital breaks even. The sponsor might however also put some weight on producer interest, i.e. on hospital profit. One rationale of such behavior can be that the hospital provides tax income and local jobs. If the sponsor puts some weight on hospital profit, the introduction of pay-for-performance might increase hospital profit.

That providers increase their gross income when a P4P-program is introduced is exactly what happened for U.K. family practitioners, where such a program was introduced in 2004. This program increased existing income according to performance with respect to 146 quality indicators including clinical care for 10 chronic diseases, organization of care, and patient experience. In 2004, the P4P-program increased the gross income of the family practitioner by about 0.32%, and in 2005/06 family practitioner income will rise even more (since quality payments have been increased; see Doran et al. 2006). Another example where the introduction of pay-for-performance will increase hospital payments is the program between Anthem Blue Cross and Blue Shield Midwest and 38 hospitals in Kentucky, Indiana and Ohio. According to the program, a quality bonus is given if certain performance measures in the dimensions outcome, process, and structure are reached. Hospitals do not risk losing money by taking part in this program. Reimbursement goes up based on a good report card, but they cannot go down for a bad one (Leapfrog, 2007).

Notice that in our model the hospital would lobby for the bounded quality level $Q^*$ in equation (9), given that the introduction of a pay-for-performance program would not adjust the block grant. This contrasts with the case where the sponsor provides the hospital with direct quality incentives, which are independent of the number of treatments (e.g. when remuneration takes the form of $R = E[z] + P \cdot E[q]$). In this case, the hospital’s profit is unbounded in $Q$. Put differently, whenever the hospital has a strong bargaining power, it can force the sponsor to offer a higher quality level than the sponsor would prefer (and the sponsor has to pay for it). By tying the quality bonus to the number of treatments, provides the sponsor with a commitment device to avoid this way of exploitation.

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9In 2003, PacifiCare began paying its California medical groups bonuses according to meeting or exceeding 10 clinical and service quality targets. The bonus payments were tied to the per member payments. The bonus potential represented about 5% of the capitation paid by
In the paper we have assumed that the sponsor cannot contract directly with the physician. One rationale for this assumption is that the manager has better information about the physician’s productivity so that hiring a manager is a way to save risk costs for the sponsor.\(^\text{10}\) A way to include a more active role for the manager in our model is to assume that the physician’s productivity on each of the two tasks is symmetrically distributed and independent of the tasks’ noise terms. Our results will continue to hold true if the timing of events is such that the manager learns the physician’s productivity after the contract with the sponsor has been signed.

We have assumed that the physician does not directly care for quality. One way to include a direct concern for quality would be to assume that the physician obtains direct utility also from quality. Her utility function will then read $u = -\exp\{ -r(w + \gamma q - c(a, b)) \}$, where $\gamma q$ represents the direct quality concern. Accordingly, the certainty equivalent reads $CE = E[w] + \gamma E[q] - c(a, b) - \frac{\beta + \gamma}{\tau} \left( \sigma_w^2 + (\beta + \gamma)^2 \sigma_q^2 \right)$. The term $\beta + \gamma$ represents the total incentive, i.e. the sum of monetary incentive and the implicit incentive resulting from the physician’s direct quality concern. It follows that the physician’s ideal quality effort is some positive level $\gamma > 0$. Like in Holmstrom and Milgrom (1991), the physician prefers to exert some effort in the absence of any financial incentive rather than being totally idle at work. By normalizing the physician’s quality concern to zero, we obtain the model presented in this paper.

We end this section by noting that if the sponsor contracts with more than one hospital, the possibility to use relative-performance-pay arises. This type of approach is used in Medicare’s Premier Hospital Quality Incentive Demonstration, where all hospitals are ranked and bonuses are paid to hospitals in the top two deciles of performance. Performance-pay based on relative performance reduces the financial risk for the sponsor because the number of hospitals that will receive an incentive is predictable. However, the level of performance required to trigger an incentive payment is unknown at the start of the year, thus creating uncertainty for hospitals in their own budgeting. In this respect relative-performance-pay may increase the risk costs, which in the end the sponsor has to compensate.\(^\text{11}\)

### 6 Conclusion

In this paper we have investigated how hospital reimbursement affects the internal wage contracts between physicians and the hospital. To this end, we have examined a particular frequent form of pay-for-performance program, which rewards hospitals financially whenever they meet a prespecified quality standard.\(^\text{14}\)

\(^{10}\) One other rationale is that this is exactly what we observe in reality. Hospitals have managers who are responsible for contracting with the staff.

\(^{11}\) Note that relative performance evaluation typically reduces the risk if the agents’ actions are correlated (this follows from the informativeness principle, Holmstrom, 1979). It seems however reasonable to assume that the production of quality is uncorrelated among hospitals.
We have labeled this form of remuneration *bonus-for-quality reimbursement.*

For the hospital, bonus-for-quality reimbursement has two effects. On the one hand, the incentive to treat patients is raised. On the other, an incentive to meet the quality standard is created. The hospital will adjust the linear wage contract offered to the physician accordingly. First, the wage contract must incentivize quality (and treatment) effort by the physician. Otherwise, the physician will not exert quality (nor treatment) effort. Second, the uncertainty associated with quality (and treatment) production results in wage uncertainty for which the physician has to be compensated for. Third, the equilibrium quality incentive is the stronger, the higher the prespecified quality standard. Fourth and finally, the equilibrium treatment incentive increases in the quality standard and in the uncertainty of quality, but decreases in the uncertainty of treatments.

The sponsor has to take this into account when specifying the bonus. Any positive level of quality standard (or quality improvement) requires a bonus above the 100 per cent level of baseline reimbursement. The higher the quality standard, the higher the bonus has to be. A higher uncertainty of quality or treatments necessitates a higher bonus.

Notice that the sponsor must set the price sufficiently high to implement the quality standard. For, the hospital manager has to offer the agent a quality bonus to ensure that the agent exerts effort on quality. The bonus contract exposes the agent to risk, which the hospital management has to compensate for. If the sponsor did not compensate the hospital for these two types of extra risk cost, its manager would head for a lower level of expected quality. Failing to set the right reimbursement incentive by just a narrow margin results in too low quality incentives \((\beta < \beta^*)\). Empirically, this implies that even if one observes that the sponsor includes rewards for quality in hospitals’ remuneration contracts, their managers may choose not to implement the desired quality standard.

In his editorial, Epstein (2006) reminds us that policy chances might lead to unexpected consequences, such as higher payments to physicians and increased budget deficits. From the model outlined in this paper we learn that the sponsor should adjust all elements in the reimbursement contract when introducing P4P-programs. Failing to do so will lead to higher cost for the sponsor and increased profitability of health care providers (in most cases).
References


7 Appendix

The appendix contains the proofs.

Proof (Existence of equilibrium incentives)
Recall equations (2). First consider $P = 1$. It follows from (2) that $\beta = 0$ and hence $\alpha = f^2/K_z$.

Now consider $P > 1$. Since $g(\cdot)$ is strictly positive, the first equation in (2) implies

$$\alpha = \frac{\beta K_q}{f^2 (P - 1) g(Q - \beta)}.$$ 

Inserting this into the second equation in (2), we obtain

$$f^4 (P - 1) g(Q - \beta) [1 + (P - 1) (1 - G(Q - \beta))] = \beta K_q K_z. \quad (12)$$

Evaluating both sides of the equation at $\beta = 0$, we have

$$f^4 (P - 1) g(Q) [1 + (P - 1) (1 - G(Q))] > 0$$

because of $P > 1$, $g(Q) > 0$, and $G(Q) \in [0, 1]$. Taking the limit $\beta \to \infty$ in (12), the left hand side converges to zero, whereas the right hand side approaches infinity. Since both sides of equation (12) are continuous in $\beta$, it follows from the intermediate value theorem that a solution $\beta^* > 0$ to (12) exists. \hfill $\blacksquare$

Proof of Proposition 2
The payer’s reimbursement price needed to implement an arbitrary quality standard $Q$ is given by

$$P^* = \frac{\sqrt{f^4 + 2\sqrt{2}\pi \sigma_q K_q K_z Q}}{f^2}$$

Part (i):

$$\frac{\partial P^*}{\partial Q} = \frac{\sqrt{2\pi \sigma_q K_q K_z}}{f^2 \sqrt{f^4 + 2\sqrt{2}\pi \sigma_q K_z Q}} > 0$$

Part (ii):

$$\frac{\partial P^*}{\partial K_z} = \frac{\sqrt{2\pi \sigma_q K_q Q}}{f^2 \sqrt{f^4 + 2\sqrt{2}\pi \sigma_q K_q K_z Q}} \geq 0$$

The inequality holds strictly if $Q > 0$. Because of $\partial K_z / (\partial \sigma_z) = r \geq 0$, it follows that

$$\frac{\partial P^*}{\partial \sigma_z^2} = \frac{\partial P^*}{\partial K_z} \frac{\partial K_z}{\partial \sigma_z^2} \geq 0,$$

with strict inequality if $r > 0$ and $Q > 0$. 

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Part (iii):
\[
\frac{\partial P^*}{\partial q} = \frac{\sqrt{2\pi} \left(3\sigma_q^2 + 1\right) K_z Q}{2\sigma_q f^2 \sqrt{f^4 + 2\sqrt{2\pi} \sigma_q K_q K_z Q}} \geq 0,
\]
with strict inequality if and only if \( Q > 0 \).

\[\blacksquare\]

Proof of Proposition 3.

Derivation of the block grant (8). Inserting the \( Q \)-implementing bonus, \( P^* \), and the equilibrium incentive, \( \alpha = (1 + P^*) f^2 / (2K_z) \), into equation (7), we obtain:

\[
-H = E[z] \cdot E[P(q)] - c(a^*, b^*) - \frac{r}{2} \text{var}(w^*)
\]

\[
= f^2 \alpha^* P + 1 - \frac{1}{2} (\alpha^*)^2 - \frac{1}{2} (b^*)^2 - \frac{r}{2} \left( (\alpha^*)^2 \sigma_q^2 + (\beta^*)^2 \sigma_q^2 \right)
\]

\[
= f^2 \alpha^* P + 1 - \frac{1}{2} (\alpha^*)^2 - \frac{1}{2} Q^2 - \frac{r}{2} \left( (\alpha^*)^2 \sigma_q^2 + Q^2 \sigma_q^2 \right)
\]

\[
= \frac{f^4}{8K_z} (P + 1)^2 - \frac{1}{2} Q^2 K_q
\]

\[
= \frac{f^4}{4K_z} + \frac{f^2}{4K_z} \sqrt{f^4 + 2\sqrt{2\pi} \sigma_q K_q K_z Q} + \frac{\sqrt{2\pi} \sigma_q}{4} K_q Q - \frac{K_q}{2} Q^2,
\]

where the last equality follows from

\[
\frac{f^4}{8K_z} (P + 1)^2 - \frac{1}{2} Q^2 K_q = -\frac{1}{4f^4K_z} \left( C_1 + C_1 \cdot \sqrt{C_2 + C_4} \right)
\]

with

\[
C_1 = f^6 - 2f^4K_z + f^4r\sigma_q^2 = -K_z f^4,
\]

\[
C_2 = f^4 \left( f^4 + 2\sqrt{2\pi} \sigma_q K_q K_z Q \right),
\]

\[
C_3 = -2f^8K_z + f^{10} + f^8r\sigma_q^2 = -f^8K_z,
\]

and

\[
C_4 = 2Q^2 f^4K_z^2K_q + \sqrt{2\pi} Q f^6 \sigma_q K_q K_z
\]

\[
-2\sqrt{2\pi} Q f^4 \sigma_q K_q K_z^2 + \sqrt{2\pi} Q f^4 \sigma_q \sigma_q^2 K_q K_z
\]

\[
= 2Q^2 f^4K_z^2K_q - \sqrt{2\pi} \sigma_q K_q K_z^2 Q f^4.
\]

Properties of the block grant. From equation (8) we obtain

\[
\frac{\partial H}{\partial Q} = -\frac{f^2\sqrt{2\pi} \sigma_q K_q}{4\sqrt{f^4 + 2\sqrt{2\pi} \sigma_q K_q K_z Q}} - \frac{\sqrt{2\pi}}{4} \sigma_q K_q + Q K_q
\]

\[
\frac{\partial^2 H}{\partial Q^2} = +K_q + \frac{1}{2\pi f^2 \sigma_q^2 K_q^2} \frac{K_z}{\left(f^4 + 2\sqrt{2\pi} Q \sigma_q K_q K_z \right)^2} > 0.
\]

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Furthermore,
\[ \frac{\partial H}{\partial Q} \bigg|_{Q=0} = \frac{\sqrt{2\pi} \sigma_q K_q}{2} > 0. \]

Correspondingly, the first order condition of minimizing the block grant with regard to \(Q\) reads
\[ f^2 \sqrt{2\pi} \sigma_q + (\sqrt{2\pi} \sigma_q - 4Q) \sqrt{f^4 + 2\sqrt{2\pi} \sigma_q K_q K_z} Q = 0, \quad (13) \]
which implies
\[ (4Q - \sqrt{2\pi} \sigma_q)^2 \left( f^4 + 2\sqrt{2\pi} \sigma_q K_q K_z \right) - f^4 2\pi \sigma_q^2 = 0. \quad (14) \]
The solutions of the latter are
\[ Q_1 = 0 \quad \text{and} \quad Q_{2,3} = \frac{2\pi \sigma_q^2 K_q K_z \pm f^2 \sqrt{f^4 + 4\pi \sigma_q^2 K_q K_z - f^4}}{4\sqrt{2\pi} \sigma_q K_q K_z}. \]
We check which of the solutions of (14) also solve (13). To this end define
\[ \varphi(Q) = \left( \sqrt{2\pi} \sigma_q - 4Q \right) \sqrt{f^4 + 2\sqrt{2\pi} \sigma_q K_q K_z Q + f^2 \sqrt{2\pi} \sigma_q}. \]
We obtain \( \varphi(Q_1) = 2f^2 \sqrt{2\pi} \sigma_q > 0 \) and
\[ \varphi(Q_{2,3}) = \left( \sqrt{2\pi} \sigma_q - \frac{2\pi \sigma_q^2 K_q K_z \pm f^2 \sqrt{f^4 + 4\pi \sigma_q^2 K_q K_z - f^4}}{\sqrt{2\pi} \sigma_q K_q K_z} \right) \]
\[ \sqrt{\frac{\pi \sigma_q^2 K_q K_z}{2} \pm \frac{1}{2} f^2 \sqrt{f^4 + 4\pi K_q K_z \sigma_q^2 + \frac{1}{2} f^4 + f^2 \sqrt{2\pi} \sigma_q}} \]
\[ = \mp f^2 \sqrt{2\pi} \sigma_q + f^2 \sqrt{2\pi} \sigma_q, \quad (15) \]
i.e. \( \varphi(Q_2) = 0 \) and \( \varphi(Q_3) = 2f^2 \sqrt{2\pi} \sigma_q > 0 \).

To see the second equality in (15), we first transform the first term in parentheses:
\[ \sqrt{2\pi} \sigma_q - \frac{2\pi \sigma_q^2 K_q K_z \pm f^2 \sqrt{f^4 + 4\pi K_q K_z \sigma_q^2 - f^4}}{\sqrt{2\pi} \sigma_q K_q K_z} \]
\[ = f^2 \frac{\mp \sqrt{f^4 + 4\pi K_q K_z \sigma_q^2} + \frac{1}{2} f^4}{\sqrt{2\pi} \sigma_q K_q K_z}. \]
Second, because of
\[ \pi \sigma_q^2 K_q K_z \pm \frac{1}{2} f^2 \sqrt{f^4 + 4\pi K_q K_z \sigma_q^2 + \frac{1}{2} f^4} = \frac{1}{4} \left( f^2 \pm \sqrt{f^4 + 4\pi K_q K_z \sigma_q^2} \right)^2, \]
we can rewrite the second term as
\[ \sqrt{\frac{1}{2}} f^2 \sqrt{f^4 + 4\pi K_q K_z \sigma_q^2 + \frac{1}{2} f^4 + \pi \sigma_q^2 K_q K_z} = \frac{1}{2} \left( \sqrt{f^4 + 4\pi K_q K_z \sigma_q^2} \pm f^2 \right). \]

The product of the first and the second term thus reduces to
\[ \frac{f^2}{2\sqrt{2\pi \sigma_q K_q K_z}} \left( f^2 \mp \sqrt{f^4 + 4\pi K_q K_z \sigma_q^2} \right) \left( \sqrt{f^4 + 4\pi K_q K_z \sigma_q^2} \pm f^2 \right) \]
\[ = \frac{f^2}{2\sqrt{2\pi \sigma_q K_q K_z}} \left( \mp 4\pi K_q K_z \sigma_q^2 \right) \]
\[ = \mp f^2 \sqrt{2\pi \sigma_q}, \]

which completes the proof. 

**Proof of Proposition 4**

We only have to establish part (a). Inserting the optimal bonus $P^*$ from equation (6) into
\[ \alpha^* = \frac{1 + P^* f^2}{2 K_z}, \]
we obtain
\[ \alpha^* = \frac{f^2 + \sqrt{f^4 + 2\pi \sigma_q K_q K_z Q}}{2 K_z}. \]

Ad (i): Because of
\[ \frac{\partial \alpha^*}{\partial Q} = \frac{f^2}{2 K_z}, \]
the claim follows from Proposition 2.

Ad (ii): Because of
\[ \frac{\partial \alpha^*}{\partial \sigma_q^2} = \frac{\partial \alpha^*}{\partial K_z} \frac{\partial K_z}{\partial \sigma_q^2} = r \frac{\partial \alpha^*}{\partial K_z}, \]
it suffices to determine $\frac{\partial \alpha^*}{\partial K_z}$:
\[ \frac{\partial \alpha^*}{\partial K_z} = -\frac{1}{2 K_z^2} \left( \frac{f^4 + \sqrt{2\pi \sigma_q K_q K_z Q}}{\sqrt{f^4 + 2\sqrt{2\pi \sigma_q K_q K_z Q}}} + 1 \right) < 0. \]
Since the inequality holds strictly for arbitrary $Q \geq 0$, the incentive $\alpha^*$ is (strictly) decreasing in $\sigma_q^2$ for $r(> 0)$.

Ad (iii): Because of
\[ \frac{\partial \alpha^*}{\partial \sigma_q^2} = \frac{f^2}{2 K_z}, \]
the claim follows from Proposition 2.
Proof of Proposition 5.
What is left to prove is the last claim, i.e. the reimbursement price increases with the hospital’s quality concern $\xi$ if and only if $QK_q > \frac{(\xi+1)v}{2}$. This claim follows from
\[
\frac{\partial P^*}{\partial \xi} = \frac{\sqrt{2\pi}\sigma_qK_z(-\xi v + 2QK_q - v)}{f^4(1-\xi)^3 \sqrt{1 + \frac{2\sqrt{2\pi}\sigma_qK_z(QK_q - \xi v)}{f^2(1-\xi)^2}}}
\]

Proof of Proposition 6.
Inserting the $Q$-implementing price $P^*_\xi$ from equation (11) in the equilibrium incentive for treatments
\[
\alpha^*_\xi = \frac{(1-\xi)f^2 1 + P^*_\xi}{K_z},
\]
we obtain
\[
\alpha^*_\xi = \frac{f^2(1-\xi)}{2K_z} \left( \sqrt{1 + \frac{2\sqrt{2\pi}\sigma_qK_z(QK_q - \xi v)}{f^4(1-\xi)^2}} + 1 \right).
\]
Differentiating this expression with respect to $\xi$, we get
\[
\frac{\partial \alpha^*_\xi}{\partial \xi} = \frac{f^4 \left( P^*_\xi + 1 \right) - f^4 \xi \left( P^*_\xi + 1 \right) + \sqrt{2\pi}v\sigma_qK_z}{2f^2K_z(\xi-1)P^*_\xi} = - \frac{(1-\xi)f^4 \left( P^*_\xi + 1 \right) + \sqrt{2\pi}v\sigma_qK_z}{2f^2K_z(1-\xi)P^*_\xi},
\]
which is negative. The second equality in Proposition 6 follows from the chain rule. $\blacksquare$
Figure 1. The block grant